Projecting Temporal Properties, Events and Actions

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Abstract

Temporal notions based on a finite set A of properties are represented in strings, on which projections are defined that vary the granularity A. The structure of properties in A is elaborated to describe statives, events and actions, subject to a distinction in meaning (advocated by Levin and Rappaport Hovav) between what the lexicon prescribes and what a context of use supplies. The projections proposed are deployed as labels for records and record types amenable to finite-state methods.

1   Introduction

Reflecting on years of work on discourse semantics, Hans Kamp writes

> when we interpret a piece of discourse — or a single sentence in the context in which it is being used — we build something like a model of the episode or situation described; and an important part of that model are its event structure, and the time structure that can be derived from that event structure by means of Russell's construction (Kamp, 2013, page 13).

The event structure Kamp has in mind is "made up of those comparatively few events that figure in this discourse" (page 9). Let us put aside for the moment how to extract from a discourse D the set E_D of events that figure in D, and observe that if the set E_D is finite (as typically happens in practice), so is the linear order returned by the Russell construction for time (details in section 2 below). This is in sharp contrast to the continuum $\mathbb{R}$, with which "real" time is commonly identified (Kamp and Reyle, 1993), or to any unbounded linear order supporting the temporal interval structure defined in Allen and Ferguson (1994), where a different perspective on events is adopted.

> We take the position that events are primarily linguistic or cognitive in nature. That is, the world does not really contain events. Rather, events are the way by which agents classify certain useful and relevant patterns of change (Allen and Ferguson, 1994, page 533).

Allen and Ferguson specify temporal structure *before* introducing events (or, for that matter, properties and actions), reversing the conceptual priority events enjoy over time in the Russell construction mentioned by Kamp. Without embracing this reversal, the present paper builds on elements of Allen and Ferguson (1994) and other works to construct time from not only events, but also properties and actions. The aim is to find a temporal ontology of finite strings that is not *too big* (which $\mathbb{R}$ or any infinite linear order arguably is) and not *too small* (which the linear order from Russell's construction can be, depending on the event structure it is fed as input). Insisting on temporal structure that is *just right* is reminiscent of Goldilocks, and perhaps more germanely, the *Goldilocks effect* observed in Kidd et al. (2012) as the tendency of human infants to look away from events that are overly simple or overly complex. Whether or not any useful link can be forged between that work and the present paper, I am not able to say. But I do claim that the notions of projections brought out below provide helpful handles on granularity, especially when granularity is varied.

That granularity is given, in the simplest case, by a finite set A of properties, expressing in section 2 events, as conceived in the Russell construction. More sophisticated pictures of events are considered and "relevant patterns of change" captured through an explicit account of action and incremental change in section 3. Strings and languages are presented in section 4 as records and record types labeled with projections, bringing out certain affinities with *Type Theory with Records* (Cooper and Ginzburg, 2015).

2 Strings from properties and changes

Leibniz's law, decreeing that any difference $x \neq y$ be discernible via some property, can be expressed in monadic second-order logic (MSO, e.g. Libkin (2010)) as the implication

$$x \neq y \;\supset\; (\exists P)\neg(P(x) \equiv P(y)) \tag{LL}$$

with $\neg(P(x) \equiv P(y))$ asserting P separates x from y. A special case of inequality $\neq$ is the successor relation S that specifies a notion of step. We link that step to a set $\{P_a\}_{a \in A}$ of properties P_a named with a finite set A (conflating the property P_a with its name $a \in A$ when convenient), and adopt the abbreviation $x \equiv_A y$ for the conjunction expressing the inseparabilty in A of x and y

$$x \equiv_A y \;:=\; \bigwedge_{a \in A} (P_a(x) \equiv P_a(y)).$$

Two substitutions in (LL), S for $\neq$, and the negation of $x \equiv_A y$ for its consequent, turn (LL) into

$$xSy \;\supset\; x \not\equiv_A y \tag{LL$_A$}$$

(pronounced "S-steps require change$_A$"). If we represent x by its *A-profile*

$$A[x] \;:=\; \{a \in A \mid P_a(x)\}$$

specifying the properties in A that hold of x, we can can study S-chains

$$x_1 S x_2 \text{ and } x_2 S x_3 \text{ and } \cdots \text{ and } x_{n-1} S x_n$$

through strings $A[x_1]A[x_2]\cdots A[x_n]$ of subsets of A. In model-theoretic terms, this suggests construing a string $s = \alpha_1 \cdots \alpha_n$ of subsets α_i of A as the model

$$\mathrm{Mod}(s) \;:=\; \langle [n], S_n, \{[\![P_a]\!]_s\}_{a \in A}\rangle$$

with domain/universe

$$[n] := \{1, \ldots, n\}$$

of string positions, interpreting S as the successor relation

$$S_n \;:=\; \{(i, i+1) \mid i \in [n-1]\}$$

$+1$ on $[n]$, and P_a as the set

$$[\![P_a]\!]_{\alpha_1 \cdots \alpha_n} \;:=\; \{i \in [n] \mid a \in \alpha_i\}$$

of positions in s where a occurs (for each $a \in A$). For example, the string $\boxed{}\,\boxed{a}\,\boxed{a, a'}\,\boxed{a'}\,\boxed{}$ of length 5 (with string symbols drawn as boxes) corresponds to the model with universe $[5] = \{1,2,3,4,5\}$, interpreting P_a as $\{2,3\}$ and $P_{a'}$ as $\{3,4\}$. (Note $\boxed{}$ is the empty set $\emptyset$ *qua* string of length 1, not to be confused with the null string of length 0 or the empty language.) The *vocabulary of* s, $voc(s)$, is the smallest set A' such that s is a string of subsets of A'

$$voc(\alpha_1 \cdots \alpha_n) \;=\; \bigcup_{i=1}^{n} \alpha_i$$

(making, for example, $\{a, a'\}$ the vocabulary of $\boxed{}\,\boxed{a}\,\boxed{a, a'}\,\boxed{a'}\,\boxed{}$).

Rather than fixing A once and for all, we let A vary, keeping it finite for bounded granularity (restricting our attention to finite strings of finite sets). If $A = \emptyset$, then $x \equiv_A y$, which is to say, the strings that satisfy (LL$_\emptyset$) are exactly those of length 1 (or 0, if we allow a model with empty universe). Evidently, the

space of models of (LL$_A$) increases as we enlarge A. Given a string s of sets that may or not be subsets of A, we define the *A-reduct of s* to be the string obtained by intersecting s componentwise with A

$$\rho_A(\alpha_1 \cdots \alpha_n) := (\alpha_1 \cap A) \cdots (\alpha_n \cap A)$$

(Fernando, 2016). For instance, the $\{a\}$-reduct of the string $\boxed{}\boxed{a}\boxed{a, a'}\boxed{a'}\boxed{}$ is

$$\rho_{\{a\}}(\boxed{}\boxed{a}\boxed{a, a'}\boxed{a'}\boxed{}) = \boxed{}\boxed{a}\boxed{a}\boxed{}\boxed{}.$$

Whereas $\boxed{}\boxed{a}\boxed{a, a'}\boxed{a'}\boxed{}$ satisfies (LL$_{\{a,a'\}}$), its $\{a\}$-reduct satisfies neither (LL$_{\{a,a'\}}$) nor (LL$_{\{a\}}$). The problem is that $\boxed{}\boxed{a}\boxed{a}\boxed{}\boxed{}$ stutters. In general, a *stutter of* a string $\alpha_1 \cdots \alpha_n$ is a position $i \in [n-1]$ such that $\alpha_i = \alpha_{i+1}$. $\boxed{}\boxed{a}\boxed{a}\boxed{}\boxed{}$ has two stutters, 2 and 4. It is easy to see that a string s is stutterless iff it satisfies (LL$_{voc(s)}$). The consequent $x \not\equiv_A y$ of (LL$_A$) is equivalent to the disjunction

$$\bigvee_{a \in A} ((\neg P_a(x) \wedge P_a(y)) \vee (P_a(x) \wedge \neg P_a(y))$$

where each $a \in A$ can separate x from y in one of two ways, corresponding to a's left and right borders, $l(a)$ and $r(a)$, respectively. We introduce predicates $P_{l(a)}$ saying: P_a is false but S-after true

$$P_{l(a)}(x) \equiv \neg P_a(x) \wedge (\exists y)(xSy \wedge P_a(y)) \tag{1}$$

and $P_{r(a)}$ saying: P_a is true but not S-after

$$P_{r(a)}(x) \equiv P_a(x) \wedge \neg(\exists y)(xSy \wedge P_a(y)). \tag{2}$$

Then $x \not\equiv_A y$ is equivalent under xSy to $\bigvee_{a \in A}(P_{l(a)}(x) \vee P_{r(a)}(x)))$

$$xSy \supset (x \not\equiv_A y \equiv \bigvee_{a \in A}(P_{l(a)}(x) \vee P_{r(a)}(x))).$$

Hence, (LL$_A$) is equivalent to

$$(\exists y)(xSy) \supset \bigvee_{a \in A}(P_{l(a)}(x) \vee P_{r(a)}(x)) \tag{3}$$

assuming (1), (2), and

$$xSy \wedge xSz \supset y = z. \tag{4}$$

(4) expresses the determinism of S, which is built into strings. As for (1) and (2), let $A_\bullet$ be the set of borders in A

$$A_\bullet := \{l(a) \mid a \in A\} \cup \{r(a) \mid a \in A\}$$

and define the *border translation $b(s)$ of* a string $\alpha_1 \cdots \alpha_n$ to be the string $\beta_1 \cdots \beta_n$ of subsets of $voc(s)_\bullet$ specified by (1) and (2)

$$\beta_i := \{l(a) \mid a \in \alpha_{i+1} - \alpha_i\} \cup \{r(a) \mid a \in \alpha_i - \alpha_{i+1}\} \text{ for } i < n \tag{5}$$
$$\beta_n := \{r(a) \mid a \in \alpha_n\}$$

(Fernando, 2018). For example,

$$b(\boxed{}\boxed{a, a'}\boxed{a'}) = \boxed{l(a), l(a')}\boxed{r(a)}\boxed{r(a')}.$$

In general, (5) says that for a non-final position i,

$$\beta_i \neq \square \iff (\alpha_{i+1} - \alpha_i) \cup (\alpha_i - \alpha_{i+1}) \neq \square$$
$$\iff \alpha_{i+1} \neq \alpha_i.$$

That is, s is stutterless iff $b(s)$ is $\square$-lite, where by definition, a string $\alpha_1 \cdots \alpha_n$ is *$\square$-lite* if for each $i \in [n-1]$, α_i is not $\square$. For the record, we have

Proposition 1. *For any sets A and X, and for any string $s \in (2^X)^*$, the following are equivalent.*

(i) *$Mod(s)$ satisfies (LL_A)*

(ii) *$\rho_A(s)$ is stutterless*

(iii) *$Mod(b(\rho_A(s)))$ satisfies (3)*

(iv) *the $A_\bullet$-reduct of $b(s)$ is $\square$-lite.*

Implicit in Proposition 1 are two notions of string compression,

$$s\alpha\alpha s' \rightsquigarrow s\alpha s' \tag{6}$$

for strings over the alphabet 2^A to satisfy (LL_A), and

$$s\square s' \rightsquigarrow ss' \qquad \text{for } s' \neq \epsilon \tag{7}$$

for strings over the alphabet $2^{A_\bullet}$ to satisfy the border translation (3) of (LL_A). Destuttering (6) is implemented fully by *block compression bc*

$$bc^{-1}\alpha_1\cdots\alpha_n = \alpha_1^+\cdots\alpha_n^+ \qquad \text{for stutterless } \alpha_1\cdots\alpha_n$$

while $\square$-removal $d_\square$ implements (7) with*out* the proviso $s' \neq \epsilon$

$$d_\square^{-1}\alpha_1\cdots\alpha_n = \square^*\alpha_1\square^*\cdots\square^*\alpha_n\square^* \qquad \text{for } \square\text{-less } \alpha_1\cdots\alpha_n$$

where a $\square$-*less* string is a string of non-empty sets. In Durand and Schwer (2008), $\square$-less strings are called *S-words* ("S for Set"), and the *S-projection over A of s* defined to be $d_\square(\rho_A(s))$. To make room for bc and link up with Allen and Ferguson (1994) and the Russell construction mentioned in the Introduction, let us agree that, given strings s and s' of sets,

(i) *s bc-projects to s'* if the $voc(s')$-reduct of s without stutters is s'

$$bc(\rho_{voc(s')}(s)) = s'$$

(ii) *s $\square$-projects to s'* if the $voc(s')$-reduct of s without any $\square$ is s'

$$d_\square(\rho_{voc(s')}(s)) = s'$$

(iii) an *s-period* is an $a \in voc(s)$ such that s *bc*-projects to $\boxed{\;}\boxed{a}\boxed{\;}$.

The occurrences of $\square$ to the left and right in $\boxed{\;}\boxed{a}\boxed{\;}$ represent the left and right bounds on a period in Allen and Ferguson (1994). As with intervals, periods a and a' can be related by exactly one element of the set

$$\mathcal{AR} := \{\text{b, bi, o, oi, m, mi, d, di, s, si, f, fi, e}\}$$

of Allen relations (Allen, 1983). Each $R \in \mathcal{AR}$ is pictured as a stutterless string $\mathfrak{s}_{aRa'}$ in Table 1 so that for any string s of sets, and all distinct a, a',

$$a \text{ and } a' \text{ are both } s\text{-periods} \iff (\exists R \in \mathcal{AR}) \; s \; bc\text{-projects to } \square\mathfrak{s}_{aRa'}\square.$$

Table 1. Allen relations as stutterless strings

R	$\mathfrak{s}_{aRa'}$	R^{-1}	$\mathfrak{s}_{aR^{-1}a'}$	R	$\mathfrak{s}_{aRa'}$	R^{-1}	$\mathfrak{s}_{aR^{-1}a'}$
b	$\boxed{a}\;\boxed{a'}$	bi	$\boxed{a'}\;\boxed{a}$	d	$\boxed{a'}\boxed{a,a'}\boxed{a'}$	di	$\boxed{a}\boxed{a,a'}\boxed{a}$
o	$\boxed{a}\boxed{a,a'}\boxed{a'}$	oi	$\boxed{a'}\boxed{a,a'}\boxed{a}$	s	$\boxed{a,a'}\boxed{a'}$	si	$\boxed{a,a'}\boxed{a}$
m	$\boxed{a}\boxed{a'}$	mi	$\boxed{a'}\boxed{a}$	f	$\boxed{a'}\boxed{a,a'}$	fi	$\boxed{a}\boxed{a,a'}$
e	$\boxed{a,a'}$						

Let us call a string s an *A-timeline* if s is stutterless and every $a \in A$ is an s-period. For $a \neq a'$, the $\{a, a'\}$-timelines are exactly the strings $\Box \mathfrak{s}_{aRa'} \Box$, for $R \in \mathcal{AR}$. How do these $\{a, a'\}$-timelines compare to the linear orders obtained by the Russell construction on event structures over $\{a, a'\}$?

Without entering into all the details of the event structure $\langle A, \prec, \bigcirc \rangle$ on which the Russell construction is applied, suffice it to say we can derive $\mathfrak{s}_{a\,m\,a'}$ from $a \prec a'$, $\mathfrak{s}_{a\,mi\,a'}$ from $a' \prec a$, and $\mathfrak{s}_{a\,e\,a'}$ from $a \bigcirc a'$, while every other string $\mathfrak{s}_{aRa'}$ is ruled out by the following fact about a linear order $<$ obtained via Russell

(†)   the instants related by $<$ are certain subsets of A, no two of which are related by $\subseteq$.

For example, for $A = \{a, a'\}$, $<$ cannot describe $\mathfrak{s}_{a\,o\,a'} = \boxed{a}\;\boxed{a, a'}\;\boxed{a'}$ since $\boxed{a} \subseteq \boxed{a, a'}$. But can we not get around the antichain condition (†) by fleshing $\mathfrak{s}_{a\,o\,a'}$ out as

$$\boxed{a, \text{pre}(a')}\;\boxed{a, a'}\;\boxed{\text{post}(a), a'}$$

and similarly for all other strings $\mathfrak{s}_{aRa'}$? In general, the idea would be for any set A and string s of sets, to form the *A-closure of s*, $cl_A(s)$, by setting $cl_A(\alpha_1 \cdots \alpha_n)$ to $\beta_1 \cdots \beta_n$ where

$$\beta_i \;:=\; \alpha_i \cup \{\text{pre}(a) \mid a \in (A - \alpha_i) \cap \bigcup_{k=i+1}^{n} \alpha_k\} \cup \{\text{post}(a) \mid a \in (A - \alpha_i) \cap \bigcup_{k=1}^{i-1} \alpha_k\}$$

adding two negations, $\text{pre}(a)$ and $\text{post}(a)$, for every $a \in A$ (familiar in the A-series of McTaggart (1908) as past and future). The difficulty with $cl_A(s)$ is that if a is an s-period, then neither $\text{pre}(a)$ nor $\text{post}(a)$ can be a $cl_A(s)$-period, as

$$bc(\rho_{\{\text{pre}(a)\}}(cl_A(s))) = \boxed{\text{pre}(a)}\;\boxed{} \quad \text{and} \quad bc(\rho_{\{\text{post}(a)\}}(cl_A(s))) = \boxed{}\;\boxed{\text{post}(a)}.$$

To cover $\text{pre}(a)$ and $\text{post}(a)$, infinitely many periods are assumed in Allen and Ferguson (1994), each bounded to the left and right.

An alternative is to drop the bounds on periods, and work with *semi-intervals* (Freksa, 1992). Or rather than introducing $\text{pre}(a)$ and $\text{post}(a)$ through the A-closure $cl_A(s)$, we might apply the border translation $b(s)$ for left and right borders $l(a)$ and $r(a)$ that capture moments of change (as opposed to instants, under the Russell construction, of pairwise overlapping events). Table 2 records $\Box$-less strings $b(\Box \mathfrak{s}_{aRa'})$, depicting how R orders $l(a), l(a'), r(a), r(a')$. For example, $\boxed{l(a)}\;\boxed{r(a)}\;\boxed{l(a')}\;\boxed{r(a')}$ depicts b's ordering $l(a) < r(a) < l(a') < r(a')$ while $\boxed{l(a), l(a')}\;\boxed{r(a), r(a')}$ depicts e's ordering $l(a) = l(a') < r(a) = r(a')$.

Table 2. Allen relations as $\Box$-less strings, after Durand and Schwer (2008)

R	$b(\Box \mathfrak{s}_{aRa'})$				R^{-1}	$b(\Box \mathfrak{s}_{aR^{-1}a'})$			
b	$l(a)$	$r(a)$	$l(a')$	$r(a')$	bi	$l(a')$	$r(a')$	$l(a)$	$r(a)$
d	$l(a')$	$l(a)$	$r(a)$	$r(a')$	di	$l(a)$	$l(a')$	$r(a')$	$r(a)$
o	$l(a)$	$l(a')$	$r(a)$	$r(a')$	oi	$l(a')$	$l(a)$	$r(a')$	$r(a)$
m	$l(a)$	$r(a), l(a')$	$r(a')$		mi	$l(a')$	$r(a'), l(a)$	$r(a)$	
s	$l(a), l(a')$	$r(a)$	$r(a')$		si	$l(a), l(a')$	$r(a')$	$r(a)$	
f	$l(a')$	$l(a)$	$r(a), r(a')$		fi	$l(a)$	$l(a')$	$r(a), r(a')$	
e	$l(a), l(a')$	$r(a), r(a')$			e				

Table 2 with $l(a)$ and $r(a)$ replaced both by a, and $l(a')$ and $r(a')$ replaced both by a' leads to Figure 4 in (Durand and Schwer, 2008, page 3288). These replacements simplify, for example, $b(\Box \mathfrak{s}_{a\,b\,a'})$ to

$$\boxed{a}\;\boxed{a}\;\boxed{a'}\;\boxed{a'}$$

with the first occurrence of a understood as a's left border, and the second as a's right. Insofar as these simplifications suffice to represent Allen relations in strings, MSO is overkill. The "relevant patterns of change" associated with events in Allen and Ferguson (1994) are, however, another matter, or so the next section argues, pointing to action and activity left out of $l(a)$ and $r(a)$.

3   Border action and activity

Box-removal $d_\square$ implements the Aristotelian slogan *no time without change* under the assumption that

(B)   all predicates appearing in a box (string symbol) express change.

While (B) holds for the strings in Table 2, it fails for those in Table 1, the approriate compression in which is destuttering bc, or *be* cumulative. By definition, a predicate P on intervals is *cumulative* if whenever an interval i meets (abuts) an interval i' for the combined interval $i \sqcup i'$,

$$P(i) \text{ and } P(i') \implies P(i \sqcup i').$$

The converse

$$P(i \sqcup i') \implies P(i) \text{ and } P(i') \qquad \text{whenever } i \text{ meets } i'$$

is the defining condition for *divisive* predicates P. Cumulativity and divisiveness combine for the condition (H) for *homogeneity*

(H)   for all intervals i and i' whose union $i \cup i'$ is an interval,

$$P(i \cup i') \iff P(i) \text{ and } P(i').$$

If $d_\square$ assumes (B), bc assumes strings are built from homogeneous predicates. Stative predicates are commonly assumed to be homogeneous, as in the well-known aspect hypothesis from Dowty (1979) claiming

> the different aspectual properties of the various kinds of verbs can be explained by postulating a single homogeneous class of predicates — stative predicates — plus three or four sentential operators or connectives. (page 71)

Developing Dowty's aspect hypothesis in terms of strings arguably runs counter to assumption (B) above. Many non-statives are given by *result verbs* that center around some prescribed post-state, as opposed to some *manner* of change (Levin and Rappaport Hovav, 2013, for example). It is natural to identify that post-state with the *consequent state* in Moens and Steedman (1988), where the Aristotle-Ryle-Kenny-Vendler verb classification (Dowty, 1979) is reworked according to Table 3.

Table 3. Moens and Steedman (1988)'s reconstruction of ARKV, annotated with strings

	atomic	extended	
+conseq	culmination (achievement)	culminated process (accomplishment)	
a	$\boxed{\text{pre}(a)\,\vert\,a}$	$\boxed{\text{pre}(a), \text{ap}(f)\,\vert\,\text{pre}(a), \text{ap}(f), \text{ef}(f)\,\vert\,\text{ef}(f), a}$	
−conseq	point (semelfactive)	process (activity)	
f	$\boxed{\text{ap}(f)\,\vert\,\text{ef}(f)}$	$\boxed{\text{ap}(f)\,\vert\,\text{ap}(f), \text{ef}(f)\,\vert\,\text{ef}(f)}$	

Table 3 formulates the culimination resulting in consequent state a as the string $\boxed{\text{pre}(a)\,\vert\,a}$, which is associated with the left border $l(a)$ by the border translation b and closure cl_A from the previous section. Line (1) in that section implies

$$\neg P_a(x) \land (\exists y)(xSy \land P_a(y)) \supset P_{l(a)}(x) \tag{8}$$

which can be read as a law of *inertia* (Dowty, 1986) saying $\mathrm{pre}(a)$ persists (forward) unless a *force* is applied, $l(a)$. If we associate a result verb with a force, it is not surprising that a force f should represent a *manner verb* lacking a lexically prescribed post-state (Levin and Rappaport Hovav, 2013), marked $-\mathrm{conseq}$ in Table 3 (with f below). The point (semelfactive) string $\boxed{\mathrm{ap}(f)}\ \boxed{\mathrm{ef}(f)}$ is built from two properties, $\mathrm{ap}(f)$ saying f is applied, and $\mathrm{ef}(f)$ representing a contextually supplied effect of that application. We are borrowing here a basic distinction drawn in Levin and Rappaport Hovav (2013) between the meaning of a verb that is lexically specified (before the verb is used) and the meaning inferred from a specific context of use. When $\mathrm{ef}(f)$ is a, it is tempting to reduce $\mathrm{ap}(f)$ to $l(a)$, except that the lexical/contextual distinction tells us to resist that reduction. Whereas the contextually supplied effect of a manner verb may vary with the use of the verb, the lexically prescribed post-state of a result verb does not. Moreover, while a point (semelfactive) can apply successively (for a process/activity), the implication

$$P_{l(a)}(x) \supset \neg P_a(x)$$

(saying $l(a)$ cannot co-exist with a in the same box) blocks successive culminations.

How is it possible that $\mathrm{ap}(f)$ and $\mathrm{ef}(f)$ can be boxed together, as in the rightmost column in Table 3 (when $\mathrm{pre}(a)$ and a cannot)? An instructive example, given by incremental change tracked by a scale $\prec$ on a set D of degrees, is a force $\uparrow D$ for a $\prec$-increase, with the effect at y

$$P_{\mathrm{ef}(\uparrow D)}(y) \ \approx \ (\exists d \in D)\, P_d(y) \wedge (\exists x S y)(\exists d' \prec d) P_{d'}(x) \ . \tag{9}$$

Unfortunately, the right side of $\approx$ in (9) quantifies over d and d', which appear as subscripts in $P_d(y)$ and $P_{d'}(x)$, not as arguments y and x. Working instead with any finite subset $D_\circ$ of D (which may well be infinite), we turn (9) into the MSO formula

$$P_{\mathrm{ef}(\uparrow D)}(y) \ \equiv \ \bigvee_{d \in D_\circ} P_{\succ d}(y) \wedge (\exists x)(x S y \wedge P_{\approx d}(x)) \tag{10}$$

built with predicates $P_{\approx d}$ approximating D by $D_\circ$. Given $D_\circ$, (10) says the $D_\circ$-degree at y is greater than the $D_\circ$-degree d at the S-predecessor x of y. Now, whereas $l(a)$ and $r(a)$ cannot co-occur

$$P_{l(a)}(x) \supset \neg P_{r(a)}(x),$$

we should look out for an opposing force $\downarrow D$ before leaping from $\mathrm{ap}(\uparrow D)$ to $\mathrm{ef}(\uparrow D)$

$$P_{\mathrm{ap}(\uparrow D)}(x) \wedge x S y \wedge \neg P_{\mathrm{ap}(\downarrow D)}(x) \ \supset \ P_{\mathrm{ef}(\uparrow D)}(y). \tag{11}$$

If we unwind the disjunction characterizing $\mathrm{ef}(\uparrow D)$ in (10), (11) gives

$$P_{\approx d}(x) \wedge P_{\mathrm{ap}(\uparrow D)}(x) \wedge x S y \wedge \neg P_{\mathrm{ap}(\downarrow D)}(x) \ \supset \ P_{\succ d}(y) \qquad (d \in D_0). \tag{12}$$

To allow $P_{\succ d}(x)$ in place of $P_{\approx d}(x)$ in (12), we modify (10) slightly to

$$P_{\mathrm{ef}(\uparrow D)}(y) \ \equiv \ \bigvee_{d \in D_\circ} P_{\succ d}(y) \wedge (\exists x)(x S y \wedge (P_{\succ d}(x) \vee P_{\approx d}(x))) \tag{13}$$

which means $\uparrow D$ may have the effect not of change but rather preservation (of $P_{\succ d}$). Pressure to change $P_{\succ d}$ comes from $\downarrow D$, for which we have $\downarrow$-counterparts to (11)

$$P_{\mathrm{ap}(\downarrow D)}(x) \wedge x S y \wedge \neg P_{\mathrm{ap}(\uparrow D)}(x) \ \supset \ P_{\mathrm{ef}(\downarrow D)}(y) \tag{14}$$

and to (13)

$$P_{\mathrm{ef}(\downarrow D)}(y) \ \equiv \ \bigvee_{d \in D_\circ} P_{\prec d}(y) \wedge (\exists x)(x S y \wedge (P_{\prec d}(x) \vee P_{\approx d}(x))).$$

The implications (11) and (14) reveal shortcomings that the borders $l(a)$ and $r(a)$ have as pictures of transitions associated with events. The account of inertia from the half of line (1) expressed by (8) is unproblematic enough: change requires force. But the other half of (1), the converse of (8), misrepresents how complicated determining the effects of forces can be. Incrementality (the possibility of more than two degrees) opens the door to competition, necessitating the "no-intervention" provisos, $\neg P_{\mathrm{ap}(\downarrow D)}(x)$ and $\neg P_{\mathrm{ap}(\uparrow D)}(x)$, in the antecedents of (11) and of (14). In Allen and Ferguson (1994), thwarted forces lead to a predicate $Try(f, t)$ that takes an *action* (or force) term f and time period t, corresponding above to $P_{\mathrm{ap}(f)}(t)$.[1]

Whether we refer to f as a force or an action, what are we to make of the property $\mathrm{ap}(f)$? As far as the point (semelfactive) entry $\boxed{\mathrm{ap}(f) \mid \mathrm{ef}(f)}$ in Table 3 is concerned, $\mathrm{ap}(f)$ is clearly non-stative — i.e., subject to $\square$-removal, as opposed to destuttering $d_\square$. But turning to a force f given by incremental change, our revision (13) of (10) has the effect beyond (12) of adding (via (11)) the implications

$$P_{\succ d}(x) \wedge P_{\mathrm{ap}(\uparrow D)}(x) \wedge xSy \wedge \neg P_{\mathrm{ap}(\downarrow D)}(x) \;\supset\; P_{\succ d}(y) \qquad (d \in D_0).$$

Conservative forces that guard against change are left out of $l(a)$,[2] along with incrementality and competition. If $\uparrow D$ can have the effect of *not* changing $P_{\succ d}$, what becomes of the assumption (B) above behind box-removal $d_\square$? In Moens and Steedman (1988), the difference between a state and a process (activity)

$$\boxed{\mathrm{ap}(f) \mid \mathrm{ap}(f), \mathrm{ef}(f) \mid \mathrm{ef}(f)} \tag{15}$$

is blurred by a *progressive state*. Arguably, that progressive state pertains to the second box $\boxed{\mathrm{ap}(f), \mathrm{ef}(f)}$ in (15), perhaps with $\mathrm{ap}(f)$ replaced by a stative variant, $\mathrm{ap}_s(f)$. Aspectual type shifts are commonly associated with reconstruals, and rather than attempt to resolve the aspectual character of $\mathrm{ap}(f)$ definitively, suffice it to repeat Levin and Rappaport Hovav (2013)'s claim that context is required to spell out the effect $\mathrm{ef}(f)$ of a manner verb f. That wrinkle is a sign of, in Robin Cooper's words, "semantics in flux," challenging a legacy from Montague (1974)

> the impression of natural languages as being regimented with meanings determined once and for all by an interpretation (Cooper, 2012, page 271).

This impression is congenial with Allen and Ferguson (1994)'s avowed position that temporal structure is prior to properties, events and actions — a position open to dispute (harking back to Russell).

4   Projections within records and record types

Semantic flux is an important motivation for Type Theory with Records (TTR), against which it is instructive to understand the present paper's

Main Claim   *Temporal notions such as those in Allen and Ferguson (1994) and Moens and Steedman (1988) can be represented in strings structured by MSO and finitary projections, on which we can reason through finite-state methods.*

The promise of finite-state methods (mentioned in the Main Claim) rests on (i) a classic theorem due to Büchi, Elgot and Trakhtenbrot (Libkin, 2010) mapping MSO-sentences to finite automata checking satisfaction (and back), and (ii) the computability by finite-state transducers of the projections proposed. These projections operate between finite sets A and A', composing $f \in \{bc, d_\square, id\}$ (where id is the identity function) with ρ_A for the function $f_{A,A'} = \rho_A; f : (2^{A'})^* \to (2^A)^*$ mapping a string s of subsets of A' to the string $f(\rho_A(s))$ of subsets of A that f returns when fed the A-reduct $\rho_A(s)$ of s.

Proposition 2.   *Given any set Θ, let $Fin(\Theta)$ be the set of finite subsets of Θ. For $f \in \{bc, d_\square, id\}$, the family $\{f_{A,A'} : (2^{A'})^* \to (2^A)^*\}_{A, A' \in Fin(\Theta)}$ is a projective system — i.e., $f_{A,A}$ is the identity on $(2^A)^*$ and $f_{A,A''}$ is the composition $f_{A',A''}; f_{A;A'}$ for all $A \subseteq A' \subseteq A'' \in Fin(\Theta)$.*

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[1] Talk of "forces" complements inertia, while "action" is in the title of Davidson (1967) and is likened in Allen and Ferguson (1994) to a *program* (quite natural to *apply*). Programs in *Dynamic Logic* (Harel et al., 2000) underlie yet another approach to verb semantics (Naumann, 2001; Pustejovsky and Moszkowicz, 2011), relations with which I hope to take up elsewhere.

[2] A force that resists change is old hat to readers familiar with, for instance, Talmy (1988).

Recall from section 2 the introduction of strings and the projections $\rho_A, b\!c$ and $d_\square$ through a bounded form of Leibniz's law in MSO (linking stutterless and $\square$-less strings according to Proposition 1). MSO properties are restricted to unary predicates over string positions, compelling us in section 3 to sidestep the formula

$$(\exists d \in D)\, P_d(y) \wedge (\exists x S y)(\exists d' \prec d) P_{d'}(x) \tag{16}$$

(in (9)) saying the D-degree at y is greater than at its predecessor. Logical hygiene around $P_a(x)$ dictates separating the temporal entities over which the property-argument x ranges from the bits incorporated into the property-index a. Among the latter bits are degrees d in $P_{\succ d}$ and $P_{\approx d}$, as well as actions/forces f in $P_{\mathrm{ap}(f)}$ and $P_{\mathrm{ef}(f)}$. That said, any finite $\prec$-chain

$$d_1 \prec d_2 \prec \cdots \prec d_n \quad \text{in } D$$

yields an approximation of (16) as the finite disjunction

$$\bigvee_{i=1}^{n} P_{>i}(y) \wedge (\exists x)(x S y \wedge P_i(x)) \tag{17}$$

much as time is sampled in section 2 by a string s, with string positions populating the MSO-model $\mathrm{Mod}(s)$.[3] A basic flaw, however, in (17) is that the indices $> i$ and i (appearing as subscripts in $P_{>i}$ and P_i) leave out the attribute that is being graded. That is, the degree d in P_d ought properly to be fleshed out as an attribute-value pair (ℓ, v) with a grade or value v that a force $\uparrow D$ can raise (and $\downarrow D$ lower). The letter ℓ for attribute can also be understood as a ℓabel in a record $\{\langle \ell_i, v_i \rangle\}_{i \in [k]}$ or record-type $\{\langle \ell_i, T_i \rangle\}_{i \in [k]}$. In the remainder of this paper, we decompose strings that capture changes in $\{P_a\}_{a \in A}$ in terms of records and record types with labels equal to subsets of A, approaching MSO (under the projections above) bottom-up and perhaps even probabilistically.

Given a finite set A, and $f \in \{b\!c, d_\square, id\}$, an (A, f)-*string* is a string s over the alphabet 2^A such that $f(s) = s$ (meaning s is stutterless for $f = b\!c$, or s is $\square$-less for $f = d_\square$). An (A, f)-*record* is a record $\{\langle \ell_i, v_i \rangle\}_{i \in [k]}$ such that each label ℓ_i is a subset of A, and each v_i is an (ℓ_i, f)-string. We can decompose a string s over 2^A into its $\{a\}$-reducts for the (A, id)-record $\{\langle \{a\}, \rho_{\{a\}}(s) \rangle\}_{a \in A}$, from which we can reconstruct s by componentwise union $\&_\circ$ of strings of the same length

$$\alpha_1 \cdots \alpha_n \;\&_\circ\; \alpha'_1 \cdots \alpha'_n \;:=\; (\alpha_1 \cup \alpha'_1) \cdots (\alpha_n \cup \alpha'_n)$$
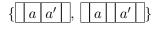
by repeatedly appealing to

$$\rho_{A_1 \cup A_2}(s) \;=\; \rho_{A_1}(s) \;\&_\circ\; \rho_{A_2}(s) . \tag{18}$$

For $f = b\!c$ or $d_\square$, however, (18) will not do,[4] assuming the (A, f)-record $\{\langle \ell_i, v_i \rangle\}_{i \in [k]}$ is understood as describing the set $\mathcal{L}(\{\langle \ell_i, v_i \rangle\}_{i \in [k]})$ of (A, f)-strings that f-project to each v_i

$$\mathcal{L}(\{\langle \ell_i, v_i \rangle\}_{i \in [k]}) \;:=\; \{ f(s) \mid s \in (2^A)^* \text{ and } (\forall i \in [k])\, f(\rho_{\ell_i}(s)) = v_i \}.$$

Under this assumption, the $(A, b\!c)$-record $\{\langle \{a\}, \boxed{\;\boxed{a}\;} \rangle\}_{a \in A}$ describes the set of A-timelines (as defined in section 2). To specify an Allen relation R between a and a', we form the label $\{a, a'\}$ and pair it with the string $\square \mathfrak{s}_{aRa'} \square$ from Table 1. But what if say, we know only that the Allen relation between a and a' is either meet, m, or before, b? Then we should pair the label $\ell = \{a, a'\}$ with the set

$$\{\; \boxed{\;\boxed{a}\,\boxed{a'}\;}\;,\; \boxed{\;\boxed{a}\;\boxed{a'}\;}\;\}$$

of $(b\!c, \{a, a'\})$-strings picturing $a \ \mathrm{m} \ a'$ and $a \ \mathrm{b} \ a'$. Mildly generalizing the notions above, let us agree

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[3] In terms familiar from, for example, Grenon and Smith (2004), strings that structure occurrents/perdurants along temporal S-steps may arise from strings that structure continuants/endurants along a $\prec$-scale. See also Jackendoff (1996).

[4] While any finite string is too short to serve as a timeline, it can be extended indefinitely through inverse limits relative to the composition of ρ_A with $b\!c$ or $d_\square$. MSO under these projections has a formulation, spelled out in Fernando (2016), as an *institution* in the sense of Goguen and Burstall (1992). So too does a finite-state fragment of TTR (Fernando, 2017), although how to relate these institutions category-theoretically remains (as far as I know) to be worked out.

(i) an (A, f)-*record type* is a record type $\{\langle \ell_i, T_i \rangle\}_{i \in [k]}$ such that each label ℓ_i is a subset of A, and each T_i is a set of (ℓ_i, f)-strings

(ii) the *language described by* an (A, f)-record type $\{\langle \ell_i, T_i \rangle\}_{i \in [k]}$ is the set $\mathcal{L}(\{\langle \ell_i, T_i \rangle\}_{i \in [k]})$ of (A, f)-strings that for each $i \in [k]$, f-project to some string in T_i

$$\mathcal{L}(\{\langle \ell_i, T_i \rangle\}_{i \in [k]}) \ := \ \{f(s) \mid s \in (2^A)^* \text{ and } (\forall i \in [k]) \ f(\rho_{\ell_i}(s)) \in T_i\}.$$

Different (A, f)-record types can describe the same language, as illustrated by the $[k+1]$-timelines in

$$\mathcal{L}(\{\langle \{i\}, \boxed{\ i\ } \rangle\}_{i \in [k+1]}) \ = \ \mathcal{L}(\{\langle \{i, i+1\}, L_i \rangle\}_{i \in [k]}) \tag{19}$$

where $k \geq 1$ and L_i is the set of 13 strings, $\square \mathfrak{s}_{iRi+1} \square$, one per Allen relation R

$$L_i \ = \ \{\square \mathfrak{s}_{iRi+1} \square \mid R \in \mathcal{AR}\}.$$

What is gained by complicating the $([k+1], \textit{bc})$-record type on the left side of (19) to that to its right? Labels with two intervals (such as i and $i+1$) allow us to represent information updates that eliminate strings from L_i. Indeed, this is the basis of interval networks which operate around a transitivity table (Allen, 1983, Figure 4) that specifies for every pair (R_1, R_2) of Allen relations, the set $t(R_1, R_2)$ of Allen relations R such that under some $\{1, 2, 3\}$-timeline, $1 R_1 2$, $2 R_2 3$ and $1 R 3$

$$t(R_1, R_2) \ = \ \{R \in \mathcal{AR} \mid \text{there is a } \{1, 2, 3\}\text{-timeline that } \textit{bc}\text{-projects to}$$
$$\square \mathfrak{s}_{1R_1 2} \square \text{ and } \square \mathfrak{s}_{2R_2 3} \square \text{ and } \square \mathfrak{s}_{1R3} \square\}.$$

For example, $t(m,m) = \{b\}$ since $\boxed{\ \ |1|2|3|\ \ }$ is the one string in the language described by

$$\{\langle \{1, 2\}, \boxed{\ \ |1|2\ |}\ \rangle, \langle \{2, 3\}, \boxed{\ \ |2|3\ |}\ \rangle\}$$

whereas $t(m,d) = \{o,d,s\}$ means exactly three strings belong to the language described by

$$\{\langle \{1, 2\}, \boxed{\ |1|2\ |}\ \rangle, \langle \{2, 3\}, \boxed{\ |3|2,3|3\ |}\ \rangle\}$$

(where $\mathfrak{s}_{a\,\text{d}\,a'} = \boxed{a' \mid a, a' \mid a'}$). The challenge, in general, is, given a set L of (A, f)-strings, to describe L through an (A, f)-record type $\{\langle \ell_i, T_i \rangle\}_{i \in [k]}$ such that, if possible,

(†) no two labels in the set $\{\ell_i\}_{i \in [k]}$ are $\subseteq$-comparable   (minimizing redundancy)

(‡) each T_i is a singleton $\{v_i\}$ (minimizing branching).

The antichain condition (†) on labels mirrors one for Russell instants in section 2, and can be satisfied by keeping only the labels that are $\subseteq$-maximal. (‡) can be a more difficult, if not impossible, demand (Woods and Fernando, 2018). A measure of non-determinism being unavoidable, L may serve as a sample space on which to define a probability mass function (Fernando and Vogel, 2019). The strings in L are finite, and hold no mysteries. To make this point forcefully, I close on an aspirational note, brazenly quoting the physicist John Archibald Wheeler on *it from bit*

> every *it* – every particle, every field of force, even the space-time continuum itself – derives its function, its meaning, its very existence entirely – even if in some contexts indirectly – from the apparatus-elicited answers to yes-or-no questions, binary choices, bits. *It from bit* symbolizes the idea that every item of the physical world has at bottom – a very deep bottom, in most instances – an immaterial source and explanation; that which we call reality arises in the last analysis from the posing of yes-no questions and the registering of equipment-evoked responses; in short, that all things physical are information-theoretic in origin and that this is a participatory universe (Wheeler, 1990, page 5).

Here, *it* is the value/string v_i (or type/language T_i), linked by ℓ_i in records (or record types), and based (at a shallow bottom) on "yes-no questions" P_a, the responses to which are registered by the apparatus of MSO in S-steps.

Acknowledgments

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