Different Tunes Played with Equal Skill: Exploring a Unified Optimization Subspace for Delta Tuning

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Abstract

Delta tuning (DET, also known as parameter-efficient tuning) is deemed as the new paradigm for using pre-trained language models (PLMs). Up to now, various DETs with distinct design elements have been proposed, achieving performance on par with fine-tuning. However, the mechanisms behind the above success are still under-explored, especially the connections among various DETs. To fathom the mystery, we hypothesize that the adaptations of different DETs could all be reparameterized as low-dimensional optimizations in a unified optimization subspace, which could be found by jointly decomposing independent solutions of different DETs. Then we explore the connections among different DETs by conducting optimization within the subspace. In experiments, we find that, for a certain DET, conducting optimization simply in the subspace could achieve comparable performance to its original space, and the found solution in the subspace could be transferred to another DET and achieve non-trivial performance. We also visualize the performance landscape of the subspace, and find that, there exists a substantial region where different DETs all perform well. Finally, we extend our analysis and show the strong connections between fine-tuning and DETs. The codes are publicly available at https://github.com/thunlp/Unified-DeltaTuning.

1 Introduction

Serving as the critical backbone for NLP, pre-trained language models (PLMs) achieve superior performance when adapted to downstream tasks (Han et al., 2021). Conventionally, the dominant way for such an adaptation is fine-tuning, which requires updating and storing all the parameters in PLMs. Consequently, with ever-larger PLMs continually being proposed (Raffel et al., 2019; Brown et al., 2020), fine-tuning becomes extremely computationally expensive. As an alternative, various delta tuning algorithms (DETs) spring up, which freeze most of the parameters and only optimize minimal adaptive parameters (Ding et al., 2022). Up to now, various DETs have been proposed, including introducing extra tunable neuron modules (Houlsby et al., 2019a), specifying partial parameters to be tunable (Ben Zaken et al., 2021) and re-parameterizing part of existing modules in PLMs (Hu et al., 2021b), etc. DETs extensively reduce the number of tunable parameters, and still achieves comparable downstream performance to fine-tuning.

Despite the success of DETs, the mechanism behind it remains unclear. An essential question is: how could the PLM adaptation using different DETs relate to each other? To answer this question, a direct exploration of the connections among different DETs is needed, but this would run into a problem: due to the versatile designs of DETs, the parameter space of various DETs is inherently different. To address the issue and investigate the above research question, we hypothesize that the adaptations of different DETs could be re-parameterized as low-dimensional optimizations in a **unified optimization subspace**. In this sense, optimizing various DETs can all be viewed as finding optimal solutions within the same subspace. Our hypothesis is inspired by recent findings that despite owning huge amounts of parameters, PLMs have an extremely low intrinsic dimension (Aghajanyan et al., 2021; Qin et al., 2021). In this regard, optimizing a certain PET, which is typically a high-

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dimensional optimization problem, could be equivalently re-parameterized as a low-dimensional optimization problem, while achieving non-trivial performance.

To find evidence for our hypothesis, we design an analysis pipeline as follows: we first independently obtain solutions for different DETs on a set of tasks. Then we learn to project these solutions to a desired subspace. Meanwhile, we also define a mapping from the subspace to each DET's original space. We contend that if the found subspace is indeed shared among various DETs, then two conditions should be satisfied: (1) the optimizations of different DETs could be equivalently conducted in the found subspace and achieve non-trivial performance, and (2) the local optima of various DETs have a substantial intersection in the subspace, which means the solution obtained in the subspace using a certain DET could be directly transferred to other DETs. If both conditions are well-established for the found subspace, then we could validate the existence of the unified optimization subspace for DETs.

We conduct experiments on a series of representative NLP tasks, and demonstrate that in the found subspace:

- **Solutions are transferable.** The solution of a DET in the found subspace not only achieves comparable performance to that in its original DET space, but can be directly transferred to another DET, achieving non-trivial performance.
- **Local optima of DETs greatly overlap.** When visualizing the performance landscape, we find that there exists a substantial region where different DETs all perform well, indicating the close connections among different DETs.
- **Fine-tuning has strong connection with DETs.** We extend the above analysis to fine-tuning and show the strong connections between fine-tuning and DETs.

In general, our study is the first work to reveal the connections among different DETs and fine-tuning from the perspective of subspace optimization, and uncovers the underlying mechanism of PLMs' downstream adaptation. We believe many applications such as the ensemble and transfer among various DETs can be well empowered by the unified optimization subspace. Our findings can be of interest to researchers who are working on designing better DETs, and may provide some guidance for using DETs in many real-world scenarios.

2 Background

Delta Tuning. DET has been regarded as the new paradigm for PLM adaptation. By training lightweight parameters, DET yields a compact and extensible model, and could achieve comparable performance to full-parameter fine-tuning. Up to now, various DET designs have sprung up. For instance, some introduce additional tunable modules after the feed-forward and attention modules in a PLM (Houlsby et al., 2019a; Pfeiffer et al., 2021); others prepend tunable prompt tokens into each attention layer (Li and Liang, 2021a) or only the embedding layer (Lester et al., 2021). Another line of work re-parameterizes existing modules with low-rank decompositions (Hu et al., 2021b). Recently, researchers demonstrate that existing DET algorithms can be combined simultaneously and achieve better performance (He et al., 2021; Mao et al., 2021).

To fathom the mechanisms behind DET, He et al. (2021) pioneered to explore the connections among different DETs. They formalize various DETs as different ways to compute the modifications on the hidden states and unify different DETs in terms of *formulas*. However, the unification in the formula does not reveal the essence of DETs' success, and does not indicate that their internal mechanisms are unified. Our paper differs from theirs in that we explore whether DETs can be unified in terms of internal mechanisms through the lens of *optimization*. Specifically, we investigate whether the optimization of different DETs can be unified in a certain subspace.

Intrinsic Dimension. Intrinsic dimension (Li et al., 2018) estimates the minimum number of tunable parameters needed to reach a satisfying performance for neural networks. Instead of training networks in their native parameter space, they linearly re-parameterize all the tunable parameters θ_0 in a randomly oriented subspace: $\theta \leftarrow \theta_0 + \texttt{Proj}(\theta_\text{I})$, where $\texttt{Proj} : \mathbb{R}^{|\theta_\text{I}|} \rightarrow \mathbb{R}^{|\theta_0|}$ denotes a random projection ($|\theta_\text{I}| \ll |\theta_0|$). During optimization, only the low-dimensional vector θ_I is tuned. Considering that $|\theta_0|$ could be extremely large, making computation of the projection intractable, Aghajanyan

et al. (2021) reduce the computational complexity using Fastfood transformation (Le et al., 2013). In experiments, they find that for PLMs, a low-dimensional (e.g., $|\theta_{\mathrm{I}}| \sim 10^3$) re-parameterization could achieve over 85% performance of fine-tuning ($|\theta_0|$ exceeds millions or even billions). Further, Qin et al. (2021) extend the tuning method from fine-tuning to prompt tuning (Lester et al., 2021). They demonstrate that the projection `Proj` can be trained in order to approximate a better optimization subspace. Based on previous explorations of intrinsic subspace, we aim to validate the existence of a unified subspace for various tuning methods.

3 Preliminary

Following He et al. (2021), we investigate three representative DET algorithms to validate our hypothesis, including Adapter (Houlsby et al., 2019a), Prefix-tuning (Li and Liang, 2021a), and LoRA (Hu et al., 2021b). We will first recap the Transformer layer (Vaswani et al., 2017), and then give a brief review of the three DETs.

Transformer layer. PLMs generally have multiple Transformer layers, each consisting of a multi-head attention (MHA) and a feed-forward network (FFN). MHA is composed of N_h attention heads, each containing a query / key / value weight matrix $\mathbf{W}_q^{(i)}$ / $\mathbf{W}_k^{(i)}$ / $\mathbf{W}_v^{(i)} \in \mathbb{R}^{d\times d_h}$, where d denotes the model dimension and $d_h = d/\mathrm{N}_h$. Given a sequence of n vectors $\mathbf{X} \in \mathbb{R}^{n\times d}$, MHA parameterizes them into queries ($\mathbf{Q}^{(i)}$), keys ($\mathbf{K}^{(i)}$) and values ($\mathbf{V}^{(i)}$) as follows:

$$\mathbf{Q}^{(i)} = \mathbf{X}\mathbf{W}_q^{(i)}, \mathbf{K}^{(i)} = \mathbf{X}\mathbf{W}_k^{(i)}, \mathbf{V}^{(i)} = \mathbf{X}\mathbf{W}_v^{(i)}.$$

Each ($\mathbf{Q}^{(i)}$, $\mathbf{K}^{(i)}$, $\mathbf{V}^{(i)}$) triple is then fed into a self-attention function to obtain the i-th head's representation $\mathbf{H}_i$. All head representations are then concatenated and combined using an output weight matrix $\mathbf{W}_o \in \mathbb{R}^{d\times d}$:

$$\mathbf{H}_i = \mathrm{softmax}(\frac{\mathbf{Q}^{(i)}(\mathbf{K}^{(i)})^T}{\sqrt{d_h}}\mathbf{V}^{(i)}),$$
$$\mathbf{H} = \mathrm{concat}(\mathbf{H}_1, ..., \mathbf{H}_{\mathrm{N}_h})\mathbf{W}_o.$$

The FFN module is a two-layer MLP:

$$\mathrm{FFN}(\mathbf{H}) = \sigma(\mathbf{H}\mathbf{W}_1 + \mathbf{b}_1)\mathbf{W}_2 + \mathbf{b}_2,$$

where $\mathbf{W}_1 \in \mathbb{R}^{d\times d_m}$, $\mathbf{d} \in \mathbb{R}^{d_m}$, $\mathbf{W}_2 \in \mathbb{R}^{d_m\times d}$ and $\mathbf{b}_2 \in \mathbb{R}^d$. d_m is often chosen larger than d.

Adapter. Adapter (Houlsby et al., 2019a) plugs in light-weight feed-forward networks in Transformer layers (after the MHA module and the FFN module). Every adapter layer typically consists of a down-projection matrix $\mathbf{W}_{\text{down}} \in \mathbb{R}^{d\times r_{\mathrm{A}}}$, a non-linear activation function $f(\cdot)$, and an up-projection matrix $\mathbf{W}_{\text{up}} \in \mathbb{R}^{r_{\mathrm{A}}\times d}$, where r_{A} denotes the bottleneck dimension. Denote the input as $\mathbf{X} \in \mathbb{R}^{n\times d}$, adapter applies a residual connection as follows:

$$\mathbf{X} \leftarrow \mathbf{X} + f(\mathbf{X}\mathbf{W}_{\text{down}})\mathbf{W}_{\text{up}}.$$

Prefix-tuning. Prefix-tuning (Li and Liang, 2021a) extends the queries $\mathbf{K}^{(i)}$ / the values $\mathbf{V}^{(i)}$ in every MHA module by prepending learnable prefix vectors $\mathbf{P}_{\mathrm{K}}^{(i)}$ / $\mathbf{P}_{\mathrm{V}}^{(i)} \in \mathbb{R}^{m\times d_h}$ before them, where m denotes the number of *virtual tokens*. The output of an attention head $\mathbf{H}_i$ can be re-formulated as:

$$\mathbf{H}'_i = \mathrm{ATT}(\mathbf{Q}^{(i)}, [\mathbf{P}_{\mathrm{K}}^{(i)}; \mathbf{K}^{(i)}], [\mathbf{P}_{\mathrm{V}}^{(i)}; \mathbf{V}^{(i)}]),$$

where $[\cdot;\cdot]$ denotes concatenation.

LoRA. LoRA (Hu et al., 2021b) re-parameterizes the weight updates $\Delta\mathbf{W}$ of the weight matrix $\mathbf{W}$ in the MHA module with low-rank decompositions, i.e., $\Delta\mathbf{W} = \mathbf{W}_{\mathrm{A}}\mathbf{W}_{\mathrm{B}}$, where $\mathbf{W}_{\mathrm{A}} \in \mathbb{R}^{d\times r_{\mathrm{L}}}$ and $\mathbf{W}_{\mathrm{B}} \in \mathbb{R}^{r_{\mathrm{L}}\times d}$ are two learnable low-rank matrices, with r_{L} being typically a small integer. For an input $\mathbf{X} \in \mathbb{R}^{n\times d}$, LoRA is formulated as:

$$\mathbf{X} \leftarrow \mathbf{X} + s \cdot \mathbf{X}\mathbf{W}_{\mathrm{A}}\mathbf{W}_{\mathrm{B}},$$

where $s \geq 1$ is a scaling hyper-parameter.

4 Analysis Pipeline

As mentioned before, we consider three representative DETs: Adapter (t_{A}), Prefix-tuning (t_{P}), and LoRA (t_{L}). Each DET t_* defines a set of tunable parameters θ_{t_*}. To adapt a PLM to a specific downstream task $\mathcal{T}_i$, we optimize $\theta_{t_*}^i$ to minimize the loss function $\mathcal{L}_{\text{task}}^i(\theta_{t_*}^i|\theta_0)$ defined by $\mathcal{T}_i$, where θ_0 denotes the pre-trained weights. To verify our hypothesis that there exists a unified optimization subspace where all DETs can achieve non-trivial performance, we propose a three-stage analysis pipeline (visualized in Figure 1), where the first stage is designed to approximate the desired subspace, so that in the second stage, the optimizations for different DETs could all be conducted in this subspace. This makes it possible to explore the connections of different DETs in the third stage. Following Qin et al. (2021), to validate the generality of the found subspace and avoid information

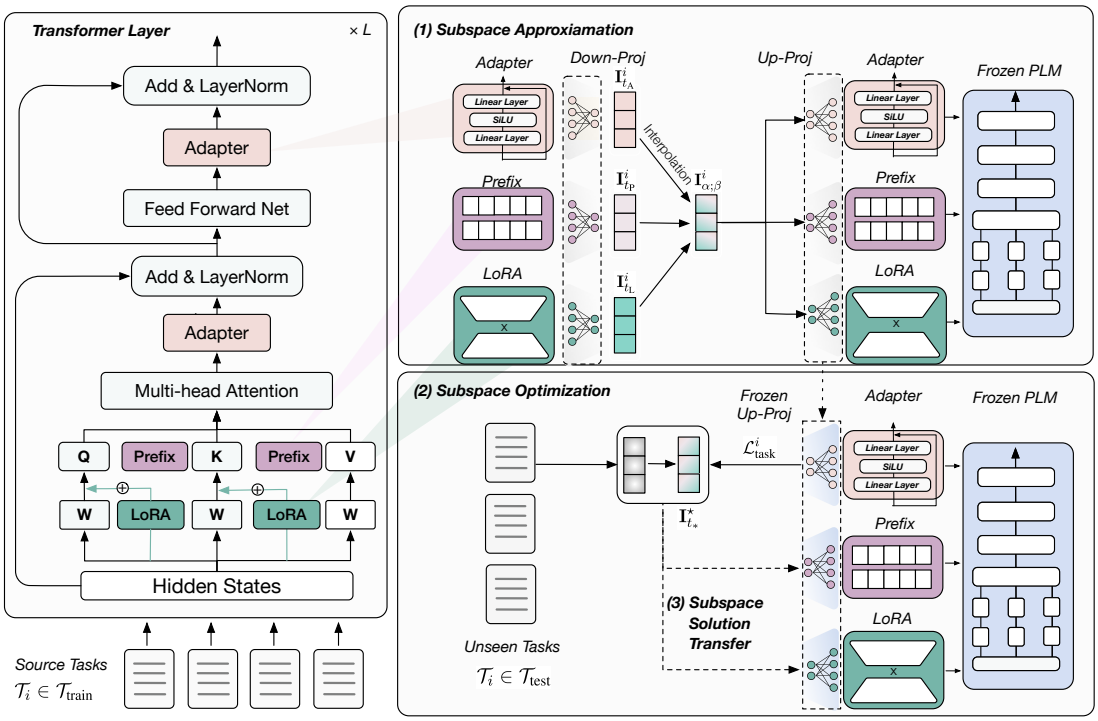


Figure 1: Illustration of our analysis pipeline, consisting of (1) **subspace approximation**, which jointly decomposes DET solutions into a shared subspace, (2) **subspace optimization**, which finds subspace solutions for a specific DET, and (3) **subspace solution transfer**, which transfers the subspace solution from a source DET to other DETs.

leakage, we approximate the subspace with a series of training tasks $\mathcal{T}_{\text{train}}$, and conduct subsequent subspace optimization on unseen tasks $\mathcal{T}_{\text{test}}$.

Subspace Approximation. To approximate the desired subspace, we decompose and then reconstruct *independent* DET solutions of $\mathcal{T}_{\text{train}}$. We first train DETs in their original space, and for each task $\mathcal{T}_i \in \mathcal{T}_{\text{train}}$, we obtain three independent solutions: $\theta^i_{t_{\text{A}}}$, $\theta^i_{t_{\text{P}}}$, and $\theta^i_{t_{\text{L}}}$. Then we assign a down-projection $\texttt{Proj}^{\downarrow}_{t_*} : \mathbb{R}^{|\theta^i_{t_*}|} \rightarrow \mathbb{R}^y$ and an up-projection $\texttt{Proj}^{\uparrow}_{t_*} : \mathbb{R}^y \rightarrow \mathbb{R}^{|\theta^i_{t_*}|}$ for each DET t_*, where y is the dimension of the intrinsic subspace. In practice, both down-projection and up-projection are MLP layers. Each down-projection decomposes a DET solution into a low-dimensional intrinsic vector $\mathbf{I}^i_{t_*} \in \mathbb{R}^y$:

$$\mathbf{I}^i_{t_*} = \texttt{Proj}^{\downarrow}_{t_*}(\theta^i_{t_*}).$$

Three intrinsic vectors $\mathbf{I}^i_{t_{\text{A}}}$, $\mathbf{I}^i_{t_{\text{P}}}$, $\mathbf{I}^i_{t_{\text{L}}}$ represent different local minima of $\mathcal{T}_i$ in the same subspace. Ideally, if three DETs can be unified in the subspace, then each vector $\mathbf{I}^i_{t_*}$ could be used to reconstruct any DET solution ($\theta^i_{t_{\text{A}}}$, $\theta^i_{t_{\text{P}}}$, or $\theta^i_{t_{\text{L}}}$). Therefore, to approximate such a subspace, we facilitate the interaction among different DETs efficiently by dynamically sampling two random ratios $\alpha \in [0, 1], \beta \in [0, 1-\alpha]$, and computing an interpolation of three intrinsic vectors of $\mathcal{T}_i$:

$$\mathbf{I}^i_{\alpha;\beta} = \alpha \cdot \mathbf{I}^i_{t_{\text{A}}} + \beta \cdot \mathbf{I}^i_{t_{\text{P}}} + (1-\alpha-\beta) \cdot \mathbf{I}^i_{t_{\text{L}}}.$$

The interpolation is mapped by each up-projection $\texttt{Proj}^{\uparrow}_{t_*}$ to reconstruct the task solution for each DET by minimizing the following loss function:

$$\mathcal{L}^i_{\text{dist}}(\overline{\theta^i_{t_*}}) = ||\overline{\theta^i_{t_*}} - \theta^i_{t_*}||^2, \quad \overline{\theta^i_{t_*}} = \texttt{Proj}^{\uparrow}_{t_*}(\mathbf{I}^i_{\alpha;\beta}).$$

To properly guide the reconstructed $\overline{\theta^i_{t_*}}$ to solve task $\mathcal{T}_i$, we also incorporate the original task loss $\mathcal{L}^i_{\text{task}}$. The overall training objective can be formulated as follows:

$$\mathcal{L}_{\text{pet}} = \sum_{i=1}^{|\mathcal{T}_{\text{train}}|} \sum_{t_* \in \{t_{\text{A}}, t_{\text{P}}, t_{\text{L}}\}} \mathcal{L}^i_{\text{dist}}(\overline{\theta^i_{t_*}}) + \mathcal{L}^i_{\text{task}}(\overline{\theta^i_{t_*}}|\theta_0).$$

During this stage, only the down-projections and up-projections are optimized, and other parameters are kept frozen. When this stage finishes, the two projections can be seen as mappings between the unified subspace and each DET's original space.

Subspace Optimization. In the second stage, we investigate whether the optimization in the subspace could achieve comparable performance to

the optimization in the original space for unseen tasks $\mathcal{T}_{\text{test}}$. If this holds, then we could empirically validate that the optimizations of different DETs could be equivalently mapped in this subspace with a low level of error, and it is possible to explore the connections among DETs in the next stage.

Specifically, we only retain the up-projection $\texttt{Proj}^{\uparrow}_{t_*}$ trained in the first stage. $\texttt{Proj}^{\uparrow}_{t_*}$ defines the mapping from the found subspace to the original DET space. We keep both PLM and $\texttt{Proj}^{\uparrow}_{t_*}$ frozen during subspace optimization. After that, for each task $\mathcal{T}_i \in \mathcal{T}_{\text{test}}$, the optimization of t_* can be conducted within the subspace defined by $\texttt{Proj}^{\uparrow}_{t_*}$ by merely tuning a randomly initialized intrinsic vector $\mathbf{I}_{t_*}$, which is formulated as:

$$\mathbf{I}^{\star}_{i,t_*} = \arg\min_{\mathbf{I}^{i}_{t_*}} \mathcal{L}^{i}_{\text{task}}(\texttt{Proj}^{\uparrow}_{t_*}(\mathbf{I}^{i}_{t_*})|\theta_0).$$

Subspace Solution Transfer. If the found subspace is shared among different DETs, then the found solution $\mathbf{I}^{\star}_{i,t_*}$ in the subspace could be directly transferred to another DET and achieve non-trivial performance. Taking the transferring between t_{A} and t_{P} as an example, for a task $\mathcal{T}_i \in \mathcal{T}_{\text{test}}$, we first conduct subspace optimization for t_{A} and obtain a well-tuned intrinsic vector $\mathbf{I}^{\star}_{i,t_{\text{A}}}$. Then we directly transfer $\mathbf{I}^{\star}_{i,t_{\text{A}}}$ to t_{P} utilizing its up-projection $\texttt{Proj}^{\uparrow}_{t_{\text{P}}}$, and obtain a t_{P}'s solution $\theta_{i,t_{\text{A}}\rightarrow t_{\text{P}}}$ in the original DET space:

$$\theta_{i,t_{\text{A}}\rightarrow t_{\text{P}}} = \texttt{Proj}^{\uparrow}_{t_{\text{P}}}(\mathbf{I}^{\star}_{i,t_{\text{A}}}).$$

5 Experiment

We conduct experiments on representative NLP tasks. We first introduce the experimental setups in §5.1, next we approximate the subspace and present the analysis in §5.2. Lastly, we explore the connection between DETs and fine-tuning in §5.3.

5.1 Experimental Setups

Training Setups. We conduct experiments with both single-task and multi-task settings.

In the *single-task setting*, we approximate the unified optimization subspace using only one dataset, i.e., $|\mathcal{T}_{\text{train}}|=1$. Then we perform the subspace optimization and subspace solution transfer on unseen tasks. However, the subspace approximated with only one task may not generalize well to diverse unseen tasks (Qin et al., 2021). For example, the subspace approximated using a NLI task can hardly be generalized to a QA task. Therefore, for the single-task setting, we only evaluate the found subspace using the unseen tasks belonging to the same category of $\mathcal{T}_{\text{train}}$.

Besides, we also experiment on the *multi-task setting*, where the unified subspace is approximated with diverse training tasks, i.e., $|\mathcal{T}_{\text{train}}| > 1$. The unified DET subspace found in the multi-task setting is expected to generalize to more diverse tasks than that in the single-task setting.

During subspace solution transfer, we choose the subspace solution that achieves the best transferring performance using the development set, and report its performance on the test set.

Tasks and Datasets. In the single-task setting, we experiment with 6 types of tasks, including:

- **Sentiment Analysis (SA)**: SST-2 (Socher et al., 2013), Rotten Tomatoes (Pang and Lee, 2005), and Amazon Review (McAuley and Leskovec, 2013).
- **Natural Language Inference (NLI)**: SciTail (Khot et al., 2018), MNLI (Williams et al., 2018), and RTE (Dagan et al., 2005).
- **Text Classification (TC)**: WiC (Pilehvar and Camacho-Collados, 2019), and WSC (Levesque et al., 2012).
- **Paraphrase Detection (PD)**: QQP(link), and MRPC (Dolan and Brockett, 2005).
- **Long-form QA (LF-QA)**: ELI5-ELI5, ELI5-Askh, and ELI5-Asks (Fan et al., 2019).
- **Multiple-choice QA (MC-QA)**: CoPA (Gordon et al., 2012), DREAM (Saha et al., 2018), QuaRTz (Tafjord et al., 2019b) and CODAH (Chen et al., 2019).

We include more diverse datasets in the multi-task setting, and randomly partition them into 60 training tasks $\mathcal{T}_{\text{train}}$ and 9 test tasks $\mathcal{T}_{\text{test}}$. More details are left in Appendix A.1.

Evaluation Metrics. For each dataset, we use the common evaluation metric, e.g., ROUGE-L for LF-QA , F1 for SA and NLI, ACCURACY for TC, PD and MC-QA. Denote E_{ori} as the performance achieved by DET in the original space, and E_{sub} as the performance achieved by optimization within the subspace, we report the relative recovering performance (%), i.e., $\frac{E_{\text{sub}}}{E_{\text{ori}}}$ in all experiments.

$\mathcal{T}_{\text{train}}$	$\mathcal{T}_{\text{test}}$	Adapter	LoRA	Prefix	Avg.
SST-2	Rotten Tomatoes	101.8	100.1	99.3	100.4
	Amazon Review	98.0	96.9	98.2	97.7
MNLI	SciTail	82.9	79.8	84.4	82.4
	RTE	95.4	68.2	80.2	81.3
WiC	WSC	70.6	57.6	77.1	68.4
QQP	MRPC	85.4	84.3	83.1	84.3
ELI5-ELI5	ELI5-Askh	91.4	87.6	80.1	86.4
	ELI5-Asks	96.6	95.0	94.9	95.5
DREAM	CODAH	77.4	70.4	74.0	73.9
	QuaRTz	75.6	78.5	74.7	76.3
	CoPA	98.3	71.6	92.5	87.5
	Avg.	88.5	80.9	85.3	84.9

Table 1: Relative performance (%) for subspace optimization under the single-task setting.

$\mathcal{T}_{\text{train}}$	$\mathcal{T}_{\text{test}}$	A→L	A→P	L→A	L→P	P→A	P→L	Avg.
SST-2	R. Tomatoes	100.6	99.0	100.7	98.8	101.0	100.8	100.2
	A. Review	97.1	97.7	97.7	98.1	97.6	96.7	97.5
MNLI	SciTail	81.7	83.0	83.2	83.6	83.4	80.9	82.6
	RTE	62.7	74.3	81.7	78.2	55.0	80.0	72.0
WiC	WSC	72.7	54.3	58.8	57.1	76.5	69.7	64.9
QQP	MRPC	83.7	65.7	83.7	69.1	84.8	85.4	78.7
ELI5-ELI5	ELI5-Askh	88.0	79.4	91.3	78.1	90.3	87.0	85.7
	ELI5-Asks	95.9	95.4	91.3	92.5	97.8	96.1	94.8
DREAM	QuaRTz	76.2	71.9	76.0	74.5	76.4	77.2	75.4
	CODAH	63.0	61.6	74.3	58.1	83.9	69.0	68.3
	CoPA	77.3	100.3	96.0	103.1	91.7	65.6	89.0
	Avg.	81.7	80.2	85.0	81.0	85.3	82.6	82.6

Table 2: Relative performance (%) for subspace solution transfer under the single-task setting. **A**, **P**, and **L** refer to Adapter, Prefix-tuning, and LoRA, respectively. As an example, **A**→**L** means we obtain $\mathbf{I}^{\star}_{t_{\text{A}}}$ by conducting subspace optimization with Adapter (source DET), and then transfer the subspace solution to LoRA (target DET) with the fixed up-projection, i.e., $\texttt{Proj}^{\uparrow}_{t_{\text{P}}}(\mathbf{I}^{\star}_{t_{\text{A}}})$.

Models. We use $\text{T5}_{\texttt{BASE}}$ (Raffel et al., 2020) as the backbone model, and unify all tasks into a text-to-text format without loss of generality. We set the dimension of the subspace to 4 in single-task setting and 100 in multi-task setting. During subspace optimization, only 4 or 100 free parameters are tuned, compared with 220M for fine-tuning. We choose the intrinsic dimension according to our preliminary experiment. The single-task performances of different intrinsic dimensions in $\{4, 8, 16\}$ do not vary much. The multi-task performance gets better when the intrinsic dimension increases. Practically, we find a dimension of 100 strikes a satisfying balance between performance and computational resources. The details of implementation are shown in Appendix A.2.

5.2 Experimental Results

5.2.1 Single-task Setting

Subspace Optimization. The results of subspace optimization are presented in Table 1. On average, for all three DETs, optimization within the subspace can recover more than 80% performance of the original space. Among three DETs, Adapter achieves the best recovering performance ($\approx$ 90%), despite only tuning 4 free parameters. This indicates that we have found a satisfying optimization subspace that could recover most of the performance of the original space[1], and the subspace can be generalized to unseen tasks $\mathcal{T}_{\text{test}}$ belonging to the same category of $\mathcal{T}_{\text{train}}$.

[1] Aghajanyan et al. (2021) deem 85% as a satisfying recovering performance for an intrinsic subspace. Although the performance of our method could be a bit lower under certain cases, we contend that the performance is already non-trivial.

Subspace Solution Transfer. Then we transfer the solution found with a source DET to other DETs. The results are presented in Table 2. On 6 out of the 11 tasks, transferring the subspace solution from a source DET to a target DET achieves more than 80% recovering performance, and achieves 82.6% on average across all tasks. This demonstrates that the transferred intrinsic vector yields DETs with non-trivial performance. In particular, in the category of sentiment analysis, the subspace of three DETs approximated on SST-2 serves as an excellent optimization subspace for similar tasks (R. Tomatoes and A. Review). Performing optimization in this subspace with an arbitrary source DET and transfer the found intrinsic vector to other DETs yield performance comparable to or even surpass the original DET space.

However, we also observe that the transferred DET on WSC does not perform very well, achieving only 64.9% recovering performance. We argue that this may be due to the inherent difference between WiC ($\mathcal{T}_{\text{train}}$) and WSC ($\mathcal{T}_{\text{test}}$): WiC evaluates the quality of context-sensitive representations, while WSC is a coreference resolution task, which requires slightly distinct language skills from WiC. Besides, the performances of the transferred DET on DREAM are also slightly below the expectation, this may due to the domain differences between DREAM and the target tasks. Although they all belong to multi-choice QA, their domains differ significantly. In fact, these unwanted transferred performance can be substantially improved

$\mathcal{T}_{\text{test}}$	Adapter	LoRA	Prefix	Avg.
Rotten Tomatoes	99.7	98.2	100.3	99.4
Yelp Polarity	99.5	99.5	98.7	99.2
WSC	88.2	75.8	80.0	81.3
AI2 ARC	93.2	87.9	78.8	86.6
QASC	99.3	71.4	90.9	87.2
QuaRTz	96.6	86.9	77.3	86.9
BLiMP-ANA	100.0	100.0	51.0	83.7
ELI5-Asks	99.9	99.6	94.3	97.9
ETHOS-Gender	79.6	88.9	59.0	75.8
Avg.	95.1	89.8	81.1	88.7

Table 3: Relative performance (%) for subspace optimization under the multi-task setting.

$\mathcal{T}_{\text{test}}$	A→L	A→P	L→A	L→P	P→A	P→L	Avg.
Rotten Tomatoes	97.5	96.6	98.5	95.9	98.3	96.2	97.2
Yelp Polarity	99.1	97.5	99.4	98.1	98.5	98.4	98.5
WSC	90.9	85.7	97.1	94.3	91.2	93.9	92.2
AI2 ARC	78.7	79.3	87.7	76.9	85.2	87.9	82.6
QASC	65.1	63.6	98.7	72.1	105.2	68.0	78.8
QuaRTz	77.4	71.7	90.9	73.9	83.4	78.1	79.2
BLiMP-ANA	98.0	47.0	95.0	49.0	92.0	95.0	79.3
ELI5-Asks	90.6	89.7	89.6	95.0	95.8	87.4	91.4
ETHOS-Gender	55.5	57.8	53.0	73.3	68.7	69.8	63.0
Avg.	83.6	76.5	90.0	80.9	90.9	86.1	84.7

Table 4: Relative performance (%) for subspace solution transfer under the multi-task setting.

in multi-task setting, as we will see in the next section.

In addition, we do not find a significant difference in the transferability among different DETs. In general, when serving as the source DET, Adapter has slightly worse transferability than other two DETs. Besides, the transferability of a DET seems to have a weak correlation to its performance of subspace optimization.

5.2.2 Multi-task Setting

To improve the subspace's task-level generalization, we propose to approximate the subspace in a multi-task manner (60 training tasks in total), and test the generalization ability of the approximated subspace on 6 categories of unseen tasks. Note for the multi-task setting, the subspace optimization and subspace solution transfer are carried out within the same subspace for *all* the unseen tasks.

Subspace Optimization. The results of subspace optimization under the multi-task setting are shown in Table 3. In general, three DETs achieve non-trivial (88.7%) performance during subspace optimization on unseen tasks. Among three DETs, Adapter still performs the best, achieving 95.1% of its original performance. However, the performance of Prefix-tuning is about 10% poorer than Adapter and LoRA. We observe that when approximating the subspace, the loss of Prefix-tuning converges much slower than Adapter and LoRA, which may partially explain the poorer performance of Prefix-tuning. We leave further exploration of this phenomenon as future work.

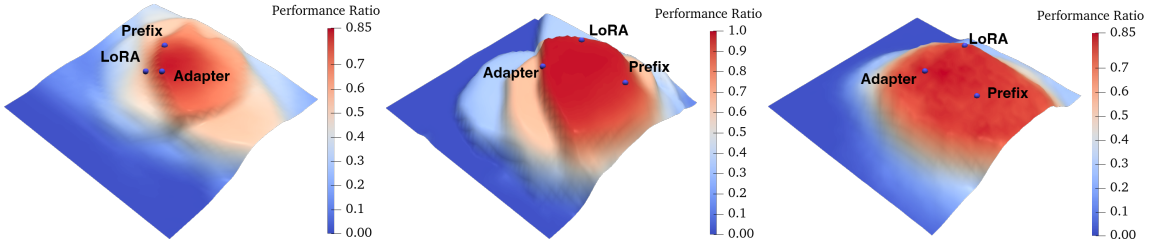
Subspace Solution Transfer. The results are presented in Table 4. On 8 out of 9 tasks, DETs recover around or more than 80% their performance in the original space. The non-trivial results demonstrate that (1) for most of the investigated unseen tasks, the local optima found by a source DET can be directly transferred to a target DET and achieve non-trivial performance; (2) the subspace approximated with multiple training tasks can be well generalized to diverse unseen tasks. Both findings provide strong evidence for our hypothesis that different DETs can be re-parameterized into a unified optimization subspace.

We also observe that, the transferring performance on WSC is far better than that in the single-task setting, demonstrating the benefits of including diverse training tasks in subspace approximation. However, we still find that there are cases where the subspace solution of a source DET has poor transferability to another DET. For instance, the transferring performance of different DETs on ETHOS-Gender is only 63.0%. We conjecture that it is due to the gap between ETHOS-Gender and the training tasks $\mathcal{T}_{\text{train}}$. As demonstrated by Qin et al. (2021), increasing the diversity and number of training tasks could significantly improve the generalization ability of the subspace on unseen tasks. We expect future works to apply our analysis to more diverse training tasks.

Furthermore, comparing the transferability of different DETs, we find that similar to the single-task setting, Adapter is still slightly worse than the other two DETs. We also find there is no symmetry in the transferability. For example, the average transferring performance from Prefix-tuning to Adapter achieves 90.9% of its performance, while the performance in the opposite direction only reaches 76.5%.

5.2.3 Performance Landscape Visualization

From Tables 2 and 4, we observe non-trivial transferring performance among different DETs. These



(a) SciTail (single-task) (b) Yelp Polarity (multi-task) (c) AI2 ARC (multi-task)

Figure 2: Performance landscape visualization on three datasets (SciTail, Yelp Polarity, and AI2 ARC). We highlight the subspace solutions ($\mathbf{I}_{t_\text{A}}, \mathbf{I}_{t_\text{L}}$ and $\mathbf{I}_{t_\text{P}}$) found independently by the three DETs.

results demonstrate that the local optima of different DETs have a substantial overlap in the approximated subspace for the investigated unseen tasks. It is natural to be concerned about how large this overlap area is since a larger overlap may indicate closer connection of different DETs. Therefore, for both single-task and multi-task settings, we visualize the performance landscape to understand to what extent the local optima of different DETs in intrinsic subspace overlap with each other.

Specifically, denote $\mathbf{I}_0$ as an origin, and $\mathbf{u}, \mathbf{v}$ as two orthogonal directions. Let α, β be two coordinates in the 2-dimensional space spanned by $\mathbf{u}$ and $\mathbf{v}$. Each solution $\mathbf{I}_0 + \alpha\mathbf{u} + \beta\mathbf{v}$ in the subspace can be mapped by the up-projection of DET t_* to the solution $\texttt{Proj}^{\uparrow}_{t_*}(\mathbf{I}_0 + \alpha\mathbf{u} + \beta\mathbf{v})$ in the DET space. Denote $E(\texttt{Proj}^{\uparrow}_{t_*}(\mathbf{I}_0 + \alpha\mathbf{u} + \beta\mathbf{v}))$ as the performance of the recovered DET, and E_PET as the average performance of the three DETs in their original space, we plot the relative performance along these two directions as follows:

$$\mathcal{P} = \frac{1}{3} \sum_{t_* \in \{t_\text{A}, t_\text{P}, t_\text{L}\}} \frac{E(\texttt{Proj}^{\uparrow}_{t_*}(\mathbf{I}_0 + \alpha\mathbf{u} + \beta\mathbf{v}))}{E_\text{PET}}.$$

If $\mathcal{P}$ is high at (α, β), then it means three DETs all correspond to high performance at this point. Let $\mathbf{I}_{t_\text{A}}, \mathbf{I}_{t_\text{P}}, \mathbf{I}_{t_\text{L}}$ denote the optimal solution obtained by tuning each DET in the subspace independently. We visualize the performance landscape around these optimal solutions. Without loss of generality, we choose $\mathbf{I}_{t_\text{A}}$ as the origin, and select two orthogonal axes $\mathbf{u}, \mathbf{v}$ as follows:

$$\mathbf{u} = \frac{\mathbf{I}_{t_\text{P}} - \mathbf{I}_{t_\text{A}}}{\|\mathbf{I}_{t_\text{P}} - \mathbf{I}_{t_\text{A}}\|}, \tilde{\mathbf{v}} = \mathbf{I}_{t_\text{L}} - \mathbf{I}_{t_\text{A}}, \mathbf{v} = \frac{\tilde{\mathbf{v}} - \langle \tilde{\mathbf{v}}, \mathbf{u} \rangle \mathbf{u}}{\|\tilde{\mathbf{v}} - \langle \tilde{\mathbf{v}}, \mathbf{u} \rangle \mathbf{u}\|}.$$

We traverse α and β from -4 to 4 with a step size of 0.4. Due to the length limit, we only show in Fig. 2 the performance landscape on (1) SciTail of the single-task setting, (2) Yelp Polarity and AI2 ARC under the multi-task setting.

Src. \ Tgt.	Adapter	Prefix	LoRA	Fine-tune
Adapter	100.6	100.1	97.7	95.5
Prefix	101.2	100.4	97.7	95.0
LoRA	101.1	100.2	97.5	95.8
Fine-tune	101.1	99.7	97.1	96.1

Table 5: Relative performance (%) for subspace optimization and subspace solution transfer for different tuning methods on Rotten Tomatoes. The subspace is approximated on SST-2. We transfer the subspace solution from a source tuning method to a target one.

We observe that the subspace solutions of different DETs almost lie in the same optimal region for each task. Comparing the landscape of both single-task and multi-task settings, the highland area is much wider in the multi-task setting. This may explain the better transferability under the multi-task setting. In general, the above results demonstrate that the optimal solutions of different DETs indeed have a large overlap, otherwise there should not exist such a flat performance highland.

5.3 Extension to Fine-tuning

Finally, we extend our analysis pipeline to fine-tuning, and investigate its connection with DETs. However, directly training a down-projection and up-projection for fine-tuning encounters difficulty: if we still use an MLP layer (as introduced in §4) to implement the projections, the number of trainable parameters will be $\mathcal{O}(yN)$, where N is the number of parameters of the PLM, and y is the dimension of our intrinsic subspace. Since PLMs generally contain tremendous parameters, it is intractable to train such an MLP. To alleviate the problem, we turn to using Fastfood transformation (Yang et al., 2015; Aghajanyan et al., 2021) as an alternative. It is an approximation for the linear projection, but requires far fewer parameters. Specifically, the up-

projection using Fastfood transformation can be formalized as follows:

$$\tilde{\theta} = \theta_0 + \mathbf{I}M, \quad M = HG\Pi HB,$$

where $\tilde{\theta}$ denotes the tunable parameters in the original space, θ_0 is the pre-trained weights, and $\mathbf{I}$ is the intrinsic vector. The Fastfood matrix M can be factorized as a Hadamard matrix H, a random permutation matrix Π, a diagonal matrix G with each element sampled from a standard normal distribution, and a diagonal matrix B with each element being ± 1 with equal probability. Unlike previous work that uses a random and frozen Fastfood matrix (Yang et al., 2015; Aghajanyan et al., 2021), we optimize the matrix G to better approximate the desired subspace. For DETs, the projection is implemented the same as before. More detailed implementations are described in Appendix A.4.

We perform subspace approximation on SST-2, and report the results of subspace optimization and subspace solution transfer on Rotten Tomatoes. As shown in Table 5, we find that: (1) all the DETs and fine-tuning can achieve satisfactory results in subspace optimization, and the solution found by any source tuning method can be transferred to other target tuning methods and achieve non-trivial performance. This demonstrates the close connection between DETs and fine-tuning in the approximated subspace. (2) Since the Fastfood transformation is an approximation for linear projection, its representation ability may be limited. Therefore, the transferring performance of fine-tuning is slightly inferior to other DETs.

In general, the above finding implicates that all tuning methods may be re-parameterized to a unified optimization subspace, which also sheds light on the reason why different DETs optimize distinct sets of parameters, but all achieve comparable downstream performance to fine-tuning.

6 Conclusion

In this work, we explore the hypothesis that the adaptations of different delta tuning methods could all be re-parameterized as low-dimensional optimizations in a unified optimization subspace. The empirical results provide strong evidence for our hypothesis. We also extend our analysis to find the connection between fine-tuning and delta tuning. We hope our findings could provide insights for future research in designing better tuning methods and understanding the mechanisms behind PLM adaptation.

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Limitations

The limitations of this paper are listed as follows:

- Under some settings, the recovering performance of the subspace optimization and subspace solution transfer can still be improved.
- We only conduct the experiments using T5_{BASE} model. We expect future works to apply our analysis pipeline to other kinds of PLMs, and PLMs with larger sizes.

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A Appendices

A.1 Datasets

All the datasets used in the multi-task setting are listed in Table 12 and Table 13. All these datasets are downloaded from Huggingface Datasets (Lhoest et al., 2021).

A.2 Hyper-parameters and Network Structure for §5.2

The down-projection $\texttt{Proj}_{t_*}^{\downarrow}$ is a two-layer MLP, with the first linear layer $f_{\text{down-1}} : \mathbb{R}^{|\theta_{t_*}|} \rightarrow \mathbb{R}^y$, and the second linear layer $f_{\text{down-2}} : \mathbb{R}^y \rightarrow \mathbb{R}^y$, where y is the dimension of intrinsic subspace. We use `tanh` as the activation function between $f_{\text{down-1}}$ and $f_{\text{down-2}}$. The up-projection $\texttt{Proj}_{t_*}^{\uparrow}$ is a single linear layer $f_{\text{up}} : \mathbb{R}^y \rightarrow \mathbb{R}^{|\theta_{t_*}|}$. Note for all the linear layers of the projections, we do not include the bias term.

To ensure that the number of parameters is consistent across the three DETs, we set r_{A} as 12, r_{L} as 10, m as 120, and d_{P} as 24. d_{P} refers to the hidden dimension of the two-layer MLP that is used to re-parameterize Prefix vectors $\mathbf{P}_{\text{K}}^{(i)}$ / $\mathbf{P}_{\text{V}}^{(i)}$, see Li and Liang 2021b for more details. The meanings of other notations are the same as those in §3. In this way, the number of parameters of $\theta_{t_{\text{A}}}^i$, $\theta_{t_{\text{P}}}^i$ and $\theta_{t_{\text{L}}}^i$ are all 1105920. Moreover, following Houlsby et al. 2019b, we choose a SiLU activation function for Adapter. Following Hu et al. 2021a, we set the scaling factor s in LoRA as 1.6. In our implementation, LoRA is applied to $\mathbf{Q}$ and $\mathbf{V}$ matrices in the MHA module.

During subspace approximation, for the multi-task setting, we randomly sample 20000 instances from the original training set of each dataset, and blend them together to form the multi-task training set. Similarly, we sample another 240 instances from each dataset to form the validation set. We set the learning rate as 1×10^{-4}, batch size as 4. We train the model for 1 epoch and evaluate on validation set for every 1000 steps. For the single-task setting, we perform grid search using the learning rates in $\{1 \times 10^{-5}, 5 \times 10^{-5}\}$ and set the batch size as 8. We train the model for a maximum step of 100000, and evaluate on validation set for every 1000 steps. When conducting subspace optimization, we perform grid search using the learning rate in $\{1 \times 10^{-2}, 5 \times 10^{-2}\}$ and set the batch size as 8. We train the model for a maximum of 5000 steps and evaluate on validation set every 500 steps.

$\mathcal{T}_{\text{train}}$	$\mathcal{T}_{\text{test}}$	Adapter	LoRA	Prefix	Avg.
SST-2	Rotten Tomatoes	99.3	98.5	95.5	97.8
	Amazon Review	96.7	96.5	97.1	96.8
MNLI	SciTail	82.0	74.1	73.3	76.5
	RTE	71.5	80.0	71.3	74.3
WiC	WSC	61.8	84.8	85.7	77.4
QQP	MRPC	85.4	81.5	68.5	78.5
ELI5-ELI5	ELI5-Askh	89.1	85.1	77.5	83.9
	ELI5-Asks	93.9	94.4	88.2	92.2
DREAM	CODAH	63.6	63.9	61.6	63.0
	QuaRTz	74.2	72.4	73.9	73.5
	CoPA	90.4	66.0	101.4	85.9
	Avg.	82.5	81.6	81.3	81.8

Table 6: Relative performance (%) for subspace optimization under single-task setting with constructed subspace.

$\mathcal{T}_{\text{test}}$	Adapter	LoRA	Prefix	Avg.
Rotten Tomatoes	92.9	88.5	66.4	82.6
Yelp Polarity	97.0	96.2	90.0	94.4
WSC	94.1	90.9	82.9	89.3
AI2 ARC	77.2	73.4	75.9	75.5
QASC	73.8	64.0	59.8	65.9
QuaRTz	78.0	78.5	74.3	76.9
BLiMP-ANA	94.0	98.0	47.0	79.7
ELI5-Asks	83.9	84.6	79.5	82.7
ETHOS-Gender	54.6	55.4	74.3	61.4
Avg.	82.8	81.1	72.2	78.7

Table 7: Relative performance (%) for subspace solution transfer under under multi-task setting with constructed subspace.

We train the model using Adafactor (Shazeer and Stern, 2018) with a constant learning rate in all experiments. Intrinsic dimension y (the dimension of the approximated subspace) is set to 4 in single-task setting and 100 in multi-task setting. We additionally set a ratio $\alpha = 10$ to balance the reconstruction loss $\mathcal{L}_{\text{dist}}^i(\overline{\theta_{t_*}^i})$ and original task loss $\mathcal{L}_{\text{task}}^i(\overline{\theta_{t_*}^i}|\theta_0)$, i.e., $\mathcal{L}_{\text{pet}} = 10 * \mathcal{L}_{\text{dist}}^i(\overline{\theta_{t_*}^i}) + \mathcal{L}_{\text{task}}^i(\overline{\theta_{t_*}^i}|\theta_0)$. All experiments are carried out on NVIDIA 32GB V100 GPU.

A.3 Simplification of Subspace Approximation

From a standpoint of analysis, it is necessary to start our pipeline from independent solutions of different DETs and then explore their connections. Now that we have validated the existence of the unified optimization subspace, for practical uses, we could simplify the original pipeline by enforcing different DETs to share the same intrinsic vector. Specifically, we *jointly* train three DETs, and

$\mathcal{T}_{\text{train}}$	$\mathcal{T}_{\text{test}}$	A→L	A→P	L→A	L→P	P→A	P→L	Avg.
SST-2	R. Tomatoes	99.7	98.5	100.9	98.8	101.2	99.7	99.8
	A. Review	98.2	98.2	98.3	98.2	98.1	97.3	98.1
MNLI	SciTail	74.9	77.4	83.5	77.0	83.9	74.8	78.6
	RTE	89.1	80.2	58.7	87.1	70.6	87.3	78.8
WiC	WSC	84.8	74.3	70.7	74.3	82.3	72.8	76.5
QQP	MRPC	84.8	67.4	79.8	76.4	78.0	5.6	65.3
ELI5-ELI5	ELI5-Askh	85.1	79.9	93.4	79.9	92.6	85.9	86.1
	ELI5-Asks	93.1	91.5	97.9	91.5	97.3	93.8	94.2
DREAM	QuaRTz	80.0	74.9	77.7	74.2	78.1	80.2	77.5
	CODAH	73.0	77.8	75.9	78.2	80.3	70.9	76.0
	CoPA	67.7	107.5	86.8	103.8	84.4	63.8	85.7
	Avg.	84.6	84.3	84.0	85.4	86.1	75.6	83.3

Table 8: Relative performance (%) for subspace solution transfer under single-task setting with constructed subspace.

$\mathcal{T}_{\text{test}}$	A→L	A→P	L→A	L→P	P→A	P→L	Avg.
Rotten Tomatoes	93.6	88.0	83.5	87.9	84.6	95.0	88.8
Yelp Polarity	97.5	92.0	98.0	87.0	97.9	97.1	94.9
WSC	93.9	94.3	58.9	94.3	64.7	93.9	83.3
AI2 ARC	82.7	78.3	88.4	77.7	83.6	81.4	82.0
QASC	85.2	68.2	103.2	70.7	99.9	85.2	85.4
QuaRTz	79.1	76.9	81.4	76.4	81.4	77.4	78.8
BLiMP-ANA	99.0	51.0	96.0	52.0	96.0	96.0	81.7
ELI5-Asks	90.5	89.5	89.9	88.8	87.9	90.5	89.5
ETHOS-Gender	52.8	0.0	51.1	0.0	50.1	57.4	35.2
Avg.	86.0	70.9	83.4	70.5	82.9	86.0	80.0

Table 9: Relative performance (%) for subspace solution transfer under multi-task setting with constructed subspace.

generate the parameters of each DET via a *shared* intrinsic vector and three individual up-projections. Both the intrinsic vector and the up-projections are trainable. Denote the intrinsic vector shared among DETs on the i-th task as $\mathbf{I}^i_{\text{shared}}$, then the parameters of DET t_* for the i-th task are generated as $\theta^i_{t_*} = \texttt{Proj}^{\uparrow}_{t_*}(\mathbf{I}^i_{\text{shared}})$. During the joint training, we minimize the loss:

$$\mathcal{L} = \frac{1}{3}\sum_{i=1}^{|\mathcal{T}_{\text{train}}|}\sum_{t_*\in\{t_{\text{A}},t_{\text{P}},t_{\text{L}}\}} \mathcal{L}^i_{\text{task}}(\overline{\theta^i_{t_*}} \mid \theta_0).$$

In this way, we can directly approximate the desired unified subspace, omitting the procedure of first obtaining solutions for different DETs, and we do not need to assign a down-projection for each DET. Subspace optimization and subspace solution transfer can then be carried out using this subspace.

We present the results of subspace optimization and subspace solution transfer for both the single-task and multi-task settings in Tables 6 to 9. We find that in general, all DETs still achieve non-trivial performance in both subspace optimization and subspace solution transfer, which means the simplification does not influence the representation ability of the found subspace. This simplified pipeline is the cornerstone of our analysis of the connection between fine-tuning and DETs. Since the simplified procedure does not require training a down-projection, the total number of tunable parameters can be further reduced.

$\mathcal{T}_{\text{train}}$	$\mathcal{T}_{\text{test}}$	Adapter	LoRA	Prefix	Fine-tune
SST-2	Rotten Tomatoes	89.2	89.3	90.0	89.8
	Amazon Review	96.2	96.6	96.6	97.0
MNLI	SciTail	94.0	93.8	93.0	94.8
	RTE	78.4	79.1	72.7	80.6
WiC	WSC	65.4	63.5	67.3	67.3
QQP	MRPC	87.3	87.3	87.3	89.7
ELI5-ELI5	ELI5-Askh	11.5	12.0	12.6	13.0
	ELI5-Asks	15.0	15.2	15.1	15.3
DREAM	CODAH	41.2	43.0	45.0	45.2
	QuaRTz	67.0	67.1	68.5	69.4
	CoPA	60.4	56.4	58.4	59.2
	Avg.	65.6	65.2	65.0	66.6

Table 10: Absolute performance for different tuning methods under the single-task setting.

A.4 HyperParameters of simplified approximation experiments

In the simplified approximation experiments, we use different learning rates for the shared intrinsic vector and DETs in subspace approximation. We set the learning rate as 5×10^{-5} for the shared intrinsic vector, and 1×10^{-4} for DETs. The batch size is 16 in the single-task setting and 8 in the multi-task setting. We train the model for a maximum of 100000 and validate every 1000 steps. For subspace optimization, we perform grid search on learning rate in $\{5 \times 10^{-2}, 1 \times 10^{-2}, 5 \times 10^{-3}, 1 \times 10^{-3}\}$. We set batch size as 16. To keep 3 DETs' number of parameters consistent, we set r_{A} as 12, r_{L} as 10, m as 24, and d_{P} as 120. Other hyperparameters are kept consistent with the main experiments.

A.5 Implementation Details for Extension to Fine-tuning

We use the simplified pipeline introduced in Appendix A.3 to further reduce the number of trainable parameters. That is, an intrinsic vector $\mathbf{I}^i$ for the i-th task is set to be a trainable parameter, and is shared among fine-tuning and different DETs. The steps of analysis is the same as in §5.2. In the experiment of subspace approximation on glue-

$\mathcal{T}_{\text{test}}$	Adapter	LoRA	Prefix	Fine-tune
Rotten Tomatoes	89.2	89.3	90.0	89.8
Yelp Polarity	97.3	97.4	97.8	97.9
WSC	65.4	63.5	67.3	67.3
AI2 ARC	31.2	32.4	32.2	31.3
QASC	33.0	37.8	33.3	43.6
QuaRTz	67.0	67.1	68.5	69.4
BLiMP-ANA	100.0	100.0	100.0	100.0
ELI5-Asks	15.0	15.2	15.1	15.3
ETHOS-Gender	79.9	79.9	77.4	74.5
Avg.	65.6	66.2	65.4	67.0

Table 11: Absolute performance for different tuning methods under the multi-task setting.

sst2. We set learning rate as 1e-4, batch size as 8, max steps as 100000 and validate every 1000 steps. For subspace optimization, we perform grid search on learning rate in $\{1 \times 10^{-1}, 5 \times 10^{-2}, 1 \times 10^{-2}, 5 \times 10^{-3}, 1 \times 10^{-3}, 5 \times 10^{-4}, 1 \times 10^{-4}\}$. We set batch size as 8 and validate every 100 steps. Other hyper-parameters are the same as Appendix A.4.

A.6 Absolute Performance for Different Tuning Methods

In the main paper, we report the relative performance of subspace optimization. In this section, we list the absolute performance of different tuning methods (Adapter, Prefix-Tuning, and LoRA) in Table 10 and Table 11 for reference.

Table 12: The training tasks involved in our multi-task setting.

Split	Task Name	Reference
Training tasks	amazon review	McAuley and Leskovec 2013
	financial_phrasebank	Malo et al. 2014
	glue-sst2	Socher et al. 2013
	imdb	Maas et al. 2011
	emotion	Saravia et al. 2018
	tweet_eval-offensive	Barbieri et al. 2020
	tweet_eval-stance_climate	Barbieri et al. 2020
	ethos-directed_vs_generalized	Mollas et al. 2020
	ethos-race	Mollas et al. 2020
	hatexplain	Mathew et al. 2020
	glue-mnli	Williams et al. 2018
	glue-qnli	Rajpurkar et al. 2016
	glue-wnli	Faruqui and Das 2018
	superglue-rte	Dagan et al. 2005; Bar-Haim et al. 2006 Giampiccolo et al. 2007; Bentivogli et al. 2009
	health_fact	Kotonya and Toni 2020
	liar	Wang 2017
	glue-qqp	(link)
	medical_questions_pairs	McCreery et al. 2020
	paws	Zhang et al. 2019
	circa	Louis et al. 2020
	onestop_english	Vajjala and Lučić 2018
	trec-finegrained	Li and Roth 2002; Hovy et al. 2001
	wiki_auto	Jiang et al. 2020
	google_wellformed_query	Faruqui and Das 2018
	sms_spam	Almeida et al. 2011
	superglue-wic	Pilehvar and Camacho-Collados 2019
	lama-google_re	Petroni et al. 2019, 2020
	numer_sense	Lin et al. 2020
	search_qa	Dunn et al. 2017
	web_questions	Berant et al. 2013
	boolq	Clark et al. 2019
	codah	Chen et al. 2019
	commonsense_qa	Talmor et al. 2019
	cosmos_qa	Huang et al. 2019
	dream	Saha et al. 2018
	hellaswag	Zellers et al. 2019
	sciq	Welbl et al. 2017
	quail	Rogers et al. 2020
	quarel	Tafjord et al. 2019a
	race-high	Lai et al. 2017
	superglue-copa	Gordon et al. 2012
	wino_grande	Sakaguchi et al. 2020
	eli5-eli5	Fan et al. 2019
	hotpot_qa	Yang et al. 2018
	quoref	Dasigi et al. 2019
	superglue-record	Zhang et al. 2018
	multi_news	Fabbri et al. 2019
	xsum	Narayan et al. 2018
	spider	Yu et al. 2018
	wikisql	an 2017
	blimp-anaphor_gender_agreement	Warstadt et al. 2020
	blimp-ellipsis_n_bar_1	Warstadt et al. 2020
	blimp-irregular_past_participle_adjectives	Warstadt et al. 2020
	blimp-wh_questions_object_gap	Warstadt et al. 2020
	cos_e	Rajani et al. 2019
	acronym_identification	Pouran Ben Veyseh et al. 2020
	crawl_domain	Zhang et al. 2020
	proto_qa	Boratko et al. 2020
	qa_srl	He et al. 2015

Table 13: The test tasks involved in our multi-task setting.

Split	Task Name	Reference
Test tasks	rotten_tomatoes	Pang and Lee 2005
	yelp_polarity	Zhang et al. 2015
	ethos-gender	Mollas et al. 2020
	superglue-wsc	Levesque et al. 2012
	ai2_arc	Clark et al. 2018
	qasc	Khot et al. 2020
	quartz-no_knowledge	Tafjord et al. 2019b
	eli5-asks	Fan et al. 2019
	blimp-anaphor_number_agreement	Warstadt et al. 2020