Appendix II. Properties of Lexical Relations.

a. Reflexivity, Symmetry, Transitivity.

Certain properties of lexical-semantic relations can be very useful in deductive inference. For instance, if we know that a cheetah is a kind of mammal and a mammal is a kind of vertebrate then we can deduce that a cheetah is a kind of vertebrate. Writing T for the taxonomy relation, we can abbreviate this sentence: if cheetah T mammal and mammal T vertebrate then cheetah T vertebrate. Whenever bTc and cTd, it follows that bTd. This fact can be described much more efficiently by the statement that the taxonomy relation is transitive. Two other commonly mentioned properties of relations are reflexivity and symmetry. These properties may apply to predicates formed from lexical entries as well as to lexical-semantic relations.

To be precise, a relation R defined on a set S is said to be a *transitive* relation if whenever b and c are R-related and also c and d are R related then b and d stand in a relation R also. Synonymy is a transitive relation just as transitivity is. The preposition *in* behaves in the same way. If Sam is in the kitchen and the kitchen is in the hotel, then we know that Sam is in the hotel. The time interrelation *before* behaves like this, too. If Zorro arrived before the posse did and the posse arrived before the explosion, then we know that Zorro arrived before the explosion.

A relation R defined on a set S is said to have the *reflexive* property if all the elements of S are R-related to themselves, that is, if mRm is true for all members m of the set S. The synonymy relation has this property: a word means the same thing as itself. The antonymy relation ANTI does not have this property. It is not true that *hot* ANTI *hot*, for example.

A relation R defined on a set S is said to be *symmetric* if whenever b and c are R-related then so are c and b; that is, R is symmetric if and only if bRc always implies cRb. Synonymy also has this property. If b is synonymous with c, then c is synonymous with b. So has antonymy. Given that *hot* ANTI *cold*, we immediately know that *cold* ANTI *hot*. Taxonomy is not symmetric, however. A lion is a kind of mammal, but a mammal is not a kind of lion.

In question answering we may be just as interested in drawing negative conclusions as positive ones. Thus it may be important to know that if bRc is true then cRb must be false. The term *asymmetric* is used to describe a relation R for which bRc and cRb are never both true, at least when b and c are different elements of the set S. Taxonomy is asymmetric and so is the time interrelation *before*. If the question asks, "Did c happen before b?" and we know that b happened before c, we can answer with a confident no. For want of a better term we will say that the relation R is *non-symmetric* if it is neither symmetric or asymmetric. In this case bRc and cRb are sometimes both true and sometimes not. Similarly, the term *irreflexive* is used for the case in which mRm is never true, while the term *nonreflexive* is used for the case in which mRm is sometimes true and sometimes not. In the same way *intransitive* is taken to mean that if bRc and cRd, we can conclude that b and d are not R-related, while *nontransitive* will mean that bRd is sometimes true if bRc and cRd, but not always.

Each lexical relation itself has a lexical entry. The reflexivity, symmetry, and transitivity properties of the relation are listed in this entry, as they are in the entries for interrelational operators and prepositions and other lexical items for which they are relevant. There are also lexical entries under the property names, *reflexive*, *irreflexive*, etc. listing the appropriate axioms. The motivation behind lexical entries for properties is first of all greater generality. Secondly, it makes it much easier to add lexical relations and to add other properties which turn out to be useful.

At this stage of development there are several transitivity axioms:

For lexical relations Rel, like taxonomy

$$b \text{ Rel } c \wedge c \text{ Rel } d \rightarrow b \text{ Rel } d$$

For interrelations J, like *before*

$$Holds(I(J,Z_1,Z_2)) \wedge Holds(I(J,Z_2,Z_3)) \rightarrow Holds(I(J,Z_1,Z_3))$$

For prepositions Q like *in* or *above*

$$Holds((F(\text{location},Z_1,Prep(Q,Z_2))) \wedge Holds(P(\text{location},Z_2, Prep(Q,Z_3))) \rightarrow Holds(P(\text{location},Z_1,Prep(Q,Z_3)))$$

Intuitively these are all instances of the same concept, transitivity. There should be some single way of expressing it. It is a defect of this representation system that there is not.

A relation that is reflexive, symmetric, and transitive is called an equivalence relation. The synonymy relation is an equivalence relation since it has all three properties. If R is an equivalence relation, then a subset consisting of all the elements which are R-related to a particular element x by the equivalence relation is called an equivalence class. In an equivalence class all the elements are R-related to each other. An equivalence relation partitions a set into equivalence classes; each element of the set belongs to exactly one equivalence class. The synonymy relation partitions the items in the lexicon in just this way. There is a class consisting of *suspicion* and all the words synonymous with *suspicion,* like *mistrust* and *doubt.* These synonymy classes are disjoint; each word sense in the lexicon belongs to exactly one of them (cf. Edmundson and Epstein 1972, Palmer 1976).

With this as a basis an equivalence relation of paraphrasability between sentences can be established. Sentence S_1 is a paraphrase of sentence S_2 if one is obtained from the other by substituting synonyms for each other.[1]

Mr. Kennedy viewed Lady Laura with suspicion.

Mr. Kennedy regarded Lady Laura with mistrust.

We might also allow substitution of conversives, nominalizations, etc.

Nancy was Sally's student.

Sally was Nancy's teacher.

Sally taught Nancy.

The equivalence classes of this relation, each one of which is the set of all paraphrases of a given sentence have a definite theoretical importance and some practical significance in question answering. One member of a class might well be part of the story; another the right answer to a question.

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1 This representation system can be viewed as defining a relation P such that S_1 P S_2 if and only if S_1 and S_2 have the same representation. If the representation system is well defined, then P should define the same equivalence classes as the paraphrasability relation

b. *Inverses.*

The inverse R of the relation R is the relation which "goes in the opposite direction" from R; that is, bRc if and only if cRb. Thus, *bake* T *make* and *make* T *bake* are two ways of saying the same thing. Both pieces of information are stated in the lexicon. However, the lexical entry for *bake* includes T *make;* the lexical entry for *make* includes T *bake.* Why bother to say the same thing in different places? There are two reasons for this. First of all, the inverse relation may be a relation that is commonly and easily verbalized, worth naming in its own right. This is certainly true of the CHILD relation, as in *puppy* CHILD *dog.* Instead of asking "What is a baby dog called?", we could ask "What is a grownup puppy called?" or "What does a puppy grow up to be?" The second reason is that putting this information in both entries can make searches easier and much faster. We may only have one half of the pair and need the other. We may have *dog* and *puppy.* This is easy if we have the information CHILD *puppy* in the *dog* entry. Otherwise we might have to search the whole lexicon. In other situations we have two words but no direct connection between them. For example, suppose the system knows *lion* T *mammal* and *mammal* T *vertebrate* and is then asked, "Is a lion a vertebrate?" The connection between *lion* and *vertebrate* can be found much more quickly if the search starts from both the *vertebrate* end and the *lion* end of the chain at the same time, but to do this there must be a pointer to *mammal* in the *vertebrate* entry. Another question comes to mind. Why call the inverse relation to CHILD by the clumsy name $\overline{\text{CHILD}}$ instead of its proper name PARENT? The ECD uses two different names for a relation and its inverse (S_0 and V_0 are inverses, for example). If this were done here, two versions of the appropriate axiom schemes would be needed, one in the CHILD entry and one in the PARENT entry.

Since a relation R is called symmetric if bRc always implies cRb, it follows that a symmetric relation is its own inverse. The synonymy relation S and antonymy relation ANTI are both self-inverse in this sense. For this reason we never need the symbol $\overline{\text{ANTI}}$, etc. $\overline{\text{ANTI}}$ is ANTI The entry for *hot* includes ANTI *cold,* the entry for *cold* includes ANTI *hot.*

c. Unique Linkage.

Raphael (1968) has proposed a property which seems extremely useful. He calls it *unique-linkage* (U). Mathematicians usually refer to such relations as one-to-one. A relation R has the unique-linkage property if whenever xRy then bRy is false for any $b \neq x$ and xRc is false for any $c \neq y$, i.e. any object is R-related to at most one other. Raphael's example of unique-linkage is the relation "just to the right of". The behavior is especially characteristic of the queuing relation, e.g. with days of the week, Monday Q Tuesday, etc.Some relations may be uniquely linked on one side only, e.g. mother-child is uniquely linked on the left. We can define U_L unique-linkage on the left and U_R unique linkage on the right. (A relation which is U_R is a single-valued function. If R has the U_L property, then its inverse is a single-valued function.)

Raphael also proposed for SIR-1 (ibid, p. 101) a property which he calls irreflexive. R is set-nonreflexive if

$$(\forall X \quad M) \sim (\forall \beta \subset X)\ (\exists \alpha \subset X)\ (\alpha \ R\ \beta)$$

In the SIR model both the 'X is a part of Y' and the 'X is owned by Y' relations have this property. What it says is that every set in the model has a minimal element with respect to the relation R. A simpler version of this property is sufficient for our purposes.

Minimum Condition	$(\forall X \subset M) \sim (\forall Y \in X)\ (\exists Z \in X)\ (ZRY)$ Every nonempty subset has a minimum.
Maximum Condition	$(\forall X \subset M) \sim (\forall Y \in X)\ (\exists Z \in X)\ (YRZ)$ Every nonempty subset has a maximum.

The part-whole relation has both properties in our model. In any nonempty subset in the model there is something in it that is not a proper subpart of anything else in that subset, and also something that has no proper subpart. A relation that has this property stops somewhere. It is not reflexive and not circular. A search that goes on looking for links of this kind will stop somewhere. The relation 'is an ancestor of' has this property. We will eventually run out of ancestors in one direction and descendants in the other, at least, inside a finite model.

The properties of relations are summarized in Table 4.

Table 4. Properties of Relations

Property	*Definition*
symmetric	$(\forall X \in M)(\forall Y \in M)(XRY \rightarrow YRX)$
asymmetric	$(\forall X \in M)(\forall Y \in M)(XRY \rightarrow \sim YRX)$
reflexive	$(\forall X \in M)\ XRX$
irreflexive	$(\forall X \in M) \sim (XRX)$
transitive	$(\forall X \in M)(\forall Y \in M)\ (\forall Z \in M)(XRY \wedge YRZ \rightarrow XRZ)$
intransitive	$(\forall X \in M)(\forall Y \in M)(\forall Z \in M)(XRY \wedge YRZ \rightarrow \sim (XRZ))$
uniquely linked	$(\forall X \in M)(\forall Y \in M)(XRY \rightarrow (\forall Z \in M)((ZRY \rightarrow X=Z)$ $(XRZ \rightarrow Y=Z)))$
uniquely linked on the left	$(\forall X \in M)(\forall Y \in M)(XRY \rightarrow (\forall Z \in M)(ZRY \rightarrow Z=X))$
uniquely linked on the right	$(\forall X \in M)(\forall Y \in M)XRY \rightarrow (\forall Z \in M)(XRZ \rightarrow Z=Y))$

d. Partial Ordering.

Any transitive relation defines a partial ordering. Several of the lexical relations discussed earlier are transitive; many lexical items are transitive too. One important reason for representing time in terms of the transitive interrelation *before* is to allow one to make the same kinds of simple deductions about time that one can make about taxonomy. Some transitive relations, like taxonomy, are also reflexive. In this case we talk about a *weak ordering*. ($X \leq Y$ for numbers is a weak ordering.) Some are not reflexive, these are called *strong* ordering relations. ($X < Y$ for numbers is a strong ordering.) The time relation *before* is a strong ordering relation. For any weak ordering there is a strong ordering and conversely. Starting with the taxonomy relation T, for example, a relation T_1 or "proper taxonomy" can be defined consisting of the pairs x and y for which xTy but x and y are different. Then xT_1y means that x is a kind of y but different from y. If instead one starts with a strong ordering relation *before*, one can define a weak relation "before_1" for which x before_1 y means that either x before y or x cooccured with y.

The queuing relation Q is not itself a partial ordering but a partial ordering can be derived from it. Monday Q Tuesday and Tuesday Q Wednesday, but it is false that Monday Q Wednesday. Queuing is an 'immediate successor relation like the relation between a natural number n and the next number n+1. A relation Q' can be defined such that xQ'y if either xQy or there are some objects $z_1, z_2, \ldots, z_n$ such that xQz_1, z_1Qz_2, ... z_nQy. It follows immediately that if bQc and cQd then bQ'd. Q', the 'successor' relation, is now transitive, for if bQ'c and cQ'd, then one can find a chain of Q-related objects linking b and d just bv concatenating the chain linking c and d. Raphael's pair of relations jright and right behave this way. The relations "is a child of" and "is a descendant of" are also paired in this way.