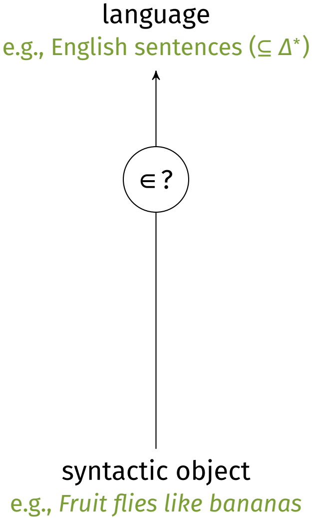
Weighted parsing for grammar-based language models

FSMNLP 2019

Richard Mörbitz   Heiko Vogler

2019-09-25

The weighted parsing problem



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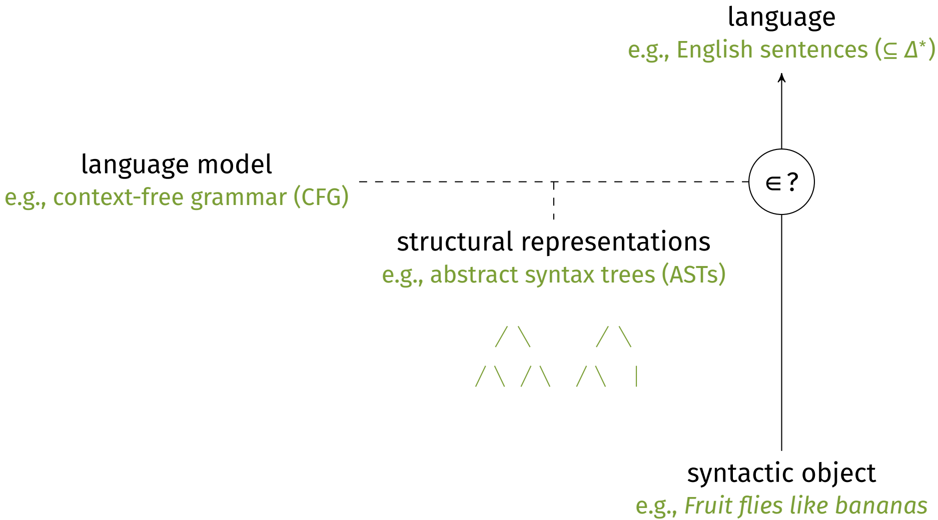

language

e.g., English sentences ($\subseteq \Delta^*$)

language model

e.g., context-free grammar (CFG)

$\in$?

structural representations

e.g., abstract syntax trees (ASTs)

syntactic object

e.g., *Fruit flies like bananas*

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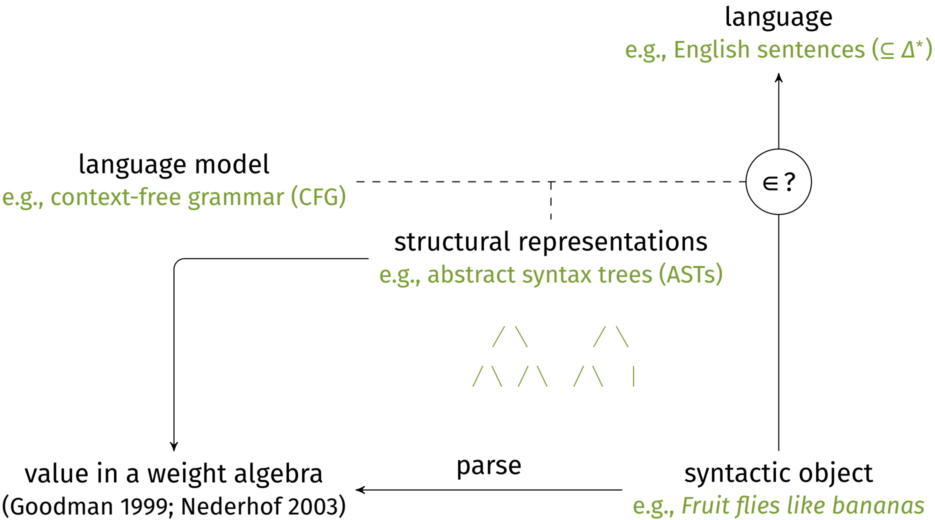

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structural representations

e.g., abstract syntax trees (ASTs)

value in a weight algebra

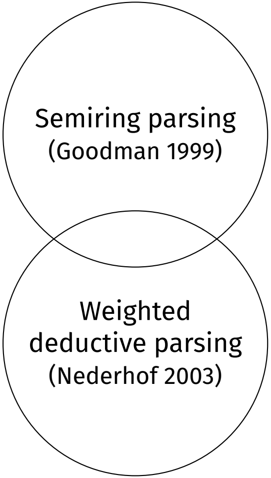
(Goodman 1999; Nederhof 2003)

parse

syntactic object

e.g., *Fruit flies like bananas*

Overview



Semiring parsing
(Goodman 1999)

Weighted
deductive parsing
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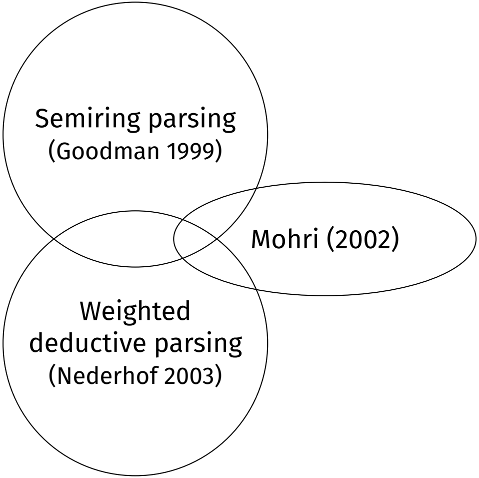

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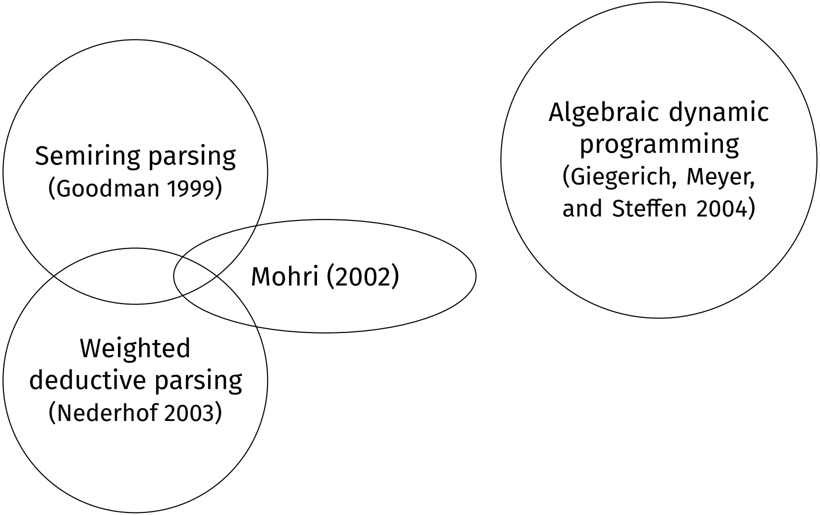

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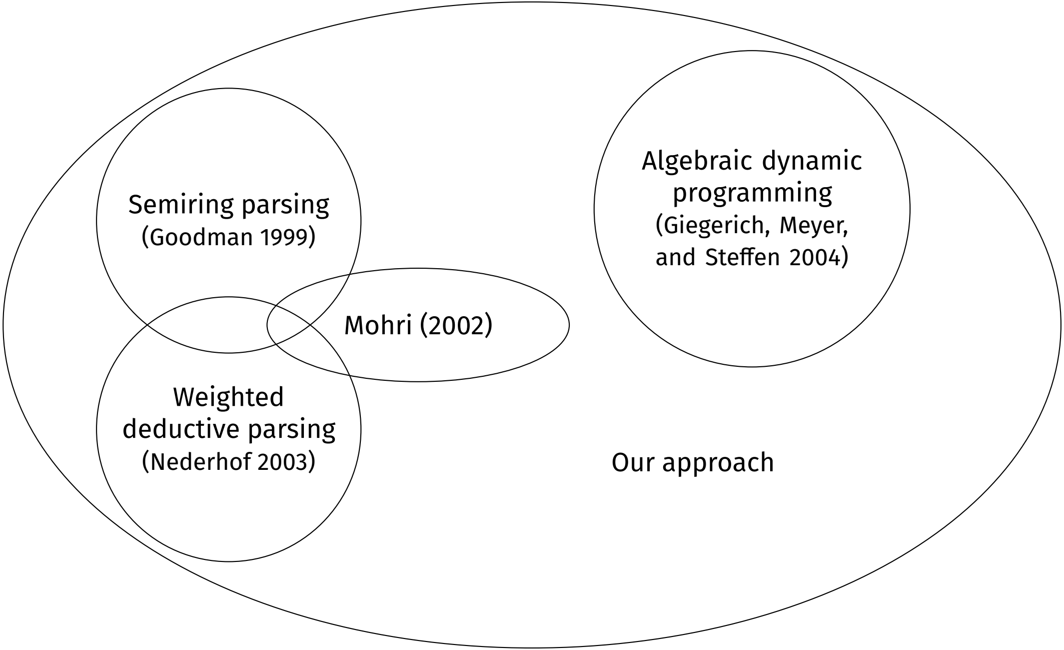

Semiring parsing (Goodman 1999)

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Our approach

Outline

1. Weighted RTG-based language models
2. The weighted parsing problem
3. The weighted parsing algorithm

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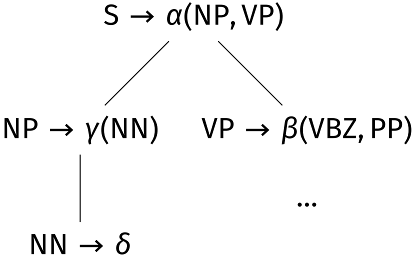
Regular tree grammars (RTG)

Tuple $G = (N, \Sigma, A_0, R)$

Example rules:

$$\begin{aligned} \mathrm{S} &\rightarrow \alpha(\mathrm{NP}, \mathrm{VP}) \\ \mathrm{VP} &\rightarrow \beta(\mathrm{VBZ}, \mathrm{PP}) \\ \mathrm{NP} &\rightarrow \gamma(\mathrm{NN}) \\ \mathrm{NN} &\rightarrow \delta \\ &\dots \end{aligned}$$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...

abstract syntax tree $d \in \mathrm{AST}(G)$

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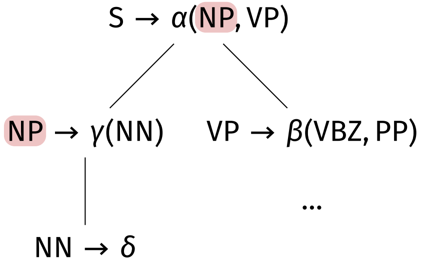

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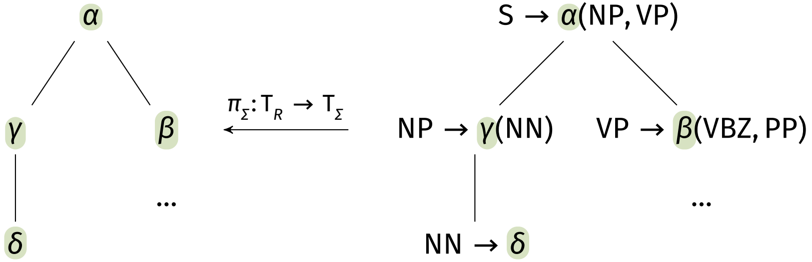

α

γ β

δ …

$\pi_\Sigma \colon \mathrm{T}_R \to \mathrm{T}_\Sigma$

$\mathrm{S} \to \alpha(\mathrm{NP}, \mathrm{VP})$

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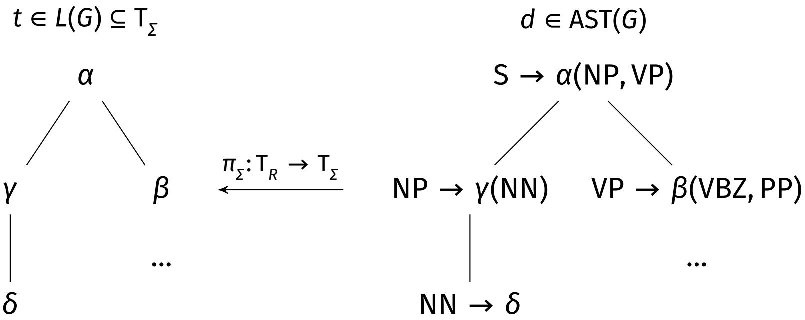
$t \in L(G) \subseteq \mathrm{T}_\Sigma$

abstract syntax tree $d \in \mathrm{AST}(G)$

Language algebras



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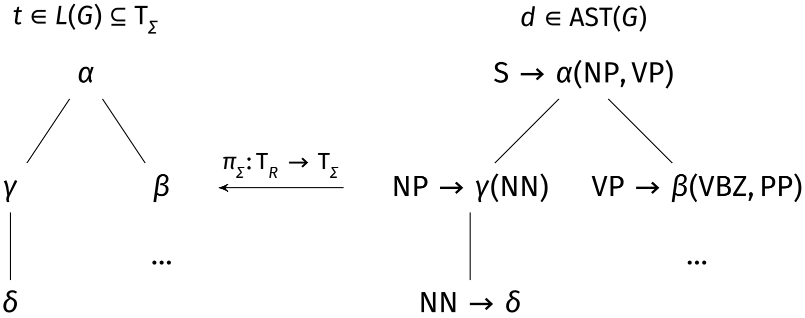

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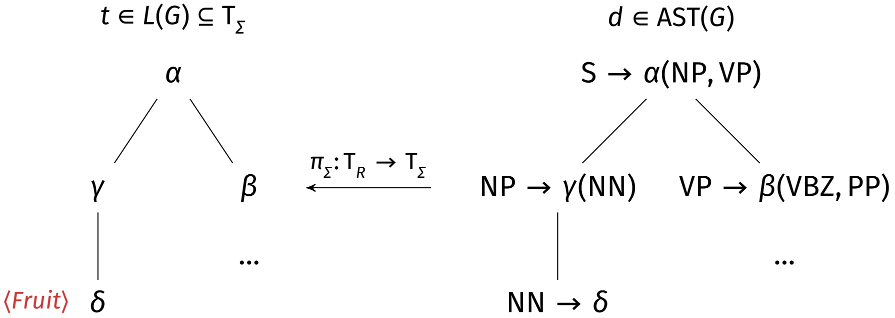

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$\langle Fruit \rangle$ δ …

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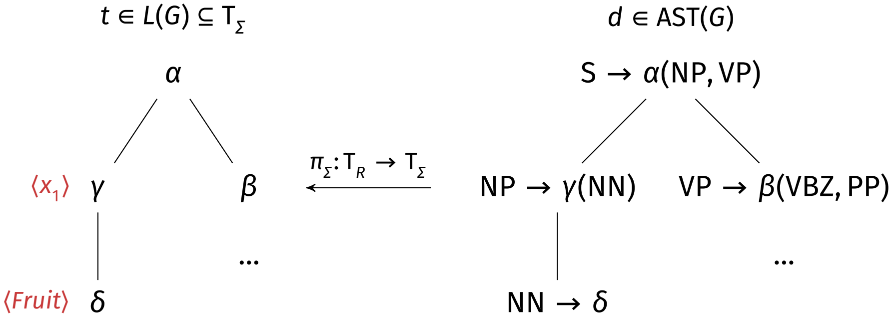

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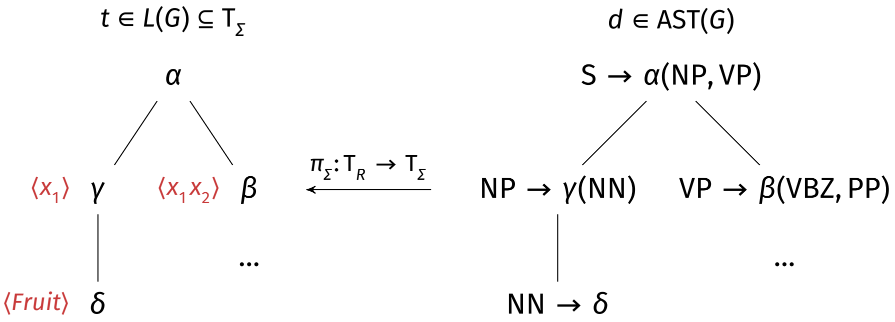

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$\langle x_1 \rangle$ γ $\langle x_1 x_2 \rangle$ β

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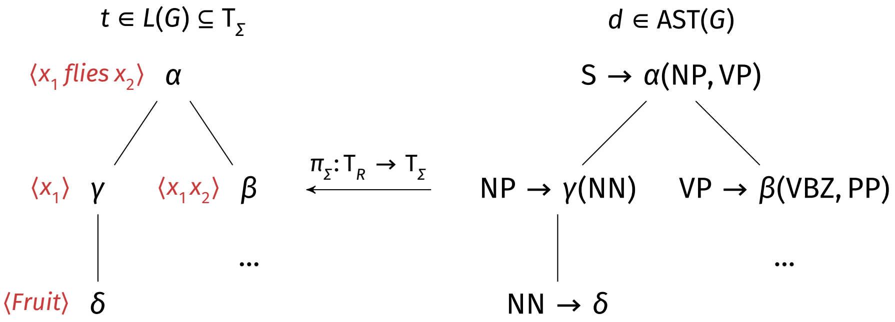

$t \in L(G) \subseteq \mathrm{T}_\Sigma$

$\langle x_1 \textit{ flies } x_2 \rangle \quad \alpha$

$\langle x_1 \rangle \quad \gamma \qquad \langle x_1 x_2 \rangle \quad \beta$

$\langle \textit{Fruit} \rangle \quad \delta \qquad \ldots$

$\pi_\Sigma \colon \mathrm{T}_R \to \mathrm{T}_\Sigma$

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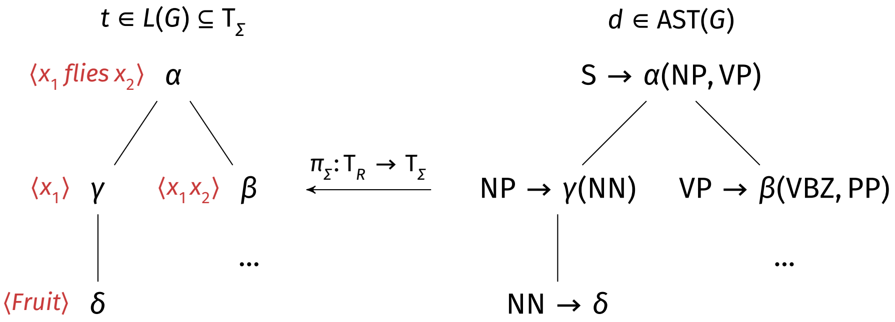

- interpretation of Σ as operations on the set of syntactic objects $\mathcal{L} = \Delta^*$
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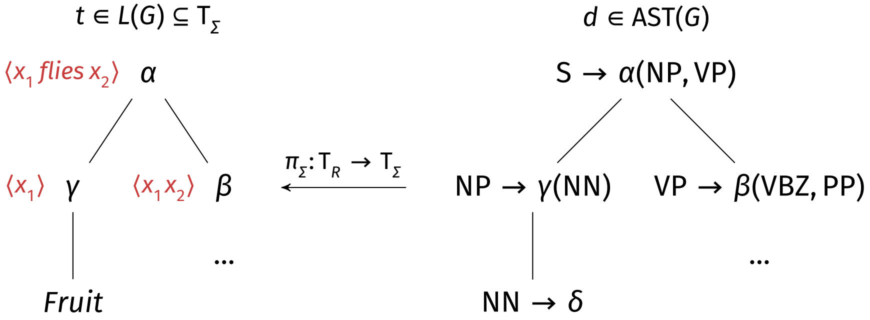
Language algebras



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$\langle x_1\ \mathit{flies}\ x_2 \rangle$ α

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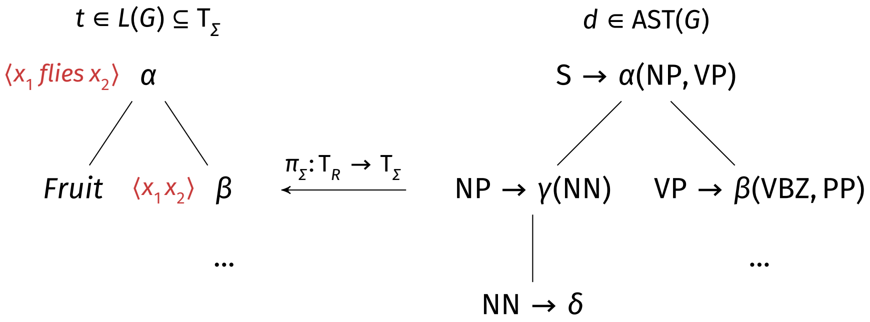

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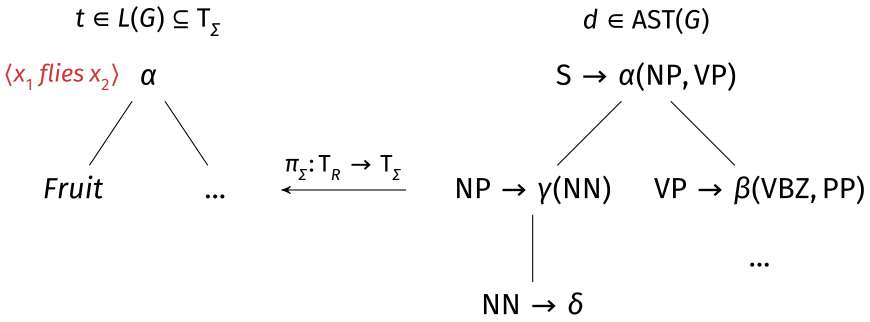

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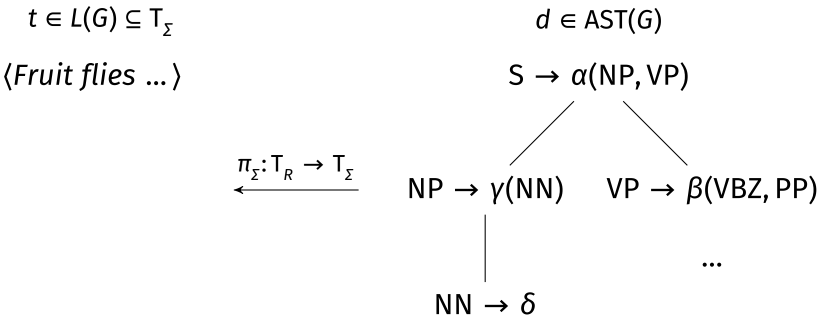

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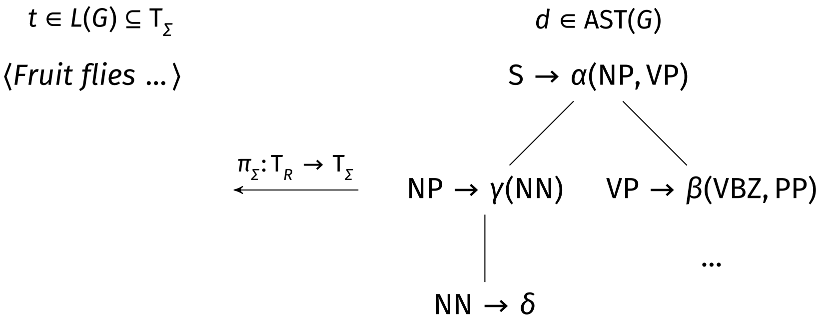

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$\langle \textit{Fruit flies} \ldots \rangle$

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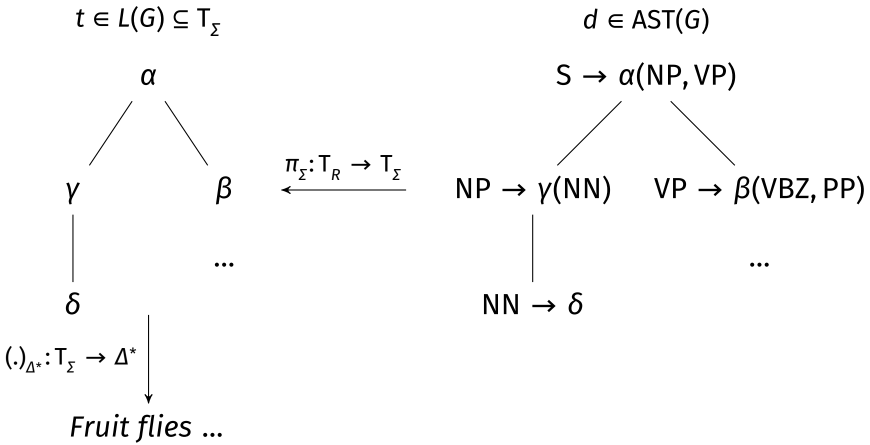

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δ

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Semirings

Algebraic structure $(\mathbb{K}, \oplus, \otimes, \mathbb{0}, \mathbb{1})$

- $\otimes$ is used to evaluate an AST to a weight
- $\oplus$ accumulates the weights of several ASTs

Semirings

Algebraic structure $(\mathbb{K}, \oplus, \otimes, \mathbb{0}, \mathbb{1})$

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Examples

- $(\mathbb{B}, \vee, \wedge, \text{false}, \text{true})$ the *Boolean semiring* with $\mathbb{B} = \{\text{false}, \text{true}\}$
- $(\mathbb{N}^{\infty}, +, \cdot, 0, 1)$ the *semiring of natural numbers*
- $(\mathbb{N}^{\infty}, \min, +, \infty, 0)$ the *tropical semiring*
- $(\mathbb{R}_0^1, \max, \cdot, 0, 1)$ the *Viterbi semiring*

Multioperator monoids (M-monoids)

Generalization of semirings

$$(\mathbb{K}, \oplus, \otimes, \mathbb{0}, \mathbb{1}) \quad \longrightarrow \quad (\mathbb{K}, \oplus, \mathbb{0}, \Omega)$$

binary $\otimes$ $\longrightarrow$ set of m-ary operations Ω (here: distributive)

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Semiring $(\mathbb{K}, \oplus, \otimes, \mathbb{0}, \mathbb{1}) \rightsquigarrow$ M-monoid $(\mathbb{K}, \oplus, \mathbb{0}, \Omega_\otimes)$ where

- $\Omega_\otimes = \{\mathrm{mul}_{\mathbb{k}}^{(m)} \mid \mathbb{k} \in \mathbb{K}, m \in \mathbb{N}\}$
- $\mathrm{mul}_{\mathbb{k}}^{(m)}(\mathbb{k}_1, \dots, \mathbb{k}_m) = \mathbb{k} \otimes \mathbb{k}_1 \otimes \cdots \otimes \mathbb{k}_m$

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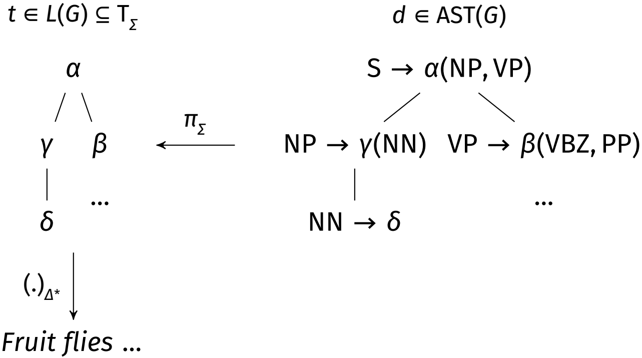
Examples

- *Viterbi M-monoid* $(\mathbb{R}_0^1, \max, 0, \Omega_{\mathrm{mul}})$
- *Minimum edit distance M-monoid* $(\{\{n\} \mid n \in \mathbb{N}\}, \min \circ \cup, \emptyset, \Omega_{\mathrm{med}})$ with $\Omega_{\mathrm{med}} = \{\mathrm{del}, \mathrm{ins}, \mathrm{rep}_=, \mathrm{rep}_\neq, \mathrm{nil}\}$

Weight algebras



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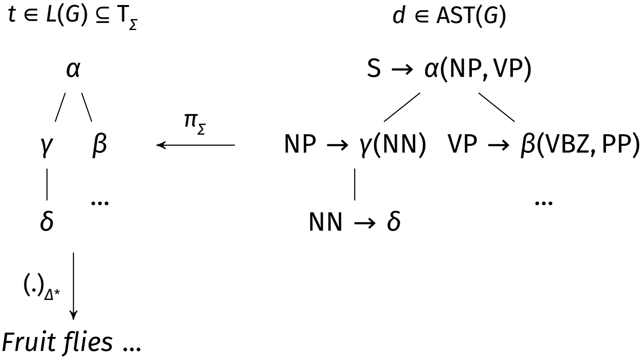
$d \in \mathrm{AST}(G)$

π_Σ

$(.)_{\Delta^*}$

Fruit flies …

Weight algebras



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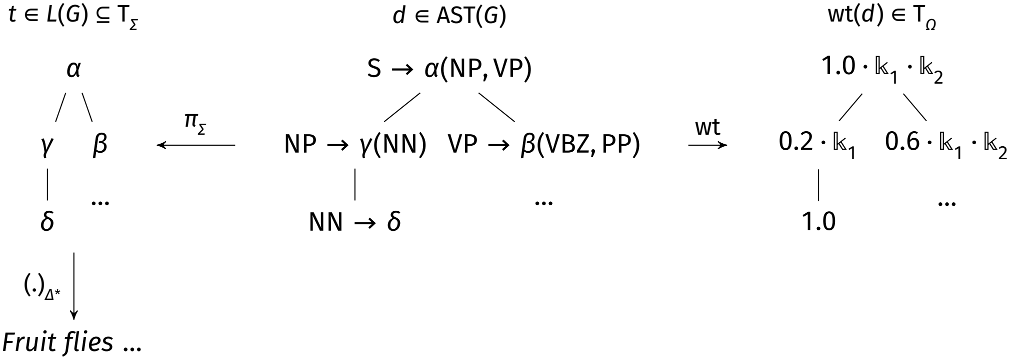
Fruit flies …

- wt: R (set of rules) → Ω (set of operations)

$(\mathbb{R}_0^1, \max, 0, \Omega_{\mathrm{mul}})$

Weight algebras



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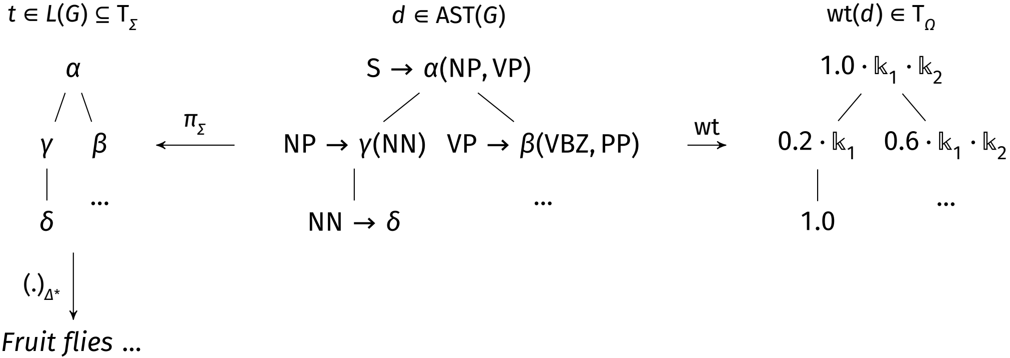

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wt

$\mathrm{wt}(d) \in \mathrm{T}_\Omega$

$1.0 \cdot \Bbbk_1 \cdot \Bbbk_2$

$0.2 \cdot \Bbbk_1$ $0.6 \cdot \Bbbk_1 \cdot \Bbbk_2$

1.0 …

- wt: R (set of rules) → Ω (set of operations)
- $(.)_{\mathbb{R}_1^0}$: T_Ω (terms) → $\mathbb{R}_0^1$ (weight algebra)

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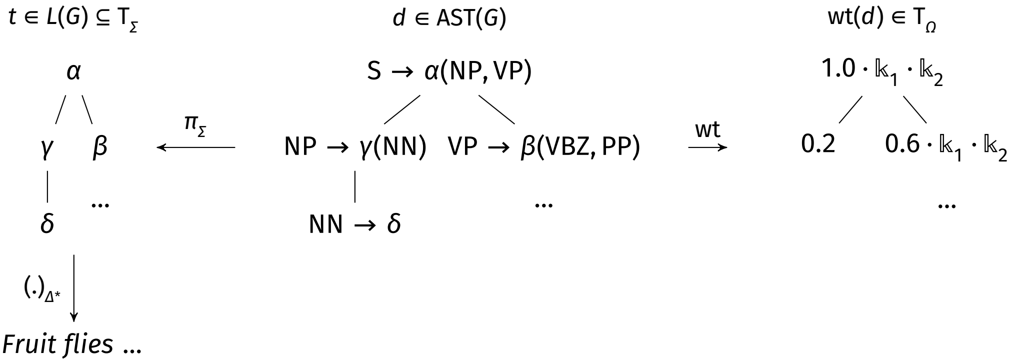
Weight algebras



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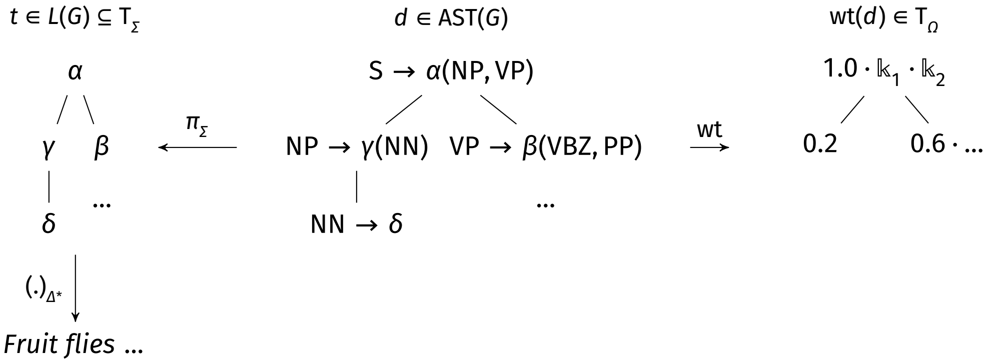

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$d \in \mathrm{AST}(G)$

$\mathrm{wt}(d) \in \mathrm{T}_\Omega$

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δ …

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wt

$1.0 \cdot \Bbbk_1 \cdot \Bbbk_2$

0.2 $0.6 \cdot \ldots$

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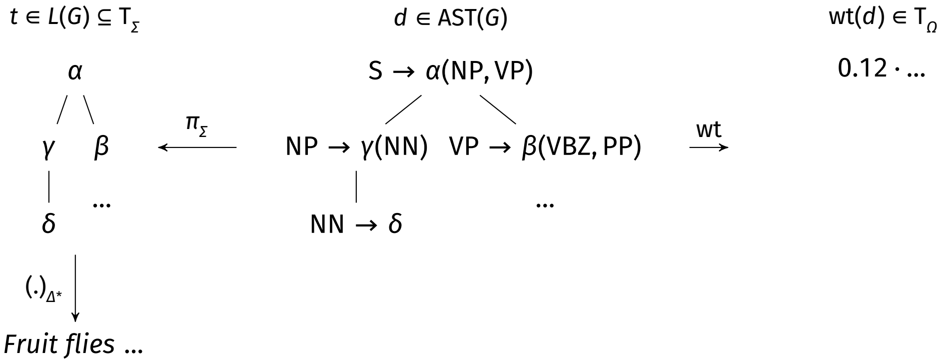
Fruit flies …

- wt: R (set of rules) → Ω (set of operations)
- $(.)_{\mathbb{R}^0_1}$: T_Ω (terms) → $\mathbb{R}^1_0$ (weight algebra)

$(\mathbb{R}^1_0, \max, 0, \Omega_{\mathrm{mul}})$

Weight algebras



$t \in L(G) \subseteq \mathrm{T}_\Sigma$

α

γ β

δ …

$\xrightarrow{(.)_{\Delta^*}}$

Fruit flies …

$\xleftarrow{\pi_\Sigma}$

$d \in \mathrm{AST}(G)$

$\mathrm{S} \to \alpha(\mathrm{NP}, \mathrm{VP})$

$\mathrm{NP} \to \gamma(\mathrm{NN})$ $\mathrm{VP} \to \beta(\mathrm{VBZ}, \mathrm{PP})$

$\mathrm{NN} \to \delta$ …

$\xrightarrow{\mathrm{wt}}$

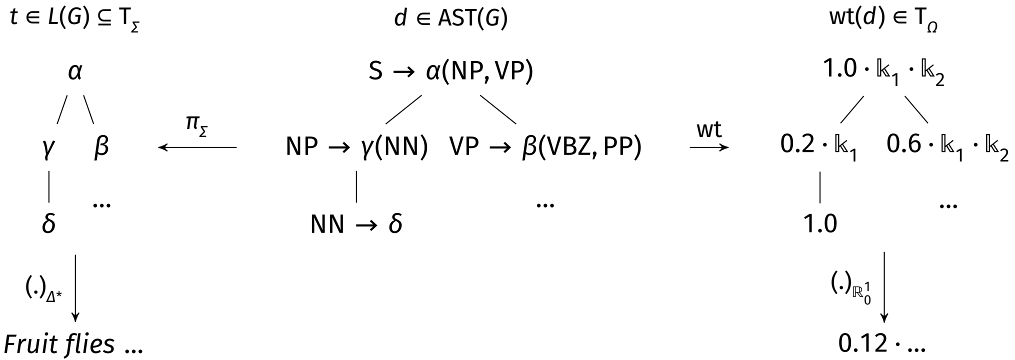
$\mathrm{wt}(d) \in \mathrm{T}_\Omega$

$0.12 \cdot \ldots$

- $\mathrm{wt}\colon R$ (set of rules) → Ω (set of operations)
- $(.)_{\mathbb{R}_1^0}\colon \mathrm{T}_\Omega$ (terms) → $\mathbb{R}_0^1$ (weight algebra)

$(\mathbb{R}_0^1, \max, 0, \Omega_{\mathrm{mul}})$

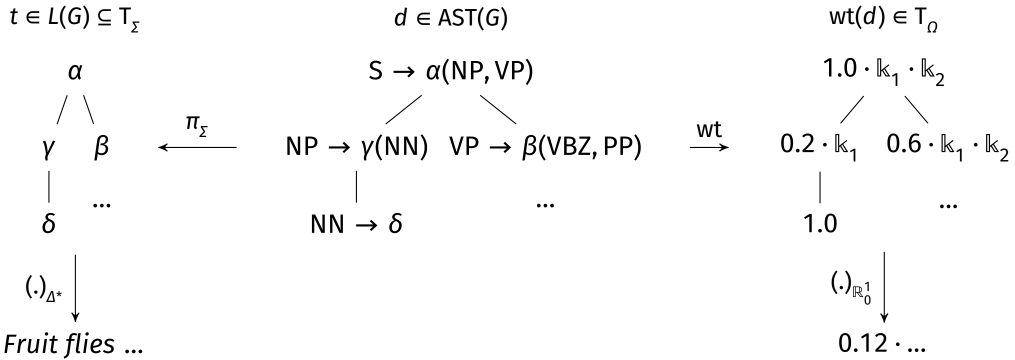
Weight algebras



- wt: R (set of rules) → Ω (set of operations)
- $(.)_{\mathbb{R}^0_1}$: T_Ω (terms) → $\mathbb{R}_0^1$ (weight algebra)

$(\mathbb{R}_0^1, \max, 0, \Omega_{\mathrm{mul}})$

Weighted RTG-based language models



$t \in L(G) \subseteq \mathrm{T}_\Sigma$

$d \in \mathrm{AST}(G)$

$\mathrm{wt}(d) \in \mathrm{T}_\Omega$

α

γ β

δ …

$\xleftarrow{\pi_\Sigma}$

$S \to \alpha(NP, VP)$

$NP \to \gamma(NN)$ $VP \to \beta(VBZ, PP)$

$NN \to \delta$ …

$\xrightarrow{\mathrm{wt}}$

$1.0 \cdot \Bbbk_1 \cdot \Bbbk_2$

$0.2 \cdot \Bbbk_1$ $0.6 \cdot \Bbbk_1 \cdot \Bbbk_2$

1.0 …

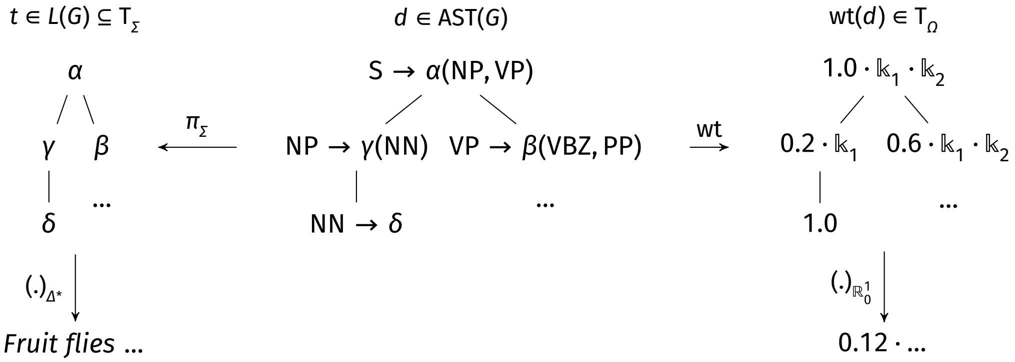
$(.)_{\Delta^*}$

Fruit flies …

$(.)_{\mathbb{R}_0^1}$

$0.12 \cdot \ldots$

Weighted RTG-based language models



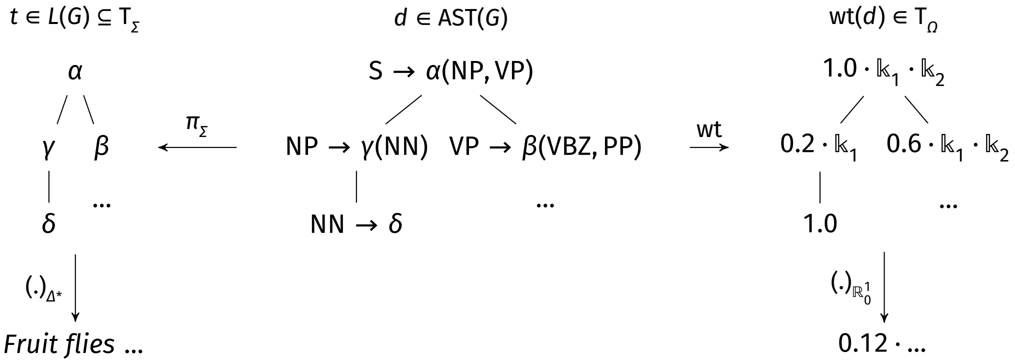
Definition (weighted RTG-based language model)

A wRTG-LM is a tuple $\Big(\underbrace{(G = (N, \Sigma, A_0, R))}_{\text{RTG}}, \underbrace{\mathcal{L}}_{\text{language algebra}}), \underbrace{(\mathbb{K}, \oplus, \mathbb{0}, \Omega)}_{\text{M-monoid}}, \underbrace{\text{wt}}_{\text{wt}: R \to \Omega}\Big)$.

Outline

1. Weighted RTG-based language models
2. The weighted parsing problem
3. The weighted parsing algorithm

The weighted parsing problem



$t \in L(G) \subseteq \mathrm{T}_\Sigma$

α

γ β

δ ...

$(.)_{\Delta^*}$

Fruit flies ...

$\xleftarrow{\pi_\Sigma}$

$d \in \mathrm{AST}(G)$

$\mathrm{S} \rightarrow \alpha(\mathrm{NP}, \mathrm{VP})$

$\mathrm{NP} \rightarrow \gamma(\mathrm{NN})$ $\mathrm{VP} \rightarrow \beta(\mathrm{VBZ}, \mathrm{PP})$

$\mathrm{NN} \rightarrow \delta$ ...

$\xrightarrow{\mathrm{wt}}$

$\mathrm{wt}(d) \in \mathrm{T}_\Omega$

$1.0 \cdot \mathbb{k}_1 \cdot \mathbb{k}_2$

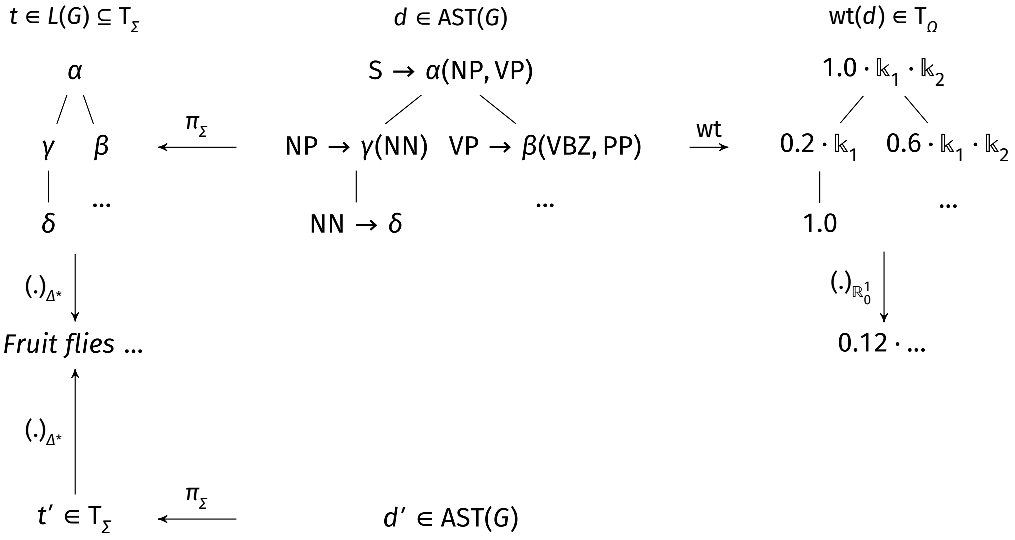
$0.2 \cdot \mathbb{k}_1$ $0.6 \cdot \mathbb{k}_1 \cdot \mathbb{k}_2$

1.0 ...

$(.)_{\mathbb{R}^1_0}$

$0.12 \cdot \ldots$

The weighted parsing problem



$t \in L(G) \subseteq T_\Sigma$

$d \in \mathrm{AST}(G)$

$\mathrm{wt}(d) \in T_\Omega$

α

$S \to \alpha(NP, VP)$

$1.0 \cdot \Bbbk_1 \cdot \Bbbk_2$

γ β

π_Σ

$NP \to \gamma(NN)$ $VP \to \beta(VBZ, PP)$

wt

$0.2 \cdot \Bbbk_1$ $0.6 \cdot \Bbbk_1 \cdot \Bbbk_2$

δ …

$NN \to \delta$ …

1.0 …

$(.)_{\Delta^*}$

$(.)_{\mathbb{R}^1_0}$

Fruit flies …

$0.12 \cdot \ldots$

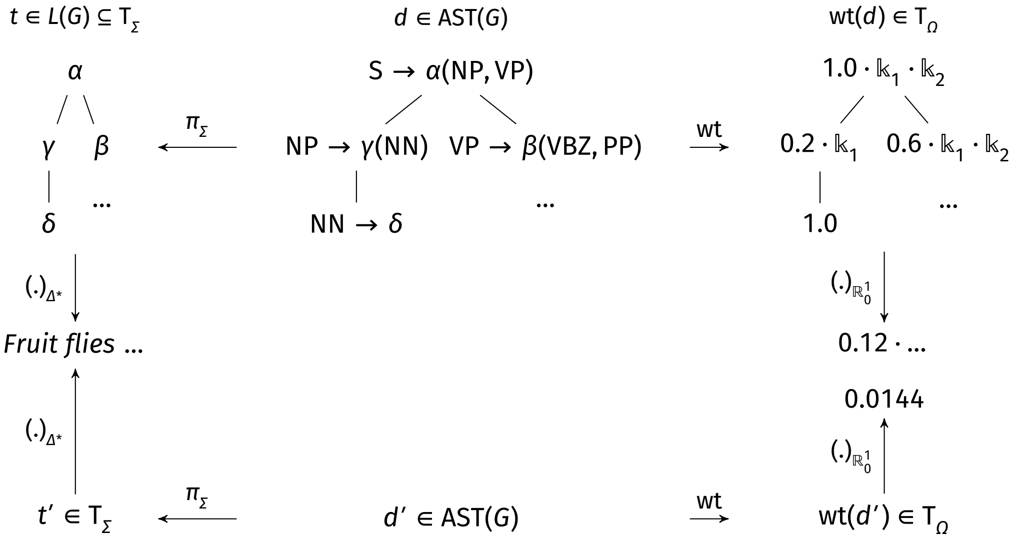
$(.)_{\Delta^*}$

$t' \in T_\Sigma$

π_Σ

$d' \in \mathrm{AST}(G)$

The weighted parsing problem



$t \in L(G) \subseteq \mathrm{T}_\Sigma$ $d \in \mathrm{AST}(G)$ $\mathrm{wt}(d) \in \mathrm{T}_\Omega$

α $\mathrm{S} \to \alpha(\mathrm{NP}, \mathrm{VP})$ $1.0 \cdot \Bbbk_1 \cdot \Bbbk_2$

γ β $\xleftarrow{\pi_\Sigma}$ $\mathrm{NP} \to \gamma(\mathrm{NN})$ $\mathrm{VP} \to \beta(\mathrm{VBZ}, \mathrm{PP})$ $\xrightarrow{\mathrm{wt}}$ $0.2 \cdot \Bbbk_1$ $0.6 \cdot \Bbbk_1 \cdot \Bbbk_2$

δ … $\mathrm{NN} \to \delta$ … 1.0 …

$(.)_{\Delta^*}$ $(.)_{\mathbb{R}_0^1}$

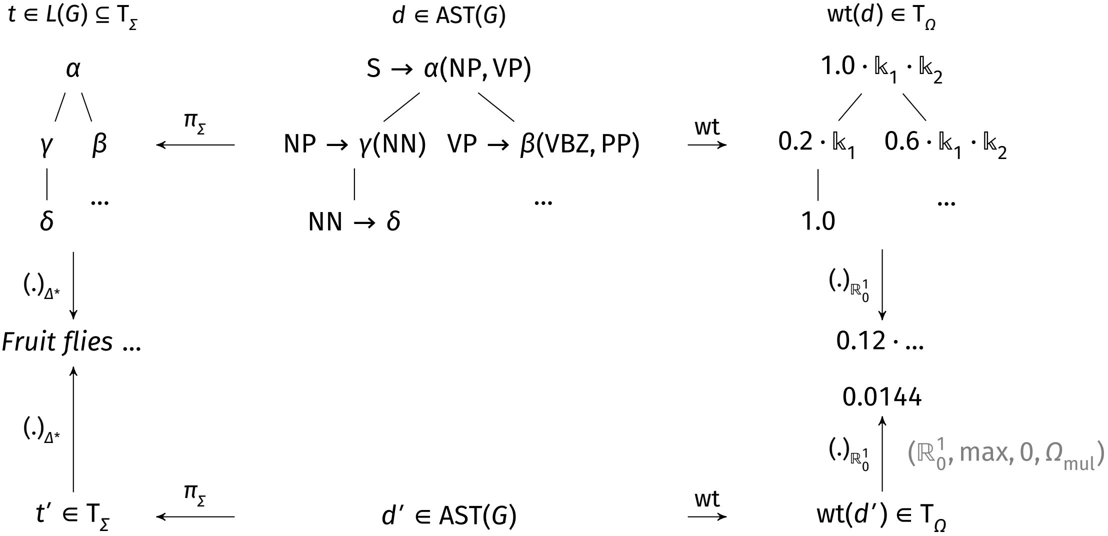
Fruit flies … $0.12 \cdot \ldots$

0.0144

$(.)_{\Delta^*}$ $(.)_{\mathbb{R}_0^1}$

$t' \in \mathrm{T}_\Sigma$ $\xleftarrow{\pi_\Sigma}$ $d' \in \mathrm{AST}(G)$ $\xrightarrow{\mathrm{wt}}$ $\mathrm{wt}(d') \in \mathrm{T}_\Omega$

The weighted parsing problem



$t \in L(G) \subseteq T_\Sigma$

$d \in \mathrm{AST}(G)$

$\mathrm{wt}(d) \in T_\Omega$

α

$S \to \alpha(NP, VP)$

$1.0 \cdot \Bbbk_1 \cdot \Bbbk_2$

γ β

$\xleftarrow{\pi_\Sigma}$

$NP \to \gamma(NN)$ $VP \to \beta(VBZ, PP)$

$\xrightarrow{\mathrm{wt}}$

$0.2 \cdot \Bbbk_1$ $0.6 \cdot \Bbbk_1 \cdot \Bbbk_2$

δ …

$NN \to \delta$ …

1.0 …

$(.)_{\Delta^*}$

$(.)_{\mathbb{R}^1_0}$

Fruit flies …

$0.12 \cdot \ldots$

0.0144

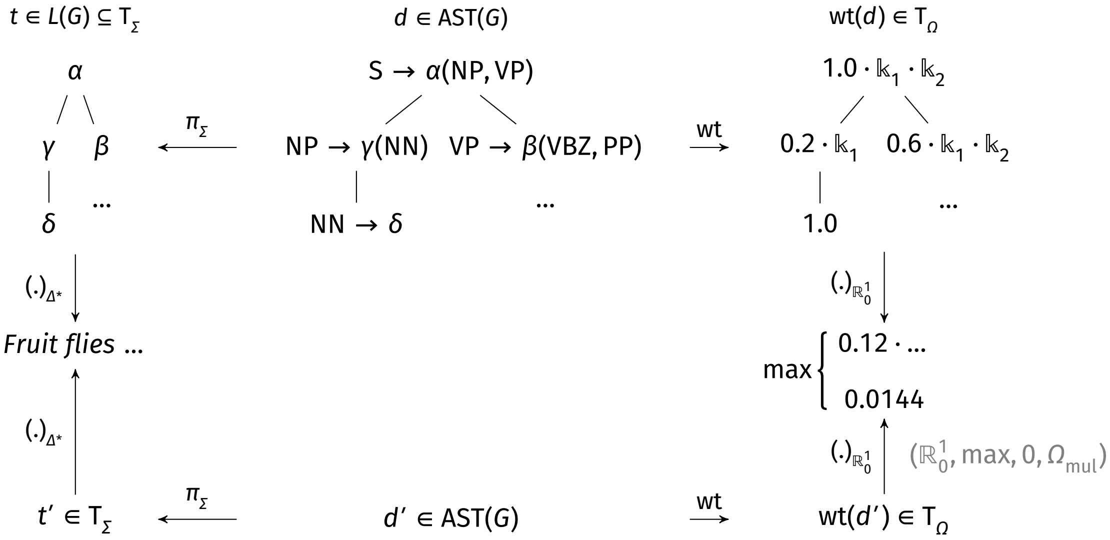
$(.)_{\Delta^*}$

$(.)_{\mathbb{R}^1_0}$

$(\mathbb{R}^1_0, \max, 0, \Omega_{\mathrm{mul}})$

$t' \in T_\Sigma$ $\xleftarrow{\pi_\Sigma}$ $d' \in \mathrm{AST}(G)$ $\xrightarrow{\mathrm{wt}}$ $\mathrm{wt}(d') \in T_\Omega$

The weighted parsing problem



$t \in L(G) \subseteq \mathrm{T}_\Sigma$

$d \in \mathrm{AST}(G)$

$\mathrm{wt}(d) \in \mathrm{T}_\Omega$

α

γ β

δ …

π_Σ

$S \to \alpha(\mathrm{NP}, \mathrm{VP})$

$\mathrm{NP} \to \gamma(\mathrm{NN})$ $\mathrm{VP} \to \beta(\mathrm{VBZ}, \mathrm{PP})$

$\mathrm{NN} \to \delta$ …

wt

$1.0 \cdot \Bbbk_1 \cdot \Bbbk_2$

$0.2 \cdot \Bbbk_1$ $0.6 \cdot \Bbbk_1 \cdot \Bbbk_2$

1.0 …

$(.)_{\Delta^*}$

Fruit flies …

$(.)_{\Delta^*}$

$(.)_{\mathbb{R}_0^1}$

$$\max \begin{cases} 0.12 \cdot \ldots \\ 0.0144 \end{cases}$$

$(.)_{\mathbb{R}_0^1}$

$(\mathbb{R}_0^1, \max, 0, \Omega_{\mathrm{mul}})$

$t' \in \mathrm{T}_\Sigma$

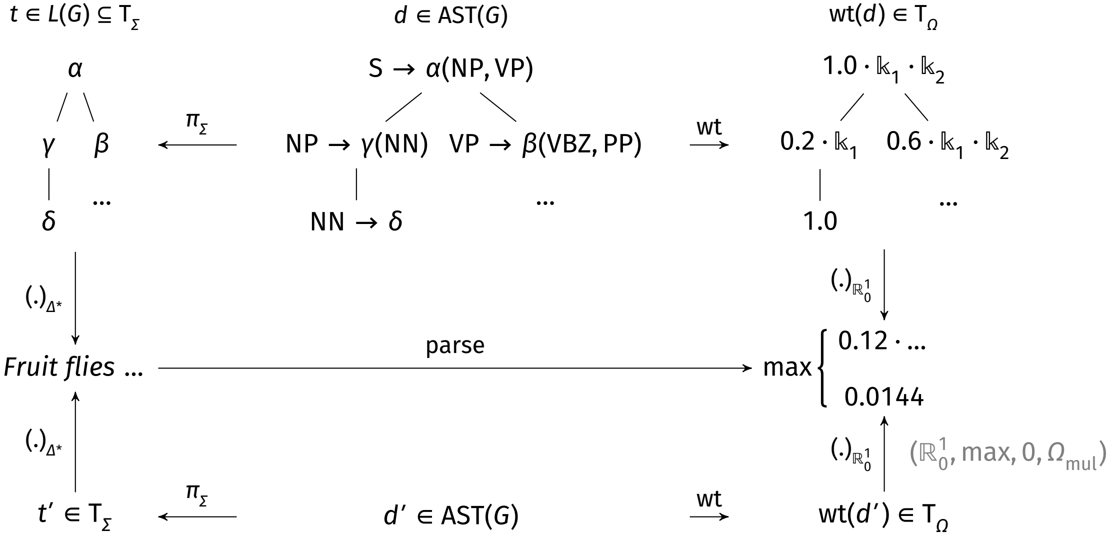
π_Σ

$d' \in \mathrm{AST}(G)$

wt

$\mathrm{wt}(d') \in \mathrm{T}_\Omega$

The weighted parsing problem



$t \in L(G) \subseteq \mathrm{T}_\Sigma$

α

$\gamma \quad \beta$

$\delta \quad \ldots$

$\xleftarrow{\pi_\Sigma}$

$d \in \mathrm{AST}(G)$

$S \rightarrow \alpha(\mathrm{NP}, \mathrm{VP})$

$\mathrm{NP} \rightarrow \gamma(\mathrm{NN}) \quad \mathrm{VP} \rightarrow \beta(\mathrm{VBZ}, \mathrm{PP})$

$\mathrm{NN} \rightarrow \delta \quad \ldots$

$\xrightarrow{\mathrm{wt}}$

$\mathrm{wt}(d) \in \mathrm{T}_\Omega$

$1.0 \cdot \mathbb{k}_1 \cdot \mathbb{k}_2$

$0.2 \cdot \mathbb{k}_1 \quad 0.6 \cdot \mathbb{k}_1 \cdot \mathbb{k}_2$

$1.0 \quad \ldots$

$(.)_{\Delta^*}$

$(.)_{\mathbb{R}_0^1}$

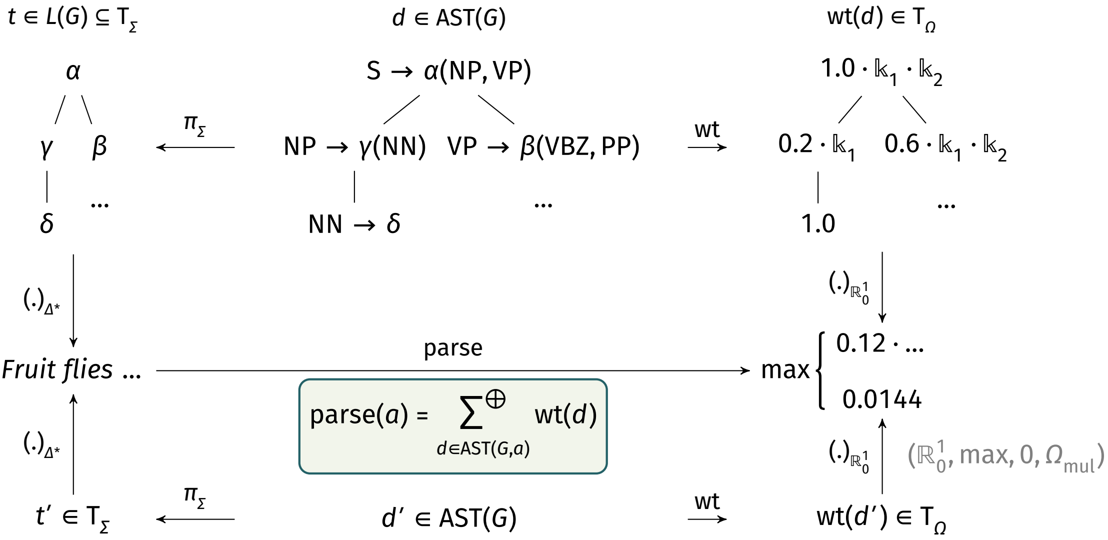
Fruit flies … $\xrightarrow{\text{parse}}$ $\max \begin{cases} 0.12 \cdot \ldots \\ 0.0144 \end{cases}$

$(.)_{\Delta^*}$

$(.)_{\mathbb{R}_0^1}$ $(\mathbb{R}_0^1, \max, 0, \Omega_{\mathrm{mul}})$

$t' \in \mathrm{T}_\Sigma \xleftarrow{\pi_\Sigma} d' \in \mathrm{AST}(G) \xrightarrow{\mathrm{wt}} \mathrm{wt}(d') \in \mathrm{T}_\Omega$

The weighted parsing problem



$$\begin{tikzcd}
t \in L(G) \subseteq \mathrm{T}_\Sigma \arrow[d, "(.)_{\Delta^*}"'] & d \in \mathrm{AST}(G) \arrow[l, "\pi_\Sigma"'] \arrow[r, "\mathrm{wt}"] & \mathrm{wt}(d) \in \mathrm{T}_\Omega \arrow[d, "(.)_{\mathbb{R}_0^1}"'] \\
\textit{Fruit flies} \ldots \arrow[rr, "\mathrm{parse}"] & & \max \begin{cases} 0.12 \cdot \ldots \\ 0.0144 \end{cases} \\
t' \in \mathrm{T}_\Sigma \arrow[u, "(.)_{\Delta^*}"] & d' \in \mathrm{AST}(G) \arrow[l, "\pi_\Sigma"'] \arrow[r, "\mathrm{wt}"] & \mathrm{wt}(d') \in \mathrm{T}_\Omega \arrow[u, "(.)_{\mathbb{R}_0^1}"']
\end{tikzcd}$$

$$\boxed{\mathrm{parse}(a) = \sum^{\oplus}_{d \in \mathrm{AST}(G,a)} \mathrm{wt}(d)}$$

The weighted parsing problem

Examples

- Semiring parsing (Goodman 1999)
    - recognition
    - string probability
    - probability of best derivation
    - derivation forest
    - best derivation(s)
    - *n* best derivation(s)

- Parsing with superior grammars (Knuth 1977; Nederhof 2003)

- Algebraic dynamic programming (Giegerich, Meyer, and Steffen 2004)
    - minimum edit distance
    - matrix chain multiplication

- Reduct of a grammar and a syntactic object (cf. Bar-Hillel, Perles, and Shamir 1961)

Outline

1. Weighted RTG-based language models
2. The weighted parsing problem
3. The weighted parsing algorithm

Weighted parsing algorithm

Two-phase pipeline (Goodman 1999; Nederhof 2003)

Weighted parsing algorithm

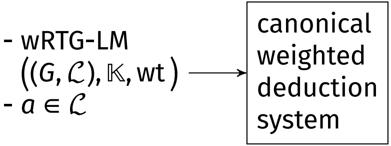
Two-phase pipeline (Goodman 1999; Nederhof 2003)

- wRTG-LM
  $\big((G, \mathcal{L}), \mathbb{K}, \mathrm{wt}\big)$
- $a \in \mathcal{L}$

$$\mathrm{parse}(a) = \sum_{d \in \mathrm{AST}(G,a)}^{\oplus} \mathrm{wt}(d)$$

Weighted parsing algorithm

Two-phase pipeline (Goodman 1999; Nederhof 2003)



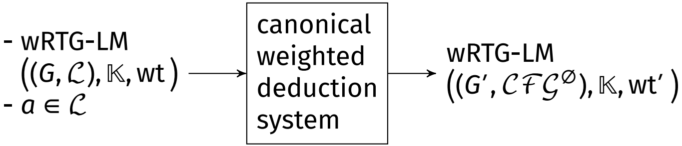
- wRTG-LM $\big((G, \mathcal{L}), \mathbb{K}, \mathrm{wt}\big)$
- $a \in \mathcal{L}$

$$\xrightarrow{\quad} \boxed{\text{canonical weighted deduction system}}$$

$$\mathrm{parse}(a) = \sum_{d \in \mathrm{AST}(G,a)}^{\oplus} \mathrm{wt}(d)$$

Weighted parsing algorithm

Two-phase pipeline (Goodman 1999; Nederhof 2003)



- wRTG-LM $\big((G,\mathcal{L}),\mathbb{K},\mathrm{wt}\big)$
- $a \in \mathcal{L}$

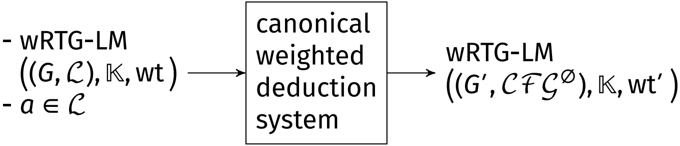
canonical weighted deduction system

wRTG-LM $\big((G',\mathcal{CFG}^{\emptyset}),\mathbb{K},\mathrm{wt}'\big)$

$$\mathrm{parse}(a) = \sum^{\oplus}_{d\in \mathrm{AST}(G,a)} \mathrm{wt}(d)$$

Weighted parsing algorithm

Two-phase pipeline (Goodman 1999; Nederhof 2003)



- wRTG-LM $\big((G, \mathcal{L}), \mathbb{K}, \mathrm{wt}\big)$
- $a \in \mathcal{L}$

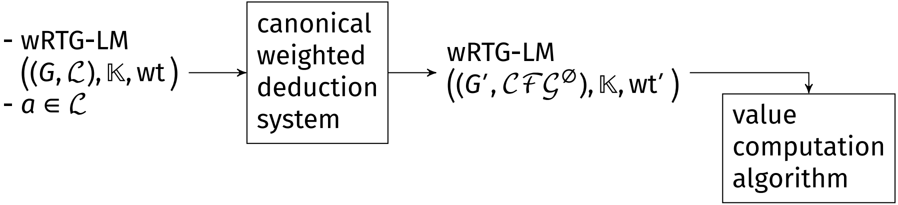
canonical weighted deduction system

wRTG-LM $\big((G', \mathcal{CFG}^{\varnothing}), \mathbb{K}, \mathrm{wt}'\big)$

$$\mathrm{parse}(a) = \sum^{\oplus}_{d \in \mathrm{AST}(G,a)} \mathrm{wt}(d) \overset{?}{=} \sum^{\oplus}_{d \in \mathrm{AST}(G')} \mathrm{wt}'(d)$$

Weighted parsing algorithm

Two-phase pipeline (Goodman 1999; Nederhof 2003)



- wRTG-LM $\big((G,\mathcal{L}),\mathbb{K},\mathrm{wt}\big)$
- $a \in \mathcal{L}$

canonical weighted deduction system

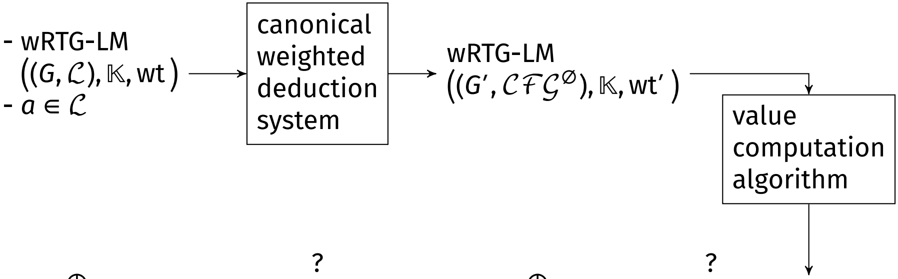
wRTG-LM $\big((G',\mathcal{CFG}^{\varnothing}),\mathbb{K},\mathrm{wt}'\big)$

value computation algorithm

$$\mathrm{parse}(a) = \sum^{\oplus}_{d \in \mathrm{AST}(G,a)} \mathrm{wt}(d) \overset{?}{=} \sum^{\oplus}_{d \in \mathrm{AST}(G')} \mathrm{wt}'(d)$$

Weighted parsing algorithm

Two-phase pipeline (Goodman 1999; Nederhof 2003)



- wRTG-LM $\big((G,\mathcal{L}),\mathbb{K},\mathrm{wt}\big)$
- $a \in \mathcal{L}$

canonical weighted deduction system

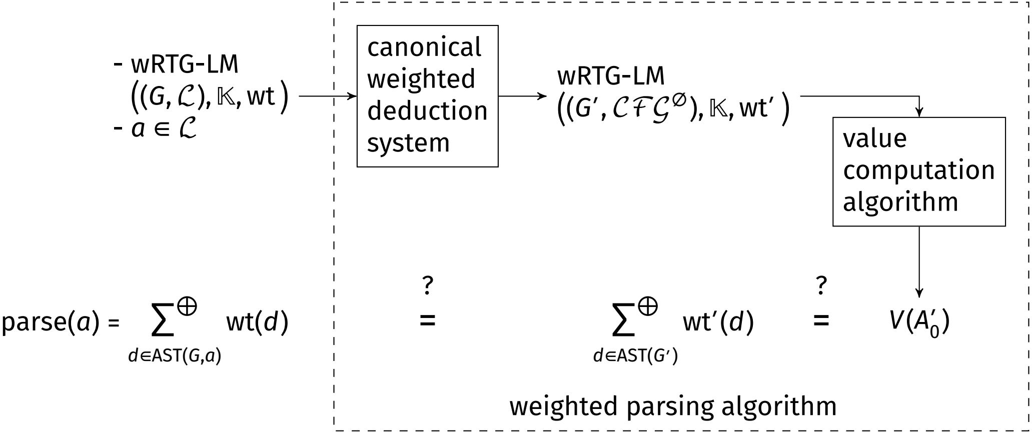
wRTG-LM $\big((G',\mathcal{CFG}^{\varnothing}),\mathbb{K},\mathrm{wt}'\big)$

value computation algorithm

$$\mathrm{parse}(a) = \sum^{\oplus}_{d\in \mathrm{AST}(G,a)} \mathrm{wt}(d) \overset{?}{=} \sum^{\oplus}_{d\in \mathrm{AST}(G')} \mathrm{wt}'(d) \overset{?}{=} V(A'_0)$$

Weighted parsing algorithm

Two-phase pipeline (Goodman 1999; Nederhof 2003)



- wRTG-LM $\big((G, \mathcal{L}), \mathbb{K}, \mathrm{wt}\big)$
- $a \in \mathcal{L}$

canonical weighted deduction system

wRTG-LM $\big((G', \mathcal{CFG}^{\varnothing}), \mathbb{K}, \mathrm{wt}'\big)$

value computation algorithm

$$\mathrm{parse}(a) = \sum^{\oplus}_{d \in \mathrm{AST}(G,a)} \mathrm{wt}(d) \overset{?}{=} \sum^{\oplus}_{d \in \mathrm{AST}(G')} \mathrm{wt}'(d) \overset{?}{=} V(A'_0)$$

weighted parsing algorithm

Canonical weighted deduction system

- wRTG-LM $\big((G, \mathcal{L}), \mathbb{K}, \mathrm{wt}\big)$ $\xrightarrow{\text{cwds}}$ wRTG-LM $\big((G', \mathcal{CFG}^{\emptyset}), \mathbb{K}, \mathrm{wt}'\big)$
- $a \in \mathcal{L}$

Parsing as deduction (Shieber, Schabes, and Pereira 1995)

$$\frac{[A_1, a_1] \;\dots\; [A_m, a_m]}{[A, a_0]} \quad \begin{cases} A \to \sigma(A_1, \dots, A_m) \text{ is a rule} \\ a_0, a_1, \dots, a_m \in \mathrm{factors}(a) \\ a_0 = \sigma(a_1, \dots, a_m) \end{cases}$$

Canonical weighted deduction system

$$\begin{array}{l} \text{- wRTG-LM } \big((G, \mathcal{L}), \mathbb{K}, \mathrm{wt}\big) \\ \text{- } a \in \mathcal{L} \end{array} \xrightarrow{\text{cwds}} \text{wRTG-LM } \big((G', \mathcal{CFG}^{\varnothing}), \mathbb{K}, \mathrm{wt}'\big)$$

Parsing as deduction (Shieber, Schabes, and Pereira 1995)

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Canonical weighted deduction system

$$\begin{array}{l} \text{- wRTG-LM } \big((G, \mathcal{L}), \mathbb{K}, \mathrm{wt}\big) \\ \text{- } a \in \mathcal{L} \end{array} \xrightarrow{\text{cwds}} \text{wRTG-LM } \big((G', \mathcal{CFG}^{\emptyset}), \mathbb{K}, \mathrm{wt}'\big)$$

Parsing as deduction (Shieber, Schabes, and Pereira 1995)

$$\frac{[A_1, a_1] \;\ldots\; [A_m, a_m]}{[A, a_0]} \quad \begin{cases} A \to \sigma(A_1, \ldots, A_m) \text{ is a rule} \\ a_0, a_1, \ldots, a_m \in \text{factors}(a) \\ a_0 = \sigma(a_1, \ldots, a_m) \end{cases}$$

Canonical weighted deduction system

$$\begin{array}{l}\text{- wRTG-LM } \big((G, \mathcal{L}), \mathbb{K}, \mathrm{wt}\big) \\ \text{- } a \in \mathcal{L}\end{array} \xrightarrow{\text{cwds}} \text{wRTG-LM } \big((G', \mathcal{CFG}^{\emptyset}), \mathbb{K}, \mathrm{wt}'\big)$$

Parsing as deduction (Shieber, Schabes, and Pereira 1995)

$$\frac{[A_1, a_1] \ \dots \ [A_m, a_m]}{[A, a_0]} \quad \begin{cases} A \to \sigma(A_1, \dots, A_m) \text{ is a rule} \\ a_0, a_1, \dots, a_m \in \text{factors}(a) \\ a_0 = \sigma(a_1, \dots, a_m) \end{cases}$$

Canonical weighted deduction system

$$\begin{array}{l} \text{- wRTG-LM } \big((G,\mathcal{L}),\mathbb{K},\mathrm{wt}\big) \\ \text{- } a \in \mathcal{L} \end{array} \xrightarrow{\text{cwds}} \text{wRTG-LM } \big((G',\mathcal{CFG}^{\emptyset}),\mathbb{K},\mathrm{wt}'\big)$$

Parsing as deduction (Shieber, Schabes, and Pereira 1995)

$$\frac{[A_1\,,a_1]\ \ldots\ [A_m\,,a_m]}{[A\,,a_0]} \quad \begin{cases} A \to \sigma(A_1,\ldots,A_m) \text{ is a rule} \\ a_0,a_1,\ldots,a_m \in \text{factors}(a) \\ a_0 = \sigma(a_1,\ldots,a_m) \end{cases}$$

Canonical weighted deduction system

$$\begin{array}{l} \text{- wRTG-LM}\left((G, \mathcal{L}), \mathbb{K}, \mathrm{wt}\right) \\ \text{- } a \in \mathcal{L} \end{array} \xrightarrow{\text{cwds}} \text{wRTG-LM}\left((G', \mathcal{CFG}^{\varnothing}), \mathbb{K}, \mathrm{wt}'\right)$$

Parsing as deduction (Shieber, Schabes, and Pereira 1995)

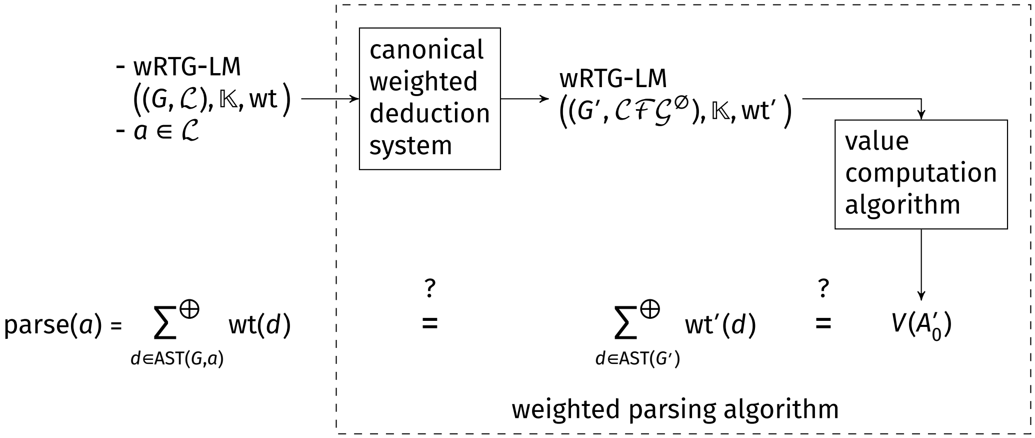
$$\frac{[A_1\,, a_1] \ \dots \ [A_m\,, a_m]}{[A\,, a_0]} \quad \begin{cases} A \to \sigma(A_1, \dots, A_m) \text{ is a rule} \\ a_0, a_1, \dots, a_m \in \mathrm{factors}(a) \\ a_0 = \sigma(a_1, \dots, a_m) \end{cases}$$

Weight preserving

1. Bijection $\psi\colon \mathrm{AST}(G, a) \to \mathrm{AST}(G')$
2. $\mathrm{wt}(d) = \mathrm{wt}'(\psi(d))$ for every $d \in \mathrm{AST}(G, a)$

Weighted parsing algorithm

Two-phase pipeline (Goodman 1999; Nederhof 2003)



- wRTG-LM $((G, \mathcal{L}), \mathbb{K}, \mathrm{wt})$
- $a \in \mathcal{L}$

canonical weighted deduction system

wRTG-LM $((G', \mathcal{CFG}^{\emptyset}), \mathbb{K}, \mathrm{wt}')$

value computation algorithm

$$\mathrm{parse}(a) = \sum^{\oplus}_{d \in \mathrm{AST}(G,a)} \mathrm{wt}(d) \overset{?}{=} \sum^{\oplus}_{d \in \mathrm{AST}(G')} \mathrm{wt}'(d) \overset{?}{=} V(A'_0)$$

weighted parsing algorithm

Value computation algorithm

Input: a wRTG-LM $\big((G', \mathcal{CFG}^{\varnothing}), (\mathbb{K}, \oplus, \mathbb{0}, \Omega), \mathrm{wt}'\big)$ with $G' = (N', \Sigma', A'_0, R')$
Variables: $V\colon N' \to \mathbb{K}$, $V_{\mathrm{new}} \in \mathbb{K}$, $\mathit{changed} \in \mathbb{B}$
Output: $V(A'_0)$

```
for each $A \in N'$ do
    $V(A) \leftarrow \mathbb{0}$
repeat
    $\mathit{changed} \leftarrow \mathrm{false}$
    for each $A \in N'$ do
        $V_{\mathrm{new}} \leftarrow \mathbb{0}$
        for each $r = (A \to \langle x_1 \dots x_m \rangle(A_1, \dots, A_m))$ in $R'$ do
            $V_{\mathrm{new}} \leftarrow V_{\mathrm{new}} \oplus \mathrm{wt}'(r)\big(V(A_1), \dots, V(A_m)\big)$
        if $V(A) \neq V_{\mathrm{new}}$ then
            $\mathit{changed} \leftarrow \mathrm{true}$
        $V(A) \leftarrow V_{\mathrm{new}}$
until $\mathit{changed} = \mathrm{false}$
```

Value computation algorithm (example)

$$\big((G,\ \mathcal{CFG}^{\varnothing}),\ (\mathbb{K}, \mathbb{0}, \oplus, \Omega),\ \mathrm{wt}\big)$$

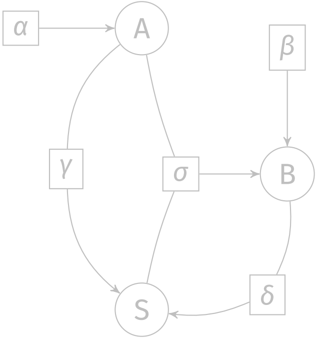
$$\mathrm{S} \xrightarrow{\omega_1} \gamma(\mathrm{A})$$

$$\mathrm{S} \xrightarrow{\omega_2} \delta(\mathrm{B})$$

$$\mathrm{A} \xrightarrow{\omega_3} \alpha$$

$$\mathrm{B} \xrightarrow{\omega_4} \sigma(\mathrm{A}, \mathrm{S})$$

$$\mathrm{B} \xrightarrow{\omega_5} \beta$$



Value computation algorithm (example)

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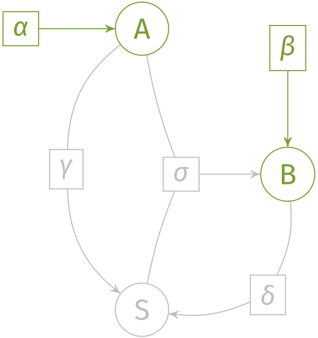

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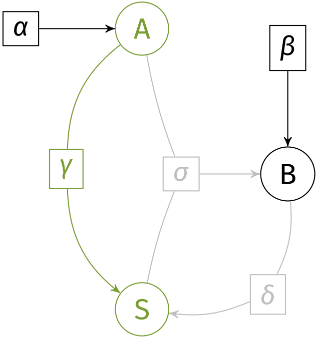

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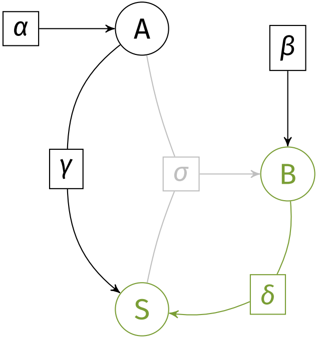

Value computation algorithm (example)

$$\Big((G,\ \mathcal{CFG}^{\emptyset}),\ (\mathbb{K}, \mathbb{0}, \oplus, \Omega),\ \mathrm{wt}\Big)$$

$$\mathrm{S} \xrightarrow{\omega_1} \gamma(\mathrm{A})$$

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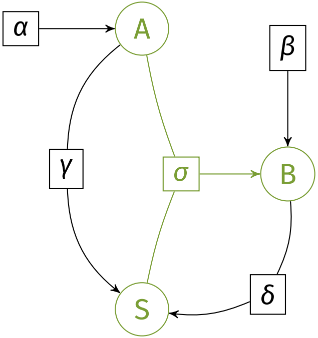

Value computation algorithm (example)

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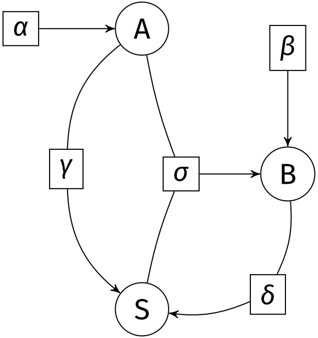

Value computation algorithm (example)

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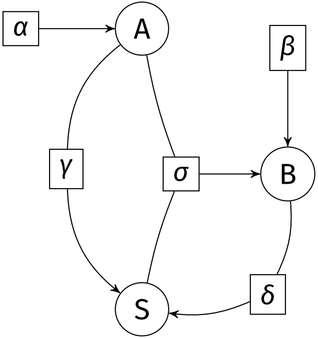

$$\begin{array}{c} S \\ A \\ B \end{array} \begin{pmatrix} \mathbb{0} \\ \mathbb{0} \\ \mathbb{0} \end{pmatrix}$$

Value computation algorithm (example)

$$\big((G,\ \mathcal{CFG}^{\varnothing}),\ (\mathbb{K}, \mathbb{0}, \oplus, \Omega),\ \mathrm{wt}\big)$$

$$\mathrm{S} \xrightarrow{\omega_1} \gamma(\mathrm{A})$$

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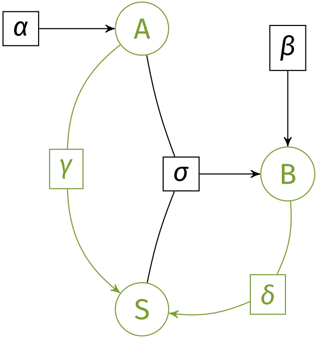

$$\begin{array}{c} \mathrm{S} \\ \mathrm{A} \\ \mathrm{B} \end{array} \begin{pmatrix} \mathbb{0} \\ \mathbb{0} \\ \mathbb{0} \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{0}) \oplus \omega_2(\mathbb{0}) \\ \\ \\ \end{pmatrix}$$

Value computation algorithm (example)

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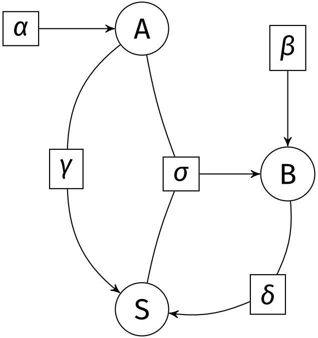

$$\begin{array}{c} S \\ A \\ B \end{array} \begin{pmatrix} \mathbb{0} \\ \mathbb{0} \\ \mathbb{0} \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{0}) \oplus \omega_2(\mathbb{0}) \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \Bbbk_1 \\ \\ \\ \end{pmatrix}$$

Value computation algorithm (example)

$$\Big((G,\ \mathcal{CFG}^{\varnothing}),\ (\mathbb{K}, \mathbb{0}, \oplus, \Omega),\ \mathrm{wt}\Big)$$

$$S \xrightarrow{\omega_1} \gamma(A)$$

$$S \xrightarrow{\omega_2} \delta(B)$$

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$$B \xrightarrow{\omega_4} \sigma(A, S)$$

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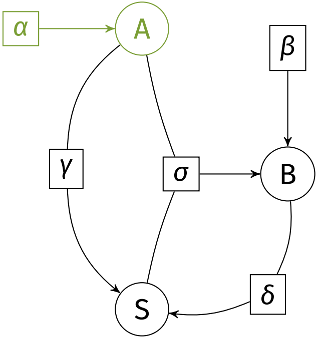

$$\begin{array}{c} S \\ A \\ B \end{array}\begin{pmatrix} \mathbb{0} \\ \mathbb{0} \\ \mathbb{0} \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{0}) \oplus \omega_2(\mathbb{0}) \\ \omega_3() \\ \end{pmatrix} = \begin{pmatrix} \Bbbk_1 \\ \\ \end{pmatrix}$$

Value computation algorithm (example)

$$\big((G,\ \mathcal{CFG}^{\varnothing}),\ (\mathbb{K}, \mathbb{0}, \oplus, \Omega),\ \mathrm{wt}\big)$$

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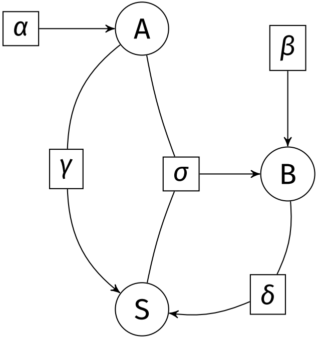

$$\begin{array}{c} S \\ A \\ B \end{array} \begin{pmatrix} \mathbb{0} \\ \mathbb{0} \\ \mathbb{0} \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{0}) \oplus \omega_2(\mathbb{0}) \\ \omega_3() \\ \end{pmatrix} = \begin{pmatrix} \Bbbk_1 \\ \Bbbk_2 \\ \end{pmatrix}$$

Value computation algorithm (example)

$$\big((G,\ \mathcal{CFG}^{\varnothing}),\ (\mathbb{K}, \mathbb{0}, \oplus, \Omega),\ \mathrm{wt}\big)$$

$$S \xrightarrow{\omega_1} \gamma(A)$$

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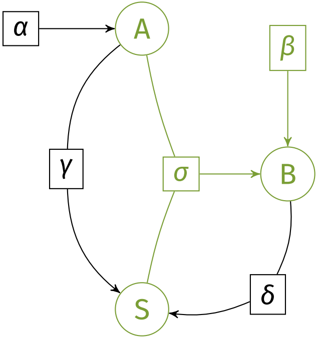

$$\begin{array}{c} S \\ A \\ B \end{array} \begin{pmatrix} \mathbb{0} \\ \mathbb{0} \\ \mathbb{0} \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{0}) \oplus \omega_2(\mathbb{0}) \\ \omega_3() \\ \omega_4(\Bbbk_2, \Bbbk_1) \oplus \omega_5() \end{pmatrix} = \begin{pmatrix} \Bbbk_1 \\ \Bbbk_2 \\ \\ \end{pmatrix}$$

Value computation algorithm (example)

$$\Big((G,\ \mathcal{CFG}^{\varnothing}),\ (\mathbb{K}, \mathbb{0}, \oplus, \Omega),\ \mathrm{wt}\Big)$$

$$S \xrightarrow{\omega_1} \gamma(A)$$

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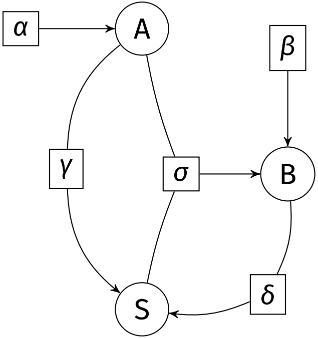

$$\begin{array}{c} S \\ A \\ B \end{array}\begin{pmatrix} \mathbb{0} \\ \mathbb{0} \\ \mathbb{0} \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{0}) \oplus \omega_2(\mathbb{0}) \\ \omega_3() \\ \omega_4(\mathbb{k}_2, \mathbb{k}_1) \oplus \omega_5() \end{pmatrix} = \begin{pmatrix} \mathbb{k}_1 \\ \mathbb{k}_2 \\ \mathbb{k}_3 \end{pmatrix}$$

Value computation algorithm (example)

$$\big((G,\ \mathcal{CFG}^{\varnothing}),\ (\mathbb{K}, \mathbb{0}, \oplus, \Omega),\ \mathrm{wt}\,\big)$$

$$S \xrightarrow{\omega_1} \gamma(A)$$

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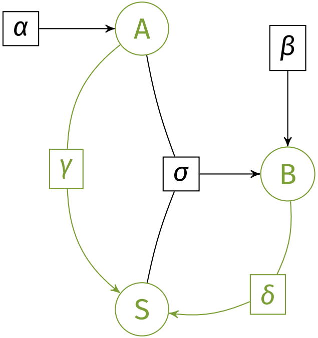
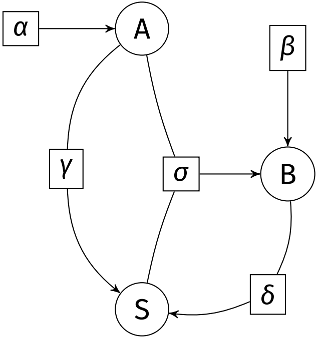


$$\begin{array}{c} S \\ A \\ B \end{array}\begin{pmatrix} \mathbb{0} \\ \mathbb{0} \\ \mathbb{0} \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{0}) \oplus \omega_2(\mathbb{0}) \\ \omega_3() \\ \omega_4(\Bbbk_2, \Bbbk_1) \oplus \omega_5() \end{pmatrix} = \begin{pmatrix} \Bbbk_1 \\ \Bbbk_2 \\ \Bbbk_3 \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\Bbbk_2) \oplus \omega_2(\Bbbk_3) \\ \\ \end{pmatrix}$$

Value computation algorithm (example)

$$\big((G,\ \mathcal{CFG}^{\varnothing}),\ (\mathbb{K}, \mathbb{0}, \oplus, \Omega),\ \mathrm{wt}\big)$$

$$\begin{aligned}
S &\xrightarrow{\omega_1} \gamma(A)\\
S &\xrightarrow{\omega_2} \delta(B)\\
A &\xrightarrow{\omega_3} \alpha\\
B &\xrightarrow{\omega_4} \sigma(A, S)\\
B &\xrightarrow{\omega_5} \beta
\end{aligned}$$



$$\begin{array}{c} S\\ A\\ B \end{array}\begin{pmatrix} \mathbb{0}\\ \mathbb{0}\\ \mathbb{0} \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{0}) \oplus \omega_2(\mathbb{0})\\ \omega_3()\\ \omega_4(\mathbb{k}_2, \mathbb{k}_1) \oplus \omega_5() \end{pmatrix} = \begin{pmatrix} \mathbb{k}_1\\ \mathbb{k}_2\\ \mathbb{k}_3 \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{k}_2) \oplus \omega_2(\mathbb{k}_3)\\ \\ \\ \end{pmatrix} = \begin{pmatrix} \mathbb{k}_1'\\ \\ \\ \end{pmatrix}$$

Value computation algorithm (example)

$$\Big((G,\ \mathcal{CFG}^{\varnothing}),\ (\mathbb{K},\mathbb{0},\oplus,\Omega),\ \mathrm{wt}\Big)$$

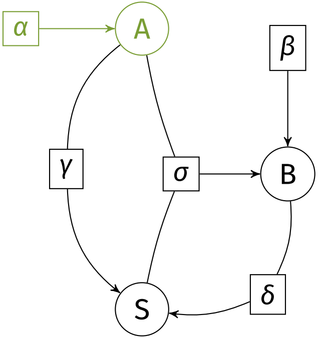
$$S \xrightarrow{\omega_1} \gamma(A)$$

$$S \xrightarrow{\omega_2} \delta(B)$$

$$A \xrightarrow{\omega_3} \alpha$$

$$B \xrightarrow{\omega_4} \sigma(A, S)$$

$$B \xrightarrow{\omega_5} \beta$$



$$\begin{array}{c} S\\ A\\ B \end{array}\!\begin{pmatrix}\mathbb{0}\\ \mathbb{0}\\ \mathbb{0}\end{pmatrix} \mapsto \begin{pmatrix}\omega_1(\mathbb{0})\oplus\omega_2(\mathbb{0})\\ \omega_3()\\ \omega_4(\mathbb{k}_2,\mathbb{k}_1)\oplus\omega_5()\end{pmatrix} = \begin{pmatrix}\mathbb{k}_1\\ \mathbb{k}_2\\ \mathbb{k}_3\end{pmatrix} \mapsto \begin{pmatrix}\omega_1(\mathbb{k}_2)\oplus\omega_2(\mathbb{k}_3)\\ \omega_3()\\ \end{pmatrix} = \begin{pmatrix}\mathbb{k}'_1\\ \\ \end{pmatrix}$$

Value computation algorithm (example)

$$\left((G,\ \mathcal{CFG}^{\varnothing}),\ (\mathbb{K}, \mathbb{0}, \oplus, \Omega),\ \mathrm{wt}\right)$$

$$S \xrightarrow{\omega_1} \gamma(A)$$

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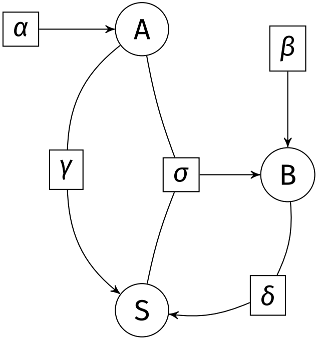

$$\begin{array}{c} S \\ A \\ B \end{array}\begin{pmatrix} \mathbb{0} \\ \mathbb{0} \\ \mathbb{0} \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{0}) \oplus \omega_2(\mathbb{0}) \\ \omega_3() \\ \omega_4(\mathbb{k}_2, \mathbb{k}_1) \oplus \omega_5() \end{pmatrix} = \begin{pmatrix} \mathbb{k}_1 \\ \mathbb{k}_2 \\ \mathbb{k}_3 \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{k}_2) \oplus \omega_2(\mathbb{k}_3) \\ \omega_3() \\ \\ \end{pmatrix} = \begin{pmatrix} \mathbb{k}'_1 \\ \mathbb{k}'_2 \\ \\ \end{pmatrix}$$

Value computation algorithm (example)

$$\big((G,\ \mathcal{CFG}^{\varnothing}),\ (\mathbb{K}, \mathbb{0}, \oplus, \Omega),\ \mathrm{wt}\big)$$

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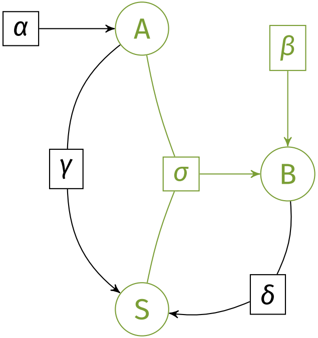

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Value computation algorithm (example)

$$\Big((G,\ \mathcal{CFG}^{\emptyset}),\ (\mathbb{K}, \mathbb{0}, \oplus, \Omega),\ \mathrm{wt}\Big)$$

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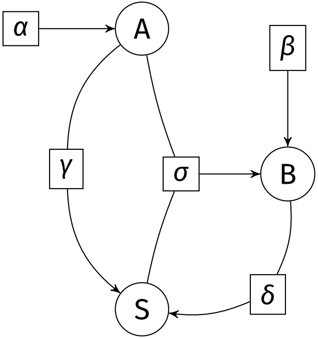

$$\begin{array}{c} S \\ A \\ B \end{array}\begin{pmatrix} \mathbb{0} \\ \mathbb{0} \\ \mathbb{0} \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{0}) \oplus \omega_2(\mathbb{0}) \\ \omega_3() \\ \omega_4(\mathbb{k}_2, \mathbb{k}_1) \oplus \omega_5() \end{pmatrix} = \begin{pmatrix} \mathbb{k}_1 \\ \mathbb{k}_2 \\ \mathbb{k}_3 \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{k}_2) \oplus \omega_2(\mathbb{k}_3) \\ \omega_3() \\ \omega_4(\mathbb{k}'_2, \mathbb{k}'_1) \oplus \omega_5() \end{pmatrix} = \begin{pmatrix} \mathbb{k}'_1 \\ \mathbb{k}'_2 \\ \mathbb{k}'_3 \end{pmatrix}$$

Value computation algorithm (example)

$$\big((G,\ \mathcal{CFG}^{\emptyset}),\ (\mathbb{K}, \mathbb{0}, \oplus, \Omega),\ \mathrm{wt}\big)$$

$$S \xrightarrow{\omega_1} \gamma(A)$$

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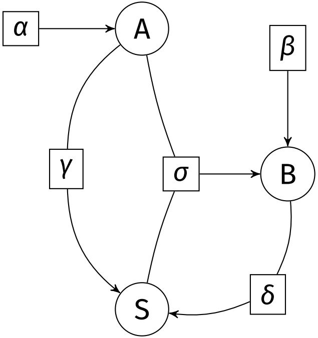


$$\begin{array}{c} S \\ A \\ B \end{array} \begin{pmatrix} \mathbb{0} \\ \mathbb{0} \\ \mathbb{0} \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{0}) \oplus \omega_2(\mathbb{0}) \\ \omega_3() \\ \omega_4(\mathbb{k}_2, \mathbb{k}_1) \oplus \omega_5() \end{pmatrix} = \begin{pmatrix} \mathbb{k}_1 \\ \mathbb{k}_2 \\ \mathbb{k}_3 \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{k}_2) \oplus \omega_2(\mathbb{k}_3) \\ \omega_3() \\ \omega_4(\mathbb{k}'_2, \mathbb{k}'_1) \oplus \omega_5() \end{pmatrix} = \begin{pmatrix} \mathbb{k}'_1 \\ \mathbb{k}'_2 \\ \mathbb{k}'_3 \end{pmatrix} \mapsto \ldots$$

Value computation algorithm (example)

$$\Big((G,\ \mathcal{CFG}^{\varnothing}),\ (\mathbb{K},\mathbb{0},\oplus,\Omega),\ \mathrm{wt}\Big) \rightsquigarrow \Big((G,\ \mathcal{CFG}^{\varnothing}),\ (\mathbb{R}^1_0,0,\max,\Omega_{\mathrm{mul}}),\ \mathrm{wt}\Big)$$

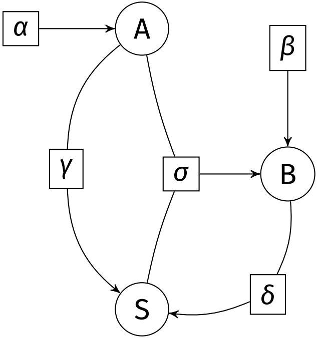
$$S \xrightarrow{\omega_1} \gamma(A)$$

$$S \xrightarrow{\omega_2} \delta(B)$$

$$A \xrightarrow{\omega_3} \alpha$$

$$B \xrightarrow{\omega_4} \sigma(A,S)$$

$$B \xrightarrow{\omega_5} \beta$$



$$\begin{array}{c} S \\ A \\ B \end{array}\begin{pmatrix} \mathbb{0} \\ \mathbb{0} \\ \mathbb{0} \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{0}) \oplus \omega_2(\mathbb{0}) \\ \omega_3() \\ \omega_4(\mathbb{k}_2,\mathbb{k}_1) \oplus \omega_5() \end{pmatrix} = \begin{pmatrix} \mathbb{k}_1 \\ \mathbb{k}_2 \\ \mathbb{k}_3 \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{k}_2) \oplus \omega_2(\mathbb{k}_3) \\ \omega_3() \\ \omega_4(\mathbb{k}'_2,\mathbb{k}'_1) \oplus \omega_5() \end{pmatrix} = \begin{pmatrix} \mathbb{k}'_1 \\ \mathbb{k}'_2 \\ \mathbb{k}'_3 \end{pmatrix} \mapsto \dots$$

Value computation algorithm (example)

$$\big((G,\ \mathcal{CFG}^{\varnothing}),\ (\mathbb{K}, \mathbb{0}, \oplus, \Omega),\ \mathrm{wt}\big) \rightsquigarrow \big((G,\ \mathcal{CFG}^{\varnothing}),\ (\mathbb{R}^1_0, 0, \max, \Omega_{\mathrm{mul}}),\ \mathrm{wt}\big)$$

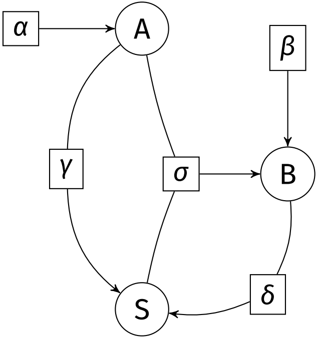
$$S \xrightarrow{0.8 \cdot \Bbbk_1} \gamma(A)$$

$$S \xrightarrow{\omega_2} \delta(B)$$

$$A \xrightarrow{\omega_3} \alpha$$

$$B \xrightarrow{\omega_4} \sigma(A, S)$$

$$B \xrightarrow{\omega_5} \beta$$



$$\begin{array}{c} S \\ A \\ B \end{array} \begin{pmatrix} \mathbb{0} \\ \mathbb{0} \\ \mathbb{0} \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{0}) \oplus \omega_2(\mathbb{0}) \\ \omega_3() \\ \omega_4(\Bbbk_2, \Bbbk_1) \oplus \omega_5() \end{pmatrix} = \begin{pmatrix} \Bbbk_1 \\ \Bbbk_2 \\ \Bbbk_3 \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\Bbbk_2) \oplus \omega_2(\Bbbk_3) \\ \omega_3() \\ \omega_4(\Bbbk'_2, \Bbbk'_1) \oplus \omega_5() \end{pmatrix} = \begin{pmatrix} \Bbbk'_1 \\ \Bbbk'_2 \\ \Bbbk'_3 \end{pmatrix} \mapsto \ldots$$

Value computation algorithm (example)

$$\Big((G,\ \mathcal{CFG}^{\varnothing}),\ (\mathbb{K}, \mathbb{0}, \oplus, \Omega),\ \mathrm{wt}\Big) \rightsquigarrow \Big((G,\ \mathcal{CFG}^{\varnothing}),\ (\mathbb{R}^1_0, 0, \max, \Omega_{\mathrm{mul}}),\ \mathrm{wt}\Big)$$

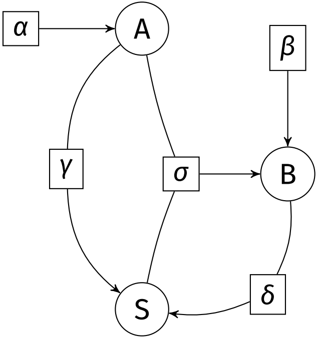
$$S \xrightarrow{0.8 \cdot \mathbb{k}_1} \gamma(A)$$

$$S \xrightarrow{0.1 \cdot \mathbb{k}_1} \delta(B)$$

$$A \xrightarrow{\omega_3} \alpha$$

$$B \xrightarrow{\omega_4} \sigma(A, S)$$

$$B \xrightarrow{\omega_5} \beta$$



$$\begin{array}{c} S \\ A \\ B \end{array} \begin{pmatrix} \mathbb{0} \\ \mathbb{0} \\ \mathbb{0} \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{0}) \oplus \omega_2(\mathbb{0}) \\ \omega_3() \\ \omega_4(\mathbb{k}_2, \mathbb{k}_1) \oplus \omega_5() \end{pmatrix} = \begin{pmatrix} \mathbb{k}_1 \\ \mathbb{k}_2 \\ \mathbb{k}_3 \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{k}_2) \oplus \omega_2(\mathbb{k}_3) \\ \omega_3() \\ \omega_4(\mathbb{k}'_2, \mathbb{k}'_1) \oplus \omega_5() \end{pmatrix} = \begin{pmatrix} \mathbb{k}'_1 \\ \mathbb{k}'_2 \\ \mathbb{k}'_3 \end{pmatrix} \mapsto \ldots$$

Value computation algorithm (example)

$$\big((G,\ \mathcal{CFG}^{\varnothing}),\ (\mathbb{K}, \mathbb{0}, \oplus, \Omega),\ \mathrm{wt}\big) \rightsquigarrow \big((G,\ \mathcal{CFG}^{\varnothing}),\ (\mathbb{R}_0^1, 0, \max, \Omega_{\mathrm{mul}}),\ \mathrm{wt}\big)$$

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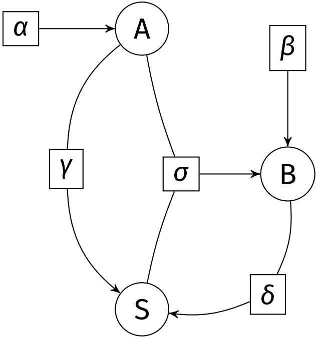

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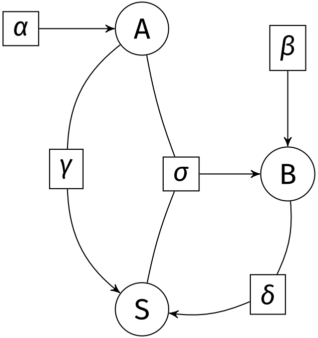

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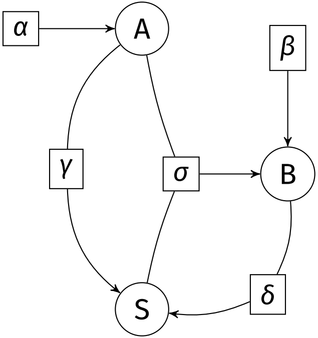

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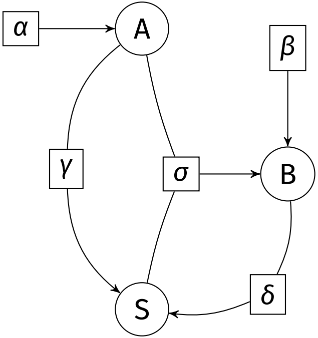

$$\begin{array}{c|c|} S & 0 \\ A & 0 \\ B & 0 \end{array} \mapsto \begin{pmatrix} 0.8\cdot 0\ \max\ 0.1\cdot 0 \\ \omega_3() \\ \omega_4(\mathbb{k}_2, \mathbb{k}_1) \oplus \omega_5() \end{pmatrix} = \begin{pmatrix} \mathbb{k}_1 \\ \mathbb{k}_2 \\ \mathbb{k}_3 \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\mathbb{k}_2) \oplus \omega_2(\mathbb{k}_3) \\ \omega_3() \\ \omega_4(\mathbb{k}'_2, \mathbb{k}'_1) \oplus \omega_5() \end{pmatrix} = \begin{pmatrix} \mathbb{k}'_1 \\ \mathbb{k}'_2 \\ \mathbb{k}'_3 \end{pmatrix} \mapsto \ldots$$

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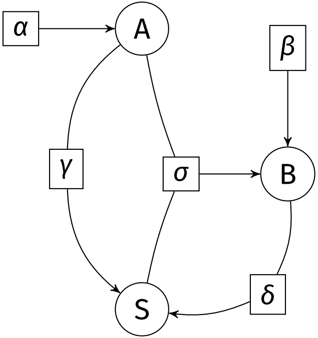

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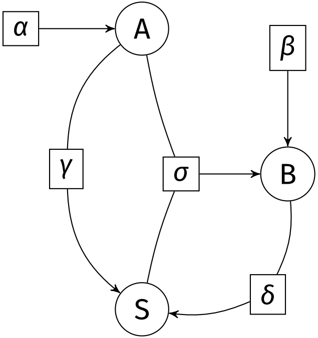

$$\begin{array}{c} S \\ A \\ B \end{array}\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 0 \\ 0.5 \\ \omega_4(\Bbbk_2, \Bbbk_1) \oplus \omega_5() \end{pmatrix} = \begin{pmatrix} \Bbbk_1 \\ \Bbbk_2 \\ \Bbbk_3 \end{pmatrix} \mapsto \begin{pmatrix} \omega_1(\Bbbk_2) \oplus \omega_2(\Bbbk_3) \\ \omega_3() \\ \omega_4(\Bbbk_2', \Bbbk_1') \oplus \omega_5() \end{pmatrix} = \begin{pmatrix} \Bbbk_1' \\ \Bbbk_2' \\ \Bbbk_3' \end{pmatrix} \mapsto \dots$$

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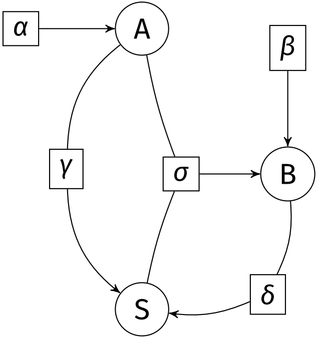

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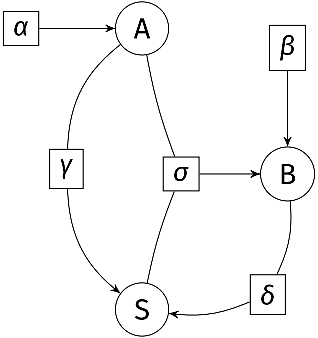

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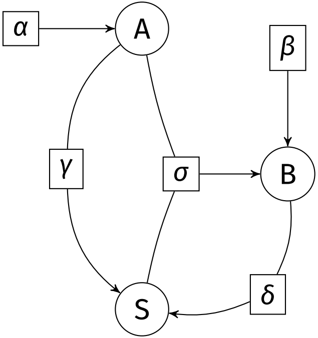

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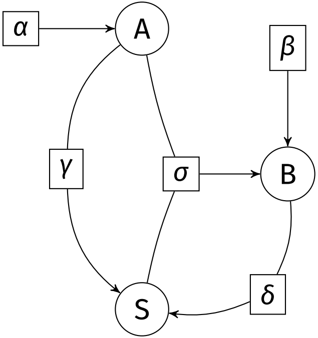

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$$\Big((G,\ \mathcal{CFG}^{\emptyset}),\ (\mathbb{K}, \mathbb{0}, \oplus, \Omega),\ \mathrm{wt}\Big) \rightsquigarrow \Big((G,\ \mathcal{CFG}^{\emptyset}),\ (\mathbb{R}_0^1, 0, \max, \Omega_{\mathrm{mul}}),\ \mathrm{wt}\Big)$$

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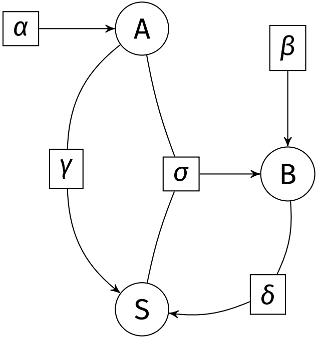

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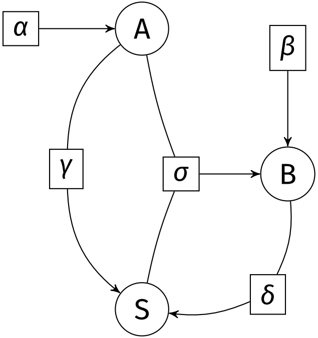

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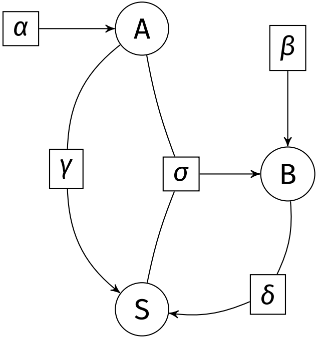

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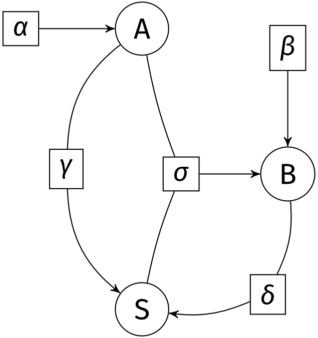

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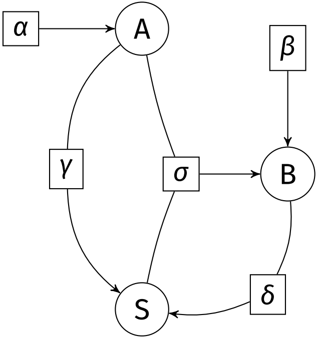

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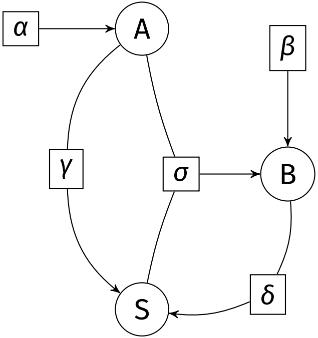

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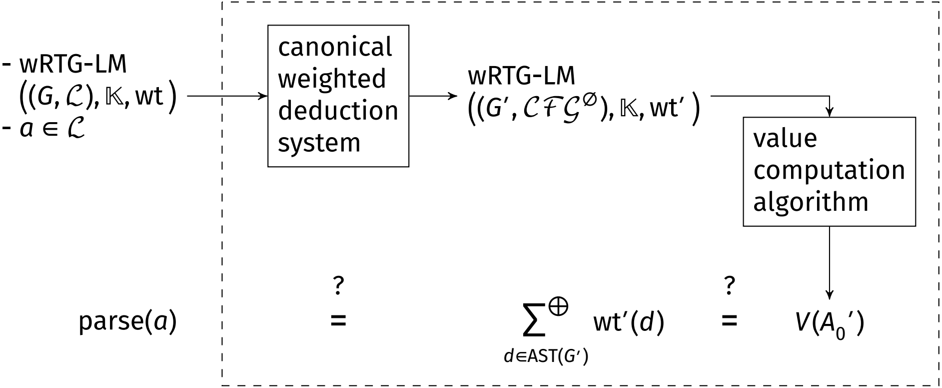
Termination and correctness



- wRTG-LM $((G,\mathcal{L}),\mathbb{K},\mathrm{wt})$
- $a \in \mathcal{L}$

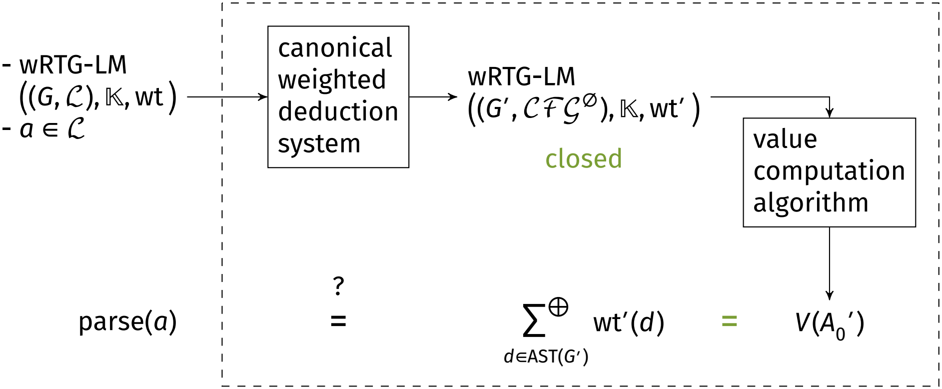
canonical weighted deduction system

wRTG-LM $((G',\mathcal{CFG}^{\emptyset}),\mathbb{K},\mathrm{wt}')$

value computation algorithm

$$\mathrm{parse}(a) \overset{?}{=} \sum^{\oplus}_{d\in\mathrm{AST}(G')} \mathrm{wt}'(d) \overset{?}{=} V({A_0}')$$

Termination and correctness



- wRTG-LM $((G,\mathcal{L}),\mathbb{K},\mathrm{wt})$
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canonical weighted deduction system

wRTG-LM $((G', \mathcal{CFG}^{\emptyset}),\mathbb{K},\mathrm{wt}')$

closed

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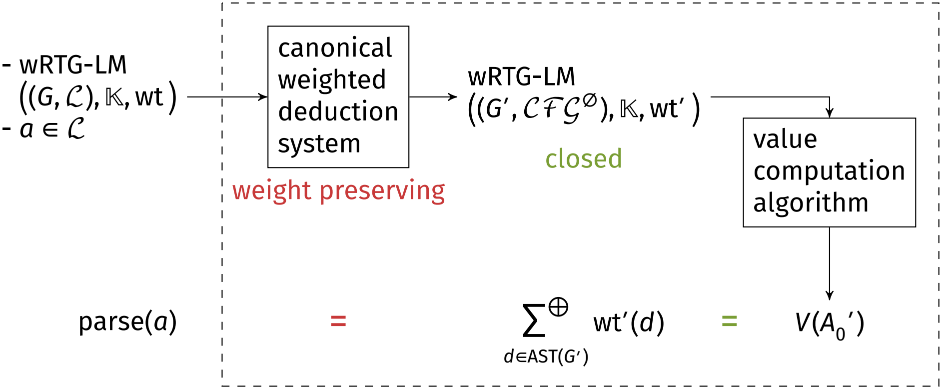

- wRTG-LM $((G, \mathcal{L}), \mathbb{K}, \mathrm{wt})$
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canonical weighted deduction system

weight preserving

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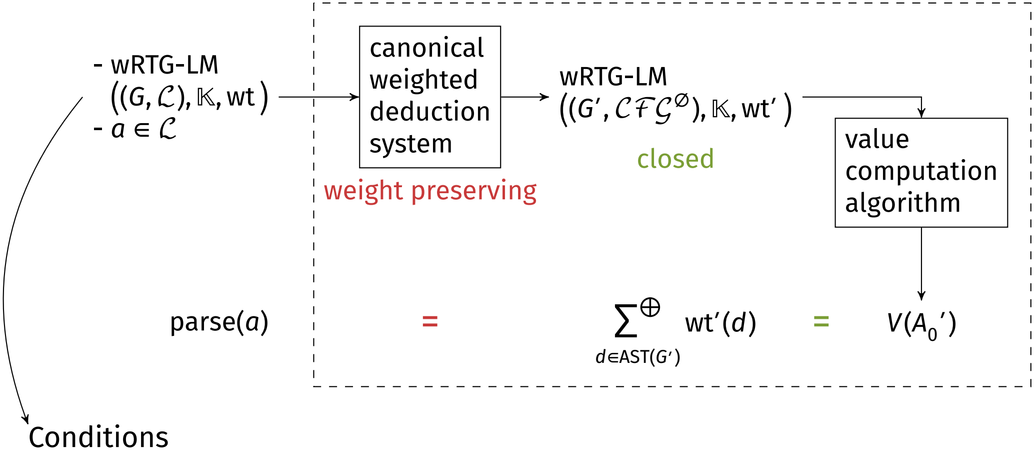

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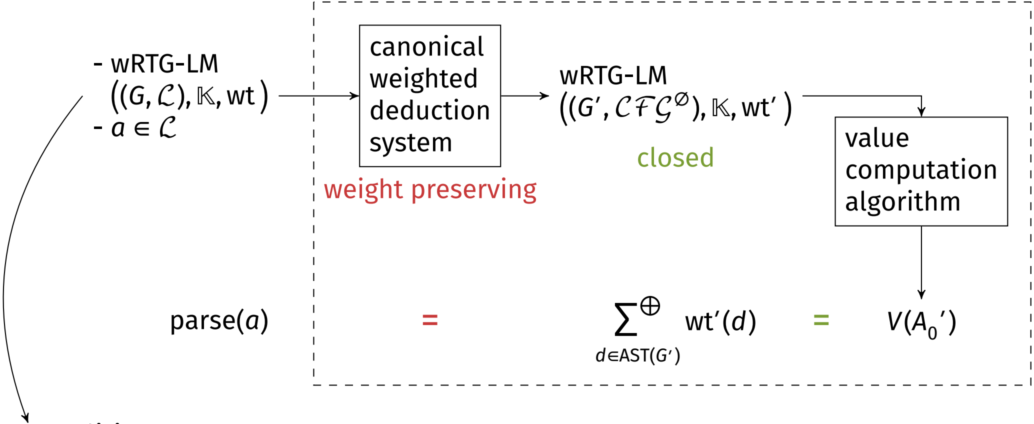
Conditions

Termination and correctness



- wRTG-LM $\big((G, \mathcal{C}), \mathbb{K}, \mathrm{wt}\big)$
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canonical weighted deduction system

weight preserving

wRTG-LM $\big((G', \mathcal{CFG}^{\emptyset}), \mathbb{K}, \mathrm{wt}'\big)$

closed

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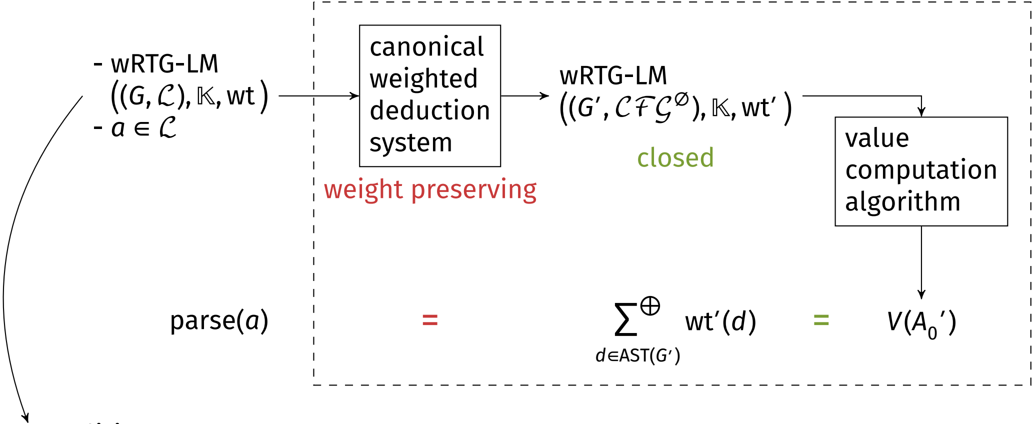
Conditions

- Sufficient: $\big((G, \mathcal{C}), \mathbb{K}, \mathrm{wt}\big)$ is closed or nonlooping

Termination and correctness



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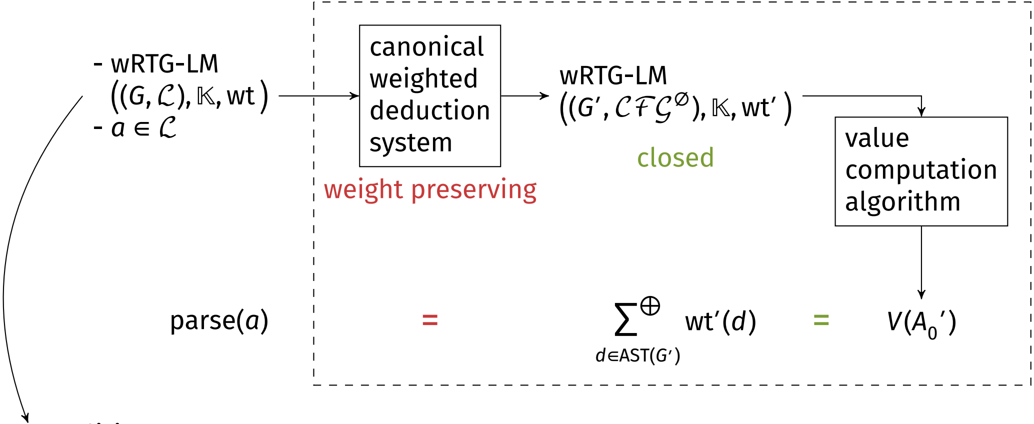
Termination and correctness



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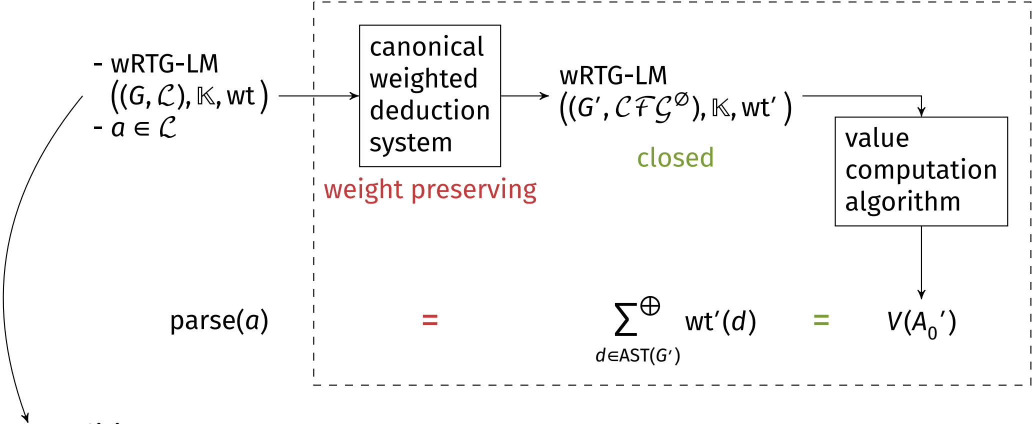

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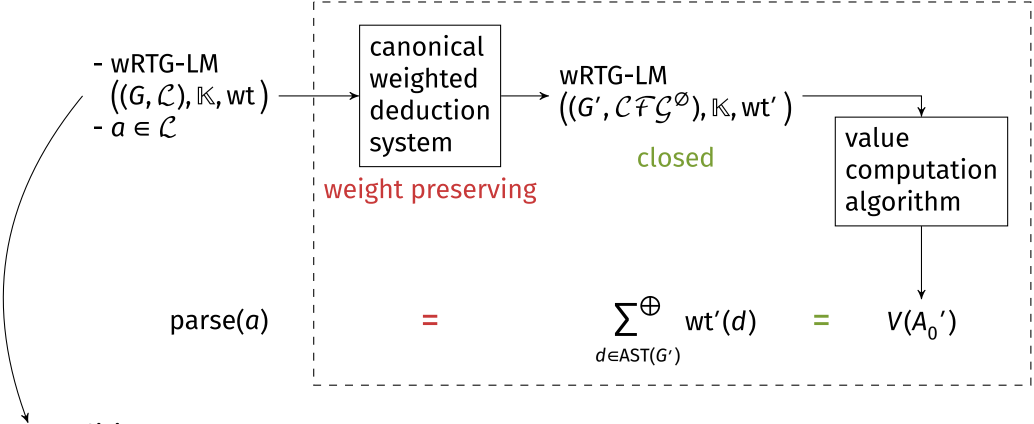
  e.g., CFG, LCFRS, TAG

Termination and correctness



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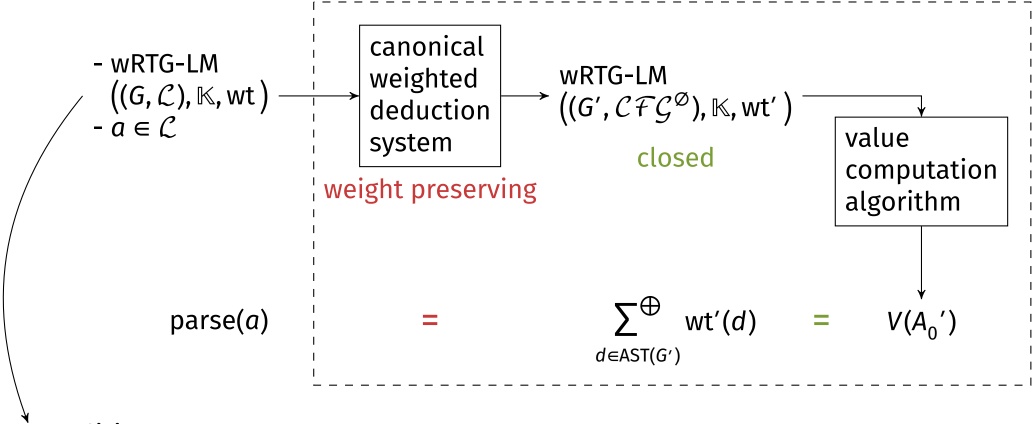
Goodman (1999)

Termination and correctness



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canonical weighted deduction system

weight preserving

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closed

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Goodman (1999)

Nederhof (2003)

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Canonical weighted deduction system

$$\text{wRTG-LM}\Big((G, \mathcal{L}), \mathbb{K}, \text{wt}\Big) \text{ and } a \in \mathcal{L} \rightsquigarrow \text{wRTG-LM}\Big((G', \mathcal{CFG}^{\varnothing}), \mathbb{K}, \text{wt}'\Big)$$

$$\frac{[A_1, \quad a_1] \dots [A_m, \quad a_m]}{[A, \quad a_0]} \quad \begin{cases} A \to \sigma(A_1, \dots, A_m) \text{ is a rule} \\ a_0, a_1, \dots, a_m \in \text{factors}(a) \\ a_0 = \sigma(a_1, \dots, a_m) \end{cases}$$

Canonical weighted deduction system

$$\text{wRTG-LM}\,\big((G,\mathcal{L}),\mathbb{K},\text{wt}\big) \text{ and } a\in\mathcal{L} \rightsquigarrow \text{wRTG-LM}\,\big((G',\mathcal{CFG}^{\emptyset}),\mathbb{K},\text{wt}'\big)$$

$$\frac{[A_1, \sigma_1, a_1] \ \dots\ [A_m, \sigma_m, a_m]}{[A, \sigma, a_0]} \quad \begin{cases} A \to \sigma(A_1,\dots,A_m) \text{ is a rule} \\ a_0, a_1, \dots, a_m \in \text{factors}(a) \\ a_0 = \sigma(a_1,\dots,a_m) \\ A_1 \to \sigma_1(\dots), \dots, A_m \to \sigma_m(\dots) \text{ are rules} \end{cases}$$

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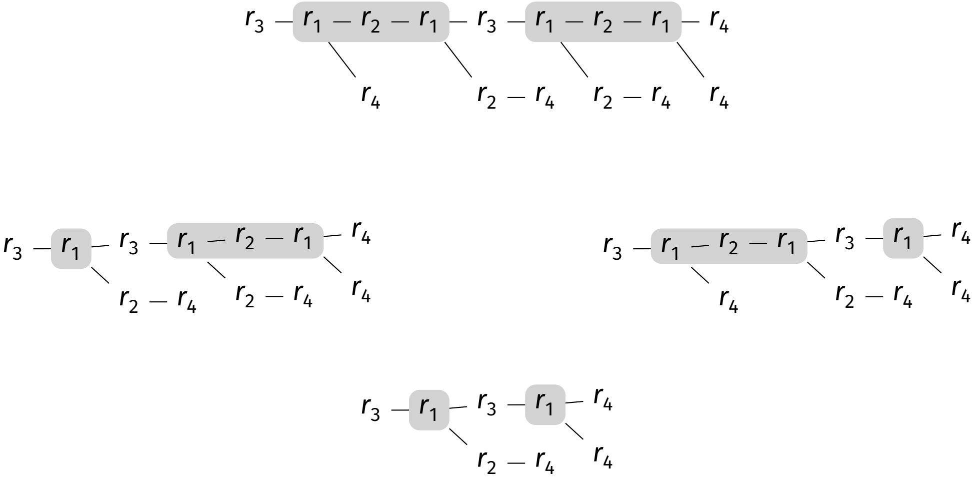
$$\frac{[A_0,\sigma,a]}{[A_0,a]} \quad \left\{A_0\to\sigma(\dots) \text{ is a rule}\right.$$

Weight preserving

1. Bijection $\psi\colon\text{AST}(G,a)\to\text{AST}(G')$
2. $\text{wt}(d)=\text{wt}'(\psi(d))$ for every $d\in\text{AST}(G,a)$

Closed wRTG-LMs

cutout(d, ρ)



Closed wRTG-LMs

Definition

Let $c \in \mathbb{N}$. A wRTG-LM $\mathcal{G} = \big((G, \mathcal{L}), \mathbb{K}, \mathrm{wt}\big)$ is *c-closed* if $\mathbb{K}$ is distributive and d-complete, and for each $d \in \mathrm{T}_R$ and cyclic string $\rho \in R^*$ the following holds: if there is a (c, ρ)-cyclic path in d, then

$$\mathrm{wt}(d)_{\mathbb{K}} \oplus \bigoplus_{d \in \mathrm{cutout}(d,\rho)} \mathrm{wt}(d)_{\mathbb{K}} = \bigoplus_{d \in \mathrm{cutout}(d,\rho)} \mathrm{wt}(d)_{\mathbb{K}} .$$

$\mathrm{AST}(G)^{(c)}$: each cycle at most c times

closed, distributive, d-complete

Theorem

For every $c \in \mathbb{N}$ and c-closed wRTG-LM $\big((G, \mathcal{L}), \mathbb{K}, \mathrm{wt}\big)$ the following holds:

$$\sum^{\oplus}_{d \in \mathrm{AST}(G')} \mathrm{wt}(d)_{\mathbb{K}} = \bigoplus_{d \in \mathrm{AST}(G)^{(c)}} \mathrm{wt}(d)_{\mathbb{K}} .$$