

# Concavity and Initialization for Unsupervised Dependency Parsing

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## Abstract

We investigate models for unsupervised learning with concave log-likelihood functions. We begin with the most well-known example, IBM Model 1 for word alignment (Brown et al., 1993) and analyze its properties, discussing why other models for unsupervised learning are so seldom concave. We then present concave models for dependency grammar induction and validate them experimentally. We find our concave models to be effective initializers for the dependency model of Klein and Manning (2004) and show that we can encode linguistic knowledge in them for improved performance.

## 1 Introduction

In NLP, unsupervised learning typically implies optimization of a “bumpy” objective function riddled with local maxima. However, one exception is IBM Model 1 (Brown et al., 1993) for word alignment, which is the only model commonly used for unsupervised learning in NLP that has a concave log-likelihood function.<sup>1</sup> For other models, such as those used in unsupervised part-of-speech tagging and grammar induction, and indeed for more sophisticated word alignment models, the log-likelihood function maximized by EM is non-concave. As a result, researchers are obligated to consider initialization in addition to model design (Klein and Manning, 2004; Goldberg et al., 2008).

For example, consider the dependency grammar induction results shown in Table 1 when training the

<sup>1</sup>It is not *strictly* concave (Toutanova and Galley, 2011).

widely used dependency model with valence (DMV; Klein and Manning, 2004). Using uniform distributions for initialization (UNIF) results in an accuracy of 17.6% on the test set, well below the baseline of attaching each word to its right neighbor (ATTACHRIGHT, 31.7%). Furthermore, when using a set of 50 random initializers (RAND), the standard deviation of the accuracy is an alarming 8.3%.

In light of this sensitivity to initialization, it is compelling to consider unsupervised models with concave log-likelihood functions, which may provide stable, data-supported initializers for more complex models. In this paper, we explore the issues involved with such an expedition and elucidate the limitations of such models for unsupervised NLP. We then present simple concave models for dependency grammar induction that are easy to implement and offer efficient optimization. We also show how linguistic knowledge can be encoded without sacrificing concavity. Using our models to initialize the DMV, we find that they lead to an improvement in average accuracy across 18 languages.

## 2 IBM Model 1 and Concavity

IBM Model 1 is a conditional model of a target-language sentence  $e$  of length  $m$  and an alignment  $a$  given a source-language sentence  $f$  of length  $l$ . The generation of  $m$  is assumed to occur with some (inconsequential) uniform probability  $\epsilon$ . The alignment vector  $a$ , a hidden variable, has an entry for each element of  $e$  that contains the index in  $f$  of the aligned word. These entries are used to define which translation parameters  $t(e_j | f_{a_j})$  are active. Model 1 assumes that the probability of the  $i$ th ele-

ment in  $\mathbf{a}$ , denoted  $a(i | j, l, m)$ , is simply a uniform distribution over all  $l$  source words plus the null word. These assumptions result in the following log-likelihood for a sentence pair  $\langle \mathbf{f}, \mathbf{e} \rangle$  under Model 1 (marginalizing  $\mathbf{a}$ ):

$$\log p(\mathbf{e} | \mathbf{f}) = \log \frac{\epsilon}{(l+1)^m} + \sum_{j=1}^m \log \sum_{i=0}^l t(e_j | f_i) \quad (1)$$

The only parameters to be learned in the model are  $\mathbf{t} = \{t(e | f)\}_{e,f}$ . Since a parameter is concave in itself, the sum of concave functions is concave, and the log of a concave function is concave, Eq. 1 is concave in  $\mathbf{t}$  (Brown et al., 1993).

IBM Model 2 involves a slight change to Model 1 in which the probability of a word link depends on the word positions. However, this change renders it no longer concave. Consider the log-likelihood function for Model 2:

$$\log \epsilon + \sum_{j=1}^m \log \sum_{i=0}^l t(e_j | f_i) \cdot a(i | j, l, m) \quad (2)$$

Eq. 2 is not concave in the parameters  $t(e_j | f_i)$  and  $a(i | j, l, m)$  because a product is neither convex nor concave in its vector of operands. This can be shown by computing the Hessian matrix of  $f(x, y) = xy$  and showing that it is indefinite.

In general, concavity is lost when the log-likelihood function contains a product of model parameters enclosed within a  $\log \sum$ . If the sum is not present, the log can be used to separate the product of parameters, making the function concave. It can also be shown that a “featurized” version (Berg-Kirkpatrick et al., 2010) of Model 1 is not concave. More generally, any non-concave function enclosed within  $\log \sum$  will cause the log-likelihood function to be non-concave, though there are few other non-concave functions with a probabilistic semantics than those just discussed.

### 3 Concave, Unsupervised Models

Nearly every other model used for unsupervised learning in NLP has a non-concave log-likelihood function. We now proceed to describe the conditions necessary to develop concave models for two tasks.

#### 3.1 Part-of-Speech Tagging

Consider a standard first-order hidden Markov model for POS tagging. Letting  $\mathbf{y}$  denote the tag

sequence for a sentence  $e$  with  $m$  tokens, the single-example log-likelihood is:

$$\log \sum_{\mathbf{y}} p(\text{stop} | y_m) \prod_{j=1}^m p(y_j | y_{j-1}) \cdot p(e_j | y_j) \quad (3)$$

where  $y_0$  is a designated “start” symbol. Unlike IBM Models 1 and 2, we cannot reverse the order of the summation and product here because the transition parameters  $p(y_j | y_{j-1})$  cause each tag decision to affect its neighbors. Therefore, Eq. 3 is non-concave due to the presence of a product within a  $\log \sum$ .

However, if the tag transition probabilities  $p(y_j | y_{j-1})$  are all constants and also do not depend on the previous tag  $y_{j-1}$ , then we can rewrite Eq. 3 as the following concave log-likelihood function (using  $C(y)$  to denote a constant function of tag  $y$ , e.g., a fixed tag prior distribution):

$$\log C(\text{stop}) + \log \prod_{j=1}^m \sum_{y_j} C(y_j) \cdot p(e_j | y_j)$$

Lacking any transition modeling power, this model appears weak for POS tagging. However, we note that we can add additional conditioning information to the  $p(e_j | y_j)$  distributions and retain concavity, such as nearby words and tag dictionary information. We speculate that such a model might learn useful patterns about local contexts and provide an initializer for unsupervised part-of-speech tagging.

#### 3.2 Dependency Grammar Induction

To develop dependency grammar induction models, we begin with a version of Model 1 in which a sentence  $e$  is generated from a copy of itself (denoted  $e'$ ):  $\log p(e | e')$

$$= \log \frac{\epsilon}{(m+1)^m} + \sum_{j=1}^m \log \sum_{i=0, i \neq j}^m c(e_j | e'_i) \quad (4)$$

If a word  $e_j$  is “aligned” to  $e'_i$ ,  $e_j$  is a root. This is a simple child-generation model with no tree constraint. In order to preserve concavity, we are forbidden from conditioning on other parent-child assignments or including any sort of larger constraints.

However, we can condition the child distributions on additional information about  $e'$  since it is fully observed. This conditioning information may include the direction of the edge, its distance, and any properties about the words in the sentence. We found that conditioning on direction improved performance: we rewrite the  $c$  distributions as  $c(e_j | e'_i, \text{sign}(j - i))$  and denote this model by Ccv1.

We note that we can also include constraints in the sum over possible parents and still preserve concavity. Naseem et al. (2010) found that adding parent-child constraints to a grammar induction system can improve performance dramatically. We employ one simple rule: roots are likely to be verbs.<sup>2</sup> We modify Ccv1 to restrict the summation over parents to exclude  $e'_0$  if the child word is not a verb.<sup>3</sup> We only employ this restriction during EM learning for sentences containing at least one verb. For sentences without verbs, we allow all words to be the root. We denote this model by Ccv2.

In related work, Brody (2010) also developed grammar induction models based on the IBM word alignment models. However, while our goal is to develop concave models, Brody employed Bayesian nonparametrics in his version of Model 1, which makes the model non-concave.

## 4 Experiments

We ran experiments to determine how well our concave grammar induction models Ccv1 and Ccv2 can perform on their own and when used as initializers for the DMV (Klein and Manning, 2004). The DMV is a generative model of POS tag sequences and projective dependency trees over them. It is the foundation of most state-of-the-art unsupervised grammar induction models (several of which are listed in Tab. 1). The model includes multinomial distributions for generating each POS tag given its parent and the direction of generation: where  $e_i$  is the parent POS tag and  $e_j$  the child tag, these distributions take the form  $c(e_j | e_i, \text{sign}(j - i))$ , analogous to the distributions used in our concave models. The DMV also has multinomial distributions for deciding whether to stop or continue generating children in each direction considering whether any children have already been generated in that direction.

The majority of researchers use the original initializer from Klein and Manning (2004), denoted here K&M. K&M is a deterministic harmonic initializer that sets parent-child token affinities inversely

<sup>2</sup>This is similar to the rule used by Mareček and Žabokrtský (2011) with empirical success.

<sup>3</sup>As verbs, we take all tags that map to V in the universal tag mappings from Petrov et al. (2012). Thus, to apply this constraint to a new language, one would have to produce a similar tag mapping or identify verb tags through manual inspection.

Model	Init.	Train $\leq 10$		Train $\leq 20$	
		Test		Test	
		$\leq 10$	$\leq \infty$	$\leq 10$	$\leq \infty$
ATTRIGHT	N/A	38.4	31.7	38.4	31.7
Ccv1	UNIF	31.4	25.6	31.0	23.7
Ccv2	UNIF	43.1	28.6	43.9	27.1
DMV	UNIF	21.3	17.6	21.3	16.4
	RAND*	41.0	31.8	-	-
	K&M	44.1	32.9	51.9	37.8
	Ccv1	45.3	30.9	53.9	36.7
	Ccv2	54.3	43.0	<b>64.3</b>	<b>53.1</b>
Shared LN	K&M	61.3	41.4		
L-EVG	RAND <sup>†</sup>	<b>68.8</b>	-		
Feature DMV	K&M	63.0	-		
LexTSG-DMV	K&M	67.7	<b>55.7</b>		
Posterior Reg.	K&M	64.3	53.3		
Punc/UTags	K&M'			-	59.1 <sup>‡</sup>

Table 1: English attachment accuracies on Section 23, for short sentences ( $\leq 10$  words) and all ( $\leq \infty$ ). We include selected results on this same test set: Shared LN = Cohen and Smith (2009), L-EVG = Headden III et al. (2009), Feature DMV = Berg-Kirkpatrick et al. (2010), LexTSG-DMV = Blunsom and Cohn (2010), Posterior Reg. = Gillenwater et al. (2010), Punc/UTags = Spitkovsky et al. (2011a). K&M' is from Spitkovsky et al. (2011b). \*Accuracies are averages over 50 random initializers;  $\sigma = 10.9$  for test sentences  $\leq 10$  and 8.3 for all. <sup>†</sup>Used many random initializers with unsupervised run selection. <sup>‡</sup>Used staged training with sentences  $\leq 45$  words.

proportional to their distances, then normalizes to obtain probability distributions. K&M is often described as corresponding to an initial E step for an unspecified model that favors short attachments.

**Procedure** We run EM for our concave models for 100 iterations. We evaluate the learned models directly as parsers on the test data and also use them to initialize the DMV. When using them directly as parsers, we use dynamic programming to ensure that a valid tree is recovered. When using the concave models as initializers for the DMV, we copy the  $c$  parameters over directly since they appear in both models. We do not have the stop/continue parameters in our concave models, so we simply initialize them uniformly for the DMV. We train each DMV for 200 iterations and use minimum Bayes risk decoding with the final model on the test data. We use several initializers for training the DMV, including the uniform initializer (UNIF), K&M, and our trained concave models Ccv1 and Ccv2.

Init.	eu	bg	ca	zh	cs	da	nl	en	de	el	hu
UNIF	24/21	32/26	27/29	44/40	32/30	24/19	21/21	21/18	31/24	37/32	23/18
K&M	<b>32/26</b>	<b>48/40</b>	24/25	38/33	31/29	<b>34/23</b>	39/33	44/33	47/ <b>37</b>	50/41	23/20
CcV1	22/21	34/27	<b>44/51</b>	<b>46/45</b>	33/31	19/14	24/24	45/31	46/31	<b>51/45</b>	32/28
CcV2	26/25	34/26	29/35	<b>46/44</b>	<b>50/40</b>	29/18	<b>50/43</b>	<b>54/43</b>	<b>49/33</b>	<b>50/45</b>	<b>60/46</b>
	it	ja	pt	sl	es	sv	tr	avg. accuracy	avg. log-likelihood		
UNIF	31/24	35/30	49/36	20/20	29/24	26/22	33/30	29.8 / 25.7	-15.05		
K&M	32/24	39/ <b>31</b>	44/28	<b>33/27</b>	19/11	<b>46/33</b>	<b>39/36</b>	36.7 / 29.4	-14.84		
CcV1	34/25	42/27	<b>50/38</b>	30/25	41/33	45/ <b>33</b>	37/29	37.5 / 30.9	-14.93		
CcV2	<b>55/48</b>	<b>49/31</b>	<b>50/38</b>	22/21	<b>57/50</b>	<b>46/32</b>	31/22	<b>43.7 / 35.5</b>	<b>-14.45</b>		

Table 2: Test set attachment accuracies for 18 languages; first number in each cell is accuracy for sentences  $\leq 10$  words and second is for all sentences. For training, sentences  $\leq 10$  words from each treebank were used. In order, languages are Basque, Bulgarian, Catalan, Chinese, Czech, Danish, Dutch, English, German, Greek, Hungarian, Italian, Japanese, Portuguese, Slovenian, Spanish, Swedish, and Turkish.

**Data** We use data prepared for the CoNLL 2006/07 shared tasks (Buchholz and Marsi, 2006; Nivre et al., 2007).<sup>4</sup> We follow standard practice in removing punctuation and using short sentences ( $\leq 10$  or  $\leq 20$  words) for training. For all experiments, we train on separate data from that used for testing and use gold POS tags for both training and testing. We report accuracy on (i) test set sentences  $\leq 10$  words and (ii) all sentences from the test set.

**Results** Results for English are shown in Tab. 1. We train on §2–21 and test on §23 in the Penn Treebank. The constraint on sentence roots helps a great deal, as CcV2 by itself is competitive with the DMV when testing on short sentences. The true benefit of the concave models, however, appears when using them as initializers. The DMV initialized with CcV2 achieves a substantial improvement over all others. When training on sentences of length  $\leq 20$  words (bold), the performance even rivals that of several more sophisticated models shown in the table, despite only using the DMV with a different initializer.

Tab. 2 shows results for 18 languages. On average, CcV2 performs best and CcV1 does at least as well as K&M. This shows that a simple, concave model can be as effective as a state-of-the-art hand-designed initializer (K&M), and that concave models can encode linguistic knowledge to further improve performance.

<sup>4</sup>In some cases, we did not use official CoNLL test sets but instead took the training data and reserved the first 80% of the sentences for training, the next 10% for development, and the final 10% as our test set; dataset details are omitted for space but are the same as those given by Cohen (2011).

Average log-likelihoods (micro-averaged across sentences) achieved by EM training are shown in the final column of Tab. 2. CcV2 leads to substantially higher likelihoods than the other initializers, suggesting that the verb-root constraint is helping EM to find better local optima.<sup>5</sup>

## 5 Discussion

Staged training has been shown to help unsupervised learning in the past, from early work in grammar induction (Lari and Young, 1990) and word alignment (Brown et al., 1993) to more recent work in dependency grammar induction (Spitkovsky et al., 2010). While we do not yet offer a generic procedure for extracting a concave approximation from any model for unsupervised learning, our results contribute evidence in favor of the general methodology of staged training in unsupervised learning, and provide a simple and powerful initialization method for dependency grammar induction.

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<sup>5</sup>However, while CcV1 leads to a higher average accuracy than K&M, the latter reaches slightly higher likelihood, suggesting that the success of the concave initializers is only partially due to reaching high training likelihood.

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