When Conditional Logic met Connexive Logic

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Abstract

Conditional logic and connexive logic are two theories whose goal is the correct formalization of the way conditionals (i.e. sentences of form *if A, C*) are used in natural language. However, both approaches are never combined in the literature. I present here the first formal system which allows modeling the ideas behind these two approaches in a unique framework. Furthermore, the resulting system allows explaining the different ways conditionals are negated in natural language.

Keywords: Conditional Logic, Connexive Logic, If, Negation

1 Introduction

It is generally agreed that the material conditional from classical logic does not correctly represent the conditional sentences from natural language. First, some critics argue that it validates too many schemas of reasoning that are intuitively incorrect. The first cases identified were called "paradoxes of material implication" and this list of defective inferences increased when classical logic was used to model natural language. In particular, counterexamples to the patterns of inference called *strengthening of the antecedent* "$A \to C \vDash (A \wedge B) \to C$", *contraposition* "$A \to C \vDash \neg C \to \neg A$" and *transitivity* "$A \to B, B \to C \vDash A \to C$" were found in English and the new systems that were devised to get rid of theses schemas were called *conditional logics* (Adams, 1965; Stalnaker, 1968; Lewis, 1973).

Second, other critics argue that some patterns of inference which are intuitively correct are not valid with the material conditional. New valid schemas should therefore be added. However, as classical logic is Post complete, it has no consistent proper extension. The new logics which are needed to solve this issue must therefore be non-classical. In particular, this argumentation is used by some logicians which consider that the notion of connection must be at the center of an analysis of conditionals and this trend is called *connexive logic*.

By taking into account these two criticisms, the conclusion is that the conditional theorems of classical logic constitute neither a superset nor a subset of the intuitively correct conditional inferences of natural language (Figure 1).

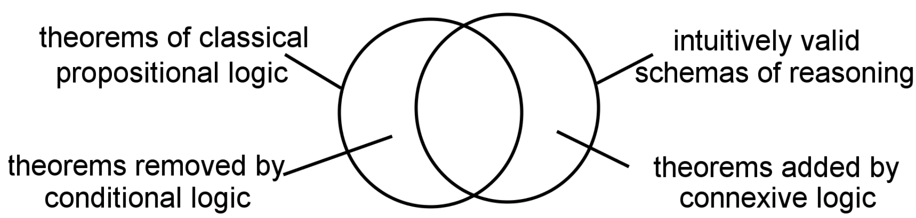

Figure 1: The partial overlapping of classical logic with intuitively correct reasoning schemas

This article is the first attempt to construct a logic which is both conditional and connexive, in order to model as closely as possible the use of conditional sentences in natural language. In section 2, a general solution validating the connexive schemas is presented. In section 3, this solution is applied to a particular conditional logic and it is formally shown that this new system validates the connexive principles. In section 4, the utility of this approach is illustrated through an explanation of the two main ways conditionals are negated in natural language. In section 5, a least drastic version of this approach is presented.

2 A Solution to Connexive Principles

In connexive logic, some schemas repelled from classical logic are deemed valid. The exact list of schemas can vary and I will here consider only the most usual. These six patterns are called AT and AT', AB and AB', and BO and BO', in reference respectively to Aristotle, Abelard and Boethius.

(AT) $\vDash \neg(\neg p \to p)$

(AT') $\vDash \neg(p \to \neg p)$

(AB) $\vDash \neg[(p \to q) \wedge (\neg p \to q)]$

(AB') $\vDash \neg[(p \to q) \wedge (p \to \neg q)]$

(BO) $p \to q \vDash \neg(p \to \neg q)$

(BO') $p \to \neg q \vDash \neg(p \to q)$

They can be illustrated with the following examples:

(AT) It is false that if it does not rain, then it rains.

(AT') It is false that if it rains, then it does not rain.

(AB) It is false that both if I trigger the alarm, then it rings and if I do not trigger the alarm, then it rings.

(AB') It is false that both if I trigger the alarm, then it rings and if I trigger the alarm, then it does not ring.

(BO) If I trigger the alarm, then it rings. Thus, it is false that if I trigger the alarm, then it does not ring.

(BO') If I trigger the alarm, then it does not ring. Thus, it is false that if I trigger the alarm, then it rings.

These six patterns of inference share a common feature: they use only two connectives, the negation and the conditional. This is why there exists three principal ways to solve this issue. Comparatively to classical logic, either the conditional (Rahman and Rückert, 2001), the negation (Priest, 1999), (Francez, 2016) or both (MacColl, 1908),(McCall, 1967; Routley, 1978; Wansing, 2005; Pizzi and Williamson, 2005) are modified.

To devise a solution, let us come back to the reasons why some logicians consider these schemas of reasoning as intuitively convincing. In a funny way, the AT sentences are directly supported not from Aristotle but from Abelard's following quotation: [1]

> No one doubts that [a statement entailing its negation] is improper and embarrassing (*inconveniens*) since the truth of one of two propositions which divide truth [i.e., contradictories] not only does not require the truth of the other but rather entirely expels and extinguishes it.

Conversely, the AB sentences seem a direct translation from the following principle expressed in Aristotle's *Prior Analytics* 57b3 (Smith, 1989):

[1] This translation is issued from Priest (1999) which also presents interesting historical observations.

> But it is impossible for the same thing to be of necessity both when a certain thing is and when that same thing is not (I mean, for example, for B to be large of necessity when A is white, and for B to be large of necessity when A is not white).

Finally, the BO principles are issued from Boethius' *De Syllogismo Hypothetico* where he defends that the negative of 'if A then B' is 'if A then not B' (Kneale and Kneale, 1962, p.191)

These three justifications turn around the reject of contradictions. In AT principles, a sentence cannot entails its negation. In AB principles, the same sentence cannot be the consequent of two contradictory sentences and the same antecedent cannot lead to two contradictory consequents. The justification for BO principles comes from the direct observation of the way conditionals are usually negated in natural language. Furthermore, if a consequent is obtained from an antecedent, its negation cannot be deduced, following the non-contradiction principle. Hence, the most direct way to interpret connexive principles through the oldest texts which justify their intuitive content is to say that their prime reason is the **rejection of contradictions**. Apart from the fact that it is the clear initial motivation for these three groups of principles, McCall (2012) notices also that it corresponds to the third variety of implication presented by Sextus Empiricus in *Outlines of Pyrrhonism*, in a passage which sums up the different positions hold during Greek Antiquity concerning the conditional:[2]

> And those who introduce connection or coherence say that a conditional holds whenever the denial of its consequent is incompatible with its antecedent.

Hence, the root principle behind the connexive conditional is the rejection of contradictions. How is it possible that such repelling which is relatively intuitive is not supported by most modern formal systems? The answer is quite simple. The advent of mathematization in modern logic gave place to sentences that are false in every interpretations. But in natural language, conditionals seldom if ever use contradictions as their components. Connexive principles, well adapted to represent conditional reasoning in our daily life, fell short as soon as contradictions are introduced by the mathematization. Notice also that because connexive principles systematically examine one sentence and its negation, they will also have difficulties to deal with tautologies which are negations of contradictions.

In support of this analysis, we can notice that tautologies and contradictions furnish counterexamples to connexive principles. Let us imagine a talk about the weather here and now. By allowing tautologies and contradictions, we could obtain the following sentences. The two first counterexamples assume the De Morgan's laws.

[2] This translation is from (Sanford, 2003).

(1) If it does not rain and it rains then it rains or it does not rain.

(Formal representation) $\neg p \wedge p \rightarrow p \vee \neg p$

(Counterexample to AT) $\neg(p \vee \neg p) \rightarrow p \vee \neg p$

(Counterexample to AT') $\neg p \wedge p \rightarrow \neg(p \wedge \neg p)$

(2) If it rains then it rains or it does not rain; and if it does not rain then it rains or it does not rain.

(Counterexample to AB) $(p \rightarrow (p \vee \neg p)) \wedge (\neg p \rightarrow (p \vee \neg p))$

(3) If it rains and it does not rain then it rains; and if it rains and it does not rain then it does not rain.

(Counterexample to AB') $((p \wedge \neg p) \rightarrow p) \wedge ((p \wedge \neg p) \rightarrow \neg p)$

(4) From "If it rains and it does not rain then it rains", we cannot conclude that it is false that "if it rains and it does not rain then it does not rain".

(Counterexample to BO) $(p \wedge \neg p) \rightarrow p \nvDash \neg((p \wedge \neg p) \rightarrow \neg p)$

(5) From "if it rains and it does not rain then it does not rain", we cannot conclude that it is false that "If it rains and it does not rain then it rains".

(Counterexample to BO') $(p \wedge \neg p) \rightarrow \neg p \nvDash \neg((p \wedge \neg p) \rightarrow p)$

One objection to this list of counterexamples could be that the ones which use contradictions are not convincing. Indeed, these sentences are not easy to understand in natural language because we have no firm intuitions about whether they can receive a truth-value and which one to attribute if they have any. On the contrary, I think that this difficulty is just another reason to forbid contradictions in the construction of conditionals, in order to judge whether connexive principles are respected in natural language. Usually, this forbidding is done at the pragmatic level. However, an investigation of the results obtained for a repelling at the semantic level is worth doing, in order to see whether a semantic validation of the principles would be possible.

The general strategy that I adopt in order to cope with the connexive schemas is therefore the following one. The semantics of the conditional will be adapted in order to forbid contradictions and tautologies in its construction. Antecedents and consequents cannot be anymore contradictory or tautological sentences. This choice is motivated by two main reasons. It falls within the spirit of the principles argued by Aristotle, Boethius and Abelard which repel contradictions and which are at the origin of connexive principles. Furthermore, the use of contradictions in natural language conditionals does not allow to obtain clear judgments about their meaning and consequences. It is therefore preferable to dismiss them and their negations (i.e. tautologies) as soon as possible, namely at the semantic level.

3 A Conditional Logic Adapted to Connexive Principles

The goal of this section is to test whether the repelling of contradictions and tautologies as constituents of hypothetical sentences in conditional logic allows validating the connexive principles.[3] Defenders of conditional logics argue that their theories offer a better representation of the way we use conditional sentences in natural language. We just saw that connexive principles are based on the rejection of contradictions in the constructions of such sentences also for natural language. Therefore, if these two hypotheses are correct, the combination of both approaches would lead to a validation of connexive schemas in conditional logic.

[3] The original exposition of this idea can be found in (Vidal, 2012).

Let us notice first that connexive principles are not valid in the most well-known conditional logics which are Stalnaker's **C2** system (Stalnaker, 1968) and Lewis's **VC** system (Lewis, 1973).[4] In these systems, a conditional is true if the 'closest' or 'most similar' possible worlds where the antecedent is true are also worlds where the consequent is true.[5] In particular, this set of the 'closest' or 'most similar' possible worlds where the antecedent holds can be the empty set, when such worlds cannot be found. In that case, the conditional is systematically true. As a consequence, a contradictory antecedent which is true nowhere will be systematically mapped to this empty set, and the resulting hypothetical sentence will automatically be true.

This aspect of their theories could be considered as a borderline case whose treatment could be modified. However, both Stalnaker and Lewis justify this choice. The first reason advanced in (Lewis, 1973, p.21) is that the 'might' counterfactual can be defined in terms of the 'would' counterfactual. In the same way that the existential quantifier can be defined through the universal quantifier in classical logic, with the consequence that the formula '$\exists x F x$' needs a non-empty domain to be true contrary to the formula '$\forall x F x$' which is true in empty domains, the 'might' counterfactual needs at least one world where the antecedent is true to be able to be true contrary to the 'would' counterfactual. (Stalnaker, 1984, p.120-121) reuses another argument already offered in (Lewis, 1973, section 1.6) to defend his choice. Any sentence is the semantic consequence of a contradiction. Therefore, the same relation must hold for the conditional connective. However, as noticed by Lewis, these "reasons are less than decisive." Finally, Unterhuber (2013) shows that the validity of (AB') is impossible in these two systems without leading to inconsistency.

We will therefore explore the consequences of the dismissal of contradictions and tautologies in hypothetical constructions for another conditional logic which is the one exposed in Vidal (2016) and Vidal (2017). We make this choice because this theory offers a basic semantics for the *if* construction which can be combined with the meaning of additional particles like *even*, *then* and *only*. This approach offers therefore a more diverse and fine-grained representation of conditionals in natural language than concurrent theories because the meaning of the forms "if A, C", "even if A, C", "if A, then C" and "only if A, C" are compositionally constructed and distinguished. Moreover and as I will explain soon, the repelling of contradictions and tautologies naturally extends the intuitive ideas behind this system. In this semantics, the evaluation of a conditional is processed along two phases. I will present here the version of the semantics for the *if then* conditional because connexive principles are generally considered to hold for this form. During the first phase of evaluation, both the antecedent and the consequent are inhibited. This means that they are no more believed true or false. This allows obtaining a neutral position concerning their truth-value. This is why this stage is called the *inhibition* or *neutralization* phase. During the second phase of the process, the antecedent is reconstructed. If in all these reconstructions, the consequent is obtained, the conditional is deemed true. This second phase is the *expansion* stage.

The first advantage of positioning such a process is that before the evaluation, three attitudes are possible concerning a sentence. It is believed either true or false or indeterminate. The inhibition allows removing all the circumstances too particular to be interesting for the evaluation and that were attached to the sentences under scrutiny. Hence, the resulting situations obtained after the reconstruction can slightly differ from the initial situation. If the antecedent was initially true, by passing through the phases of inhibition and reconstruction, we can now examine other ways it could have been true. In particular, if both the antecedent and the consequent were initially true, by inhibiting both of them and by constructing various new situations where the antecedent is true, the consequent is no more certain to be obtained if there was no connection between the two. Hence, *unconnected conditionals*, which are compounded from two independent sentences that are true, like "if Mickey has four fingers, Pacific is an ocean" are not declared true in this semantics, contrary to what is obtained in **C2**, **VC** and in most conditional logics.[6] If the antecedent was initially false, we can now consider different situations where

[4]The most well-known actual development of this trend represented by the work of Kratzer (2012) does not change this treatment.

[5]In this paper, we expose Chellas (1975)'s version of Stalnaker's semantics, where a set of possible worlds and not a single world is the result of the search of the closest worlds.

[6]Notice that several psychological experiments (Matalon, 1962; Skovgaard-Olsen et al., 2016; Vidal and Baratgin, 2017)

it would be true and avoid contradictions with the initial situation. Finally, if the antecedent was initially considered indeterminate, the neutralization phase does not change anything and we can safely consider situations where it is obtained.

This intuitive process of judgment can be turned into a formal semantics in terms of possible worlds. Starting from the initial world of evaluation which is bivalent, the antecedent and the consequent are first inhibited through what is called a *neutralization function*. Notice that the set of possible worlds resulting from this neutralization are all worlds where these two sentences are neither true nor false. They are indeterminate and the possible worlds used must therefore be trivalent (a sentence evaluated in one world receive one truth-value among three possibilities: true, false or indeterminate). To be deemed successful, this phase must conduct to a non-empty set of possible worlds. During the second phase, the antecedent is added again to these possible worlds, through what is called an *expansion function*. After that, we check whether the consequent is true in all these situations where the antecedent was reconstructed. If this is the case, the conditional is true. Different variations concerning the truth or falsity of the antecedent are considered during this second phase. However, not all possibilities are explored because some of them are too absurd or not sufficiently relevant for the case at hand. They are therefore limited to what is called a *universe of projection* which is the set of the envisaged alternatives. Notice that in the present semantics, we enforce that all the possible worlds in this universe of projection are bivalent concerning the antecedent and the consequent of the conditional judged.

This semantics is detailed in (Vidal, 2017, Appendix A) and completed in Vidal (2016) for the *if then* conditional. In Fig. 2, we depict the meaning of “if A, then C”, in which w stands for the starting world of evaluation and the square for the universe of projection.

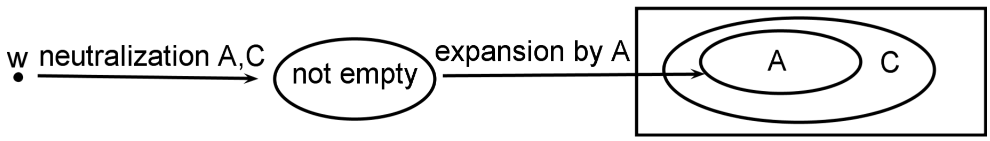

Figure 2: Semantics of the *if then* conditional

More formally, we obtain the following truth-conditions for the sentence *if A, then C*, with $[C]^U$ the set of possible worlds in the universe of projection U where C is true.

Definition 3.1 (Truth-Conditions for If A Then C)**.**
$\vDash_w A \rightarrow C$ iff in the associated universe of projection U, with n the neutralization function and e the expansion function

i) $n_w(A, C) \neq \varnothing$

ii) $e_{n_w(A,C)}(A) \subseteq [C]^U$

In this system, connexive schemas are not valid. In order to obtain this validity, let us see how to improve this semantics by removing contradictions and tautologies. We can first notice that such improvement is a natural extension of the intuitive ideas behind this approach. Indeed, during the first phase of judgment, the sentences are inhibited, which means that they are no more believed true or false. To obtain such an evaluation for contradictory and tautological sentences seems impossible and this is a good reason to repel them in the construction of conditionals. Furthermore, during the second phase, several alternatives are envisaged, some in which the antecedent is true and some in which it is false, in order to constitute the universe of projection. Again, contradictions and tautologies are not well suited for respecting the intuitions behind this constraint.[7] In order to extend this semantics, we will add now

confirmed that subjects do not validate the reasoning $A, C \vDash A \rightarrow C$.

[7]Based on different intuitions than the present proposal, classical variably strict account of conditional ((Stalnaker, 1968), (Lewis, 1973)) and strict accounts of conditionals ((Lewis, 1918), (Warmbrōd, 1981), (von Fintel, 2001)) have no real reasons to repel contradictions and tautologies from hypothetical constructions. They would have therefore more difficulties to justify such an extension of the system.

the following requirement. In the universe of projection, the set of possible worlds representing the antecedent and the consequent cannot be the empty set nor the totality of possible worlds. In that way, the two sentences whose relation is examined cannot respectively be true or false in all the alternatives envisaged. With this additional constraint, we not only repel the logical tautologies and contradictions but also the sentences that would be tautological or contradictory only relatively to the universe of projection considered. Formally, this extension of the semantics is expressed by the following truth-conditions where the items iii) and iv) are added comparatively to the previous definition:

Definition 3.2 (Connexive Truth-Conditions for If A Then C)**.**
$\vDash_w A \to C$ iff in the associated universe of projection U, with n the neutralization function and e the expansion function

i) $n_w(A, C) \neq \varnothing$

ii) $e_{n_w(A,C)}(A) \subseteq [C]^U$

iii) $[A]^U \neq \varnothing$ and $[A]^U \neq U$

iv) $[C]^U \neq \varnothing$ and $[C]^U \neq U$

Let us prove now that the connexive principles are valid with this new formal semantics for the conditional. Let us remark first that $e_{n_w(A,C)}(A) = [A]^U$, that is the set of possible worlds where the antecedent is rebuilt is the same as the set of the possible worlds where the antecedent is true in the universe of projection.

Proof of AT.

For any world w, $\neg(\neg p \to p)$ is true in w iff $\neg p \to p$ is false in w.

We prove by contradiction that $\neg p \to p$ cannot be true in w. Indeed, to be true, we would need:

i) $n_w(p, p) \neq \varnothing$ and

ii) $[\neg p]^U \subseteq [p]^U$ and $[\neg p]^U \neq \varnothing$ and $[\neg p]^U \neq U$ and $[p]^U \neq \varnothing$ and $[p]^U \neq U$

But this last relation is a set-theoretic contradiction equivalent to:

iii) $(U \setminus P) \subseteq P$ and $(U \setminus P) \neq \varnothing$ and $(U \setminus P) \neq U$ and $P \neq \varnothing$ and $P \neq U$ (contradiction)

Hence, by bivalence, $\neg p \to p$ is false in any world w. □

The validity of the other schemas is demonstrated in the same way. The negation of the semantic consequence leads to a set-theoretic contradiction. We show below these contradictions for AB and BO, the proofs for AT', AB' and BO' being totally equivalent.

Proof of AB.

iii) $P \subseteq Q$ and $(U \setminus P) \subseteq Q$ and $P \neq \varnothing$ and $P \neq U$ and $(U \setminus P) \neq \varnothing$ and $(U \setminus P) \neq U$ and $Q \neq \varnothing$ and $Q \neq U$ (set-theoretic contradiction) □

Proof of BO.

iii) $P \subseteq Q$ and $P \subseteq (U \setminus Q)$ and $P \neq \varnothing$ and $P \neq U$ and $Q \neq \varnothing$ and $Q \neq U$ and $(U \setminus Q) \neq \varnothing$ and $(U \setminus Q) \neq U$ (set-theoretic contradiction) □

Hence, by enforcing the repelling of contradictions and tautologies in our conditional logic, we managed to validate the connexive principles.

The following objection could be addressed to the present proposal. The identity principle (ID) would be lost: $\nvDash p \to p$. Indeed, we do not accept anymore tautologies and contradictions in the construction of conditional sentences. This objection can be answered in the following way. First, an antecedent of a conditional is hypothetical. By saying "if A", we consider that A could have been true. But this truth is not necessary and so A could also have been false. Obviously, this cannot be the case for tautologies and contradictions. So, their repelling in antecedent is natural and the consequence is that (ID) cannot be universally valid but only applies to contingent propositions. Second, the loss of this principle does not cause damage. Indeed, this conditional does not carry any interesting information. By learning that p holds, from $p \rightarrow p$, we can only deduce that p. Hence, we learn nothing from this conditional. This means that the identity principle has no informative utility and can be safely removed from our logical system. Finally, notice that the rejection of the identity principle is a respectable position which is as old as the philosophical discussions on conditionals. Indeed, according to Sextus Empiricus and following Sanford (2003)'s translation, "those who judge by 'suggestion' declare that a conditional is true if its consequent is in effect included in its antecedent. According to these, 'If it is day, then it is day,' and every repeated conditional will probably be false, for it is impossible for a thing itself to be included in itself."

4 How to negate a conditional

If a person says the sentence "If A, then C" in which I do not believe, I will often express my disagreement with the locution "If A, then not C." Hence, this is not the whole conditional which is negated but its sole consequent. The following sentences illustrates this point:

(6) If it's sunny, then Mary will go to the beach.

(7) It is false that if it's sunny, then Mary will go to the beach.

(8) If it's sunny, then Mary won't go to the beach.

To negate sentence (6), sentence (7) is rarely used because it is too pedantic. Sentence (8) will be preferred because it is shorter. The connexive principles and in particular the schema (BO') allows explaining why the negation of the consequent is a way to negate the whole conditional.

(BO') $p \rightarrow \neg q \vDash \neg(p \rightarrow q)$

With the schema (BO'), we can directly deduce sentence (7) from sentence (8). Hence, this principle allows explaining why negating the consequent of a conditional is a way to negate the whole conditional.

However, as noticed by Dummett (1996) and Woods (1997), there exists another way to negate a conditional, which is more nuanced. For instance, we could negate sentence (6) in the following way:

(9) If it's sunny, then it is possible that Mary won't go to the beach.

In sentence (9), the speaker lets a possibility for Mary to go or to not go to the beach, while in sentence (8), it is certain that she will not go there. Let us call these two ways to negate a conditional respectively the *weak negation* and the *strong negation*. The *strong negation* is expressed by the (BO') principle. The *weak negation* means that there exists another way to negate a conditional. As a consequence, the converse of (BO') cannot be valid in a system because it would make the *weak negation* impossible:

(ConvBO') $\neg(p \rightarrow q) \nvDash p \rightarrow \neg q$

The logical system that we described in this paper validates (BO') but invalidates (ConvBO'). It gives therefore place for both the *strong* and the *weak* negations. This can be illustrated by two pictures. In Figure 3, we see that as soon as all possible worlds where the antecedent is true are also worlds where the negation of the consequent is true, none of them can be worlds where the consequent is true, simply by bivalence.

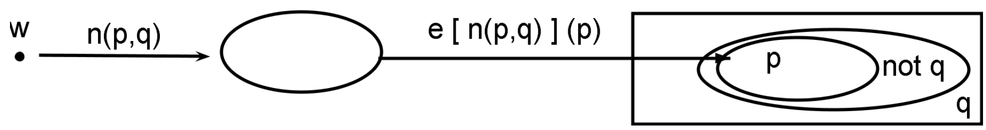

Figure 3: Strong negation of a conditional

To illustrate the *weak* negation, we just need to have a part of the possible worlds where the antecedent is true being also worlds where the consequent is true. By bivalence, the other part will be worlds where the negation of the consequent is true. This way to negate a conditional is illustrated in Figure 4.

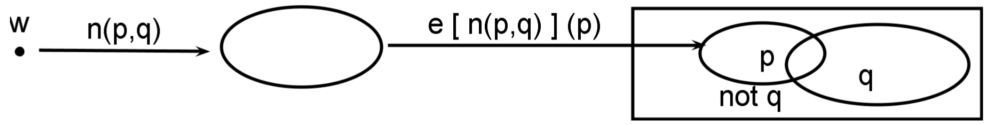

Figure 4: Weak negation of a conditional

The validation of the schema (BO') and the invalidation of its converse are therefore crucial features for a logic aiming to model the way we negate conditionals in natural language. It is therefore important for a conditional logic to be extendable both technically and intuitively in order to incorporate these two principles. We already saw that conditional logics do not validate in general connexive principles and in particular the schema (BO'). This is the case in particular for the systems of Lewis (1973) and Adams (1975). Stalnaker (1968)'s system **C2** could be seen as a notable exception because it does not validate (BO') but it contains the axiom (a4) which is very close: $\Diamond A \supset [(A \rightarrow C) \supset \neg(A \rightarrow \neg C)]$. This axiom stipulates that if the antecedent is possible, the truth of a conditional implies the negation of the same conditional with a negated consequent. Stalnaker notices rightfully that to negate the consequent is a usual way to negate the conditional. Because this feature is represented by his axiom (a4), it is an advantage of his approach. However, the default of Stalnaker's system is that it validates the schema (ConvBO') and as we saw, this schema forbids the *weak negation*. Hence, Stalnaker's solution is incomplete concerning the problem of negating conditionals in natural language.

5 A least drastic solution

The solution proposed so far repels contradictions and tautologies from conditional constructions in order to validate connexive principles, because we determined that they were only applicable to contingent propositions. But there exists another option that we will now examine. Its basic idea is that the initial truth-conditions for conditionals should be kept (Definition 3.1) and that we should check whether the connexive principles simply hold when its constituents are contingent.

The first step to devise this solution is to define what is a contingent proposition for a conditional construction. In the present system, two properties are required. First, the constituent of the conditional can be inhibited, in conjunction with the other constituents. Second, in the universe of projection obtained, the truth-set of the contingent proposition is neither the empty set nor the totality of possible worlds.

More formally, we obtain the following definition for the notion of contingency that we note $\blacklozenge$.

Definition 5.1 (Contingent proposition)**.**
$\blacklozenge A$ iff for every conditional containing A as a component, w being the starting world of evaluation of this conditional and U its associated universe of projection:

i) $n_w(A, ...) \neq \varnothing$

ii) $[A]^U \neq \varnothing$ and $[A]^U \neq U$

With this definition, tautologies and contradictions cannot be contingent. Furthermore, we obtain the following valid schemas of reasoning which are the connexive principles limited to contingent propositions.

(AT⧫) $\blacklozenge p \vDash \neg(\neg p \rightarrow p)$

(AT'⧫) $\blacklozenge p \vDash \neg(p \rightarrow \neg p)$

(AB⧫) $\blacklozenge p, \blacklozenge q \vDash \neg[(p \rightarrow q) \wedge (\neg p \rightarrow q)]$

(AB'⧫) $\blacklozenge p, \blacklozenge q \vDash \neg[(p \rightarrow q) \wedge (p \rightarrow \neg q)]$

(BO⧫) $\blacklozenge p, \blacklozenge q, p \rightarrow q \vDash \neg(p \rightarrow \neg q)$

(BO'⧫) $\blacklozenge p, \blacklozenge q, p \rightarrow \neg q \vDash \neg(p \rightarrow q)$

The additional advantage of such a definition of contingency is that we can express that the identity schema is valid for contingent propositions in conditionals, whether we adopt the first version of its semantics (Definition 3.1) or its strengthened form (Definition 3.2).

(ID⧫) $\blacklozenge p \vDash p \rightarrow p$

With this second version of our solution, we have a theory that makes sensible predictions for both contingent and non-contingent clauses. Linguistically, this seems to be the better option. But strictly speaking, we have no more a connexive logic.

6 Conclusion

Let us sum up the results obtained. The first version of the solution presented in this paper manages to validate connexive principles in the frame of a conditional logic. An important advantage of such an extension is the capacity to explain the way conditionals are negated in natural language. The price to pay is the repelling of contradictions and tautologies in conditional constructions, in line with the initial motivations given by the Greek philosophers arguing for connexive principles. If this price is too high to pay, our second version of the solution where the notion of contingency is defined must be preferred. With this new notion, it is possible to express that connexive principles are only valid for contingent propositions, which seems to be linguistically more satisfying.

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