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Axiomatic Characterization of Synonymy and Antonymy

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1. Introduction

1.1. Background

This work is a continuation of research reported in the paper Mathematical Models of Synonymy, which was presented at the 1965 International Conference on Computational Linguistics. That paper presented a historical summary of the concepts of synonymy and antonymy. It was noted that since the first book on English synonyms, which appeared in the second half of the 18th century, dictionaries of synonyms and antonyms have varied according to the particular explicit definitions of "synonym" and "antonym" that were used. The roles of part-of-speech, context of a word, and substitutability in the same context were discussed.

Traditionally, synonymy has been regarded as a binary relation between two words. Graphs of these binary relations were drawn for several sets of words based on Webster's Dictionary of Synonyms and matrices for these graphs were exhibited as an equivalent representation. These empirical results showed that the concepts of synonymy and antonymy required the use of ternary relations between two words in a specified sense rather than simply a binary relation between two words. The synonymy relation was then defined implicitly, rather than explicitly, by three axioms stating the properties of being reflexive, symmetric, and transitive. The antonymy relation was also defined by three axioms stating the properties of being irreflexive, symmetric, and antitransitive (the last term was coined for that study). It was noted that these six axioms could be expressed in the calculus of relations and that this relation algebra could be used to produce shorter proofs of theorems. However, no proofs were given. In addition, several geometrical and topological models of synonymy and antonymy were posed and examined.

It was noted that certain of these models were of more theoretical than practical interest. Each model was seen to be simple in that it could be expressed from mathematically elementary concepts, and each stressed certain aspects of the linguistic object being modeled at the expense of others. However, there seemed to be little theoretical preference among them. Their adequacy as models could be measured by their generality and predictive power. In terms of these criteria the algebraic model, whether expressed in terms of relations, graphs, or matrices, seemed to have the most usefulness. In part, this was due to the fact that one geometrical model, although highly suggestive, did not include a precise specification of the origin, axes, or coordinates for words in an n-dimensional space. Similarly, one topological model required a closure operation for each of the intensions or senses and had no linguistically interesting interpretation.

1.2 Summary

The present paper investigates more thoroughly the characterizations of synonymy and antonymy initiated in Edmundson (1965). In section 2, synonymy and antonymy are defined jointly and implicitly by a set of axioms rather than separately as before. First, it is noted that the original six axioms are insufficient to permit the proofs of certain theorems whose truth is strongly suggested by intuitive notions about synonyms and antonyms. In addition, it is discovered that certain fundamental assumptions about synonymy and antonymy must be made explicit as axioms. Some of these have to do with specifying the domain and range of the synonymy and antonymy relations. This is related to questions about whether function words, which linguistically belong to closed classes, should have synonyms and antonyms and whether content words, which linguistically belong to open classes, must have synonyms and antonyms. Several fundamental theorems of this axiom system are stated and proved. The informal interpretation of many of these theorems are intuitively satisfying. For example, it is proved that any even power of the antonymy relation is the synonymy relation, while any odd power is the antonymy relation.

In section 3, topological characterizations are posed and examined. A neighborhood topology is introduced by defining the neighborhood of a word. It is proved that this definition satisfies four neighborhood axioms. Also, a closure topology is introduced by defining the closure of a set of words. It is proved that this definition satisfies the four closure axioms.

2. Algebraic Characterization

2.1. Introduction - Relations

Before investigating antonymy and synonymy, we will establish some notions and notations for the calculus of binary relations.

Consider a set V of arbitrary elements, which will be called the universal set. A binary relation on V is defined as a set R of ordered pairs $\langle x,y \rangle$, where $x,y \in V$. The proposition that x stands in relation R to y will be denoted by xRy. The domain $\mathcal{D}(R)$, range $\mathcal{R}(R)$, and field $\mathcal{F}(F)$ of relation R are, respectively, defined by the sets

$$\{x:(\exists y)(xRy)\} \quad ; \quad \{y:(\exists x)(xRy)\} \quad ; \quad \{x:(\exists y)(xRy)\} \cup \{y:(\exists x)(xRy)\}$$

The complement, union, intersection, and converse relations are defined by

$$x\bar{R}y \equiv \sim xRy \quad ; \quad x(R \cup S)y \equiv xRy \vee xSy \quad ; \quad x(R \cap S)y \equiv xRy \wedge xSy \quad ;$$
$$xR^{-1}y \equiv yRx$$

The identity relation I and null relation $\emptyset$ are defined by

$$xIy \equiv x = y \quad ; \quad x\emptyset y \equiv (x \neq x) \vee (y \neq y)$$

The product and power relations are defined by

$$xR|Sy \equiv (\exists z)[xRz \wedge zSy] \quad ; \quad R^n \equiv R|R^{n-1} \quad n \geq 1$$

Inclusion and equality of relations are defined by

$$R \subseteq S \equiv xRy \Longrightarrow xSy \quad ; \quad R = S \equiv R \subseteq S \wedge S \subseteq R$$

Later we will use the following elementary theorems which are stated here without proof:

Theorem: $R \subseteq S \implies R^{-1} \subseteq S^{-1}$

Theorem: $R \subseteq S \implies \overline{S} \subseteq \overline{R}$

Theorem: $(R^{-1})^{-1} = R$

Theorem: $(R|S)|T = R|(S|T)$

Theorem: $(R|S)^{-1} = S^{-1}|R^{-1}$

Theorem: $I|R = R|I = R$

Theorem: $S \subseteq T \implies R|S \subseteq R|T \wedge S|R \subseteq T|R$

2.2 Axioms and Definitions

Under the assumption that synonymy and antonymy are ternary relations on the set C of all content words, the following definitions will be used:

$xS_iy \equiv$ word x is a synonym of word y with respect to the intension i (or word x is synonymous in sense i to word y)

$xA_iy \equiv$ word x is an antonym of word y with respect to the intension i (or word x is antonymous in sense i to word y)

We will assume that the synonymy and antonymy relations are defined jointly and implicitly by the following set of axioms rather than separately as in Edmundson (1965).

Axiom 1 (Reflexive): $(\forall x)[xS_ix]$

Axiom 2 (Symmetric): $(\forall x)(\forall y)[xS_iy \implies xS_i^{-1}y]$

Axiom 3 (Transitive): $(\forall x)(\forall y)(\forall z)[xS_iy \wedge yS_iz \implies xS_iz]$

Axiom 4 (Irreflexive): $(\forall x)[x\overline{A}_ix]$

Axiom 5 (Symmetric): $(\forall x)(\forall y)[xA_iy \implies xA_i^{-1}y]$

Axiom 6 (Antitransitive): $(\forall x)(\forall y)(\forall z)[xA_iy \wedge yA_iz \implies xS_iz]$

Axiom 7 (Right-identity): $(\forall x)(\forall y)(\forall z)[xA_iy \wedge yS_iz \implies xA_iz]$

Axiom 8 (Nonempty): $(\forall y)(\exists x)[xA_iy]$

The properties named in Axioms 6 and 7 were coined for this study.

The above eight axioms may be expressed in the calculus of relations as follows:

Axiom 1 (Reflexive): $I \subseteq S_i$

Axiom 2 (Symmetric): $S_i \subseteq S_i^{-1}$

Axiom 3 (Transitive): $S_i^2 \subseteq S_i$

Axiom 4 (Irreflexive): $I \subseteq \overline{A}_i$

Axiom 5 (Symmetric): $A_i \subseteq A_i^{-1}$

Axiom 6 (Antitransitive): $A_i^2 \subseteq S_i$

Axiom 7 (Right-identity): $A_i|S_i \subseteq A_i$

Axiom 8 (Nonempty): $(\forall y)[A(y) \neq \emptyset]$ where $A(y) \equiv \{\langle x,y\rangle : x \in \mathcal{D}(A)\}$

This relation algebra will be used to produce shorter proofs, although this is not necessary. The consistency of this set of axioms is shown by exhibiting a model for them; their independence will not be treated.

In addition to the synonymy and antonymy relations it will be useful to introduce the following classes that are the images of these relations. The <u>synonym class</u> of a word y is defined by

$$s_i(y) \quad \equiv \quad \{x : xS_iy\}$$

which may be extended to an arbitrary set E of words by

$$s_i(E) \quad \equiv \quad \{x : (\exists y)[y \in E \wedge xS_iy]\}$$

Similarly, the <u>antonym class</u> of a word y is defined by

$$a_i(y) \quad \equiv \quad \{x : xA_iy\}$$

which may be extended to a set E of words by

$$a_i(E) \quad \equiv \quad \{x : (\exists y)[y \in E \wedge xA_iy]\}$$

2.3 Theorems

For reasons of notational simplicity, the subscript denoting the intension i will be omitted in the sequel whenever possible. However, the theorems must be understood as if the subscript were present.

As with any symmetric relation, it is possible to get stronger results than Axiom 2 and Axiom 5.

Theorem: $S^{-1} = S$

Proof: $S \subseteq S^{-1}$ by Axiom 2. Hence $S^{-1} \subseteq (S^{-1})^{-1} = S$. Therefore $S^{-1} = S$ by definition of equality.

Theorem: $A^{-1} = A$

Proof: Same as above theorem using Axiom 5.

Also we get a stronger result than the transitivity property of Axiom 3:

Theorem: $S^2 = S$

Proof: $S^2 \subseteq S$ by Axiom 3. Hence $S = S|I \subseteq S|S = S^2$ by Axiom 1. Therefore $S^2 = S$ by definition of equality.

In fact, by induction we have the generalization:

Theorem: $S^n = S \quad n \neq 0$

Proof: $S^n = S|S^{n-1} = S|(S|S^{n-2}) = \cdots = S|(S|(S|\cdots|S)\cdots) = S$.

It can be shown that antonymy and synonymy are distinct: $A \neq S$. In fact we have the stronger result:

Theorem: $A \subseteq \overline{S}$

Proof: Assume $A \nsubseteq \overline{S}$. Hence $A \cap S \neq \emptyset$ or $(\exists x)(\exists y)[x(A \cap S)y]$. Then $xAy \wedge xSy$ implies $xAy \wedge ySx$ by Axiom 2. So xAx, which contradicts $x\overline{A}x$ by Axiom 4: $I \subseteq \overline{A}$. Therefore $A \subseteq \overline{S}$.

Only because of Axiom 8, can we get a stronger result than the anti-transitivity property of Axiom 6.

Theorem: $A^2 = S$

Proof: $A \supseteq A|S$ by Axiom 7. Hence $A^2 = A|A \supseteq A|(A|S) = A^{-1}|(A|S) = (A^{-1}|A)|S$ since $A^{-1} = A$. Now $(\forall y)(\exists x)[xAy]$ by Axiom 8. So $(\forall y)(\exists x)[yA^{-1}x \wedge xAy]$ by Axiom 5. Hence $(\forall y)[yIy \Longrightarrow yA^{-1}|Ay]$. Thus $I \subseteq A^{-1}|A$. So $A^2 \supseteq I|S = S$. Therefore $A^2 = S$ since $A^2 \subseteq S$ by Axiom 6 and $S \subseteq A^2$.

The right-identity property of Axiom 7 can be strengthened to:

Theorem: $A|S = A$

Proof: $A|S \subseteq A$ by Axiom 7. Now $A = A|I \subseteq A|S$ since $I \subseteq S$. Therefore $A|S = A$ by definition of equality.

As a corollary we get that S and A commute:

Corollary: $A|S = S|A$

Proof: $A|S = A = A^{-1} = (A|S)^{-1} = (A^{-1}|S^{-1})^{-1} = S|A$

From the above two theorems it follows that:

Theorem: $S|A = A$

Proof: $S|A = A|S = A$.

As a special case we get:

Theorem: $A^3 = A|A^2 = A|S = A$.

In fact, we have the generalization:

Theorem: $A^n = \begin{cases} S \text{ if } n \text{ even} \\ A \text{ if } n \text{ odd} \end{cases}$

Proof: For n even, $A^n = A^{2k} = (A^2)^k = S^k = S$. For n odd, $A^n = A^{2k+1} = A|(A^2)^k = A|S = A$.

Next, several theorems about synonym classes and antonym classes will be stated and proved. First, the synonym class of a word is not empty:

Theorem: $s(y) \neq \emptyset$

Proof: Now $I \subseteq S$ by Axiom 1. So $(\forall y)[ySy]$. Hence $(\exists x)[xSy]$. Therefore, $s(y) \neq \emptyset$.

Because S is a symmetric relation, we have:

Theorem: $y \in s(x) \iff x \in s(y)$

Proof: $y \in s(x) \iff ySx \iff yS^{-1}x \iff xSy \iff x \in s(y)$.

Since S is reflexive, symmetric, and transitive, S is by definition an equivalence relation on the set C of all content words. Hence, we have the important result:

Theorem: $xSy \iff s(x) = s(y)$

Proof: ($\Longrightarrow$) Assume xSy. First let $u \in s(x)$. Then $uSx \wedge xSy \Longrightarrow uS^2y \Longrightarrow uSy \Longrightarrow u \in s(y)$. Hence $s(x) \subseteq s(y)$. Also $s(y) \subseteq s(x)$ by a similar argument. Therefore $s(x) = s(y)$. ($\Longleftarrow$) Assume $s(x) = s(y)$. Then $u \in s(x) \Longrightarrow u \in s(y)$. So $uSx \Longrightarrow uSy$. Hence $xSu \wedge uSy \Longrightarrow xS^2y \Longrightarrow xSy$. Therefore xSy.

In fact, we have the stronger result:

Theorem: $s(x) \cap s(y) = \begin{cases} s(x) & \text{if } xSy \\ \emptyset & \text{if } x\bar{S}y \end{cases}$

Hence for a given intension i the equivalence relation S_i partitions the set C of all content words into subsets that are disjoint (i.e., the subsets have no word in common) and exhaustive (i.e., every word is in some subset):

Theorem: $C = \bigcup_{x \in C} s_i(x)$

Second, the antonym class of a word is not empty:

Theorem: $a(y) \neq \emptyset$

Proof: Axiom 8: $(\forall y)(\exists x)[xAy]$ implies $a(y) \neq \emptyset$.

Note that a word does not belong to its antonym class:

Theorem: $y \notin a(y)$.

Proof: Assume $y \in a(y)$ so that yAy. But this contradicts Axiom 4: $yIy \implies y\bar{A}y$. Therefore $y \notin a(y)$.

Next we will establish some relations between synonym classes and antonym classes.

Theorem: $xAy \iff a(x) = s(y)$

Proof: $(\implies)$ Assume $x \in a(y)$. First let $u \in a(x)$. Now $u \in a(x) \wedge xAy \implies uAx \wedge xAy \implies uA^2y \implies uSy \implies u \in s(y)$. Hence $a(x) \subseteq s(y)$. Also $s(y) \subseteq a(x)$ by a similar argument. Therefore $a(x) = s(y)$. $(\impliedby)$ Assume $a(x) = s(y)$. But $y \in s(y) = a(x)$. Hence yAx. Therefore xAy by Axiom 5.

In fact, we get the following necessary and sufficient condition for equality:

Theorem: $a(x) = a(y) \iff s(x) = s(y)$

Proof: $(\implies)$ Assume $a(x) = a(y)$. Now $a(x) \cap a(y) \neq \emptyset \implies (\exists z)[z \in a(x) \wedge a(y)] \implies (\exists z)[z \in a(x) \wedge z \in a(y)] \implies (\exists z)[zAx \wedge zAy] \implies (\exists z)[xAz \wedge zAy] \implies xA^2y \implies xSy$. Therefore $s(x) = s(y)$ by a previous theorem. $(\impliedby)$ Assume $s(x) = s(y)$. Then xSy. First, let $u \in a(x)$. Then uAx. Hence $uAx \wedge xSy \implies uA|Sy \implies uAy \implies u \in a(y)$. Therefore $a(x) \subseteq a(y)$. Also $a(y) \subseteq a(x)$ by an identical argument. Therefore $a(x) = a(y)$.

2.4 Comments on the Algebraic Characterization

Even though $s(y) \neq \emptyset$ since ySy by Axiom 1, it may be necessary to add the following axiom:

$$\text{Axiom 9:} \quad (\forall y)(\exists x)[x \neq y \wedge xSy]$$

to guarantee that the domain of the relation S is not trivial, i.e.,

$$s(y) - \{y\} \neq \emptyset$$

Axiom 9 is not necessary if $s(y)$ is permitted to be a unit set for certain words. Thus, we might define $s(y) = \{y\}$ for any function word y, e.g., s(and) = {and}. But this will not work for antonymy since $a(y)$ might be considered empty for certain words such as function words, e.g., a(and) = $\emptyset$. The alternative of defining $a(y) = \overline{\{y\}}$ is not reasonable since it produces more problems than it solves. Axiom 8: $(\forall y)(\exists x)[xAy]$ is reasonable if the contraries of words (e.g., nonuse, impossible, etc.) are permitted, i.e., $\bar{y} \in a(y)$.

The theorems

$$S^2 = S \quad , \quad A^2 = S \quad , \quad A|S = A \quad , \quad S|A = A$$

can be summarized in the following multiplication table for products of the relations S and A

	S	A
S	S	A
A	A	S

which is isomorphic to the table for addition modulo 2

	0	1
0	0	1
1	1	0

Note, even without Axioms 1-8, for

$$(1)\ A^2 = S \quad , \quad (2)\ A^3 = A \quad , \quad (3)\ A|S = A$$

that (1) and (2) imply (3), (1) and (3) imply (2), but (2) and (3) do not imply (1).

Suppose that for every pair $\langle x,y \rangle$ of words in the vocabulary V of a language exactly one of the following ternary relations holds:

(1) x and y are synonymous, xSy
(2) x and y are antonymous, xAy
(3) neither (1) nor (2), xMy

This can be expressed by

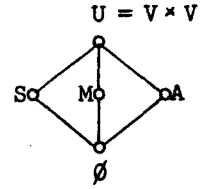
$$(\forall x)(\forall y)[x,y \in V \Longrightarrow xSy \vee xAy \vee xMy]$$

which is an exclusive disjunction. Thus the vocabulary V is partitioned as follows:

$$V = s(y) \cup a(y) \cup m(y)$$

for every word y. This also can be pictured in the lattice of relations

U = V × V



It can be shown that the multiplication table for products of the relations S,A, and M is

	S	A	M
S	S	A	M
A	A	S	M
M	M	M	M^2

3. Topological Characterizations

3.1. Introduction

We will now examine two topological models of synonymy. Being topological, they concern "semantic spaces" of words without any notion of "semantic distance" between two words. Again, we will restrict our attention to content words. Topological models for the antonymy relation will not be considered.

3.2. Neighborhood Topology

The first model considers a neighborhood topology, i.e., a topology based on neighborhoods. A set is said to have a neighborhood topology if there exist elements x called points and sets N_x called neighborhoods of x which satisfy the following axioms:

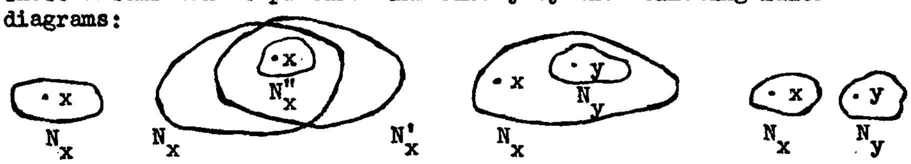
Axiom 1: $(\forall x)(\exists N_x)[x \in N_x]$

Axiom 2: $(\forall N_x)(\forall N_x')(\exists N_x'')[N_x'' \subseteq N_x \cap N_x']$

Axiom 3: $(\forall y)(\forall N_x)(\exists N_y)[y \in N_x \implies N_y \subseteq N_x]$

Axiom 4: $(\forall x)(\forall y)(\exists N_x)(\exists N_y)[x \neq y \implies N_x \cap N_y = \emptyset]$

These axioms can be pictured informally by the following Euler diagrams:



Define a neighborhood $n_i(x)$ of a word x as any subset of the synonym class $s_i(x)$ of x that contains x, i.e.,

$$x \in n_i(x) \subseteq s_i(x)$$

Again, for reasons of notational simplicity, the subscript denoting the intension i will be omitted whenever possible.

First, neighborhood Axiom 1 is satisfied.

Theorem: $(\forall x)(\exists n(x))[x \in n(x)]$

Proof: By definition s(x) is a neighborhood n(x) of x containing x.

Second, neighborhood Axiom 2 is satisfied.

Theorem: $(\forall n(x))(\forall n'(x))(\exists n''(x))[n''(x) \subseteq n(x) \cap n'(x)]$

Proof: For arbitary n(x) and n'(x), let $n''(x) = n(x) \cap n'(x)$. Then $n''(x) \subseteq s(x)$ since $n''(x) = n(x) \cap n'(x) \subseteq s(x) \cap s(x) = s(x)$. Also, $x \in n''(x)$ since $x \in n(x) \wedge x \in n'(x)$ imply $x \in n(x) \cap n'(x) = n''(x)$. Therefore, $(\forall n(x))(\forall n'(x))(\exists n''(x))[n''(x) \subseteq n(x) \cap n'(x)]$.

Third, neighborhood Axiom 3 is satisfied.

Theorem: $(\forall y)(\forall n(x))(\exists n(y))[y \in n(x) \implies n(y) \subseteq n(x)]$

Proof: For arbitrary $y \in n(x)$, let $n(y) = n(x)$. But $y \in n(x)$ implies $s(x) = s(y)$ since $y \in n(x) \subseteq s(x) = \{z : zSx\}$ implies ySx and ySx implies $s(y) = s(x)$. Then $n(y) \subseteq s(y)$ since $n(y) = n(x) \subseteq s(x) = s(y)$. Also $y \in n(y)$ since $y \in n(x) = n(y)$. Therefore, $(\forall y)(\forall n(x))(\exists n(y))[y \in n(x) \implies n(y) \subseteq n(x)]$.

In fact, the neighborhood topology satisfies Axiom 4, which is a separation axiom:

Theorem: $(\forall x)(\forall y)(\exists n(x))(\exists n(y))[x \neq y \implies n(x) \cap n(y) = \emptyset]$

Proof: Assume $x \neq y$. Let $n(x) = \{x\}$ and $n(y) = \{y\}$. Then $x \in n(x) \subseteq s(x)$ and $y \in n(y) \subseteq s(y)$. Thus $n(x) \cap n(y) = \{x\} \cap \{y\} = \emptyset$ since $x \neq y$.

Therefore, with respect to synonymy, words have a neighborhood topology since

(1) $(\forall x)(\exists n(x))[x \in n(x)]$
(2) $(\forall n(x))(\forall n'(x))(\exists n''(x))[n''(x) \subseteq n(x) \cap n'(x)]$
(3) $(\forall y)(\exists n(y))[y \in n(x) \implies n(y) \subseteq n(x)]$
(4) $(\forall x)(\forall y)(\exists n(x))(\exists n(y))[x \neq y \implies n(x) \cap n(y) = \emptyset]$

3.3. Closure Topology

The second model considers a closure topology, i.e., a topology based on a closure operation. A set is said to have a closure topology if there exists a unary operation on its subsets, denoted by ~ and called the closure, which satisfies the following axioms:

Axiom 1: $\widetilde{\emptyset} = \emptyset$

Axiom 2: $E \subseteq \widetilde{E}$

Axiom 3: $\widetilde{\widetilde{E}} \subseteq E$

Axiom 4: $\widetilde{E \cup F} = \widetilde{E} \cup \widetilde{F}$

Define the closure of a set E of words as the synonym class of E, i.e.,

$$\widetilde{E} \equiv s(E)$$

The closure axioms can be shown to be satisfied by using the original definition of synonym class

$$s(E) \equiv \{x : (\exists y)[y \in E \wedge xSy]\}$$

However, shorter proofs are possible by noting that the synonym class of a set E of words can be expressed as

$$s(E) = \bigcup_{y \in E} s(y) = \bigcup_{y \in E} \{x : xSy\}$$

First, closure Axiom 1 is satisfied:

Theorem: $s(\emptyset) = \emptyset$

Proof: $S(\emptyset) = \bigcup_{y \in \emptyset} s(y) = \emptyset$

Second, closure Axiom 2 is satisfied:

Theorem: $E \subseteq s(E)$

Proof: $s(E) = \bigcup_{y \in E} s(y) \supseteq \bigcup_{y \in E} \{y\} = E$ since $y \in s(y) \implies \{y\} \subseteq s(y)$.

Third, closure Axiom 3 is satisfied:

Theorem: $s[s(E)] \subseteq s(E)$

Proof: Now $s(s(y)) = s(\{u : uSy\}) = \{v : vS^2y\} \subseteq \{v : vSy\} = s(y)$ since $S^2 \subseteq S$. Thus $s[s(E)] = \bigcup_{x \in s(E)} s(x) = \bigcup_{x \in \bigcup_{y \in E} s(y)} s(x) =$

$$\bigcup_{y \in E} \bigcup_{x \in s(y)} s(x) = \bigcup_{y \in E} s(s(y)) \subseteq \bigcup_{y \in E} s(y) = s(E)$$

Fourth, closure Axiom 4 is satisfied:

Theorem: $s(E \cup F) = s(E) \cup s(F)$

Proof: $s(E \cup F) = \bigcup_{y \in E \cup F} s(y) = \bigcup_{y \in E} s(y) \cup \bigcup_{y \in F} s(y) =$

$s(E) \cup S(F)$.

Therefore, with respect to synonymy, words have a closure topology since

(1) $s(\emptyset) = \emptyset$
(2) $E \subseteq s(E)$
(3) $s[s(E)] \subseteq s(E)$
(4) $s(E \cup F) = s(E) \cup s(F)$

Note that from Axioms 2 and 3 we get

Theorem: $s[s(E)] = s(E)$

3.4. Comments on Topological Characterizations

Note that for the neighborhood topology a separation axiom has been added to the three axioms proposed in Edmundson (1965). Also, the neighborhood topology seems more intuitively satisfying than the closure topology. However, for the closure topology if we define the derived set of a set E of words as the set of all words that are synonymous to some word of E, but not identical to that word, i.e.,

$$E' \equiv \{x : (\exists y)[y \in E \wedge x \neq y \wedge xSy]\}$$

then we have the following result:

Theorem: $s(E) = E \cup E'$

which may be given a reasonable linguistic interpretation. An example is $\{y\}' = s(y) - \{y\}$ which was discussed in the section on algebraic characterization.

4. Conclusions

These results support the belief that the algebraic characterization is insightful and appropriate. For example, the assumption that synonymy is an equivalence relation also has been made, either directly or indirectly, by F. Kiefer and S. Ábrahám (1965), U. Weinreich (1966), and others. Since the axiom system defines the notions of synonymy and antonymy jointly and implicitly, it avoids certain difficulties that are encountered when attempts are made to define these notions separately and explicitly.

These topological characterizations provide a nonmetric representation of what has been called informally a "semantic space". Previous attempts to construct a semantic space that is metric (i.e., one for which a distance function is defined) have not met with much success. The consideration of general topological spaces avoids this difficulty.

References

R. Carnap, Introduction to Symbolic Logic and Its Applications, W. Meyer and J. Wilkinson (trs.), Dover, N. Y., 1958.

H. P. Edmundson, "Mathematical Models of Synonymy", International Conference on Computational Linguistics, 1965.

P. Kiefer and S. Ábrahám, "Some Problems of Formalization in Linguistics", Linguistics, v. 17, Oct. 1965, pp. 11-20.

V. V. Martynov, Pytannja prikladnoji lingvistyky; tezisy dopovidej mižvuzovs'koji naukovoji konferenciji, Sept. 22-28, 1960, Černivcy.

A. Naess, "Synonymity as Revealed by Intuition", Philosophical Review, v. 66, 1957, pp. 87-93.

U. Weinreich, "Explorations in Semantic Theory", in Current Trends in Linguistics, III, T. Sebeok (ed.), Mouton and Co., The Hague, 1966.

P. Ziff, Semantic Analysis, Cornell University Press, Ithica, N. Y., 1960.