DYNAMIC LOGIC WITH POSSIBLE WORLD

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Abstract

This paper introduces a semantic theory DLPW, Dynamic Logic with Possible World, which extends Groenendijk's DPL and Cresswell's Indices Semantics. The semantics can interpret the temporal and modal sense and anaphora.

Key words: Semantic Model, Dynamic Logic, Possible Worlds.

§1 Introduction

At present there are three main aspects in semantical field:

1. Transformation of sentences or discourses into formulas in high order logic.
2. Semantic interpretation of the logical formulas.
3. Semantic ambiguity.

This paper presents the semantics, dynamic logic with possible world, which combines DPL[3] with Indices semantics[1] and extends the theories, the theory can be used to interpret the temporal and modal sense and anaphoric connection. For the limitation of space we only give the definitions and examples concerned rather than present a formalization which should include the axioms and rules concerned like in DPL[3].

Following Montague semantics, the discussion on meaning of a sentence started with predicate formula, high order logical formual with lambda terms, which is translated from a sentence S by means of a set of rules and reduced to a first order predicate formula A finally.

Problem: given an expression A, meaning[A]=? Assume that a model M is an ordered pair ⟨D, F⟩ where D is a domain, a non-empty set and F an interpretation function assigning a semantic value to each non-logical constant of the language. A value assignment g is a function assigning a member of D to each variable of the language. W is a possible world. The intension of the expression A is $Int[A]=\|A\|^{M,g}$. The extension of the expression A is $Ext[A, W]=Int[A](W)=\|A\|^{M,W,g}$.

Some semantic evaluations are as follows:

1. Montague Semantics, given M, W, evaluates extension $\|A\|^{M,W,g}$.
2. Possible World Semantics (M. J. Cresswell), given M, W, finds the set of possible worlds which satisfy the extension of A. i.e., $\{W \mid \|A\|^{M,W,g}=True\}$.
3. Dynamic Predicate Logic (J. Groenendijk), given M, W, finds dynamic changes between the value assignments:

   $\|A\|^{M,W}=\{\langle g_{in}, g_{out}\rangle\}$.
4. Dynamic Logic with Possible World, given M, finds dynamic changes between the ordered pairs of value assignment and world: $\|A\|^{M}=\{\langle G_{in}, G_{out}\rangle \mid G_{in}=\langle g_{in}, W_{in}\rangle, G_{out}=\langle g_{out}, W_{out}\rangle\}$.

The evaluations 1, 2 above are in static sense and for use of interpretation of sentences without anaphora, while evaluations 3, 4 above in dynamic sense and for use of interpretation of anaphoric connection.

§ 2 DLPW, Anaphoric Connection

In this section we hope that DLPW can offer a successful application to anaphoric connection like the intepretation of the famous donkey sentence in Groenendijk's DPL[3]. The sentence

(1) There was a key, it is lost.

can be formalized as

(2) $P\exists_x key(x)$ & NOW lost(x) (in the hope that) having the same semantic interpretation as the formula

(3) $P\exists_x(key(x)$ & NOW lost(x)).

The idea will come true by means of dynamic logic DLPW. A state, denoted by π $(=\langle\sigma, g\rangle)$, may be a pair of a sequence of time $\sigma(\sigma=\{\sigma(0), \sigma(1)\})$ and a value assignment g.

There are revisions of semantic definitons of operators concerned.

(4) $\|NOW\,\alpha\|=\{\langle\pi_1, \pi_2\rangle|\pi_1=\langle\sigma_1, g_1\rangle, \pi_2=\langle\sigma_2, g_2\rangle, \langle\langle\sigma_1[1/0], g_1\rangle, \pi_2\rangle\in\|\alpha\|\}$.

(5) $\|P\alpha\|=\{\langle\pi_1, \pi_2\rangle|\pi_1=\langle\sigma_1, g_1\rangle, \pi_2=\langle\sigma_2, g_2\rangle, \exists_t(t<\sigma_1(0), \langle\langle\sigma_1[term\ t/0], g_1\rangle, \pi_2\rangle\in\|\alpha\|)\}$.

(6) $\|\varphi\&\psi\|=\{\langle\pi_1, \pi_2\rangle|\exists_{\pi,\pi'}(\pi=\langle\sigma, g\rangle, \pi'=\langle\sigma', g\rangle, \langle\pi_1, \pi\rangle\in\|\varphi\|, \langle\pi', \pi_2\rangle\in\|\psi\|)\}$.

(7) $\|\exists_x\varphi\|=\{\langle\pi_1, \pi_2\rangle|\exists_\pi, (\pi=\langle\sigma, g\rangle, g=g_1[x], \langle\pi, \pi_2\rangle\in\|\varphi\|)\}$.

(8) $\|\varphi(x)\|=\{\langle\pi_1, \pi_2\rangle|\pi_1=\pi_2, g_1=g_2, g_1(x)\in F(\varphi)\}$.

Where $F(\varphi)$ is a set of individuals, F is the function in a model M. φ is an atomic formula. Here obviously, conjunction is treated like composition (i.e., compound statement $\{S_\varphi; S_\psi\}$). Intuitively, the conjunction is treated in the sequential sense. The meanings of the formulas $\sigma_1[1/0]$, $\sigma_1[term\ t/0]$, $g_1[x]$ follow the statements in the preceding section.

Assume that initially for all i, j, $\sigma_i(j)=t_0$ and σ_i, g_i in π_i denoted components concerned. Hence

(9) $\langle\pi_1, \pi_2\rangle\in\|(2)\|$ iff, by (6), for some $\pi_3, \pi_3', g_3(=g_3')$,

(10) $\langle\pi_1, \pi_3\rangle\in\|P\exists_x key(x)\|$ and

(11) $\langle\pi_3', \pi_2\rangle\in\|NOW\ lost(x)\|$.

(10) holds iff, by (5), for some $t(<\sigma_1(0))$,

(12) $\langle\langle\sigma_1[term\ t/0], g_1\rangle, \pi_3\rangle\in \|\exists_x key(x)\|$ iff, by (7), for some $h(=g_1[x])$,

(13) $\langle\langle\sigma_1[term\ t/0], g_1[x]\rangle, \pi_3\rangle\in \|key(x)\|$ iff, by (8),

(14) $\pi_3=\langle\sigma_1[term\ t/0], g_1[x]\rangle$ (i.e. $\sigma_3=\sigma_1[term\ t/0]$, $g_3=g_1[x]$), and $g_1[x](x)\in F(key)$, where $g_1[x](x)=h(x)$.

(11) holds, iff, by (4),

(15) $\langle\langle\sigma_3'[1/0], g_3'\rangle, \pi_2\rangle\in\|lost(x)\|$, iff by (8), $\pi_2=\sigma_3'[1/0], g_3'\rangle$ (i.e., $\sigma_2=\sigma_3'[1/0]$ and $g_2=g_3'=g_3$, (by (6))$=g_1[x]$ (by (14)), and $g_3(x)\in F(lost)$.

It means that $g_3(x)(=g_1[x](x)=$individual k_0, say) is a key at $t(<t_0)$ and $g_3(x)$ $(=k_0)$ is lost at t_0.

On the other hand, for the formula (3),

(16) $\langle\pi_1, \pi_3\rangle\in\|(3)\|$ iff, by (5), for some $t(<\sigma_1(0))$,

(17) $\langle\langle\sigma_1[term\ t/0], g_1\rangle, \pi_2\rangle \in \|\exists_x(key(x)$ & NOW lost(x))$\|$, iff, by (6) for some $h(=g_1[x])$,

(18) $\langle\langle\sigma_1[term\ t/0], g_1[x]\rangle, \pi_2\rangle\in \|key(x)$ & NOW lost(x)$\|$, iff, by (6) for some $\pi_3, \pi_3', g_3(=g_3')$,

(19) $\langle\langle\sigma_1[term\ t/0], g_1[x]\rangle, \pi_3\rangle\in \|key(x)\|$ and

(20) $\langle\pi_3', \pi_2\rangle\in\|NOW\ lost(x)\|$,

(19) holds iff $\pi_3=\langle\sigma_1[term\ t/0], g_1[x]\rangle$, and $g_1[x](x)\in F(key)$, i.e., $\sigma_3=\sigma_1[term\ t/0]$ and $g_3=g_1[x]$,

(20) holds iff, by (4),

(21) $\langle\langle\sigma_3'[1/0], g_3'\rangle, \pi_2\rangle\in\|lost(x)\|$ iff,

by (8), $\pi_2 = \langle \sigma_3'[1/0], g_3' \rangle$, and $g_3'(x) \in F(lost)$, i.e., $\sigma_2 = \sigma_3'[1/0]$ and $g_2 = g_3' = g_3 = g_1[x]$.

It shows formulas (2) and (3) have identical meaning. This is just what we require.

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