The Impact of Depth on Compositional Generalization in Transformer Language Models

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Abstract

To process novel sentences, language models (LMs) must generalize compositionally—combine familiar elements in new ways. What aspects of a model's structure promote compositional generalization? Focusing on transformers, we test the hypothesis, motivated by theoretical and empirical work, that deeper transformers generalize more compositionally. Simply adding layers increases the total number of parameters; to address this confound between depth and size, we construct three classes of models which trade off depth for width such that the total number of parameters is kept constant (41M, 134M and 374M parameters). We pretrain all models as LMs and fine-tune them on tasks that test for compositional generalization. We report three main conclusions: (1) after fine-tuning, deeper models generalize more compositionally than shallower models do, but the benefit of additional layers diminishes rapidly; (2) within each family, deeper models show better language modeling performance, but returns are similarly diminishing; (3) the benefits of depth for compositional generalization cannot be attributed solely to better performance on language modeling. Because model latency is approximately linear in the number of layers, these results lead us to the recommendation that, with a given total parameter budget, transformers can be made shallower than is typical without sacrificing performance.

1 Introduction

The number of possible sentences in natural language is enormous; regardless of the size of its training set, a language model (LM) will regularly encounter sentences it has never seen before. The ability to interpret such sentences relies on compositional generalization: the capacity to combine familiar words and syntactic structures in new ways (Montague, 1970; Fodor and Pylyshyn, 1988). Transformer LMs (Vaswani et al., 2017), while highly successful in many settings, often struggle when tested on benchmarks that require compositional generalization (Kim and Linzen, 2020). What architectural factors affect a transformer's ability to generalize compositionally?

In this paper, we test the hypothesis that increasing a transformer's depth—the number of layers it has—improves its performance on tasks that require compositional generalization. This hypothesis is motivated both by theoretical work, which has shown that adding layers increases the expressive capacity of neural networks in general (Raghu et al., 2017) and transformers in particular (Merrill et al., 2021), and by experimental work suggesting that deeper models generalize more compositionally than shallower ones (Mueller et al., 2022; Murty et al., 2023).

While existing empirical work lends some credibility to this hypothesis, to directly confirm it we must address the confound between depth and size (number of parameters). As each additional layer introduces a new set of parameters, deeper models are also larger, all else being equal, and LMs' performance on a wide variety of tasks is known to be correlated with their size (Kaplan et al., 2020; Hoffmann et al., 2022; Muennighoff et al., 2023). To address this confound, we construct classes of models with an equal total number of parameters but differing depths; we do so by reducing the model's feed-forward dimension to compensate for added depth. We pretrain all models as language models and fine-tune them on four compositional generalization tasks: the semantic parsing tasks COGS (Kim and Linzen, 2020), COGS-vf (Qiu et al., 2022a) and GeoQuery (Zelle and Mooney, 1996), and the English passivization portion of Multilingual Transformations (Mueller et al., 2022). In all of these tasks, the model is trained on a particular data distribution and is expected to generalize to

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another distribution by composing together familiar elements in novel ways.

In addition to any possible direct effect on compositional generalization, depth may also be correlated with other factors which may themselves predict compositional generalization, such as language modeling loss during pretraining or in-distribution performance on the fine-tuning task. This complicates the interpretation of any relationship we might find between depth and generalization performance. To address this concern, we also investigate and correct for the effect of depth on language modeling performance and in-distribution loss.

We report the following findings, across three model size classes (41M, 134M, and 374M parameters):

1. In general, deeper models have lower perplexity (Section 3.1). The marginal increase in performance gained from additional layers diminishes rapidly as models get deeper, and performance begins to degrade when the feed-forward dimension approaches the dimensionality of the model's contextualized embeddings.

2. In general, deeper models display better compositional generalization (Section 3.2). Again, most of the benefit of depth accrues from the first few layers; for several of the compositional generalization benchmarks we use, performance saturates very quickly as models get deeper.

3. Deeper models generalize more compositionally even after correcting for the fact that their language modeling perplexity is lower and their in-distribution performance on the fine-tuning task is higher (Section 3.3).

4. Since transformers' latency is approximately linear in their depth, it is in many case more efficient to make a model wider rather than deeper, given a fixed parameter budget (Section 4).

2 Methodology

2.1 Constructing Families of Models with Equal Numbers of Parameters

To make a transformer deeper without increasing the total number of parameters, we need to also make it narrower. There are several ways to do so: we can reduce the size of the feed-forward dimension d_{ff}, reduce the size of the contextual embeddings d_{model}, or reduce the size of the attention outputs d_{attn} (see Appendix B for a diagram of a transformer layer annotated with dimensionality labels). Vaswani et al. (2017) coupled these variables at $d_{\text{model}} = d_{\text{attn}} = d_{\text{ff}}/4$. Most transformer LMs have adopted this ratio (Devlin et al., 2019; Kaplan et al., 2020; Hoffmann et al., 2022, *inter alia*), though Raffel et al. (2019) increased the size of d_{ff} relative to d_{model} and d_{attn} for their two largest models. By contrast, we vary d_{ff} with depth (while holding $d_{\text{model}} = d_{\text{attn}}$ constant). By keeping the attention mechanism identical across models of varying depths, we rule out the possibility that depth will be confounded with the capacity of the self-attention mechanism. We refer to $d_{\text{model}}/d_{\text{ff}}$, conventionally set to $1/4$, as the *feed-forward ratio*.

Deriving hyperparameter relations. As a starting point for our size classes of models, we use hyperparameters taken from T5-base and T5-large (Raffel et al., 2019) as well as a smaller model from Kim and Linzen (2020) which has identical layer-internal hyperparameters to T5-small but fewer layers.[1] We implement models using `t5x` (Roberts et al., 2022). We then calculate how much the feed-forward dimension must change to accommodate adding or removing layers. Starting from the formula in Kaplan et al. (2020), the number of parameters M in a layer is

$$M(d_{\text{ff}}) = 2d_{\text{model}}d_{\text{ff}} + 4d_{\text{model}}d_{\text{attn}} = \beta \cdot d_{\text{ff}} + A,$$

where the constant β represents the contribution of the parameters of the feed-forward block which projects vectors from $\mathbb{R}^{d_{\text{model}}}$ into $\mathbb{R}^{d_{\text{ff}}}$ and back into $\mathbb{R}^{d_{\text{model}}}$; and the constant A represents the parameters of everything aside from the feed-forward block, including the attention mechanism. The total parameter count of a full model N in terms of d_{ff} and n_{layers} is then

$$N(n_{\text{layers}}, d_{\text{ff}}) = n_{\text{layers}} \cdot M(d_{\text{ff}}) + 2d_{\text{model}}n_{\text{vocab}}.$$

Given initial values $(n^0_{\text{layers}}, d^0_{\text{ff}})$ which characterize the baseline model in each size class (e.g., T5-large), our goal is to find pairs $k, w(k)$ such that

$$N(n^0_{\text{layers}} + k, d^0_{\text{ff}} - w(k)) = N(n^0_{\text{layers}}, d^0_{\text{ff}}).$$

Solving for w as a function of k tells us how much to increase (or decrease) d^0_{ff} if we remove (or add)

[1] Unlike T5 and the original transformer, we implement GPT-style causal decoder-only language models; following Wang et al. (2022) we consider decoder-only models with half as many total layers as their encoder-decoder variants.

k layers from an existing model:

$$w(k) = \left\lfloor \left(1 - \frac{n^0_{\text{layers}}}{n^0_{\text{layers}} + k}\right)\left(d^0_{\text{ff}} + \frac{A}{\beta}\right)\right\rceil. \quad (1)$$

Since adding or removing k layers might require changing d^0_{ff} by a fractional amount, we round $w(k)$ to the nearest integer. Table 2 reports the exact hyperparameter values we use for each of our three size classes, derived from Equation 1 above, and Figure 1 shows each size class plotted as $(n_{\text{layers}}, d_{\text{ff}})$ pairs.

2.2 Datasets and Training

2.2.1 Language Modeling

We use the Colossal Clean Crawled Corpus (C4; Raffel et al. 2019) for pretraining. We use a context size n_{ctx} of 1024 tokens and a batch size of 128 sequences $\approx$ 131k tokens. We pretrain each model for 1M steps, resulting in a total training dataset of roughly 131B tokens.

2.2.2 Compositional Generalization

In compositional generalization datasets, models are tested on a distribution that contains novel combinations of pieces, each of which has been previously seen independently during training. We fine-tune our pretrained models on the training portion of the dataset for 10,000 steps with a batch size of 128. Validation loss continued to decrease throughout training runs on each dataset, so we report values from the end of each fine-tuning run without early stopping. We use the following datasets (for examples of instances of these tasks, see Table 1):

1. **COGS** (Kim and Linzen, 2020) is a semantic parsing dataset consisting of natural-language sentences paired with formal semantic representations. It is constructed such that the out-of-domain generalization distribution contains two generalization types: new combinations of familiar words (*lexical generalization*, such as using the word 'hedgehog' as the object of a sentence when this word has only been seen during training as a subject); or new combinations of familiar syntactic structures (*structural generalization*, such as relative clauses that are more deeply nested than seen in training).

2. **Variable-free COGS** (COGS-vf; Qiu et al. 2022a) is a simplified variant of COGS where the semantic representations do not use numbered variables (see Table 1 for a comparison between COGS and COGS-vf). Removing variables from the representation has the benefit of lowering the associated computational cost of training by making sequences shorter. This conversion has been previously shown to improve the performance of models by reducing the complexity of the output space (Qiu et al., 2022b), but comes at the cost of limiting the capacity of the formal language to represent phenomena that require coordination of variable identity, such as control and anaphor binding.

3. **GeoQuery** (Zelle and Mooney, 1996) contains natural-language questions about US geography paired with SQL-style database queries representing those questions. We report results on the GeoQuery Standard split.

4. **English passivization** (Mueller et al., 2022) is a dataset of English active-voice sentences paired with their passive-voice counterparts (adapted from Mulligan et al. 2021). This benchmark is designed to test whether models use shallow, positional heuristics or syntactically principled ones. While Mueller et al. (2022) implemented a number of transformations in different languages, we focus on the English Passivization task.

3 Results

3.1 Language Modeling

Deeper models have lower perplexity. Depth has a significant impact on language modeling performance. At the shallow end of the spectrum, increasing depth results in a dramatic improvement in perplexity (Figure 2). In Figure 3a we compare the perplexity of each model in a size class relative to that of the best-performing model of that size. In the extreme case, the perplexity of a single-layer model can be nearly twice that of the optimal model in the class. Moreover, as parameter count increases the disparity between the worse, shallower models and the better, deeper models increases as well: For 41M-parameter models the ratio between the perplexity of the single-layer model and that of the optimal (5-layer) model is 1.59; for the 134M-parameter models, the ratio is 1.86; and for the 374M-parameter models, the ratio is 1.99.

Performance increases most rapidly within the first few layers. While deeper models do, in general, perform better than shallower ones, the in-

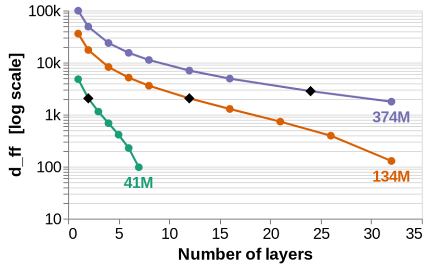


Figure 1: Models for the 41M-, 134M-, and 374M-parameter size classes. Points indicate models trained in this paper, and black diamonds represent the baseline models for each class whose hyperparameters were taken from Kim and Linzen (2020) and Raffel et al. (2019).

COGS	x: A hedgehog ate the cake . y: *cake(x_4); hedgehog(x_1) AND eat.agent(x_2, x_1) AND eat.theme(x_2, x_4)
COGS-vf	x: A hedgehog ate the cake on the bed . y: eat(agent = hedgehog, theme = *cake(nmod.on = *bed))
GeoQuery	x: which states have cities named m0 y: answer(intersection(state, loc_1(intersection(city, m0))))
English Passivization	x: our vultures admired her walrus above some zebra . y: her walrus above some zebra was admired by our vultures .

Table 1: Examples of inputs (x) and targets (y) from each compositional generalization dataset.

crease in performance that comes from adding layers diminishes rapidly as models become deeper (Figure 3a). The performance difference between 1-layer and 2-layer models is dramatic across all size classes; moving from 2 to 4 layers results in a much more modest performance improvement. We also note that as models get larger in our setup, they are able to make productive use of increasingly more layers: the optimal 41M-parameter model in our setup has 5 layers, while the optimal 134M-parameter model has 12; among 374M-parameter models, the 24-layer model had the best performance. At the same time, the pattern of the diminishing utility of depth holds even for the largest models we study.

Performance degrades when models become too narrow. At the deeper end of our scale, adding layers is not only unhelpful for performance, but begins to harm it (see the right-hand sides of each size-class curve in Figure 3a). As previously noted, the point at which trading width for depth becomes harmful is not an absolute function of depth, since the depth of the optimal model differs across classes. However, comparing the relative performance of models within a size class to the feed-forward ratio $d_{\text{model}}/d_{\text{ff}}$ shows that model performance begins to worsen once d_{ff} becomes smaller than d_{model} (to the right of the red dashed line in Figure 3b); when this happens, the affine projection of the vectors from $\mathbb{R}^{d_{\text{model}}}$ into $\mathbb{R}^{d_{\text{ff}}}$ becomes a non-injective map. In Section 5 we analyze the weight matrices of the affine transforms in the feed-forward network of each layer and demonstrate that as $d_{\text{model}}/d_{\text{ff}}$ increases the transforms become increasingly rank-deficient.

Larger models can tolerate more extreme feed-forward ratios. Varying d_{ff} while keeping d_{model} constant results in feed-forward ratios $d_{\text{model}}/d_{\text{ff}}$ which deviate significantly from the ratio of $1/4$, which is the de-facto standard in the literature (black vertical rule in Figure 3b). We find that smaller models are more sensitive to the particular value of the feed-forward ratio, and that for small models the standard ratio may not be optimal. Within the 41M-parameter size class there is a narrow range of feed-forward ratios in which model performance is within a few percentage points of the best-in-class model. As models get larger, this range expands leftward to include models which have increasingly wide feed-forward networks relative to the size of their contextual embeddings. In other words, larger models have more leeway to trade depth for width, becoming wider in proportion to their model dimension d_{model} without incurring large penalties for their perplexity. Furthermore, when $d_{\text{model}}/d_{\text{ff}} < 1$ the feed-forward ratio does not predict the relative perplexity of a model independent of its size.

3.2 Compositional Generalization

We next fine-tune the models pretrained in the previous section on the training portions of each compositional generalization dataset, and measure the full-sequence (exact match) accuracy of the models

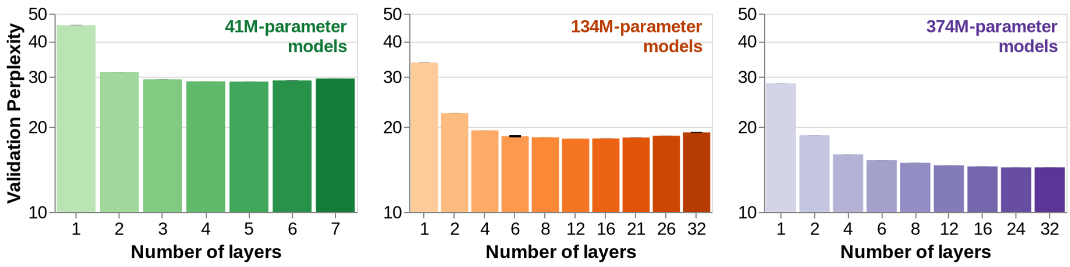
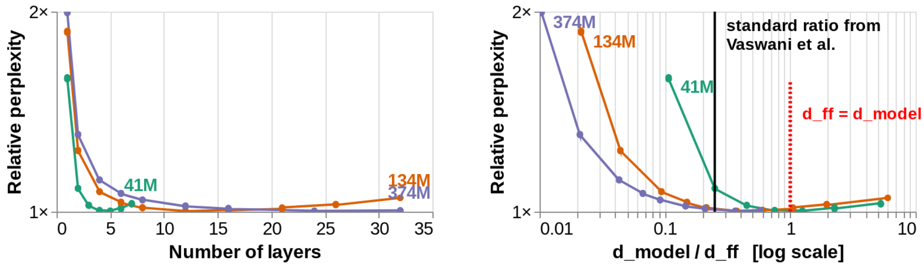


Figure 2: Deeper models achieve lower perplexities than shallower ones after equal amounts of training data regardless of size, but the benefits of adding layers diminish quickly with depth. Mean over 5 runs shown with error bars.



(a) (b)

Figure 3: Relative perplexity compared to the best model in each size class. **(a)** Perplexity goes down rapidly as models get deeper; only a few layers are needed to obtain most of the value of depth. **(b)** When $d_{\text{model}}/d_{\text{ff}} > 1$ (red dashed rule), perplexity slowly increases. As models get larger, the range of $d_{\text{model}}/d_{\text{ff}}$ ratios where performance is close-to-optimal expands leftward to include smaller and smaller values.

on the out-of-distribution generalization set.

Deeper models generalize better. As with language-modeling performance, deeper models tend to attain higher generalization accuracies than shallower models in the same size class (Figure 4). The effect of depth on compositional generalization is more variable than it is for language modeling, however: for COGS, COGS-vf, and GeoQuery there is some small non-monotonicity in the generalization accuracy as a function of depth. On English Passivization, the 41M- and 134M-parameter classes show largely-consistent trends where deeper models perform better than shallower ones; the 374M-parameter models show more significant non-monotonicity, though the deepest models do still outperform the shallowest ones.

The benefit of depth saturates quickly for some tasks. As with language modeling, most of the benefit of depth is gained by having only a few layers. For three of the tasks—COGS, COGS-vf, and GeoQuery—we see threshold depths after which generalization accuracy stays relatively constant as depth increases. These threshold depths are low and constant across model sizes, but vary by dataset: 4–6 layers for COGS, and 2–4 layers for COGS-vf and GeoQuery. Performance on COGS-vf appears to saturate with fewer layers than on COGS despite the fact that the two datasets express the same linguistic phenomena; this suggests that the saturation we observe on some datasets is closely linked to the complexity of the output representation independent from the complexity of the compositional generalization expressed in the data. On English Passivization, the impact of depth is more variable, which makes it difficult to identify a size-independent threshold.

The threshold effects suggest that some subsets of the datasets can be addressed with relatively simple models. We investigate this hypothesis by separately analyzing the models' performance on the two types of generalization cases included in COGS and COGS-vf: lexical generalization, where a familiar word needs to be interpreted in a familiar syntactic context in which it has not been observed; and structural generalization, where the syntactic structure is novel and needs to be constructed from familiar syntactic pieces. We find that even deep models at the largest model size systematically fail to generalize structurally (Figure 5); the benefit of

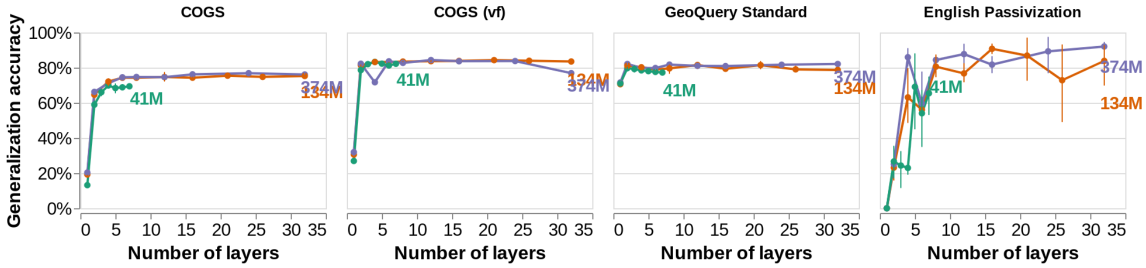


Figure 4: Deeper models generalize better than shallower models on compositional tasks across datasets and size classes. Error bars (easily visible only on the English Passivization data) report 95% confidence intervals in estimation of the mean, taken over 5 runs.

depth is largely limited to the easier lexical generalization cases. This supports the hypothesis that the saturated effect of depth is due to the existence of easier subsets of the datasets, and shows that increasing depth alone does not substantially improve the models' ability to learn the correct inductive bias for these structural tasks.

3.3 Depth Effects are Independent between Upstream and Downstream Tasks

We have shown that deeper models generalize better than shallower models do. But in Section 3.1 we also showed that deeper models attain lower validation perplexity in pretraining than shallower models; and deeper models achieve lower in-distribution loss on the fine-tuning tasks than shallower ones (Figure 7a). Both of these observations constitute potential confounds for the interpretation of the previous section: it could be that depth does not *directly* improve generalization accuracy, but only does so indirectly by improving language modeling performance or in-distribution accuracy on the fine-tuning task, which in turn lead to better generalization. To determine if this is the case, we correct for both of these potential confounds.

First, to correct for the deeper models' lower pretraining loss, we repeat our fine-tuning experiments using intermediate checkpoints of pretrained models that have equal validation perplexity within a size class. We pick the least-performant (i.e., shallowest) model within a size class as the reference model and note its validation perplexity at the end of pretraining. We then pick the intermediate checkpoints of all deeper[2] models at the point during pretraining when they achieved this reference perplexity (Figure 6a). Finally, we fine-tune each of these checkpoints on the compositional generalization tasks. We repeat this process for successively deeper reference models. We find that even when fine-tuning from checkpoints of equal validation perplexity, deeper models still generalize better than shallower models (Figure 6b).

Next, to correct for the fact that deeper models perform better than shallower ones on the in-distribution split of the compositional generalization tasks, we compare the models' generalization accuracy at points during fine-tuning when they have equal in-distribution loss. Figure 7b shows that even after adjusting for in-distribution performance, deeper models still achieve higher accuracy on the out-of-distribution generalization set than shallower models do.

4 Training and Inference Latency

What are the practical implications of the fact that the benefit of depth saturates after a handful of layers? Empirically, the compute cost of training and running our equal-parameter models exhibits a strongly linear relationship with depth. Figure 8 shows (for our largest models) that the latency during training grows linearly with the depth of the model. The causes of this penalty are two-fold. First, our choice to use narrower feed-forward dimension for deeper models to maintain constant total parameter count leads to a slightly higher floating point operation (FLOP) count for deeper models. To see this, we start from the cost formula introduced in Kaplan et al. (2020):

$$C_{\text{forward}} = 2N + 2n_{\text{layers}}n_{\text{ctx}}d_{\text{attn}},$$

where N is total parameter count. Since both n_{ctx} and d_{attn} are constant for all models of a particular

[2] We only consider models deeper than the reference model since, in general, shallower models will never attain the perplexity of the reference model at the end of its pretraining. This assumption breaks down when considering the deepest models in each size class, but these are far deeper than the depth at which compositional generalization performance saturates, so we do not extensively explore this regime.

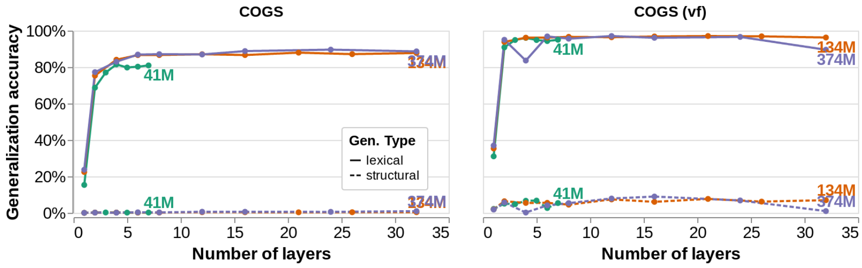
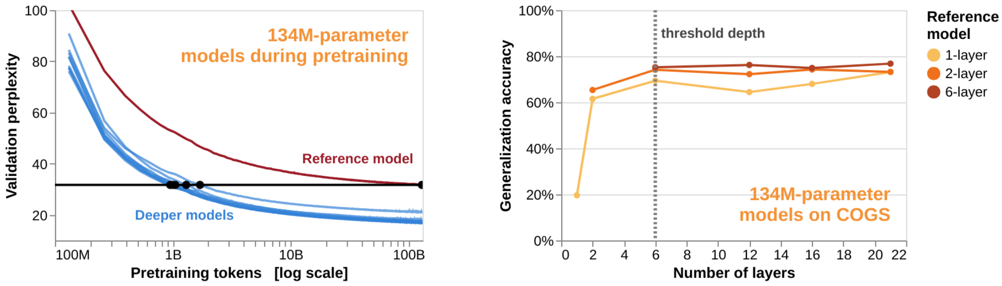


Figure 5: Increasing depth improves lexical generalization (solid lines) in both COGS and COGS-vf, but does not meaningfully improve structural generalization performance (dashed lines). Data shown is from a single run per condition.



(a)

(b)

Figure 6: **(a)** To correct for the potential effect of deeper models' lower pretraining loss on their generalization accuracy, we pick a reference model depth (red) and use checkpoints (black) from deeper models (blue) which have equal validation perplexity as the reference model does at the end of its pretraining. We then fine-tune these 'pretraining-corrected' checkpoints on the compositional tasks. **(b)** Even when fine-tuning checkpoints with equal validation perplexity, deeper models still generalize better than shallower models do up through six layers. The figure shows generalization accuracies from 134M-parameter models on COGS (single run per condition).

size (as d_{attn} and d_{ff} are decoupled from one another), the total FLOP count of a model is linear in depth, though this term is dominated by the $2N$ term unless model depth, attention size, or context length become very large.

The second and likely more important reason that deeper models are slower is because the computations at layer k depend on the results of the computations at layer $k-1$. Because of this sequential dependency, parallelism cannot be applied across layers (Tay et al., 2021).

Combined with the diminishing utility of depth for performance, the linear latency cost of depth as total size is kept constant leads us to the following practical recommendations:

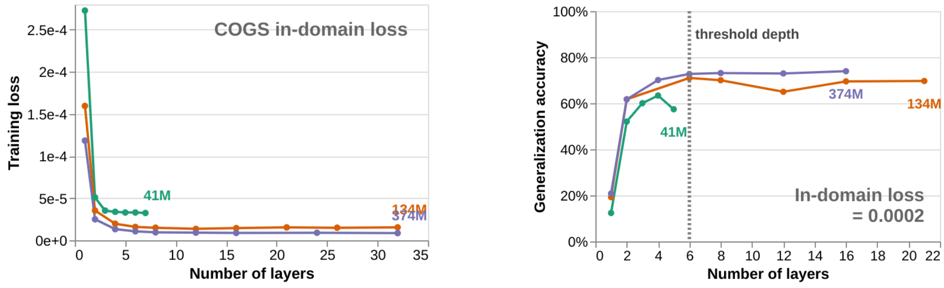
1. When trying to minimize GPU-hours for fixed data volume, shallower models can train in far less time than deeper models while still attaining acceptable levels of performance relative to the best-performing model in a given size class.

2. With a fixed budget of GPU-hours for training, shallower models can train on more data than deeper models can over any fixed amount of time, since shallower models have lower latency, potentially resulting in better performance than deeper models trained on less data.

These benefits are not confined to training: deeper models also incur a per-layer cost during inference. This means that the penalty a deeper model pays must be amortized over the lifetime cost of using a model for inference. Here too, reducing the GPU-time spent on inference by using shallower models of comparable performance could reduce compute costs, both for cloud-served models and for on-device inference (Strubell et al., 2019; Pope et al., 2022; Gupta and Agrawal, 2022).

5 Analysis of Feed-Forward Transforms

At extreme feed-forward ratios, we observe that model performance degrades. We investigate the role that the feed-forward ratio plays in the observed performance of our models. When d_{ff} is smaller than d_{model}, this transform is lossy, but small values of d_{ff} could nevertheless impact performance even when d_{ff} is still larger than d_{model}.



(a) (b)

Figure 7: **(a)** Deeper models attain lower (better) in-domain loss values on compositional tasks. **(b)** Deeper models generalize better than shallower ones on COGS, even at points during fine-tuning when models have equal loss (0.0002) on the in-distribution portion of the dataset. Data shown is from a single run per condition.

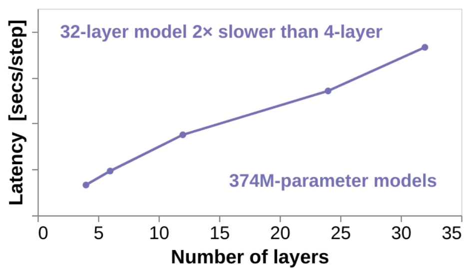


Figure 8: Deeper models train more slowly than shallower ones when controlling for total parameter count. We report relative latency [seconds per training step] for a subset of our 374M-parameter models that were trained on the same accelerator, showing a strongly-linear relationship between depth and latency. Similar relative trends are observed for other model sizes classes.

To determine if that is the case, we conduct rank analysis on the affine transforms which comprise the feed-forward block. For a given affine transform T, we compute the ordered singular values $\{\sigma_1, \sigma_2, \ldots, \sigma_k\}$ where $k = \min(d_{\text{model}}, d_{\text{ff}})$ is the rank of T and $\sigma_i \geq \sigma_{i+1}$. We then normalize each value by dividing by the ℓ_1 norm of $\{\sigma_1, \ldots, \sigma_k\}$ to calculate how much of T's image is accounted for by the best i-rank approximation of T for $i \leq k$. Figure 9 shows how in deeper models (i.e., those with increasingly large $d_{\text{model}}/d_{\text{ff}}$ ratios) the transforms become increasingly skewed away from making full-use of their available ranks.

6 Related Work

Compositionality. Previous work has explored the degree to which neural models exhibit compositional behavior by training or fine-tuning models on compositional tasks such as simple command sequences (Lake and Baroni, 2018) or semantic parsing (Kim and Linzen, 2020; Keysers et al., 2020). Other work has explored methods to improve the compositional behavior of models, including through data augmentation (Qiu et al., 2022a), larger models (Qiu et al., 2022b), and architectural changes (Gordon et al., 2019; Csordás et al., 2021; Ontanon et al., 2022). Our work complements these approaches by exploring a specific architecture change: increasing depth without changing total model size.

Impacts of depth. Theoretical work has shown that the expressive capacity of neural networks in general (Raghu et al., 2017) and transformer models in particular (Merrill et al., 2021) grows exponentially in depth. Empirical work also points to the role of depth in model performance. In a more general setting, Tay et al. (2021) found that scaling by depth is generally more helpful than scaling by width on downstream tasks, though they do not attempt to control for size. For compositional generalization in particular, Mueller et al. (2022) found that reducing depth was more harmful than reducing width for pretrained encoder-decoder models. Murty et al. (2023) observed that deeper transformer encoders often have more tree-like representations and higher parsing accuracies on some compositional tasks. Tempering these positive results, Veit et al. (2016) noted that in models with residual connections, even very deep networks leveraged only shallow subnetworks of roughly constant depth. Brown et al. (2022) also concluded that wide, shallow transformer models can attain roughly-equal performance to deeper ones. Both sets of results, however, are confounded by a lack of control for total parameter count. Very recently, Gromov et al. (2024) found that nearly half of the layers of deep language models can be pruned after training without substantially harming performance

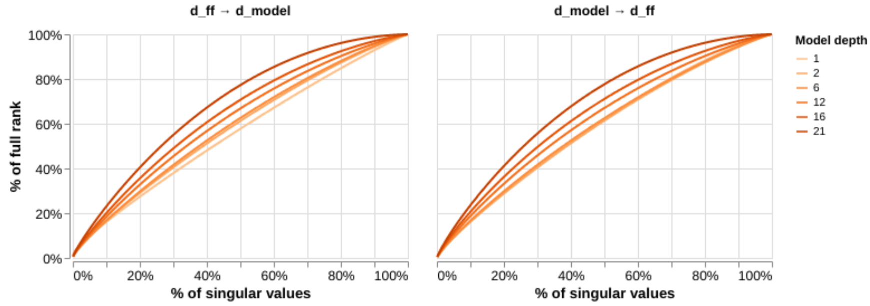


Figure 9: As models get deeper and $d_{\text{model}}/d_{\text{ff}}$ ratio gets larger (values between 0.01 for the shallowest model shown and 1 for the deepest), the input **(left)** and output **(right)** projections in the feed-forward block become increasingly close to rank-deficient transforms. A graph of $y = x$ here would indicate that models spread their rank equally across all singular values. Data from 134M-parameter models.

on downstream tasks.

Early-exit schemes. Early-exit research (Zhou et al. 2020; Schuster et al. 2022, *inter alia*) shows that deep models can dynamically be made shallower by skipping later layers of computation once a heuristic deems a computed representation "good enough." We view our work as complementing this approach; while early-exit reduces the computational cost of a model's inference on an input-dependent basis, our work shows that the cost can be reduced for all inputs during both training and inference. However, since we do not explore early-exit training or inference with our equal-parameter models here, it is possible that even our shallower models could benefit from early-exit schemes that would further reduce computational costs.

Controlling for model size. There are different possible approaches to studying the impact of hyperparameter choices without affecting the net model size. Kaplan et al. (2020) covaried number of layers n_{layers} with the contextual embedding dimension d_{model}, which they coupled to the attention-internal d_{attn} and feed-forward dimension at the standard ratio of $d_{\text{model}} = d_{\text{attn}} = d_{\text{ff}}/4$. They concluded that performance increases are largely driven by increasing the total parameter count of models, and that within "reasonable limits" language modeling perplexity is only weakly dependent on shape (though Tay et al. 2021 concluded that the same was not true for performance on downstream tasks, but did so without controlling for the impact of size). Our work investigates the role that depth plays on both pretraining and fine-tuning tasks while controlling for total parameter count.

7 Conclusion

Compositional generalization is essential for interpreting novel sentences. What aspects of the transformer LM architecture contribute to an inductive bias favoring compositional generalization? In a controlled experiment that teases apart depth from total number of parameters, we find that deeper transformers show better compositional generalization, and better language modeling performance, independent of their total number of parameters. At the same time, in most cases the usefulness of adding layers decreases rapidly as models get deeper: comparatively shallow models can achieve generalization accuracy on compositional tasks that is comparable to that of much deeper models, and language modeling perplexity within a few percentage points of the best-in-class model. Because deeper transformers have higher latency, this indicates that for a given parameter budget, shallower models can be significantly faster with a minimal sacrifice in performance.

8 Limitations

Attention heads. We do not investigate the role that attention heads play in compositional generalization broadly, nor how the function of heads changes with depth. Previous work (Michel et al. 2019, *inter alia*) showed that reducing the number of attention heads in a transformer (pre- or post-training) does not significantly harm performance. Mechanistic interpretability work has found that specific attention heads in transformers learn to compute task-specific functions (Voita et al., 2019; Htut et al., 2019; Olsson et al., 2022). Our findings here raise two questions which should be further investigated: first, does the relative unimportance of the *number* of attention heads still hold in regimes when a model is significantly shallower and wider than convention; and second, do any of the attention heads in our models learn to perform specifically compositional computations, and does this vary as models get deeper or shallower?

Alternative approaches to controlling for total size. Our approach to controlling for total parameter count necessitates making depth-width tradeoffs. An alternative approach would be to construct Universal Transformers (Dehghani et al., 2018), where each model in a size class has a transformer layer with the same parameters repeated n_{layers} times. Such a weight-sharing approach would allow for deeper models to have arbitrarily-wide feed-forward networks, mitigating the impact of making models too narrow. While such weight sharing prevents models from performing different computation in different layers, such restriction may in fact be beneficial for compositional generalization where similar computations (e.g., combining two syntactic phrases to a larger phrase) may need to apply recursively at different scales.

Pretraining corpus effects. We consider models pretrained on natural-language data. For our particular choice of compositional generalization experiments, the presence of lexical items in both the pretraining corpus and the generalization datasets represents a potential confounder of generalization performance which could be mitigated by modifying compositional datasets (Kim et al., 2022). More generally, we do not study how the distribution of pretraining data affects the inductive biases conferred to LMs (Papadimitriou and Jurafsky, 2023). As a particular area of interest for future work, we point out the hypothesis that including source code in the pretraining corpus (OpenAI, 2023; Google et al., 2023) will improve compositional generalization.

Fine-tuning vs. in-context learning. We use fine-tuning to adapt our pretrained models to the compositional tasks. Due to its computational cost and task-specificity, fine-tuning is less useful in practice than in-context learning as model size grows (Brown et al., 2020). Because in-context learning only becomes reliable at scales far larger than we are able to train, we did not explore the effect of depth on compositional generalization accuracy in in-context learning (Si et al., 2022); we point this out as an avenue for future research.

9 Ethics Statement

Throughout our experimental process, we sought to comply with best practices to mitigate any risks associated with LLM research. We use open-source datasets which are inspectable by third-parties for issues such as bias, and toxicity. We do not release any public checkpoints for the models we train, so there is no risk to misuse of any created artifacts, though we note that we derive our implemented models from existing publicly-available T5 models. We train models on English-only natural-language data, and fuller exploration should be done to explore how language impacts the results found here.

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A Design and Result Tables

Table 2 reports exact hyperparameters for the model classes trained. Table 3 displays pretraining and compositional generalization accuracy on all model sizes and tasks.

B Annotated Transformer Layer

Figure 10 shows the schematic for a single transformer layer. The layers input enters on the left and passes through the various model components (grey boxes), being combined with the residual connections before exiting right to subsequent layers. Blue boxes show the dimensionality of the vectors after transformation; we are primarily concerned with the size of the embedding vectors d_{model} and the internal dimension of the feed-forward block d_{ff}. The size of the vectors internal to the attention mechanism, d_{attn}, is not shown here but is usually set to be equal with d_{model}; we follow this convention here. Non-learned operations like addition, layer normalization, and the feed-forward network's nonlinearity are shown in grey circles.

	41M						
n_{layers}	1	**2**	3	4	5	6	7
d_{ff}	4779	**2048**	1138	682	409	227	97
	$d_{\text{model}} = d_{\text{attn}} = 512,\ n_{\text{heads}} = 8$						

	134M									
n_{layers}	1	2	4	6	8	**12**	16	21	26	32
d_{ff}	36k	17k	8193	5121	3584	**2048**	1280	731	393	128
	$d_{\text{model}} = d_{\text{attn}} = 768,\ n_{\text{heads}} = 8$									

	374M								
n_{layers}	1	2	4	6	8	12	16	**24**	32
d_{ff}	99k	49k	24k	15k	11k	6998	4907	**2816**	1770
	$d_{\text{model}} = d_{\text{attn}} = 1024,\ n_{\text{heads}} = 64$								

Table 2: Models of varying depths across three size classes. Bolded variants are the baseline models whose hyperparameters were taken from Kim and Linzen (2020) and Raffel et al. (2019).

size	n_{layers}	C4 val. PPL (↓)	COGS (↑)	COGS-vf (↑)	GeoQuery Standard (↑)	English Passivization (↑)
41M	1	45.7	12.4	25.7	68.2	0.00
	2	31.1	58.2	78.3	76.4	9.88
	3	29.3	63.1	80.8	**79.6**	26.2
	4	28.8	68.5	82.5	78.6	28.0
	5	**28.8**	63.4	82.5	76.8	**89.9**
	6	29.1	68.4	82.6	77.5	74.1
	7	29.6	**72.3**	**83.0**	77.1	78.3
134M	1	33.6	19.4	26.3	72.5	0.00
	2	22.3	65.5	83.0	81.4	29.9
	4	19.4	71.1	83.6	78.2	59.3
	6	18.7	74.3	83.2	80.0	49.4
	8	18.3	72.9	83.7	73.6	91.9
	12	**18.1**	73.0	84.7	**82.9**	87.1
	16	18.2	75.0	83.8	81.1	93.2
	21	18.3	75.1	**84.8**	80.0	88.1
	26	18.6	75.4	84.1	82.1	**98.4**
	32	19.2	**75.7**	84.0	78.9	94.8
374M	1	28.4	21.5	36.8	72.9	0.00
	2	18.6	66.2	82.2	80.7	13.6
	4	15.9	72.4	71.9	80.0	89.8
	6	15.2	75.1	83.1	78.2	18.8
	8	14.9	75.2	82.6	80.7	84.3
	12	14.6	76.3	84.3	80.0	81.0
	16	14.5	76.3	**85.1**	81.1	87.2
	24	**14.4**	78.0	83.1	83.2	89.6
	32	14.7	**78.8**	79.7	**84.6**	**90.2**

Table 3: Validation perplexity (↓, lower is better) on C4 after pretraining and generalization accuracy (%; ↑, higher is better) on compositional datasets after 10 k steps of fine-tuning. Bold values indicate best-in-size-class performance. Data is from a single run per condition.

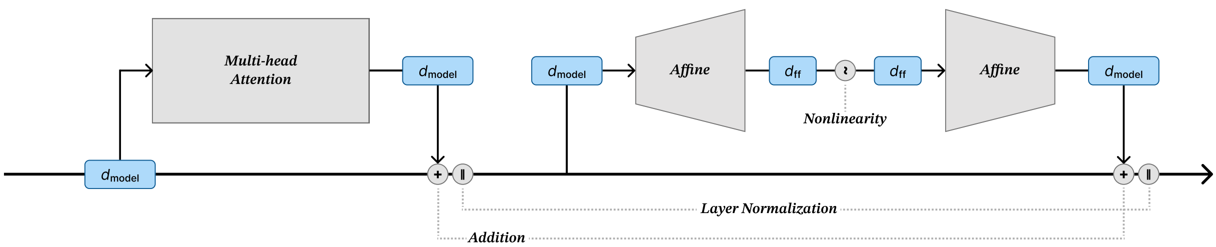

Figure 10: Diagram of a single transformer layer, annotated with the dimensions (blue) of each vector. Information is passed from left to right, through each component (grey box), and added back to the residual embeddings before normalization.