Harnessing Deep Neural Networks with Logic Rules: Supplementary Material

A Solving Problem Eq.(3), Section 3

We provide the detailed derivation for solving the problem in Eq.(3), Section 3, which we repeat here:

$$
\begin{aligned}
\min_{q,\boldsymbol{\xi}\geq 0} \ & \mathrm{KL}(q(\mathbf{Y}|\mathbf{X}) \| p_\theta(\mathbf{Y}|\mathbf{X})) + C\sum_{l,g_l}\xi_{l,g_l} \\
\text{s.t.} \ & \lambda_l(1-\mathbb{E}_q[r_{l,g_l}(\mathbf{X},\mathbf{Y})]) \leq \xi_{l,g_l} \\
& g_l = 1,\ldots,G_l, \quad l=1,\ldots,L,
\end{aligned}
\tag{A.1}
$$

The following derivation is largely adapted from (Ganchev et al., 2010) for the logic rule constraint setting, with some reformulation that produces closed-form solution.

The Lagrangian is

$$
\max_{\boldsymbol{\mu}\geq 0,\boldsymbol{\eta}\geq 0,\alpha\geq 0} \ \min_{q(\mathbf{y}),\boldsymbol{\xi}} L, \tag{A.2}
$$

where

$$
\begin{aligned}
L = & \ \mathrm{KL}(q(\mathbf{Y}|\mathbf{X}) \| p_\theta(\mathbf{Y}|\mathbf{X})) + \sum_{l,g_l}(C+\mu_{l,g_l})\xi_{l,g_l} \\
& + \sum_{l,g_l}\eta_{l,g_l}\left(\mathbb{E}_q[\lambda_l(1-r_{l,g_l}(\mathbf{X},\mathbf{Y}))] - \xi_{l,g_l}\right) + \alpha\Big(\sum_{\mathbf{Y}} q(\mathbf{Y}|\mathbf{X}) - 1\Big)
\end{aligned}
\tag{A.3}
$$

Solving Eq.(A.2), we obtain

$$
\begin{aligned}
\nabla_q L = & \log q(\mathbf{Y}|\mathbf{X}) + 1 - \log p_\theta(\mathbf{Y}|\mathbf{X}) + \sum_{l,g_l}\eta_{l,g_l}\left[\lambda_l(1-r_{l,g_l}(\mathbf{X},\mathbf{Y}))\right] + \alpha = 0 \\
\Longrightarrow \quad & q(\mathbf{Y}|\mathbf{X}) = \frac{p_\theta(\mathbf{Y}|\mathbf{X})\exp\left\{-\sum_l \eta_l\lambda_l(1-r_{l,g_l}(\mathbf{X},\mathbf{Y}))\right\}}{e\exp(\alpha)}
\end{aligned}
\tag{A.4}
$$

$$
\nabla_{\xi_{l,g_l}} L = C + \mu_{l,g_l} - \eta_{l,g_l} = 0 \qquad \Longrightarrow \qquad \mu_{l,g_l} = C - \eta_{l,g_l} \tag{A.5}
$$

$$\nabla_\alpha L = \sum_{\mathbf{Y}} \frac{p_\theta(\mathbf{Y}|\mathbf{X}) \exp\left\{-\sum_{l,g_l} \eta_{l,g_l}\lambda_l(1-r_{l,g_l}(\mathbf{X},\mathbf{Y}))\right\}}{e\exp(\alpha)} - 1 = 0$$
$$\Longrightarrow \qquad \alpha = \log\left(\frac{\sum_{\mathbf{Y}} p(\mathbf{Y}|\mathbf{X}) \exp\left\{-\sum_{l,g_l} \eta_{l,g_l}\lambda_l(1-r_{l,g_l}(\mathbf{X},\mathbf{Y}))\right\}}{e}\right) \tag{A.6}$$

Let $Z_\eta = \sum_{\mathbf{Y}} p(\mathbf{Y}|\mathbf{X}) \exp\left\{-\sum_{l,g_l} \eta_{l,g_l}\lambda_l(1-r_{l,g_l}(\mathbf{X},\mathbf{Y}))\right\}$. Plugging α into L

$$\begin{aligned} L &= -\log Z_\eta + \sum_{l,g_l}(C+\mu_{l,g_l})\xi_{l,g_l} - \sum_{l,g_l}\eta_{l,g_l}\xi_{l,g_l} \\ &= -\log Z_\eta \end{aligned} \tag{A.7}$$

Since Z_η monotonically decreases as η increases, and from Eq.(A.6) we have $\eta_{l,g_l} \leq C$, therefore:

$$\begin{aligned} &\max_{C\geq\eta\geq 0} -\log Z_\eta \\ &\qquad \Longrightarrow \qquad \eta^*_{l,g_l} = C \end{aligned} \tag{A.8}$$

Plugging Eqs.(A.6) and (A.8) into Eq.(A.5) we obtain the solution of q as in Eq.(4).

B Identifying Lists for NER

We design a simple pattern-matching based method to identify lists and counterparts in the NER task. We ensure high precision and do not expect high recall. In particular, we only retrieve lists that with the pattern "1. ... 2. ... 3. ..." (i.e., indexed by numbers), and "- ... - ... - ..." (i.e., each item marked with "-"). We require at least 3 items to form a list.

We further require the text of each item follows certain patterns to ensure the text is highly likely to be named entities, and rule out those lists whose item text is largely free text. Specifically, we require 1) all words of the item text all start with capital letters; 2) referring the text between punctuations as "block", each block includes no more than 3 words.

We detect both intra-sentence lists and inter-sentence lists in documents. We found the above patterns are effective to identify true lists. A better list detection method is expected to further improve our NER results.

References

Ganchev, K., Graça, J., Gillenwater, J., and Taskar, B. (2010). Posterior regularization for structured latent variable models. *The Journal of Machine Learning Research*, 11:2001–2049.