Investment-Driven Social Influence: A Statistical Physics Approach to Advertising Response

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Abstract

This paper explores social influence in consumer responses to advertising through investment-mediated conversational dynamics. We implement conversational engagement via advertising expenditure patterns, recognizing that marketing spend directly translates into conversational volume and reach across multi-channel ecosystems. Our approach integrates social psychology frameworks with statistical physics analogies as epistemic scaffolding following Ruse's "analogy as heuristic" idea. The model introduces three parameters—Marketing Sensitivity, Response Sensitivity, and Behavioral Sensitivity—quantifying emergent properties of investment-driven influence networks. Validation against three real-world datasets shows competitive performance compared to conventional approaches of modeling the consumer response curve like Michaelis-Menten and Hill equations, with context-dependent advantages in network-driven scenarios. These findings illustrate how advertising ecosystems operate as complex adaptive systems (CAS) where influence propagates through investment-amplified conversational networks.

1 Introduction

Advertising represents investment-mediated conversational dynamics between brands and consumers (Ballantyne and Varey, 2006), where social influence mechanisms shape response patterns through resource allocation strategies (Cialdini, 2009). Contemporary advertising functions as a recursive process calibrating individual cognition to collective signaling systems via strategic investment across conversational touchpoints (Kelman, 1958; Turner et al., 1991). Rather than simply transmitting information, effective advertising creates perturbations within social reference fields through investment allocation, where consumer decisions emerge from group identity dynamics mediated by investment-amplified dialogue volume (Hyman, 1942; Bearden and Etzel, 1989; II et al., 2002).

We align conversational engagement with advertising expenditure, aware that marketing spend drives conversational volume, reach, and persistence across channels. Following Ruse (1979)'s "analogy-as-heuristic" approach, we use statistical physics concepts not as literal equivalents but as formal frameworks revealing patterns in investment-driven influence propagation.

Our model addresses key questions: How do investment levels determine conversational reach and influence outcomes? How do cultural factors amplify or diminish investment-mediated influence? What mathematical frameworks capture advertising expenditure-to-conversational influence relationships? By combining social psychology insights with physics-inspired modeling, we extend prior work on social dynamics (Castellano et al., 2009) and consumer behavior (Farivar and Wang, 2022).

2 Related Work

Research on social influence in advertising spans psychology, marketing, and computational modeling. Social identity theory highlights how group affiliation drives behavior when sufficient conversational exposure occurs (Charness and Chen, 2020), while social proof explains peer-driven adoption emerging from investment-amplified dialogue volume (Karasawa, 1991). Opinion dynamics models describe interaction-driven attitude convergence, particularly relevant for understanding how advertising investment creates conversational conditions for influence propagation (DeGroot, 1974; Friedkin and Johnsen, 2011).

Physics-inspired approaches prove valuable for social dynamics. The Ising model describes binary state interactions producing collective behaviors (Castellano et al., 2009), while percolation theory

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models information spread through connected networks (Essam, 1980). However, their application to investment-mediated consumer response remains underexplored. Our work bridges this gap, using physics analogies to model how advertising investment drives social influence through conversational networks, contrasting with traditional approaches based on Michaelis-Menten and Hill equations (Michaelis and Menten, 1913; Hill, 1910).

3 Social Influence in Consumer Behavior

Social influence shapes consumer responses through complex investment-mediated conversational processes deeply rooted in established social psychology principles. Social identity theory suggests that individuals systematically align their behaviors with perceived group norms, enhancing engagement when advertising campaigns achieve sufficient conversational volume to effectively communicate and reinforce shared values within target communities (Charness and Chen, 2020; Foroudi, 2019). For instance, a brand endorsed by a particular social group can spur widespread adoption as consumers actively seek in-group approval and validation, but this process requires adequate advertising investment to ensure sufficient conversational reach and message persistence within that specific social network (Wachter, 2020).

Social proof mechanisms drive engagement when peers participate in brand-related conversations and advocacy behaviors, amplifying campaign impact through validation processes, with investment levels serving as the primary determinant of the frequency, persistence, and reach of these validating conversational touchpoints (Karasawa, 1991). Group cohesion, reinforced by shared preferences and common identity markers, facilitates collective decision-making processes when sufficient investment creates sustained conversational environments that closely mirror the opinion convergence processes described in social influence literature (Greer, 2012; DeGroot, 1974).

Group polarization phenomena intensify attitudes within cohesive social groups, where sustained discussions strengthen shared preferences and amplify campaign impact when investment ensures adequate conversational persistence and frequency to maintain dialogue momentum (Myers, 1982). Social Impact Theory postulates that influence effectiveness depends critically on the source's perceived strength, temporal immediacy, and the number of influencers—factors that are directly modulated by advertising investment decisions that determine conversational volume, channel diversity, and message repetition across multiple touchpoints (Latané, 1981). For example, influencer endorsements on online media platforms spread through social networks in patterns resembling epidemiological diffusion processes, but the extent, speed, and ultimate reach of propagation correlates strongly with investment levels that determine reach amplification mechanisms and message persistence within network structures (Centola, 2010).

Consider a comprehensive social media campaign promoting eco-friendly products within sustainability-focused communities. When social influencers within these communities endorse the product, social identity mechanisms and social proof dynamics drive rapid engagement among followers, but the ultimate effectiveness depends critically on investment levels that determine conversational frequency, reach amplification, and the creation of multiple reinforcing touchpoints. Higher investment enables the creation of multiple conversational threads and sustained dialogue, further amplified by group polarization effects during online discussions and community interactions.

Conversely, a campaign targeting a fragmented audience may require sophisticated targeted messaging strategies with carefully allocated investment to create sufficient conversational density within each discrete segment, as low group cohesion inherently limits influence spread unless compensated by strategic resource distribution across multiple channels and touchpoints. These dynamics underscore the fundamental need for mathematical models that adequately account for investment-mediated network effects and conversational interactions.

4 Physics as Heuristics

Following Ruse (1979)'s methodological framework, we draw on fundamental concepts from statistical physics as heuristic guides for understanding emergent behaviors in investment-driven conversational systems, while maintaining a clear distinction between mathematical analogy and literal equivalence.

Phase transitions in statistical physics represent critical transformations where complex systems undergo abrupt qualitative changes in their macroscopic properties as control parameters cross specific threshold values. Consider the canonical liquid-to-gas transition: as temperature increases beyond a critical point, the system's collective behavior shifts discontinuously from the ordered, cohesive state characteristic of liquid phases to the disordered, dispersed state characteristic of gaseous phases (Stanley, 1971). This transformation emerges not from gradual, continuous change but from the cooperative reorganization of microscopic interactions once critical thermodynamic conditions are satisfied.

In the context of investment-mediated social influence networks, viral adoption phenomena exhibit analogous structural characteristics—remaining dormant and exhibiting minimal propagation below certain investment thresholds before triggering rapid, system-wide behavioral cascades when sufficient conversational volume and network activation are achieved (Centola, 2010). Our mathematical formulation captures this threshold-dependent behavior through the term $\left(1 - e^{\beta x}\right)^{-\gamma}$, where the exponential component $e^{\beta x}$ modulates the approach to critical boundaries representing conversational saturation limits, while the negative exponent $-\gamma$ generates the characteristic divergent response that signals the onset of collective adoption processes.

5 Interdisciplinary Foundations of the Model

Understanding investment-mediated social influence requires systematic integration of insights from multiple academic disciplines. Our theoretical framework synthesizes diverse fields to capture the full complexity of how advertising expenditure drives conversational influence dynamics across contemporary media ecosystems.

From computational linguistics, we incorporate pragmatic theories of conversation as coordinated action systems where meaning emerges through dynamic contextual negotiation facilitated by investment-determined frequency, reach, and temporal persistence (Clark, 1996). The parameter γ (Behavioral Sensitivity) in our model parallels computational linguistic concepts of semantic propagation through discourse networks (Hamilton et al., 2016), where investment-amplified linguistic markers function as activation nodes that trigger cascading meaning-making processes across interconnected conversational communities. Recent advances in linguistic accommodation and synchrony within dialogue systems have demonstrated how pragmatic alignment serves as a necessary precursor to deeper influence mechanisms (Danescu-Niculescu-Mizil et al., 2012), providing robust empirical validation for our conceptualization of advertising investment as the primary driver of conversational conditions necessary for effective influence propagation.

Behavioral economics contributes complementary insights into the cognitive processes underlying social influence when mediated by investment-driven conversational exposure patterns. Our approach to modeling non-linear response curves aligns systematically with Thaler and Sunstein (2008)'s dual-process framework, where automatic (System 1) and deliberative (System 2) reasoning systems interact during preference formation. The Marketing Sensitivity parameter (α) in Equation 1 can be understood as quantifying how investment-driven conversational volume influences the critical transition point between these cognitive systems—specifically, the threshold at which sustained dialogue exposure enables social signals to override individual utility calculations (Akerlof and Kranton, 2000).

It is relevant to note that none of the parameters in Equation 1 directly correspond to channel influence values obtained from standard marketing mix models. Parameter C represents the intrinsic effectiveness of channels, requiring complex interplay with sensitivity parameters for accurate real-world performance prediction. Parameter α quantifies channels' capacity to scale conversational impact with incremental investment. Parameter γ provides insights into audience structure and viral propagation potential—information that current Marketing Mix Modeling approaches systematically lack.

6 Proposed model

We propose a comprehensive model for consumer response (y) to advertising spend (x), focusing on investment-driven complex social influence mechanisms described by the following equation:

$$y = Cx^{\alpha}\left(1 - e^{\beta x}\right)^{-\gamma} \qquad (1)$$

In Equation 1, parameter C represents the intrinsic channel effectiveness—the fundamental capacity to convert advertising investment into consumer response under standardized conditions. Marketing Sensitivity α (constrained to range 0–1) governs how conversational volume scales with incremental investment. Response Sensitivity β measures conversational saturation dynamics and can assume positive or negative values. Behavioral Sensitivity γ (range 0–1) quantifies audience clustering coefficients and viral propagation potential.

We want to note that Equation 1 shows important mathematical constraints when $\beta > 0$: for large values of x, the condition $e^{\beta x} > 1$ makes the expression $(1 - e^{\beta x})$ negative, generating complex-valued results when γ assumes non-integer values. This mathematical limitation requires careful consideration of domain restrictions for practical applications.

Our equation is similar to the one introduced by Little and Lodish (1969): $r(x) = r_0 a(1+e^{-bx})$. In this equation x is the exposure level, r is the return, r_0 is the return without advertising $r|x = 0$, and a, b are non-negative constants. This approach can be understood as a conditional expectation of the average fraction potential realized for a set of consumers at exposure level y, denoted by $r(x)$. There are relevant differences between this equation and Equation 1. In Little and Lodish (1969)'s equation, the term $r_0 a$ implies a linear growth depending on the return without advertising r_0. In practice, r_0 is very difficult to calculate. Instead, we propose a scaling law term (Cx^a) meaning that a change in the quantity x leads to a corresponding change in the quantity y, regardless of their initial sizes. Additionally, we add the exponent γ considering the scaling hypothesis (near critical points, physical quantities in complex systems show a scaling behavior that can be described using power laws).

7 Experimental setup

We use three real-world advertising campaign datasets collected from distinct companies under strict Non-Disclosure Agreements (NDAs), implementing rigorous anonymization protocols including differential privacy techniques (Dwork, 2006) and systematic channel pseudonymization (El Emam and Alvarez, 2015; Hundepool et al., 2012).

We use a Bayesian Marketing Mix Modeling approach using Google's Lightweight MMM library (Jin et al., 2017) with the following parameters: model 'carryover', seasonality degrees 4, acceptance probability 0.85, warmup samples 2000, final samples 2000. Response curves are systematically fitted using our proposed equation, Hill's model (Equation 2)(Hill, 1910), and the Michaelis and Menten equation (Equation 3) (Michaelis and Menten, 1913). We use L-BFGS-B optimization algorithms - a quasi-Newton method that approximates the Broyden–Fletcher–Goldfarb–Shanno algorithm or BFGS (Head and Zerner, 1985)- from Python's library SciPy.

$$y = \frac{1}{1 + \left(\frac{k_a}{x}\right)^n} \tag{2}$$

$$y = \frac{V_{\max} x}{k_m + x} \tag{3}$$

When optimizing parameters we have found that constraining β to negative values makes optimization more unstable. This is why in our experiments we do not set restrictions on positive β values given the relatively low spending ranges characteristic of our datasets, though broader generalization requires constraining $\beta < 0$ to avoid mathematical instability in high-investment scenarios. Another possibility to explore in future work is to adjust the term $e^{\beta x}$ in Equation 1 to $-e^{\beta x}$. We assume this is fundamentally an optimization problem.

8 Results

We evaluate model performance using both Ordinary Least Squares (OLS) regression and Restricted Total OLS (RTO) regression, which assumes zero response at zero media spend ($y = 0$ when $x = 0$). Statistical metrics include the coefficient of determination (r^2), p-values, and F-p-values to assess goodness of fit and statistical significance.

Table 1: Dataset 1: Performance across 5 retail channels

	OLS		**RTO**	
Model	r^2	p-val	r^2	p-val
Proposed	0.062	0.290	**0.535**	0.000
Michaelis-Menten	0.101	0.211	0.444	0.000
Hill	0.096	0.264	0.456	0.000

Table 2: Dataset 2: Performance across 5 SAAS channels

	OLS		**RTO**	
Model	r^2	p-val	r^2	p-val
Proposed	0.319	0.004	0.903	0.000
Michaelis-Menten	0.318	0.004	**0.939**	0.000
Hill	0.334	0.003	**0.945**	0.000

Table 3: Dataset 3: Performance across 13 consumer goods channels

Model	OLS r^2	OLS p-val	RTO r^2	RTO p-val
Proposed	0.096	0.227	**0.348**	0.000
Michaelis-Menten	0.098	0.232	0.347	0.000
Hill	0.081	0.204	0.334	0.000

Table 4: Overall performance summary across all datasets

	OLS r^2			RTO r^2		
Dataset	**Prop**	**M-M**	**Hill**	**Prop**	**M-M**	**Hill**
Data 1 (retail)	0.062	0.101	0.096	**0.535**	0.444	0.456
Data 2 (SAAS)	0.319	0.318	0.334	0.903	**0.939**	**0.945**
Data 3 (consumer)	0.096	0.098	0.081	**0.348**	0.347	0.334
Average	0.159	0.172	0.170	0.595	0.577	0.578

Table 5: Model parameters for Dataset 1 (retail channels)

Channel	α	β	γ	C	RoAS	Inf.%
TV spend	0.165	-0.072	0.000	54997	0.929	5.62
OOH spend	0.018	0.286	0.008	77805	0.451	2.01
Print ads	0.048	-1.000	1.000	77556	1.341	2.07
Google search	0.150	0.004	0.004	37877	1.865	4.55
Facebook	0.045	-0.011	1.000	90719	0.748	2.64

Table 6: Model parameters for Dataset 2 (SAAS channels)

Channel	α	β	γ	C	RoAS	Inf.%
Online 1	0.228	0.010	0.075	5079	9.65	10.43
Offline 1	0.343	-0.164	0.000	5123	8.97	8.08
Offline 2	0.041	0.082	0.223	13837	62.69	1.55
Offline 3	0.192	-0.012	0.009	13884	2.37	20.22
Offline 4	0.034	0.006	0.145	35052	86.49	4.63
Offline 5	0.378	0.002	0.069	6246	29.40	29.93

Table 7: Model parameters for Dataset 3 (consumer goods channels)

Channel	α	β	γ	C	RoAS	Inf.%
Brand Search	0.069	0.890	0.140	1.64	0.24	0.38
Partnerships	0.515	0.198	0.835	1.78	0.84	0.97
TV	0.667	0.327	0.028	1.48	0.35	2.14
Programmatic	0.458	0.984	0.298	2.88	1.17	2.40
Magazines 1	1.000	0.636	0.000	3.76	1.71	1.04
Magazines 3	0.141	0.854	0.300	41.07	11.57	5.76
Business Events 1	0.686	0.059	0.000	9.53	3.40	2.62
Business Events 2	0.671	-0.017	0.009	17.31	6.12	4.01

RTO regression consistently demonstrates superior performance compared to OLS across all datasets, improving our equation's average r^2 from 0.159 to 0.595, providing strong empirical support for theoretical assumptions about zero-intercept response characteristics. Our equation demonstrates competitive performance with distinct context-dependent advantages: superior performance in retail contexts (Dataset 1) and consumer goods markets (Dataset 3), while established biochemical analogy equations excel in SAAS environments (Dataset 2). This pattern suggests that our social influence framework demonstrates particular effectiveness in network-driven consumer markets where social proof and viral mechanisms predominate.

9 Discussion

Parameter analysis across datasets shows different channel dynamics with important practical implications. High γ values (approaching 1.0) indicate substantial viral potential through investment-amplified conversational cascades, particularly evident in Facebook and Print ads channels in Dataset 1. High α values suggest effective conversational volume scaling with investment, exemplified by Offline 1 in Dataset 2 ($\alpha = 0.343$) and Magazines 1 in Dataset 3 ($\alpha = 1.000$). High β values signal rapid conversational saturation dynamics requiring sophisticated budget management strategies.

Dataset 1 analysis reveals TV spending with the highest Marketing Sensitivity ($\alpha = 0.165$), suggesting significant responsiveness to budget changes in conversational reach and frequency. Print ads and Facebook demonstrate maximum Behavioral Sensitivity ($\gamma = 1.0$), indicating strong network clustering effects where investment drives the formation of coherent conversational communities. Dataset 2 exhibits Offline 5 with both the highest channel influence (29.93%) and Marketing Sensitivity ($\alpha = 0.378$), while Offline 2 and 4 show exceptional RoAS values (62.69 and 86.49 respectively), suggesting highly efficient conversion of investment into response through well-structured conversational pathways.

Dataset 3 shows remarkable channel diversity, with Magazines 1 showing maximum Marketing Sensitivity ($\alpha = 1.0$) and Partnerships displaying high Behavioral Sensitivity ($\gamma = 0.835$), suggesting strong viral potential when adequate investment creates sustained conversational engagement. Notably, channels with high γ values include offline channels, indicating that strong influence spread potential exists in non-digital communities when appropriate investment creates suitable conversational conditions.

Different parameter combination analysis unveils complex strategic insights for optimal investment allocation (Figure 1): high α and high γ channels show strong network effects ideal for viral campaigns where diverse audiences interconnect through investment-sustained conversations, effectively behaving as single homogeneous groups; high α and low γ channels suit targeted campaigns for fragmented audiences, particularly effective in new product launch campaigns requiring preliminary market segmentation with focused investment strategies; low α and high γ channels enable precision targeting of aggregated audiences, particularly valuable for mature products or established brands where sustained conversational engagement drives incremental adoption.

These empirical findings confirm the explanatory power of our investment-mediated social influence model, where theoretical constructs from Social Impact Theory (Latané, 1981) and group polarization phenomena (Myers, 1982) manifest as measurable parameter variations across diverse market contexts. The observed path-dependency in channel performance—captured through our model's ability to differentiate between viral-prone channels (high γ) and scaling-responsive channels (high α)—fundamentally contrasts with uniform influence propagation models (DeGroot, 1974) that assume homogeneous network effects. This differentiation enables strategic marketing decision-making based on channel-specific influence mechanisms rather than aggregate performance metrics (Friedkin and Johnsen, 2011). Moreover, the systematic variations in parameter combinations across different business contexts suggest that our framework captures the underlying complexity of investment-driven conversational dynamics as they operate within distinct market ecosystems. The model's capacity to reveal these nuanced patterns through formal mathematical representation indicates robust theoretical foundations that align with complex adaptive systems principles, where strategic investment allocation creates emergent influence properties through non-linear network interactions.

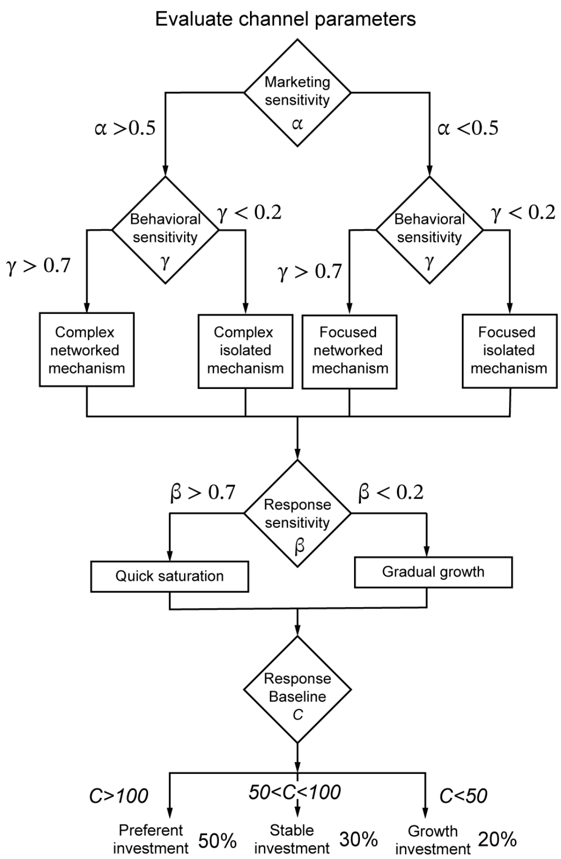

Figure 1: Strategic investment allocation framework based on model parameters. The decision tree provides systematic guidance for advertising budget allocation by evaluating channel characteristics through Marketing Sensitivity (α), Behavioral Sensitivity (γ), and Response Sensitivity (β) parameters.

10 Limitations

While our datasets represent real-world scenarios from different businesses, anonymization requirements necessarily limit detailed sector-specific analysis and prevent comprehensive data sharing for independent validation by other researchers. The three-parameter mathematical model, though comprehensive in scope, may not capture all influence mechanisms operative in highly specialized contexts where decision-making processes differ significantly from standard consumer markets, particularly in contexts where investment-driven conversational dynamics operate through fundamentally different mechanisms.

The physics analogies used in our theoretical framework, while providing valuable heuristic insights for understanding complex dynamics, should not be interpreted as literal equivalences between advertising systems and physical phenomena. The model's performance proves notable variation across different datasets, suggesting context-dependent applicability that requires careful validation for specific use cases and market conditions. The mathematical constraints inherent in our formulation when $\beta > 0$ set limitations on generalization to scenarios involving higher investment levels, requiring either systematic parameter constraints or fundamental equation modifications for broader practical applicability. Future research should incorporate larger-scale datasets, temporal dynamics to enhance generalization capabilities, sector-specific validation studies, and mathematical refinements to address these inherent limitations.

11 Conclusion

This research establishes a comprehensive theoretical and empirical framework for understanding investment-mediated social influence in consumer responses to advertising by conceptualizing marketing communications as dynamic systems where advertising expenditure systematically drives conversational volume, reach, and temporal persistence. Through systematic synthesis of statistical physics heuristics with established social psychology theories, we have developed a formal mathematical framework that captures how strategic investment translates into influence propagation through conversational networks in patterns that traditional marketing models fail to adequately represent.

Our mathematical framework provides a precise analytical language for quantifying how advertising investment drives emergent properties of conversational engagement within evolving social contexts. The model's comprehensive empirical validation through diverse real-world datasets demonstrates competitive performance with distinct context-dependent advantages over conventional approaches, particularly in capturing the network-dependent, non-linear dynamics of social influence that characterize contemporary consumer markets.

Our findings suggest a fundamental reconceptualization of advertising effectiveness: from traditional message optimization paradigms to a more complex investment-mediated conversation design, where brands must strategically allocate resources to create optimal conversational conditions necessary for influence propagation rather than simply crafting more or less persuasive content. Future research directions include systematic incorporation of linguistic markers and semantic content analysis to refine predictive capabilities, development of dynamic temporal extensions to capture conversational evolution patterns, investigation of cross-cultural variations in parameter sensitivities, and exploration of potential applications in multi-agent dialogue systems that simulate authentic social influence patterns driven by strategic resource allocation.

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