Some Formal Properties of Higher Order Anaphors

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Abstract

Formal properties of functions denoted by higher order anaphors like *each other* and syntactically complex expressions containing *each other* are studied. A partial comparison between these functions and functions denoted by (simple and complex) reflexives is draw. In particular it is shown that both types of function are predicate invariant (in a generalised sense). These results allows us to understand the anaphoric character of both reflexive and reciprocal expressions.

1 Introduction

By higher order anaphor, I mean expressions like *each other*, sometimes called basic higher order anaphors, and various complex expressions syntactically containing *each other*. These complex anaphors include Boolean compounds like *each other and most students, each other and themselves* and various modifications of *each other* like *only each other* or *at least each other*. Higher order anaphors are also expressions formed by the application of a *higher order anaphoric determiner* like *each other's* or *every ...except each other* to a common noun (CN). All such expressions will be called reciprocals and sentences containing them (in object position) will be called reciprocal sentences.

Higher order anaphors can be opposed to (logically) simple anaphors whose basic example is the reflexive pronoun *himself/herself/themselves*. This simple basic anaphora can also occur in Booleanly complex anaphors like *himself and most students* or in modified expressions like *only himself, even themselves*. So the distinction between simple and higher order anaphors is of logical nature: as we will see below functions denoted by higher order anaphors take binary relations (or binary relations and sets) as arguments and give sets of type $\langle 1 \rangle$ quantifiers as output whereas simple anaphors have arguments of the same type as higher order ones (that is their arguments are binary relations or sets and binary relations) but their output are sets (of individuals).

The semantics of reciprocal sentences is a complex matter (as shown for instance in Dalrymple *et al.*, 1998; Cable, forthcoming; Dotlačil, forthcoming; Mari, forthcoming). In fact there does not seem to be any general agreement concerning the data and the interpretation of reciprocal constructions (cf. Beck, 2000). In this paper I am not, strictly speaking, interested in the semantics of higher order anaphors but in the formal properties of functions denoted by higher order anaphors. Two types of such properties will be discussed: those which are similar to properties of functions denoted by simple anaphors and those which make them different from functions denoted by simple anaphors. Formal properties of functions denoted by simple anaphors have been studied in Keenan (2007), Zuber (2010b) and Zuber (2011) and some formal properties of higher order anaphors are given in Sabato and Winter (2012) and Peters and Westerståhl (2006). As far as I can tell, no comparison between the two types of function have been made. Moreover, only basic anaphors (that is syntactically simple anaphors) have been taken into consideration.

2 Formal preliminaries

We will consider binary relations and functions over universe E which is supposed to be finite. If a function takes only a binary relation as argument, its type is noted $\langle 2 : \tau \rangle$, where τ is the type of the output; if a function takes a set and a binary

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relation as arguments, its type is noted $\langle 1,2:\tau\rangle$. If $\tau=1$ then the output of the function is a set of individuals and thus the type of the function is $\langle 2:1\rangle$. For instance the function $SELF$, defined as $SELF(R)=\{x:\langle x,x\rangle\in R\}$, is of this type. The case we will basically consider here is when τ corresponds to a set of type $\langle 1\rangle$ quantifiers and thus τ equals, in Montagovian notation, $\langle\langle\langle e,t\rangle t\rangle t\rangle$. In short, the type of such functions will be noted either $\langle 2:\langle 1\rangle\rangle$ (functions from binary relations to sets of type $\langle 1\rangle$ quantifiers) or $\langle 1,2:\langle 1\rangle\rangle$ (functions from sets and binary relations to sets of type $\langle 1\rangle$ quantifiers).

Let R be a binary relation. Then : $dom(R)=\{x:\exists_y\langle x,y\rangle\in R\}$ and $rg(R)=\{x:\exists_y\langle y,x\rangle\in R\}$. Furthermore, for any $a\in E$, $aR=\{x:\langle a,x\rangle\in R\}$ and $Ra=\{x:\langle x,a\rangle\in R\}$. The relation R^{-1} is the converse of R (that is $R^{-1}=\{\langle x,y\rangle:\langle y,x\rangle\in R\}$) and the relation R^S is the maximal symmetric relation included in R, that is $R^S=R\cap R^{-1}$. A type $\langle 2:1\rangle$ or type $\langle 2:\langle 1\rangle\rangle$ function F is convertible iff $F(R)=F(R^{-1})$. Relation I is defined as $I=\{\langle x,x\rangle:x\in E\}$. The relation R^t is the transitive closure of the relation R, that is the smallest transitive relation in which R is included.

Basic type $\langle 1\rangle$ quantifiers are functions from sets (sub-sets of E) to truth-values. In this case they are denotations of subject NPs. However, NPs can also occur in oblique positions and in this case their denotations do not take sets (denotations of verb phrases) as arguments but rather denotations of intransitive verb phrases, that is relations, as arguments. To account for this eventuality it has been proposed to extend the domain of application of basic type $\langle 1\rangle$ quantifiers so that they apply to n-ary relations and act as arity reducers, that is have as output an (n–1)-ary relation. Since we are basically interested in binary relations, the domain of application of basic type $\langle 1\rangle$ quantifiers will be extended by adding to their domain the set of binary relations. In this case the quantifier Q can act as a"subject" quantifier or a "direct object" quantifier giving rise to the *nominative case extension* Q_{nom} and *accusative case extension* Q_{acc} respectively. They are defined as follows (Keenan, 1987; Keenan and Westerståhl, 1997):

D1: For each type $\langle 1\rangle$ quantifier Q, $Q_{nom}(R)=\{a:Q(Ra)=1\}$.

D2: For each type $\langle 1\rangle$ quantifier Q, $Q_{acc}(R)=\{a:Q(aR)=1\}$.

From now on $Q_{nom}(R)$ will be noted $Q(R)$. Nominative and accusative extensions can thus be considered as functions from binary relations to sets. By type $\langle 1\rangle$ quantifiers I will mean basic type $\langle 1\rangle$ quantifiers as well as their nominative and accusative extensions.

Given that type $\langle 1\rangle$ quantifiers and their arguments form Boolean algebras, every quantifier Q has its Boolean complement, denoted by $\neg Q$, and its post-complement $Q\neg$, defined as follows: $Q\neg=\{P:P\subseteq E\wedge P'\in Q\}$ (where P' is the Boolean complement of P). The dual Q^d of the quantifier Q is, by definition, $Q^d=\neg(Q\neg)=(\neg Q)\neg$. A quantifier Q is self-dual iff $Q=Q^d$. These definitions work also for extended type $\langle 1\rangle$ quantifiers. It easy to see for instance that $\neg(Q_{acc})=(\neg Q)_{acc}$ and $(Q^d)_{acc}=(Q_{acc})^d$. A type $\langle 1\rangle$ quantifier Q is *positive* iff $Q(\emptyset)=0$.

A special class of type $\langle 1\rangle$ quantifiers is formed by *individuals*, that is ultrafilters generated by an element of E. Thus I_a is an individual (generated by $a\in E$) iff $I_a=\{X:a\in X\}$. Ultrafilters are special (principal) filters. A (principal) filter generated by the set $A\subseteq E$ is the following quantifier: $Ft(A)=\{X:X\subseteq E\wedge A\subseteq X\}$. Thus ultrafilters are principal filters generated by singletons.

One property that we will use is the property of *living on*. The basic type $\langle 1\rangle$ quantifier lives on the set A (where $A\subseteq E$) iff for all $X\subseteq E$, $Q(X)=Q(X\cap A)$. If E is finite then there is always the smallest set on which a quantifier Q lives: it is the meet of all sets on which Q lives. The fact that A is the smallest set on which the quantifier Q lives will be noted $Li(Q,A)$. If $A\in Q$ then A is called the witness set of Q: $A=wt(Q)$. The quantifier Q is called *plural*, noted $Q\in PL$, iff $\exists_{a,b\in E}$ such that $Q\subseteq I_a\cap I_b$.

Functions from pairs of sets to truth-values or binary relations between sets are type $\langle 1,1\rangle$ quantifiers. In NLs they are denoted by (unary) nominal determiners, that is expressions which take one CN as argument and give a NP as output. Denotations of nominal determiners obey various constraints. Recall first the constraint of conservativity for type $\langle 1,1\rangle$ quantifiers. A well-known definition of conservativity is given in D5:

D3: $F \in CONS$ iff for any property X, Y one has $F(X, Y) = F(X, X \cap Y)$

Definition D3 can be generalised so that it applies to type $\langle 1, 2 : \tau \rangle$ functions (cf. Zuber 2010a):

D4: A function F of type $\langle 1, 2 : \tau \rangle$ is conservative iff $F(X, R) = F(X, (E \times X) \cap R)$

Observe that the above definition does not depend on the type τ of the result of the application of the function. So obviously it can be used with higher order functions. Type $\langle 2 : 1 \rangle$ functions can also be (predicate or argument) invariant and invariance is a property depending on the type of the output of the function. Thus (see Keenan and Westerståhl, 1997):

D5: A type $\langle 2 : 1 \rangle$ function F is predicate invariant iff $a \in F(R) \equiv a \in F(S)$ whenever $aR = aS$.

For instance the function $SELF$ is predicate invariant. The following definitions are generalisations of predicate invariance applying to type $\langle 2 : \langle 1 \rangle \rangle$ and type $\langle 1, 2 : \langle 1 \rangle \rangle$ functions:

D6: A type $\langle 2 : \langle 1 \rangle \rangle$ function F satisfies **HPI** (higher order predicate invariance) iff for any positive type $\langle 1 \rangle$ quantifier Q, any $A \subseteq E$, any binary relations R, S, if $A = Wt(Q)$ and $Ft(A)R = Ft(A)S$ then $Q \in F(R)$ iff $Q \in F(S)$.
D7: A type $\langle 1, 2 : \langle 1 \rangle \rangle$ function F satisfies **D1HPI** (higher order predicate invariance for unary determiners) iff for any positive type $\langle 1 \rangle$ quantifier Q, any $A \subseteq E$, any binary relations R and S, if $A = Wt(Q_1)$ and $Ft(A)R \cap X = Ft(A)S \cap X$ then $Q \in F(X, R)$ iff $Q \in F(X, S)$.

3 Reciprocals and reflexives

In this section I briefly present simple syntactic, or categorial, similarities and, possibly, differences, between reflexives and reciprocals, both simple and syntactically complex.

We will consider sentences of the form given in (1):

(1) NP TVP GNP

In this schema, GNP is a generalised noun phrase. GNPs are linguistic objects that can play the role of syntactic arguments of transitive verb phrases (TVPs). So ”ordinary” NPs or DPs (determiner phrases) are GNPs. However there are *genuine* GNPs which differ from ”ordinary” NPs in that they cannot play the role of all verbal arguments; in particular they cannot occur in subject position. This is the case of anaphoric expressions.

The GNPs related to reflexives and reciprocals are *anaphoric noun phrases* (ANPs). Roughly, their (”referential”) meaning depends on the meaning of another expression in the sentence, the so-called *antecedent of the anaphor*, by which it is bound. In the simplest case the antecedent is the subject NP. Thus a more specific form of sentences that we will consider of the form given in (2) instantiated in (3) and (4):

(2) NP TVP ANP.
(3) Most students washed themselves.
(4) Leo and Lea hate each other.

Thus the GNPs we consider are ANPs. In the above examples we have syntactically simple ANPs. Such ANPs can occur as syntactic parts of complex GNPs; in particular they can be parts of Boolean compounds and can be modified by categorially polyvalent modifiers such as *only, also, even, at least, let alone*, etc. :

(5a) Leo and Lea admire themselves and most teachers.
(5b) Leo and Lea admire each other, themselves and two teachers.
(6) Two monks hug each other only.

A special class of complex ANPs is formed by the application of *anaphoric determiners* (ADets), to CNs. Again, this can be done both with reflexive determiners and with reciprocal ones. Many languages have possessive anaphoric determiners. This is the case with Slavic languages which have the possessive ”determiner-pronoun” $SVOJ$ (meaning, roughly ’ones own’) which can be considered as ADet with reflexive meaning (cf. Zuber, 2011). Similarly, marking the simple reciprocal *each other* in English by the possessive marker results in a ADet with reciprocal meaning. This possibility is indicated in the following examples:

(7) Leo and Lea admire their own books.
(8) Leo and Lea admire each other's books.

Thus *their own* in (7) is a ADet with a reflexive meaning and *each other's* in (8) is an ADet with reciprocal meaning.

More interestingly it is possible to use an ordinary determiner (or its "part") and the simple ANPs *himself/herself/themselves* to form a ADet with reflexive meaning and to use an ordinary determiner and the simple ANPs *each other* to form an ADet with reciprocal meaning. Thus, roughly speaking (Zuber, 2010a), if *D* is an ordinary one place determiner, denoting monotone increasing function, then *D..., including himself* or *D...in addition to themselves* are ADets with the reflexive meaning. If *D* is a determiner denoting monotone decreasing functions then *D, not even himself* is an ADet as well. The following sentences contain various complex ADets with reflexive meaning:

(9) Two students admire most teachers in addition to themselves and Picasso
(10) Leo and Lea washed some vegetarians including at least themselves.
(11) Leo and Lea admires no philosophers, not even themselves or Socrates.

Quite similar procedure can be applied, though probably somewhat less productively, to (syntactically) simple and complex reciprocals in order to obtain ADets with reciprocal meaning. The following examples illustrate this possibility:

(12) Two students shaved most students including each other.
(13) Leo and Lea admire most logicians in addition to each other.
(14) Leo and Lea admire no philosopher, let alone each other.

As the following examples show simple and complex reflexives and reciprocals can occur also in other than direct object positions. The following example show that reflexives and reciprocals can be arguments of a verb taking three arguments:

(15) Leo protected himself/himself and Lea from Al.
(16) Leo and Lea protected every students from themselves.
(17) Most philosophers protect themselves from themselves.
(18) Most philosophers protect themselves and the president from themselves.
(19) Two monks protect themselves from the guru and themselves.
(20) Five philosophers protected each other from themselves.
(21) Leo and Lea/every student protected each other from Al.
(22) Leo and Lea protected every philosopher from each other.

This shows that reflexives can occur twice in a sentence in two different argumental positions of the verb. This is not the case with reciprocals:

?(23) Leo and Lea prevented each other from each other
?(24) Leo and Lea gave each other each other's book.

The above sentences are not acceptable, or at least not interpretable.

The difference pointed out by the above examples is related to the difference in the categorial status of reflexives on the one hand and reciprocals on the other. Thus it is usually assumed that ANPs with reflexive meaning are "argument" reducers: when applied to a di-transitive verb phrase they give a transitive verb phrase, and when applied to a transitive verb phrase they give just a VP.

The situation with reciprocals is different. Recall that ANPs are GNPs. GNPs apply to TVPs and give VPs as result. So what is the category of such VPs. Ignoring directionality, the subject NPs in the constructions we are interested in are of the category $S/(S/NP)$. This means that, in order to avoid type mismatch, verb phrases must be raised and have the category $S/(S/(S/NP))$. Then their denotational type is $\langle\langle\langle e, t\rangle t\rangle t\rangle$. Consequently, sentences of the form (1) are true iff the quantifier denoted by the NP is an element of the set denoted by $TVP\ GNP$. Thus ANPs with reciprocal meaning denote type $\langle 2 : \langle 1\rangle\rangle$ functions. This categorial difference is related to the following semantic difference. Consider the following examples:

(25a) Leo and Lea washed themselves
(25b) Bill and Sue washed themselves.

(26) Four persons, Leo, Lea, Bill and Sue washed themselves.
(27a) Leo and Lea hug each other.
(27b) Bill and Sue hug each other.
(28) Four persons, Leo, Lea, Bill and Sue hug each other.

Clearly (25a) in conjunction with (25b) entails (26) whereas (27a) in conjunction with (27b) does not entail (28). This means that the quantifiers denoted by the subject NPs in (27a) and (27b) do not apply to the predicate denoted by the complex VPs in these sentences and that the GNPs like *each other* denote type $\langle 2 : \langle 1 \rangle\rangle$ functions.

There are of course genuine type $\langle\langle\langle e,t\rangle t\rangle t\rangle$ (or type $\langle 2 : \langle 1 \rangle\rangle$ in our notation) functions, that is such that they are not lifts of simple type $\langle 2 : 1\rangle$ functions.

4 Higher order anaphors

We have seen that higher order anaphors denote type $\langle 2 : \langle 1 \rangle\rangle$ functions. Any type $\langle 2 : 1\rangle$ function whose output is denoted by a VP can be lifted to the type $\langle 2 : \langle 1 \rangle\rangle$ function. This is in particular the case with the acusative and nominative extensions of a type $\langle 1\rangle$ quantifier. For instance the accusative extension of a type $\langle 1\rangle$ quantifier can be lifted to type $\langle 2 : \langle 1 \rangle\rangle$ function in the way indicated in (29). Such functions will be called *accusative lifts*. More generally iff F is a type $\langle 2 : 1\rangle$ function, its lift F^L, a type $\langle 2 : \langle 1 \rangle\rangle$ function, is defined in (30):

(29) $Q^L_{acc}(R) = \{Z : Z(Q_{acc}(R)) = 1\}$.
(30) $F^L(R) = \{Z : Z(F(R)) = 1\}$

The variable Z above runs over the set of type $\langle 1\rangle$ quantifiers.

As we have seen, simple reflexives are interpreted by the function $SELF$. This function is of type $\langle 2 : 1\rangle$, that is a function which takes binary relations as argument and gives a set as result. Complex reflexives are interpreted by corresponding Boolean combination of $SELF$ with (lifted) denotations of NPs being a part Boolean compounds or, in the case of modification by categorially polyvalent particles, by modifications of $SELF$. Obviously, they are also of type $\langle 2 : 1\rangle$. These functions satisfy predicate invariance defined in D5. The function $SELF$, but not the functions denoted by complex reflexives, also satisfies the left predicate invariance:

D 8: A type $\langle 2 : 1\rangle$ function F is left predicate invariant iff for any $a \in E$ and any binary relations R, S, if $Ra = Sa$ then $a \in F(R)$ iff $a \in F(S)$ where $Ra = \{x : \langle x, a\rangle \in R\}$.

Accusative extensions of type $\langle 1\rangle$ quantifiers, which can also be considered as type $\langle 2 : 1\rangle$ functions, satisfy a stronger condition than predicate invariance. They satisfy so-called *accusative extension* condition AE (Keenan and Westerståhl, 1997):

D 9: A type $\langle 2 : 1\rangle$ function F satisfies AC iff for any $a, b \in E$ and any binary relations R, S, if $aR = bS$ then $a \in F(R)$ iff $b \in F(S)$.

It is important (Keenan, 2007) that functions denoted by reflexive expressions, simple and complex, do not satisfy AC and thus they are different from accusative extensions of type $\langle 1\rangle$ quantifiers denoted by ”ordinary” NPs in the object position. . In that sense, reflexive expressions are also genuine GNPs.

The corresponding higher order extension condition is defined in D10:

D10: A type $\langle 2 : \langle 1 \rangle\rangle$ function F satisfies **HEC** (higher order extension condition) iff for any positive type $\langle 1\rangle$ quantifiers Q_1 and Q_2, any $A, B \subseteq E$, any binary relations R, S, if $A = Wt(Q_1)$ and $B = Wt(Q_2)$, and $Ft(A)R = Ft(B)S$ then $Q_1 \in F(R)$ iff $Q_2 \in F(S)$.

Functions which are accusative lifts satisfy **HEC**. We will see that functions denoted by higher order anaphors do not satisfy **HEC** because functions satisfying **HEC** have the following obvious property:

Proposition 1: Let F be a type $\langle 2 : \langle 1 \rangle\rangle$ function which satisfies **HEC** and let $R = E \times C$, for $C \subseteq E$ arbitrary. Then for any $X \subseteq E$ either $Ft(X) \in F(R)$ or for any X, $Ft(X) \notin F(R)$.

In order to present various properties of functions denoted by higher order anaphors I will discuss only some such functions and not define all functions which constructions discussed in the previous section denote. Some other functions are discussed in Zuber (2012).

Consider first the function given in (31):

(31) $RFL\text{-}RECIP(R) = \{Q : \exists_{A \subseteq E} A = Wt(Q) \wedge Q(Dom(A \times A) \cap (R \cap R^{-1})) = 1\}$

Informally, this function can be considered as the denotation of an anaphor like *each other or oneself or themselves*. In other words it does not make *a priori* a distinction between "purely" reflexive and "purely" reciprocal interpretation, as apparently it happens in many languages. Observe in particular that individuals can be in the output of this function. Furthermore, the meet of two individuals can be in the output of this function even if they are in the relation R with themselves only.

The following function excludes the "reflexive part" and interprets purely reciprocal anaphors (in their strong logical reading):

(32) $SEA(R) = \{Q : A = Wt(Q) \wedge |A| \geq 2 \wedge Q(Dom((A \times A) \cap (R \cap R^{-1}) \cap I')) = 1\}$, where I' is the complement of the identity relation I.

Consider now example (33), where, clearly, a Boolean composition of two higher order functions is involved, one of which is an accusative lift:

(33) Leo and Lea admire each other and most teachers.

We want to give a function interpreting the complex anaphor *each other and most teachers*. Obviously this function has to entail the function SEA above and be completed by the part corresponding to *most teachers*. It is given in (34):

(34) $SEA_Q(R) = \{Z : Z \in SEA \wedge Z \in Q^L_{acc}\}$

The above functions are based on the relation R^S. Sentences in (35) have somewhat illogical interpretation. Functions corresponding to these interpretations are given in (36):

(35a) Five students followed each other.
(35b) All pupils followed each other and two teachers.
(36a) $ILEA(R) = \{Z : \exists_{A \subseteq E}(Li(Z, A) \wedge A \times A \cap I' \subseteq R^t\}$
(36b) $ILEA_{Qconj}(R) = ILEA(R) \cap Q^L_{acc}(R)$

Let us see now some constraints on the above functions. First we have:

Proposition 2: Functions $RFL\text{-}RECIP$, SEA and SEA_Q satisfy **HPI**.

Proof We prove only that $RFL\text{-}RECIP$ satisfies **HPI**. Suppose that $A = Wt(Q)$ and that $Q \in REF\text{-}RECIP(R)$. We have to show that if for some binary relation S (i) holds (i): $\forall_{x \in A}(xR = xS)$ then $Q \in RFL\text{-}RECIP(S)$. Given the definition of $RFL\text{-}RECIP$ this happens when $Q(Dom((A \times A) \cap (S \cap S^{-1}) = 1$. But if (i) holds then $(A \times A) \cap (R \cap R^{-1}) = (A \times A) \cap (S \cap S^{-1})$. Hence $Q \in RFL\text{-}RECIP(S)$.

It is easy to prove, using proposition 1, that:

Proposition 3: Functions $RFL\text{-}RECIP$, SEA and SEA_Q do not satisfy **HAI**.

Proof: We prove only that the function $RFL\text{-}RECIP$ does not satisfy **HAI**. Given its definition in (31) we can see that for $C \subseteq E$ arbitrary , for any C_1 such that $C \subseteq C_1$ we have $Ft(C_1) \notin RFL\text{-}RECIP(E \times C)$ and for any $C_2 \subseteq C$ we have $Ft(C_2) \in RFL\text{-}RECIP(E \times C)$. Hence, given proposition 1, $RFL\text{-}RECIP$ does not satisfy **HPI**.

Here are some other properties:

Proposition 4: Let $F \in \{RFL\text{-}RECIP, SEA, ILEA\}$ and $R = S^{-1}$. Then $F(R) = F(S)$

Proposition 4 has an interesting consequence: since $R = (R^{-1})^{-1}$, it follows from Proposition 2 that functions $RFL\text{-}RECIP, SEA$ and $ILEA$ are convertible.

The above properties do not hold for complex higher order functions that is functions denoted by syntactically complex reciprocals. For higher order functions based on the relation R^S the following proposition holds:

Proposition 5: Let $F \in \{RFL\text{-}RECIP, SEA, SEA_Q\}$, $R = S^{-1}$ and

$Dom(R) = Dom(S)$. Then $F(R) = F(S)$.

To illustrate Proposition 5 consider the following examples:

(37a) Five students followed each other.
(37b) Five students preceded each other.

If we consider that the relation expressed by *follow* is the converse of the relation expressed by *precede* that (37a) and (37b) are equivalent.

Observe that the property of functions expressed in Proposition 6 does not depend on the type of the output of the function. It is easy to see, for instance that many reflexives functions denoted by reflexives have a similar property. More precisely we have:

Proposition 6: Let $F \in \{SELF, SELF \otimes Q_{acc}\}$, where $\otimes$ is a Boolean connective, $R = S^{-1}$ and $Dom(R) = Dom(S)$. Then $F(R) = F(S)$.

Thus Propositions 5 and 6 express, informally, properties of functions sensitive to some aspects of their arguments only. Conservativity, as defined in D4 is such a property. Definition of conservativity given in D4 naturally applies to functions denoted by anaphoric determiners. The conservativity of anaphoric determiners giving rise to reflexives is discussed in Zuber (2010b). We are not directly interested here in the semantics of anaphoric determiners but it would be easy to show that the anaphoric determiner *Every...except each other* as it occurs in (38) denotes a conservative function:

(38) Two students washed every student except each other.

To conclude let us see some other properties of functions denoted by anaphors. These functions are "sensitive" to some aspects of their arguments, that is to some properties of the binary relations to which they apply. Consider the following definition:

D11: A type $\langle 2 : \tau \rangle$ function F is *symmetry sensitive*, $F \in SYMS$, iff $F(R) = F(S)$ whenever $R \cap R^{-1} = S \cap S^{-1}$.

Functions $SELF$, $RFL\text{-}RECIP$ and SEA are symmetry sensitive. Functions denoted by complex anaphors (reflexive or reciprocal) do not have this property. They have the following property:

D12: A type $\langle 2 : \tau \rangle$ function F is *symmetry and range sensitive*, $F \in SYMRS$ iff $F(R) = F(S)$ whenever $R \cap R^{-1} = S \cap S^{-1}$ and $Rg(R) = Rg(S)$.

Note that $SYMS \subseteq SYMRS$. Thus not only functions denoted by complex anaphors but also those denoted by simple anaphors are symmetry and range sensitive. This is what all anaphors have in common. In order to distinguish anaphors with purely reflexive meaning from those with purely reciprocal meaning the following definitions can be used:

D13: A type $\langle 2 : \tau \rangle$ function F is *symmetry only sensitive*, $F \in SYMOS$, iff $F(R) = F(S)$ whenever $R \cap R^{-1} \cap I' = S \cap S^{-1} \cap I'$ and $Rg(R) = Rg(S)$.
D14: A type $\langle 2 : \tau \rangle$ function F is *reflexivity and range sensitive*, $F \in REFLRS$, iff $F(R) = F(S)$ whenever $R \cap I = S \cap S \cap I$ and $Rg(R) = Rg(S)$.

For instance *only each other* denotes a symmetry only sensitive function and *himself* or *himself and most students* denote reflexivity and range sensitive functions.

Observe that $SYMOS \subseteq SYMRS$ and $REFLS \subseteq SYMRS$. Similarly $SYMS \subseteq SYNRS$. Thus purely reflexive anaphors denote functions which are not symmetry only sensitive and purely reciprocal anaphors denote functions which are not reflexivity sensitive but both classes are symmetry and range sensitive.

5 Conclusive remarks

It has been shown that it is preferable to treat simple and complex reciprocal expressions, belonging to the class of higher order anaphors, as denoting type $\langle 2 : \langle 1 \rangle \rangle$ functions (that is functions having relations as arguments and sets of type $\langle 1 \rangle$ quantifiers as result) and not as denoting type $\langle 1, 2 \rangle$ quantifiers, as usually proposed. The main reason for this treatment is the fact that the basic reciprocal expression *each other* can combine not only with NPs (which denote (extensions of) type $\langle 1 \rangle$

quantifiers) but also with expressions which denote functions which are not quantifiers (or their extensions). In that respect the reciprocals are similar to reflexives since functions interpreting reflexives (like function $SELF$, its modifications and its Boolean compounds) are neither quantifiers not extensions of a type $\langle 1 \rangle$ quantifier.

It is well-known (Keenan 2007; Zuber 2010b) that the existence of anaphors in NLs shows that the expressive power of natural languages would be less than it is if the only noun phrases we needed were those interpretable as subjects of main clause intransitive verbs. The reason is that anaphors like *himself, herself* must be interpreted by functions from relations to sets which lie outside the class of generalised quantifiers as classically defined. In this paper some preliminary results are presented to show that the existence of higher order anaphors even further extends the expressive power of NLs.

The move to consider that higher order anaphors denote genuine type $\langle 2 : \langle 1 \rangle \rangle$ functions allows us to understand the "non-Boolean" behaviour of the conjunction *and* in their context. Observe, for instance, that (39a) in conjunction with (39b) does not entail (40):

(39a) Leo and Lea hug each other.
(39b) Bill and Sue hug each other.
(40) Four people hug each other.

Functions denoted by higher order anaphors satisfy higher order invariance: they are predicate invariant in a generalised sense. They are different from quantifiers denoted by NPs on the direct object position because they do not satisfy the higher order accusative extension which accusative lifts satisfy. In that respect they similar to functions denoted by simple anaphors (reflexives) which are predicate invariant and do not satisfy the accusative extension condition.

Various conservativity-like properties of functions denoted by reciprocals have been also exhibited. Thus it has been indicated that both types of anaphoric determiners, those giving rise to reflexives and those giving rise to reciprocals, denote conservative functions. Moreover, it has been formally expressed how both types of functions are "sensitive'" only to some aspects of binary relations which are their arguments.

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