Enthymemetic Conditionals: Topoi as a guide for acceptability

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Abstract

To model conditionals in a way that reflects their acceptability, we must include some means of making judgements about whether antecedent and consequent are meaningfully related or not. Enthymemes are non-logical arguments which do not hold up by themselves, but are acceptable through their relation to a topos, an already-known general principle or pattern for reasoning. This paper uses enthymemes and topoi as a way to model the world-knowledge behind these judgements. In doing so, it provides a reformalisation (in TTR) of enthymemes and topoi as networks rather than functions, and information state update rules for conditionals.

1 Introduction

The content of the antecedent and consequent of a conditional, not just their truth or falsity, makes a difference to whether we find the conditional acceptable or not, generally rejecting those that seem disconnected (Douven, 2008). If we are to model conditionals in a way that reflects their acceptability, we must include some means of making those judgements. Enthymemes are non-logical arguments which do not hold up by themselves, but are acceptable through their relation to a topos, an already-known general principle or pattern for reasoning. Arguments and justifications in interaction tend to be underpinned by general principles and rules of thumb, rather than being truly 'logical'. For models of dialogue to be adequate then, these non-logical arguments need to be handled – namely, as proposed by Breitholtz (2014a), through incorporating enthymemes and topoi into the dialogue model. Apart from the evidence from their own acceptability conditions, which correlate strongly with judgements of high conditional probability, conditional structures are also associated with 'that kind of thinking', being used as plain-language explanations of particular topoi (e.g. "if something is a bird, then it flies" in Breitholtz, 2014b), or used as materials on reasoning in any number of experiments (e.g. Pijnacker et al., 2009). If we are going to explicitly recognise the use of such 'rule' type objects in discourse, then conditionals are one place where they show up, at least sometimes.

This paper has two aims. First, to propose a formalisation of enthymemes and topoi that is geared towards relating them to more complex rule-based world knowledge, including a distinction between knowledge about causality, non-causality, and ambiguity about causality. Second, to account for the acceptability (or not) of conditionals by proposing an enthymeme-like structure as associated with *if*-conditionals, such that topoi can enhance their content and are used in judging whether a given conditional is acceptable or not. The acceptability of conditionals is linked to perceived relationships between the antecedent and consequent cases: with enthymemes and topoi, whose presence in the model is independently justified, we can incorporate this non-arbitrarily into the dialogue state.

The rest of this section will provide some background. Section 2 is focused on enthymemes, topoi, and specification of the alternative formalism, while Section 3 uses this in a proposal of update rules associated with conditionals. Lastly, Section 4 provides a conclusion. This paper draws on work on enthymemes and topoi elsewhere in Breitholtz (2014a,b) etc., and will likewise use Type Theory with Records (Cooper, 2012, hereafter referred to by the acronym TTR) for formalisation.

1.1 Enthymemes and Topoi

Enthymemes are incomplete non-logical arguments that get treated as complete ones. They are

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incomplete in that to be accepted, they must be identified as a specific instance of a more general pattern that is already in the agent's resources – a topos. Topoi encode world knowledge that comes as a 'rule of thumb', such as characteristics typical of groups, and a speaker may hold contradictory topoi as equally valid in different scenarios, with no clash experienced unless both are used at the same time. Speakers make enthymemetic arguments by linking what on the surface might technically be non-sequiturs, but are easily identified as an argument using accepted principles. For example, a speaker might say "Let's go left here, it's a shortcut". This argument invokes the assumption that shorter routes are better, and that therefore the left turn being a shortcut is a good reason to take it – but they might equally say "it's longer", invoking an assumption that a longer route is preferable.

Topoi have been proposed to be a resource available to speakers, and consequently a means to address non-monotonic reasoning (Breitholtz, 2014b), the treatment of non-logical rules as expressing necessity, and contradictory claims being equally assertable, as in the route-taking example above (Breitholtz, 2014a).

To these ends, they have been formalised in TTR for use in dialogue (Breitholtz and Cooper, 2011), as functions from records to record types, as in this example (Breitholtz, 2014a):

(1) a. *Topos:*

$$\lambda r : \begin{bmatrix} x & : Ind \\ c_{bird} & : \text{bird}(x) \end{bmatrix} \left(\begin{bmatrix} c_{fly} & : \text{fly}(r.x) \end{bmatrix} \right)$$

b. *Enthymeme:*

$$\lambda r : \begin{bmatrix} x = \text{Tweety} & : Ind \\ c_{bird} & : \text{bird(x)} \end{bmatrix} \left(\begin{bmatrix} c_{fly} & : \text{fly(Tweety)} \end{bmatrix} \right)$$

Both are of type *Rec* → *RecType*, and the fields of the specified record types match, but fields of the enthymeme have been restricted to specific values. A function to a record type does not by itself indicate what happens once we have access to that type. For these functions to be useful, they are additionally governed by a theory of action, which will license various actions that can be performed with the type, e.g. judging that the original situation is additionally of that type, judging that there exists some situation of the type described, or creating something of that type (Cooper, in prep).

1.2 Conditionals

The assumption that conditionals express a proposition is fundamental to most linguistic work on the topic, both that which follows the commonly accepted restrictor theory of conditional semantics based on the work of Lewis (1975), Kratzer (1986) and Heim (1982), and that which does not (e.g. Gillies (2010)).

By conditionals being 'propositional', we mean that adding an *if*-clause to some indicative clause does not fundamentally change the kind of semantic object it is: for indicative clause "I'm going home", just as the conjunction "I'm going home *and I'm watching a film*" still expresses a proposition, so does "*If this doesn't get interesting soon*, I'm going home".

As mentioned at the beginning, the acceptability of conditionals correlates strongly with their conditional probability: the more likely the consequent is in the antecedent-case, the more acceptable the conditional tends to be be. Stalnaker (1970) proposed that the probability of a conditional and the conditional probability of the consequent on the antecedent are one and the same, in what is usually referred to as *the Equation*. That is, the overall probability $P($*if this doesn't get interesting then I'm going home*$)$ is the same as the conditional probability $P($*I'm going home*$|$*This doesn't get interesting*$)$. A subsequent proof by Lewis (1976) found that there is no single proposition based on the antecedent and consequent such that its probability will consistently match the conditional probability. Therefore one could have a propositional theory of conditionals, or validate the Equation – but not both.

However, conditional probability seems so important to the meaning of conditionals that in the view of some non-linguists, (e.g. Edgington, 1995; Bennett, 2003) conditionals should properly be considered be probabilistic, directly expressing the conditional probability of the consequent on the antecedent, P(*cons*|*ant*). Subsequent empirical work overwhelmingly supports the intuition behind the original Equation, and shows that conditional probability does indeed tend to correlate with acceptability (e.g. Evans et al., 2003; Oaksford and Chater, 2003). Conditional probability thus needs to be taken seriously, whether one believes it is the core content of a conditional or not: indeed, figuring out how propositional theories can accommodate its relationship to acceptability is an important issue (e.g. Douven and Verbrugge, 2013). Conditional probability is also not the only factor in acceptability: it is further moderated by whether there appears to be a connection between antecedent and consequent (Skovgaard-Olsen et al., 2016). To make these judgements, we need to know about the relationships between the antecedent and consequent states.

As a note, this paper remains technically agnostic about whether the propositional or probabilistic analysis is correct: your mileage may vary on whether the update rules in Section 3 should also add a proposition associated with *if p, q* to the agent's knowledge base, were they to be more comprehensively specified. The underlying acceptability issue, and the potential use of topoi in the metrics underlying those acceptability judgements, means that this does not impact on the core of the proposals here.

1.3 TTR: a brief overview

Since it will be used later, this section provides a very brief introduction to TTR.

A central idea in TTR is the judgement of objects as being of some type. If a is judged to be of type T, this is written as $a : T$. Several of these judgements, or requirements for judgements, can be collected in structured objects as records and record types. In a record type, fields consist of a label and type, while fields in a record consist of a label and a value. For a record r to be of a record type *RT*, it must have fields with the labels specified in *RT*, and the values in those fields in r must be the types specified by the equivalently labelled fields in *RT*. For example, the records in (2) and (3) are both of the record type (4), provided that x is of type T_1. The type of a field need not be stand-alone either: it may also be constructed from a predicate and arguments, like the field d in (5).

(2) $\begin{bmatrix} a = x \end{bmatrix}$

(3) $\begin{bmatrix} a = x \\ b = y \\ c = z \end{bmatrix}$

(4) $\begin{bmatrix} a : T_1 \end{bmatrix}$

(5) $\begin{bmatrix} a : T_1 \\ d : p(a) \end{bmatrix}$

(6) $\begin{bmatrix} a : T_1 \\ b : T_2 \end{bmatrix}$

(7) $\begin{bmatrix} a : T_1 \\ c : T_3 \end{bmatrix}$

There also exist sub- and super-type relations between types. One record type is a subtype of another if it is a more specified version of it. This means that it has at least the same fields as the supertype, whose types are the same type or subtypes of the equivalent fields in the supertype. For example, (6) and (7) are different types, but are both subtypes of the more general (4). A record of type (6) is not necessarily of type (7), but will be of type (4). Depending on whether $x : T_1$, $y : T_2$ and $z : T_3$, the record in (3) will be of all three types.

2 Enthymemes, Topoi and Other Knowledge

Given that their presence in an agent's resources has already been motivated, topoi are a natural way to account for the required knowledge about some 'dependence' between antecedent and consequent. Enthymemes and topoi are snippets of reasoning, rather than complex networks, but they should also be related explicitly to other rule-like world knowledge, which includes the possibility of multiple relationships between more than two cases, and knowledge of explicitly causal relations. If we are going to use topoi to express the kind of knowledge that also forms such networks (i.e. informative about causality or related probabilities), then they should be in the same form as that knowledge: the alternative, to keep rule-like topoi apart from knowledge about rule-based(ish) systems, is counter-intuitive.

Bayesian networks (a combination of directed acyclic graphs and probability distributions) are a common way to encode causal relations. They have two components, the first of which is a directed acyclic graph, with the various variables as nodes, and directed edges describing any direct relationships. Graphs and networks are a useful way to describe relationships, and express a more complex set of relationships than a linear chain of functions. The graph structure is in accordance with constraints about what direct parenthood in the graph can mean – that the parent is part of the minimal set of preceding nodes whose value determines the probability distribution of the child.

The second component to a Bayesian Network is a set of probability functions for determining the values of variables given the values of their parents – their conditional probabilities. Associated probabilities are also a natural means of modelling learning, by adjusting the confidence in a given rule on the basis of evidence and experience, allow us to make explicit the level of confidence in a judgement beyond a binary. For unreliable rules, a high (but below 1) probability can be used to express that they are likely to be correct in a given case, but not certain.

2.1 Graphical Topoi

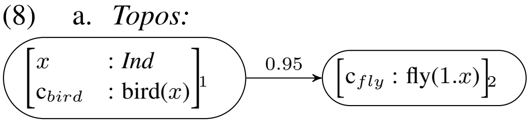
The proposal is as follows. Topoi and enthymemes are of the same type as any other 'relational' knowledge, by which I mean knowledge about causal and correlational relations. This knowledge can be encoded as a graph: topoi and enthymemes as usually discussed are minimal examples, containing only two nodes. The direction(s) of the links between connected nodes, along with additional constraints, indicate either causal or non-causal relations via directed or bi-directed links respectively.

Where there is a bi-directional link somewhere in a path between two nodes, their relationship is confirmed as non-causal. Where there is an absence of any path between two nodes, the relationship may be treated as potential independence, while where there are links in one direction only, the relationship may be treated as potential causality. However, neither the potential independence or causality is locked in: there should be a distinction between merely lacking information, and having confirmation about an absence. Certainty about independence or causality is expressed via constraints explicitly preventing the creation of any path that would violate them.

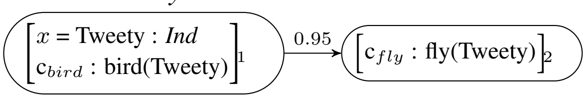
The choice of bi-directed rather than undirected edges to express non-causality is motivated by a desire for the difference in belief from potentially causal to non-causal to be something that changes easily (i.e. with the addition of information, not replacement of one thing with another of a different type), and for creation of a 'casual' (not a typo) middle-ground, where only one direction is of relevance and there is no strong commitment either way. It can be treated as potentially causal, being the only direction of interest, but whether this is the whole story between the two is not specified.

All this is meant to allow for a more complex set of relationships than expressed in your average topos which, as stated earlier, is a minimal case with just two nodes. The original example can be thought of as follows, graphs with only two nodes:

(8) a. *Topos:*



b. *Enthymeme:*



Once x is filled (as 'Tweety'), this should be reflected in any other nodes where the same variable appears. The confidence rating of 0.95 has been somewhat arbitrarily set here for topoi to imply high confidence without certainty. Generally, the confidence rating associated with a link in a known network should be subject to change on the basis of experience, increasing or decreasing as their predictions are borne out or subverted. Topoi as 'rules of thumb' are particularly robust to contradictory evidence, with the same agent in different contexts accepting and using topoi that lead to opposite conclusions: see, for example, notions *opposites attract* vs. *birds of a feather flock together*. Integration of ordinary learning with the potential for entrenched 'against all evidence' beliefs is a larger topic that is not addressed here, but will be necessary in future work.

Enthymemes are distinguished from other arguments by the fact they don't hold up by themselves, but are instead accepted on the basis of identification with a topos – this doesn't include arguments that are accepted despite being unsupported. However, the terms enthymeme and topos will continue to be used here: this is partly for convenience, but also because once the context indicates that an enthymemetic argument is being made (such as a recognisable suggestion+motivation pattern like "Let's go left here, it's a shortcut"), an unsupported 'enthymeme', once accepted, can be used to establish a potential new topos (Breitholtz, 2015).

2.2 Graphical Topoi in TTR

This subsection provides a treatment in TTR of the above proposal. The variable at each node is a $RecType$, representing a situation, with the probability of a $RecType$ being across whether it is true or false (for type *T*, whether $\exists a : T$). Let *RecType*$_i$ be a *RecType* associated with an index, and *ProbInfo* be a constraint on some probability. The supertype of enthymemes and topoi, rather than a function *Rec→RecType*, is the type *Network*:

(9) *Network* $=_{def}$

$$\left[\begin{array}{ll} \text{nodes} : \left\{\mathit{RecType}_i\right\} & \\ \text{links} : \left\{\langle \mathit{RecType}_i, \mathit{RecType}_i\rangle\right\} & \\ \text{probs} : \left\{\text{ProbInfo}\right\} & \\ \text{c}_{index} : \forall\langle x'_j, y_p\rangle, \in \text{links}, x'_j \sqsubseteq_r x_i \in \text{nodes}, i = j, & \\ \qquad\qquad \forall\langle z_q, x''_k\rangle \in \text{links}, x''_k \sqsubseteq_r x_i \in \text{nodes}, i = k. & \\ \text{c}_{links} : \forall\langle x'_i, y'_p\rangle \in \text{links}, \exists x_i, y_p \in \text{nodes}, x'_i \sqsubseteq_r x_i, y'_p \sqsubseteq_r y_p & \end{array}\right]$$

The *nodes* field is the set of nodes in the graph, while the *links* field is the set of directed edges between them, each 'link' being an ordered pair. Let $\sqsubseteq_r$ indicate a subtype relation where subtyping is through restriction of one or more fields i.e. not through the specification of extra fields. The first constraint c_{index} enforces co-indexing, that if subtypes of a node are included in members of *links*, they all share the same index. The second constraint c_{links} specifies that any members of *links* are between (potentially restricted subtypes of) members of $nodes$. For ease of reading and the sake of space, the constraints will not be repeated in further examples. In a link $\langle x_i, x_j \rangle$, the specification of member x_i may use j to indicate some $r : x_j$, and vice versa, e.g. where a is some field in x_i and b is some field in x_j, in x_i we can specify that $a = j.b$.

Causality, non-causal correlation and independence are interpreted on the basis of the members of *links*. Where a path is a sequence of indices $\langle 1, \ldots, k \rangle$ such that for each $i, i+1$ there is $\langle x_i, x_{i+1} \rangle \in$ *links*, the node indexed i is a predecessor of the node indexed j (shorthand: *predecessor*$(i, j, \textit{links})$) if there is a path from i to j, given the contents of *links*. In this way the set *links* can be checked for evidence that two nodes are in a non-causal relation (if there is a bi-directional *predecessor* relation somewhere in a path between the two, e.g. if $\langle x_i, x_j \rangle, \langle x_j, x_i \rangle \in$ *links*), are potentially independent (there is no *predecessor* relation at all between the two), or in a potentially causal relation (one is a *predecessor* of the other, but not the other way around). We can distinguish direct and indirect causality by whether a minimal path with a direct link $\langle x_i, x_j \rangle$ is possible or not. As a rule, when we talk about causality, we will mean direct causality.

For n : *Network* containing nodes x_i and x_j, independence and causality can be expressed in updated n' as follows, where $a \wedge b$ indicates the merge of two records, a record containing all fields from both, and $a \boxed{\wedge} b$ indicates their asymmetric merge (see Cooper and Ginzburg, 2015), where in event of a field appearing in both records, the field from b is the one found in the merge, effectively overwriting the field of a.

(10) *Independence of i and j:*

$$n' = n \wedge \left[c_{ind_{ij}} : \neg predecessor(i, j, links) \wedge \neg predecessor(j, i, links) \right]$$

(11) *Direct causality from i to j:*

$$n' = n \wedge \left[c_{cause_{ij}} : \langle i, j \rangle \in links \wedge \neg predecessor(j, i, links) \right]$$

(12) *Indirect causality from i to j:*

$$n' = n \wedge \left[c_{indcaus_{ij}} : predecessor(j, i, links) \wedge \neg predecessor(j, i, links) \right]$$

The original example can now be rewritten as (13):

(13) *Topos:*

$$\begin{bmatrix} \text{nodes} = \\ \left\{ \begin{bmatrix} \text{x : Ind} \\ c_{bird} : \text{bird(x)} \end{bmatrix}_1, \begin{bmatrix} \text{x : Ind} \\ c_{fly} : \text{fly(x)} \end{bmatrix}_2 \right\} : \{ RecType_i \} \\ \text{links} = \\ \left\{ \left\langle \begin{bmatrix} \text{x : Ind} \\ c_{bird} : \text{bird(x)} \end{bmatrix}_1, \begin{bmatrix} \textbf{x = 1.x} : \text{Ind} \\ c_{fly} : \text{fly(x)} \end{bmatrix}_2 \right\rangle \right\} : \{ \langle RecType_i, RecType_i \rangle \} \\ \text{probs} = \\ \left\{ P\left(\begin{bmatrix} \textbf{x = r.x} : \text{Ind} \\ c_{fly} : \text{fly(x)} \end{bmatrix}_2 \mid \text{r} : \begin{bmatrix} \text{x : Ind} \\ c_{bird} : \text{bird(x)} \end{bmatrix}_1 \right) = 0.95 \right\} : \{ \text{ProbInfo} \} \end{bmatrix}$$

(14) *Enthymeme:* as above, but all variants indexed with 1 are replaced with $\begin{bmatrix} \textbf{x = Tweety} : \text{Ind} \\ c_{bird} : \text{bird(x)} \end{bmatrix}_1$

An *Enth* is defined as a *Network* containing a node that has at least one field restricted to a specific object, removing its generality. A *Topos* is a *Network* in which no fields are restricted to a specific object.

An enthymeme e may be identified with a topos t if its nodes and links have equivalents in t, that is if for every node $x_i \in e.nodes, \exists y_p \in t.nodes$ such that $x_i \sqsubseteq y_p$ and for any links $\langle x'_i, x'_j \rangle \in e.links, \exists \langle y'_p, y'_q \rangle \in t.links$ such that $x'_i \sqsubseteq y'_p$ and $x'_j \sqsubseteq y'_q$. This may be by a clear match for the topos fields, but may also include the types of fields in the enthymeme as subtypes of fields in the topos[1].

3 Conditionals and Reasoning

Having reformalised topoi and enthymemes as an object intended for more general correlational and causal knowledge, i.e. like a Bayesian network[2] we turn back to conditionals.

[1] as in the example "Give a coin to the porter, he carried the bags all the way here" from (Breitholtz, 2014b), where carrying someone else's bags is recognised as a subtype of work, and the enthymemetic argument is on the basis of a topos like *work should be rewarded*

[2] though not strictly: the graph of a Bayesian network should be acyclic, while these do allow for cycles

Firstly, and as mentioned at the beginning, expressing this kind of relational knowledge is (both intuitively and according to empirical evidence) strongly associated with conditionals, and existence of a dependence relation and high conditional probability usually determine their acceptability. Van Rooij and Schulz (2019) suggest a way to combine these two features into a single measure, the relative difference the state of the parent in a relation makes to the likelihood of the child. Pleasingly, with some independence assumptions this measure works not only for the 'causal' direction typically expressed by conditionals (*if there's fire, there's smoke*), but for the reverse as expressed by evidential conditionals (*if there's smoke, there's fire*). However, for it to do so, the direction of the relationship still has to be recognised even when the 'usual' roles of antecedent as parent and consequent as child have flipped. This kind of structural knowledge is topoic.

Secondly, and while it feels almost trivial to point out, we use conditionals to tell each other new things, e.g. the speaker explaining their experiences with "if you done anything wrong well you get, you get the cane and anything else" (BNC, *H5G 78*). When we are informed of something through the use of a conditional, we don't necessarily know beforehand that they lie in such a relation: otherwise they would only be useful to draw attention to connections we haven't made, not to tell each other things that are entirely new. Indeed, Skovgaard-Olsen et al. (2016) found evidence that when faced with a conditional, people assume that there is a positive connection between antecedent and consequent unless they have reason to believe otherwise. It is not so much that an acceptable conditional has to be backed up by pre-existing knowledge about the relation between the antecedent and consequent cases, but at the very least it should not *clash* with any.

Breitholtz (2014a) mentions how an enthymemetic argument can be recognised on the basis of the current conversational game/expected rules (with the specific example of knowledge that a suggestion may be followed by the speaker providing a motivation), or by an explicit lexical cue. With the above in mind, I will suggest that use of an *if*-conditional is one such linguistic cue.

3.1 Enthymemetic Conditionals

The overall suggestion is as follows. *If*-conditionals are associated with the making of enthymeme-like arguments. Note that I say "enthymeme-*like* arguments", not "enthymemetic arguments". Enthymemes depend on identification with a previously-known topos, while conditionals can be used to teach new relations, rather than just make statements that rely on existing knowledge to make sense. Although they are structured like the characterisation of enthymemes and topoi above, they are not all strictly speaking 'enthymemetic'. The content of a conditional can be checked against the topoi in the agent's resources. Given a match with a topos, an enhanced version can be added to the agent's knowledge.

Even without a guiding topos, conditionals allow us to express or learn information via an assumption that there is a positive connection between antecedent and consequent – provided we do not already know that the two are independent, or that the consequent shouldn't follow from the antecedent. If no supporting topos is found, a more minimal version can be added without the benefit of any extra details a topos might have provided.

The direction of antecedent as parent is 'default' in the sense that it should be preferred if distinct topoi in both directions are available, and is the direction assumed in case neither a supporting topos nor a conflicting one is found. The topoi in an agent's resources may conflict with each other, and by necessity one of them was learned first: despite this, a conditional does not lead to formation of an acceptable enthymeme when such a clashing topos is already present. If there only exists a potential match for the nodes in a topos that specifies there is definitely no link, or is a conflicting link, then the conditional should be rejected.

The processes of comparing a potential enthymeme with a topos and of updating structured knowledge on the basis of a conditional can be thought of algorithmically as follows:

(15) *Match between an enthymeme and topos:*
Search known topoi for topos with a node matching the first enthymeme node
If none: no match, `false`.
If found: check topos for nodes matching each further node in enthymeme.
If any failure: resume searching topoi.

If found: check each edge in enthymeme has an equivalent in topos.
If any failure: resume searching topoi.
If found: check any constraints in enthymeme have an equivalent in topos.
If found: match, `true`.
If any failure: resume searching topoi.

(16) *Enhancing an enthymeme with a topos:*
Make new copy of topos.
For each node in topos with an equivalent in enthymeme, add any further specification.
For any node in topos with no equivalent node in enthymeme, but with elements also found in a node that was further specified, update accordingly.

(17) *Updating with a conditional:*
Check for conflicting topos.
If found, `reject`.
If not found, check for topos matching *ant→cons* equivalent link.
If found, `enhance` *ant→cons* and `add`.
If not found, check for topos matching *ant←cons* equivalent link.
If found, `enhance` *ant←cons* and `add`.
If not found, `add` *ant→cons*.

Below are illustrations of what should be understood from the evidential conditional "If the glass fell, the cat pushed it", given knowledge of a topos equivalent to *if someone pushes something, the thing falls*.

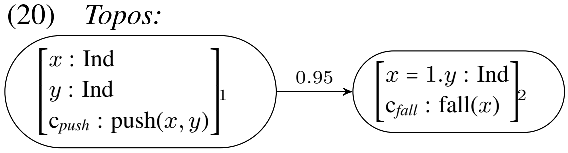
(18) *Ant. content:*

$$\begin{bmatrix} x = \text{obj}_3 \\ \text{c}_{glass} = \text{glass}(\text{obj}_3) \\ \text{c}_{fall} = \text{fall}(\text{obj}_3) \end{bmatrix}$$

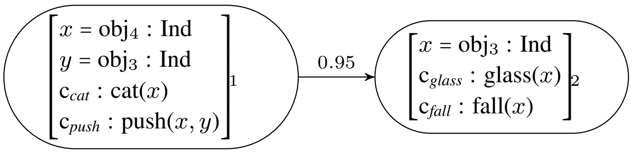
(19) *Cons. content:*

$$\begin{bmatrix} x = \text{obj}_4 \\ y = \text{obj}_3 \\ \text{c}_{cat} = \text{cat}(\text{obj}_4) \\ \text{c}_{push} = \text{push}(\text{obj}_4, \text{obj}_3) \end{bmatrix}$$

(20) *Topos:*



(21) *Enhanced enthymeme:*



The following subsections describe dialogue state update rules associated with conditionals, characterised in TTR.

3.2 Enthymemetic Conditionals in TTR

To begin with, the type of an information state is minimally given as (22), broadly following the decisions for the place of enthymemes and topoi in Breitholtz (2014a) etc.

(22) *InfoState* $=_{def}$

$$\begin{bmatrix} \text{priv} & : \begin{bmatrix} \text{Topoi} & : \{\text{Topos}\} \end{bmatrix} \\ \text{shrd} & : \begin{bmatrix} \text{enths} & : \{\text{Enth}\} \\ \text{Topoi} & : \{\text{Topos}\} \\ \text{Moves} & : \text{list(LocProp)} \end{bmatrix} \end{bmatrix}$$

(23) *Update rule* $=_{def}$

$$\begin{bmatrix} \text{pre} & : \text{InfoState} \\ \text{effects} & : \text{Infostate} \end{bmatrix}$$

The information state has two parts: the agent's private resources, and their representation of the shared context. The *private* resources include a set of general topoi which they can use as resources. A public *Topoi* field tracks which topoi have been introduced onto the dialogue gameboard. The general form for update rules is given in (23): *pre* describes the preconditions for states to which the rule can be applied, and *effects* the changes.

Next we will add a few useful functions on the basis of some of the content of Section 2.1: a means to describe whether there is a successful match between an enthymeme and a topos, and a means to reference the result of an enthymeme that has been enriched by the content of a topos.

(24) *enthMatch*(e : Enth, t : Topos) : Bool, **true** iff all of the following hold
(i) All e's nodes are subtypes of t's nodes: $\forall x_i \in e.nodes, \exists y_p \in t.nodes$ such that $x_i \sqsubseteq y_p$,
(ii) All e's links are subtypes of t's links: $\forall \langle x'_i, x'_j \rangle \in e.links, \exists \langle y'_p, y'_q \rangle \in t.links$ such that $x'_i \sqsubseteq y'_p$ and $x'_j \sqsubseteq y'_q$,
(iii) For any constraints on links in e, the same constraints hold for equivalent links in t: $\forall c_{ind_{ij}} \in e, \exists c_{ind_{pq}} \in t$ or $\text{c}_{ind_{qp}} \in t$, $x_i \in e.nodes, y_p \in t.nodes, x_i \sqsubseteq y_p$ and $x_j \in e.nodes, y_q \in t.nodes, x_j \sqsubseteq y_q$. Likewise for all $\text{c}_{cause_{ij}} \in e$, there is an equivalent $\text{c}_{cause_{pq}} \in t$.

(25) *enhanceEnth*(e : Enth, t : Topos) : Enth, e' such that e' is an asymmetric merge of t and e,

where the sets in *nodes*, *links* and *probs* undergo asymmetric union such that for any nodes $x_i \in e.nodes, y_p \in t.nodes$, $x_i \sqsubseteq y_p$, the corresponding node $z_u \in e'.nodes = y_p \boxed{\wedge} x_i$.

Likewise for any subtypes x'_i and y'_p, $x'_i \sqsubseteq y'_p$ in members of $e.links$, $t.links$, $e.probs$ and $t.probs$.

That is, the asymmetric aspect of the merge is at the level of the indexed nodes, not the fields containing them.

The update rules for each case are given in the subsections below. There are three rules given: where there is a supporting topos in the 'default' direction, where there is not but there is a supporting topos in the reverse direction, and where there is neither support nor a clash.

3.2.1 Recognising a supporting topos

First are the update rules for when the agent has a topos linking the two parts of the conditional: The update in case of a supporting topos in the *ant→cons* direction is given in (26):

(26) *default direction, ant→cons:*

$$\left[\begin{array}{l}
\text{pre}: \\
\quad \left[\begin{array}{l}
\text{priv}: \\
\quad \left[\begin{array}{ll}
\text{Topoi} & : \{\text{Topos}\} \\
t & : \text{Topos} \\
c_{member} & : t \in \text{Topoi} \\
c_{def} & : \textit{enthMatch}(x : X, t)
\end{array}\right] \\
\text{shrd}: \left[\text{Moves[0]} = \text{Assert(if(a, b))} \quad : \text{LocProp}\right]
\end{array}\right] \\
\text{effects}: \\
\quad \left[\begin{array}{l}
\text{shrd}: \\
\quad \left[\begin{array}{l}
\text{enths} = \text{pre.shrd.enths} \\
\qquad \cup\ \textit{enhanceEnth}(x : X, t) : \{\text{Enth}\} \\
\text{Topoi} = \text{pre.shrd.Topoi} \cup \text{pre.priv.}t : \{\text{Topos}\}
\end{array}\right]
\end{array}\right]
\end{array}\right]$$

where X is the type

$$\left[\begin{array}{l}
\text{nodes} = \{\text{a.sit-type}_1, \text{b.sit-type}_2\} : \textit{RecType}_i \\
\text{links} = \{\langle \text{a.sit-type}_1, \text{b.sit-type}_2 \rangle\} : \langle \textit{RecType}_i, \textit{RecType}_i \rangle \\
\text{probs} = \{\text{P(b.sit-type}_2 | \text{ r : a.sit-type}_1) = 0.95\} : \text{ProbInfo}
\end{array}\right]$$

This rule may be applied following assertion of a conditional, where an agent knows a topos t that matches an enthymeme based on the content of the conditional, with a link from antecedent to consequent. In this case, the agent may add such an enthymeme enhanced with the topos to their *enths*, and add the underlying topos to the set of currently active topoi in the conversation.

Where such an option does not exist, a topos with only a link from consequent to antecedent can be used, as described in (27). The enthymeme added to *enths* in this case will contain a link only in the *ant←cons* direction.

(27) *alternative direction, ant←cons:*

$$\left[\begin{array}{l}
\text{pre}: \\
\quad \left[\begin{array}{l}
\text{shrd}: \left[\text{Moves[0]} = \text{Assert(if(a, b))} \quad : \text{LocProp}\right] \\
\text{priv}: \left[\begin{array}{ll}
\text{Topoi} & : \{\text{Topos}\} \\
t & : \text{Topos} \\
c_{member} & : t \in \text{Topoi} \\
c_{no\text{-}def} & : \nexists t', t' \in \textit{Topoi} \wedge \textit{enthMatch}(x : X, t') \\
c_{alt} & : \textit{enthMatch}(y : Y, t)
\end{array}\right]
\end{array}\right] \\
\text{effects}: \\
\quad \left[\begin{array}{l}
\text{shrd}: \left[\begin{array}{l}
\text{enths} = \text{pre.shrd.enths} \\
\qquad \cup\ \textit{enhanceEnth}(y : Y, t) : \{\textit{Enth}\} \\
\text{Topoi} = \text{pre.shrd.Topoi} \cup \text{pre.priv.}t : \{\textit{Topos}\}
\end{array}\right]
\end{array}\right]
\end{array}\right]$$

where X is as defined in (26), and Y is the type

$$\left[\begin{array}{l}
\text{nodes} = \{\text{a.sit-type}_1, \text{b.sit-type}_2\} : \textit{RecType}_i \\
\text{links} = \{\langle \text{b.sit-type}_2, \text{a.sit-type}_1 \rangle\} : \{\langle \textit{RecType}_i, \textit{RecType}_i \rangle\} \\
\text{probs} = \{\text{P(a.sit-type}_1 | \text{ r : b.sit-type}_2) = 0.95\} : \{\text{ProbInfo}\}
\end{array}\right]$$

Relative to (26), the update rule for this case has a constraint in its preconditions that there are no topoi with a link in the *ant→cons* direction, and the enthymeme is instead enhanced by a topos that does support the alternative order.

3.2.2 New information

The last rule describes the case where the agent's known topoi have neither evidence about a link between the antecedent or consequent, the definite absence of one, or a conflicting one. In this case, an 'enthymeme' with a link in the *ant→cons* direction may be added to *enths* solely on the basis of the conditional content. No additional topos is added to the list of active topoi – the process for generalising an acceptable enthymeme to a reusable topos is not addressed here.

The shorthand for presence of a clashing topos is given in (28) as *enthClash*. An enthymeme clashes with a topos where the equivalent parent nodes lead to mutually exclusive child nodes, i.e. child nodes where a true type cannot be formed from their meet.

(28) *enthClash*(e : Enth, t : Topos) : Bool,
true iff
$\exists x_i, y_j \in e.nodes, p_i, q_j \in b.nodes, x_i \sqsubseteq p_i$,
$\exists \langle x'_i, y'_j \rangle \in e.links, x'_i \sqsubseteq x_i, y'_j \sqsubseteq y_j$,

$$\exists \langle p_i', q_j' \rangle \in t.links,\ p_i' \sqsubseteq p_i, q_j' \sqsubseteq q_j,$$
$$\text{and } \neg T, \text{ where } T = y_j' \wedge q_j'$$

(29) *neither support nor opposing knowledge:*

$$\begin{bmatrix} \text{pre}: \begin{bmatrix} \text{shrd}: \begin{bmatrix} \text{Moves[0] = Assert(if(a, b))} : \text{LocProp} \end{bmatrix} \\ \text{priv}: \begin{bmatrix} \text{Topoi}: \{\text{Topos}\} \\ c_{\textit{no-clash}} : \nexists t, t \in \textit{Topoi} \wedge \textit{enthClash}(x : X, t) \\ c_{\textit{no-def}} : \nexists t, t \in \textit{Topoi} \wedge \textit{enthMatch}(x : X, t) \\ c_{\textit{no-alt}} : \nexists t, t \in \textit{Topoi} \wedge \textit{enthMatch}(y : Y, t) \\ c_{\textit{no-ind}} : \nexists t, t \in \textit{Topoi} \wedge \textit{enthMatch}(z : Z, t) \end{bmatrix} \end{bmatrix} \\ \text{effects}: \begin{bmatrix} \text{shrd}: \begin{bmatrix} \text{enths = pre.shrd.enths} \cup x : X \end{bmatrix} \end{bmatrix} \end{bmatrix}$$

where X, Y are as in (26), (27), and Z is the type

$$\begin{bmatrix} \text{nodes} = \{\text{a.sit-type}_1, \text{b.sit-type}_2\} : \textit{RecType}_i \\ \text{links} = \varnothing : \{\langle \textit{RecType}_i, \textit{RecType}_i \rangle\} \\ \text{probs} = \varnothing : \{\textit{ProbInfo}\} \\ c_{\textit{ind}_{12}} : \neg \textit{predecessor}(1, 2, \text{links}) \\ \qquad \wedge \neg \textit{predecessor}(2, 1, \text{links}) \end{bmatrix}$$

Relative to the previous two update rules, the preconditions in this rule specify that there is no known topos that supports an enthymeme with a link between the antecedent and consequent in either direction, which has an explicit constraint enforcing independence between the two, or which otherwise clashes with the possible conditional enthymeme.

4 Conclusion

The acceptability of a conditional is often determined by the conditional probability of the consequent on the antecedent, and recognition of some meaningful link between the two. However, both intuitively and according to experimental evidence, positive acceptability judgements can still be made without fore-knowledge of such a connection. This paper presented two proposals on the basis that the knowledge enabling these judgements is topoic, integrating these factors into the representation of the dialogue state and agent resources. First, a formalisation of enthymemes and topoi as graphs was presented, on the grounds that they should be in the same form as other knowledge about causal and correlational relationships. Second, update rules for conditionals using topoi and enthymemes were presented, drawing on topoi to recognise the presence and direction of a 'meaningful' connection between antecedent and consequent, and making an assumption of one in the absence of any evidence.

There are several avenues for further work. Most work focuses on declarative conditionals, the most common form by far. However, conditional clauses are also used to form conditionalised questions and directives. The proposals here should be related to these forms, whether because to an extent they apply in those cases too, or because this topoic association is exclusive to declarative conditionals. This paper has also said nothing about more standard propositional aspects of conditionals. The proposals here about structural knowledge associated with conditionals should be integrated with this more standard fare.

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