

Span-based discontinuous constituency parsing: a family of exact chart-based algorithms with time complexities from $\mathcal{O}(n^6)$ down to $\mathcal{O}(n^3)$

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Abstract

We introduce a novel chart-based algorithm for span-based parsing of discontinuous constituency trees of block degree two, including ill-nested structures. In particular, we show that we can build variants of our parser with smaller search spaces and time complexities ranging from $\mathcal{O}(n^6)$ down to $\mathcal{O}(n^3)$. The cubic time variant covers 98% of constituents observed in linguistic treebanks while having the same complexity as continuous constituency parsers. We evaluate our approach on German and English treebanks (Negra, Tiger, and DPTB) and report state-of-the-art results in the fully supervised setting. We also experiment with pre-trained word embeddings and Bert-based neural networks.

1 Introduction

Syntactic parsing aims to recover the latent syntactic relations between words in a sentence, expressed in a given syntactic formalism. In this paper, we focus on constituency trees where the syntactic structure is described by the means of a hierarchical structure composed of nodes: words are leaf nodes whereas internal nodes represent labeled constituents or phrases, see Figure 1. Constituency trees can broadly be classified into two categories. On the one hand, in a *continuous* constituent tree, each node must dominate a contiguous sequence of words.¹ On the other hand, in a *discontinuous* constituent tree, a node can dominate a non-contiguous sequence of words. It has been argued that modeling discontinuity is unavoidable, see for example McCawley (1982) and Bunt et al. (1987) for English and Müller (2004) for German.

Phrase-structure grammars have been proposed to parse and generate constituency trees. For ex-

ample, Context-Free Grammars (CFG) are able to process continuous constituent trees whereas Multiple Context Free Grammars (Seki et al., 1991, MCFG) and Linear Context-Free Rewriting System (Vijay-Shanker et al., 1987, LCFRS) are able to process discontinuous constituent trees. CFGs have been widely studied for practical parsing due to the availability of time-efficient *chart-based* parsing algorithms based on dynamic programming: parsing a sentence of length n is a $\mathcal{O}(gn^3)$ problem where g is a grammar related constant (Kasami, 1966; Younger, 1967; Cocke, 1969). However, parsing algorithms for MCFGs and LCFRSs are deemed to be impractical despite their polynomial-time complexity (see Section 2). Therefore, most of the experimental work in this field has been limited to parsing short sentences, e.g. sentences that contain less than 40 words (Kallmeyer and Maier, 2010; Evang and Kallmeyer, 2011; Maier et al., 2012; Kuhlmann and Nivre, 2006).

Advances in machine learning led to the development of constituency parsers that are no longer based on phrase-structure grammars. Instead, the prediction step only ensures the well-formedness of the resulting structure and does not enforce compliance of the syntactic content represented by the structure. For example, a verbal phrase is not constrained to contain a verb. As such, they can be assimilated to the mainstream approach to bi-lexical dependency parsing where one considers candidate outputs only in a restricted class of graphs: non-projective (McDonald et al., 2005), projective (Eisner, 1997) or bounded block degree and well-nested spanning aborescences (Gómez-Rodríguez et al., 2009, 2011; Corro et al., 2016), among others (Kuhlmann and Nivre, 2006; Satta and Kuhlmann, 2013; Pitler et al., 2012). These approaches assume that intricate relations in the syntactic content can be implicitly learned by the scoring function.

Span-based parsing is a grammarless approach

*Work partially done while the author was a postdoc at University of Amsterdam with Ivan Titov.

¹The set of words that a node dominates is the set of leaf nodes in the subtree for which this node is the root.

to constituency parsing that decomposes the score of a tree solely into the score of its constituents, originally proposed for continuous constituency parsing (Hall et al., 2014; Stern et al., 2017; Cross and Huang, 2016).² Recovering the highest scoring tree can be done exactly using a slightly updated CYK algorithm or using inexact³ methods like top-down or transition based algorithms. This approach has obtained state-of-the-art results for continuous constituency parsing (Stern et al., 2017; Kitaev and Klein, 2018; Kitaev et al., 2019). In this work, we propose the first span-based parser with an exact decoding algorithm for discontinuous constituent parsing. To this end, we introduce a novel exact chart-based algorithm based on the parsing-as-deduction formalism (Pereira and Warren, 1983) that can parse constituent trees with a block degree of two, including ill-nested structures (see Section 3), which have been argued to be unavoidable to model natural languages (Chen-Main and Joshi, 2010). Therefore, our constituency parser is closely related to the dependency parser of Gómez-Rodríguez et al. (2009, 2011). Despite its $\mathcal{O}(n^6)$ time-complexity, where n is the length of the input sentence, our algorithm is reasonably fast: in our experiments, all treebanks can be parsed without removing long sentences. Moreover, we observe that several deduction rules are of little use to retrieve trees present in treebanks. Therefore, we experiment with variants of the algorithm where we remove specific deduction rules. This leads to parsing algorithms with lower asymptotic complexity that experimentally produce accurate parses. Importantly, we show that a specific form of discontinuity can be parsed in $\mathcal{O}(n^3)$, that is with the same asymptotic complexity as continuous constituency parsing.

Our main contributions can be summarized as follows:

- we propose a new span-based algorithm for parsing discontinuous constituency trees of block degree two with exact decoding and reasonable average execution time;
- we propose a cubic-time algorithm that can parse a significant portion of discontinuous

²In contrast, for example, to several transition systems that can incorporate scores related to actions that were executed during derivation, or to split point decision and left-right span scores in the parser of Stern et al. (2017).

³The term inexact refers to the fact that these methods are not guaranteed to recover the highest scoring structure.

constituents in various corpora while having the same theoretical complexity as continuous constituency parsers, without requiring any preprocessing or post-processing steps contrary to the approaches of van Cranenburgh et al. (2016) and Boyd (2007);

- we report state-of-the-art parsing results on these treebanks in a fully supervised setting and experiment with pre-trained word embeddings, including Bert based models.

We release the C++/Python implementation of the parser.⁴

2 Related Work

Phrase-structure grammars: The LCRFS formalism has been widely used in the context of discontinuous constituency parsing, although MCFG and Simple Range Concatenation Grammars (Boullier, 1998) have been shown to be equivalent, see Seki et al. (1991) and Boullier (2004). Kallmeyer and Maier (2010) introduced the first practical chart-based LCFRS parser for German, which was subsequently applied to English (Evang and Kallmeyer, 2011). However, they restrict their data to sentences that contain less than 25 words. To improve parsing time, Maier et al. (2012) proposed an experimentally faster parser based on the A^* search algorithm together with a block degree two restriction. However, they still limit the sentence size to a maximum of 40 words. A single sentence of 40 words takes around 3 minutes to be parsed, an impressive improvement over the parser of Kallmeyer and Maier (2010) that needs several hours, but still prohibitively slow for large scale parsing. Concurrently to this work, Stanojević and Steedman (2020) proposed a span based grammar-less LCFRS parser. They also explored the well-nested variants.

Graph based parsing: A different line of work proposed to explore constituency parsing as a dependency parsing problem.⁵ In other words, even if it is straightforward to represent constituency trees as hierarchical phrase structures, the same syntactic content can be represented with different mathematical objects (Rambow, 2010), including directed graphs commonly used for de-

⁴<https://github.com/FilippoC/disc-span-parser-release>

⁵Note that opposite line of work also exists, that is reducing dependency parsing to constituency parsing, see for example Maier and Kallmeyer (2010).

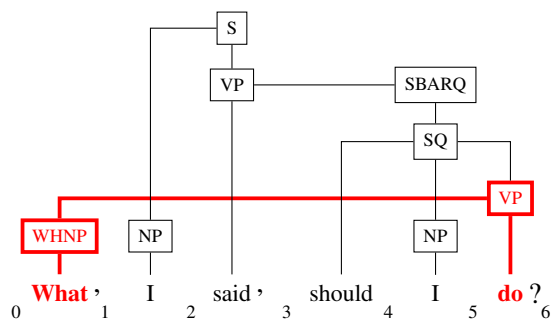


Figure 1: Example of a discontinuous constituency tree. The bold red VP node dominates two sequences of words: “What” and “do”. All other nodes are continuous. Numbers below the sentence are interstice indices used in the algorithm description.

pendency parsing. Fernández-González and Martins (2015) reduced the (lexicalized) constituency parsing task to dependency parsing where the constituency structure is encoded into arc labels. Then, discontinuous constituency parsing is reduced to the labeled *Spanning Arborescence* problem which can be solved in quadratic time. The same reduction has also been used in a sequence-to-sequence framework (Fernández-González and Gómez-Rodríguez, 2020). Corro et al. (2017) proposed a joint supertagging and dependency parsing reduction where vertices represent supertags⁶ and labeled arcs encode combination operations (substitution and adjunction). The problem is then reduced to the labeled *Generalized Spanning Arborescence* problem which is known to be NP-hard (Myung et al., 1995). One benefit of these approaches is that they do not assume any restriction on the constituency structure: they can parse ill-nested structures and have no block degree restriction. However, they cannot impose such constraints, which may be beneficial or required and they factor the score of a tree into dependency, supertag and/or label scores, which means that the learning objective is not directly related to the evaluation metric which focuses on constituents. Moreover, the factorization relies on possibly erroneous heuristics (head-percolation tables) to lexicalize the original structure if the information is not present in the treebank. On the contrary, in this work, we directly score parts of the syntactic content (i.e. labeled constituents). Therefore, at training time we can optimize an objective directly related to the

⁶A supertag is an elementary tree that encodes the sequence of lexicalized constituents for which a given word is the head, see Bangalore and Joshi (1999)

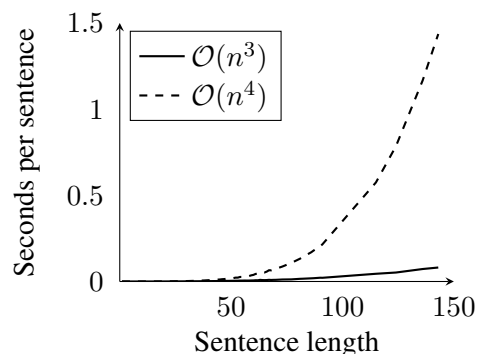


Figure 2: Execution time per sentence length of the chart-based algorithm for the $O(n^3)$ (solid line) and $O(n^4)$ (dashed lines) variants.

end-goal evaluation.

Transition systems: Lastly, transition-based parsers have been proposed, based on the idea of the SWAP transition for non-projective dependency parsing (Nivre, 2009), see Versley (2014) and following work based on shift-reduce strategy (Maier, 2015; Maier and Lichte, 2016; Stanojević and Alhama, 2017a). These systems rely on the fact that a discontinuous tree can be transformed into a continuous one by changing word order in the input sentence. They do not require strong independence assumption on the scoring model which can be useful to encode richer information, especially for long-distance relationships. However, the number of transitions required to parse discontinuities can impact prediction accuracy. To alleviate this problem, two different approaches have been explored: Coavoux and Crabbé (2017) introduced a two-stack system coupled with a GAP transition and Maier and Lichte (2016) proposed the SHIFT-I transition to access non-local elements directly, therefore reducing the number of transitions. In exchange for a rich parameterization, transition systems lose optimality guarantees with respect to the scoring model and rely on greedy or beam-search decoding. These approaches achieve state-of-the-art results while being fast at test time (Coavoux and Cohen, 2019; Coavoux et al., 2019). On the contrary, our approach is exact with respect to the scoring model, i.e. it will always return the highest scoring structure.

Search space restriction: Previous work in constituency and dependency parsing proposed to restrict the search space to improve time complexity while at the same time ensuring a good coverage of data. Satta and Schuler (1998) proposed an al-

gorithm that parses a subclass of Tree Adjoining Grammar in $\mathcal{O}(n^5)$ instead of $\mathcal{O}(n^6)$. Our parsing algorithm is highly related to the dependency parser of Gómez-Rodríguez et al. (2011) and the restricted variant of the latter introduced by Pitler et al. (2012).

3 Parsing Algorithm

We describe our algorithm using the *parsing-as-deduction* framework (Pereira and Warren, 1983). As such, our description is independent of the value one wants to compute, whether it be the (k-)best derivation(s), the partition function or span marginals (Goodman, 1999).⁷ However, we will focus on argmax decoding.

We are interested in constituency trees of block degree two, including ill-nested trees. The block degree two constraint is satisfied if each node dominates at most two disjoint sequences of words. Let $w_1 \dots w_n$ be a sentence. A constituency tree for this sentence is ill-nested if it contains two nodes dominating disjoint sets of words $W^{(1)}$ and $W^{(2)}$ such that there exists $w_i, w_j \in W^{(1)}$ and $w_k, w_l \in W^{(2)}$ such that $i < k < j < l$ or $k < i < l < j$.

Filtering: Contrary to CFGs and LCFRS CKY-style parsers, there is no side-condition constraining allowed derivations in span-based parsers. The label of a constituent is independent of the label of its children.

Binarization: Interestingly, span-based parsers do not require explicit binarization of the constituency structure. Although grammar based parsers require binarization of the grammar production rules and therefore of the constituency structure to ensure tractable complexity, span-based parsers can take care of binarization *implicitly* by introducing a supplementary constituency label with a fixed null score.

Unary rules: We follow Stern et al. (2017) and merge unary chains into a single constituent with a new label, e.g. the chain $\text{SBARQ} \rightarrow \text{SQ}$ will result in a single constituent labeled SBARQ_SQ .

3.1 Items

Let \mathcal{N} be the set of non-terminals (labels) and n the length of the input sentence. We define spans with interstice indices instead of word indices, see Figure 1. Items manipulated by our deduction rules

⁷Note that parsing without grammatical constraints results in all sentences having a non-empty parse forest, therefore the recognition problem is ill-defined.

are 5-tuples $[A, i, k, l, j]$ where $A \in \mathcal{N} \cup \{\emptyset\}$ is the constituent label with value \emptyset indicating a null label used for implicit binarization. Given that each item represents a constituent, we will use the same notation to refer to the chart item and to the linguistic structure interchangeably. Indices $i, j \in \mathbb{N}, k, l \in \mathbb{N} \cup \{-\}$ define the constituent span:

- if the constituent is continuous, then $k = l = -$ and $0 \leq i < j \leq n$;
- otherwise, the constituent is discontinuous (with a single gap) and $0 \leq i < k$ and $l < j \leq n$, with $k < l$, define its left and right spans, respectively.

For example, the tree in Figure 1 contains the discontinuous constituent $[\text{VP}, 0, 1, 5, 6]$.

3.2 Axioms and goal

Axiom items are word level constituents, i.e. items of the form $[A, i, -, -, i + 1]$ with $0 \leq i < n$ and $A \in \mathcal{N} \cup \{\emptyset\}$. In our experiments, axioms can have a null label, i.e. $A = \emptyset$, because we do not include part-of-speech tags as leaves of the constituency tree.

The goal item is defined as $[A, 0, -, -, n]$ with $A \in \mathcal{N} \cup \{\emptyset\}$. The goal can have a null label, so we can parse empty trees and disconnected structures occurring in treebanks without further pre/post-processing steps.

3.3 Deduction rules

The deduction rules used to derive the goal from axioms are listed on Figure 3. Each rule takes exactly two premises. Note that rule (a) is the only rule needed for continuous constituency parsing.

Rule (b) creates a discontinuous constituent from two continuous constituents. The set of rules (e)-(f)-(g)-(h) (resp. (c)) allow to combine one discontinuous and one continuous constituent to produce a discontinuous one (resp. a continuous one).

Finally, there are rules that combine two discontinuous antecedents. Rule (d) is the only such rule that is allowed for building well-nested trees. The other four rules (i)-(j)-(k)-(l) are used for the construction of ill-nested trees. As such, it is easy to control whether ill-nested structures are permitted or not by including or excluding them.

3.4 Soundness and completeness

The algorithm is sound by definition because items cannot represent constituents with a gap degree

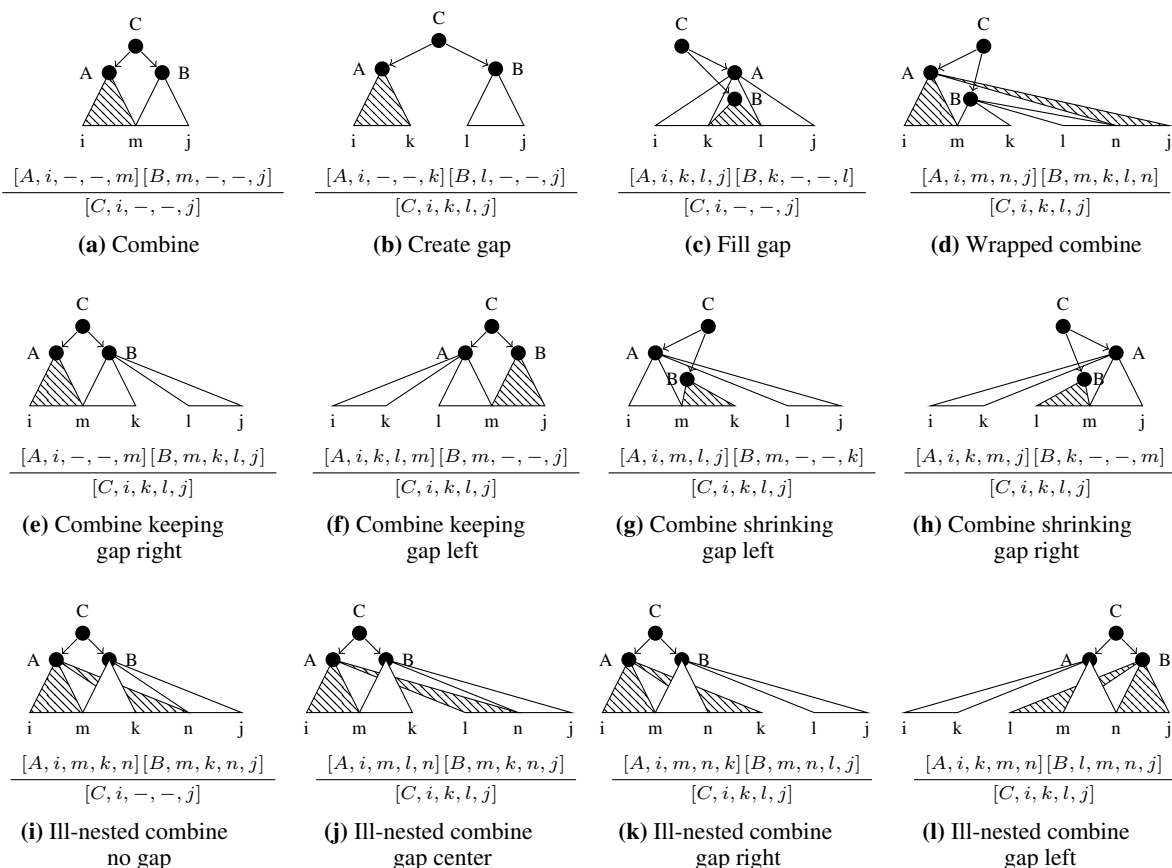


Figure 3: Deduction rules of our algorithm.

strictly greater than two and every rule deduces an item representing a constituent spanning a greater number of words, therefore they cannot construct invalid trees where a parent constituent spans fewer words than one of its children.

Completeness can be proved by observing that every possible binary parent-children combination can be produced by one of the rules. For the non-binary case, completeness follows from the fact that a constituent with 3 or more children can be built by first deriving intermediary constituents with label \emptyset .

3.5 Complexity

The space and time complexity can be inferred from item structures and deduction rules: the space complexity is $\mathcal{O}(|\mathcal{N}|n^4)$ and time complexity is $\mathcal{O}(|\mathcal{N}|^3n^6)$. In practice, we decompose the score of a tree into the sum of the score of its constituents only and there are no constraints between antecedents and consequent labels. Therefore, we can build intermediary unlabeled items of the form $[i, k, l, j]$ which replace antecedents in every rule in Figure 3. With this update, the time complexity is linear in the number of labels, that is, $\mathcal{O}(|\mathcal{N}|n^6)$.

We instantiate variants of the algorithm that cannot parse the full family of block degree two trees but that can still fit most actual linguistic structures present in treebanks, with a lower time complexity. By using only rules (a), (b) and (c) we can build a parser with a $\mathcal{O}(n^4)$ time complexity. In the next section, we show that this specific variant can be optimized into a $\mathcal{O}(n^3)$ time parser. By adding rules (e), (f), (g), (h) and (i) we build a $\mathcal{O}(n^5)$ parser. Finally, we construct $\mathcal{O}(n^5)$ and $\mathcal{O}(n^6)$ well-nested parsers by excluding rules (i), (j), (k) and (l).

3.6 Cubic time discontinuous constituency parser

A specific variant uses only deduction rules (a), (b) and (c) from Figure 3, leading to a $\mathcal{O}(n^4)$ space and time complexity. In this setting, there is no way to combine two items representing discontinuous constituents or to have a discontinuous constituent that has a discontinuous child in the resulting parse tree. In this section, we prove that the family of trees induced by this variant of the parser can actually be parsed with a $\mathcal{O}(n^3)$ time complexity, that is equivalent to continuous constituency parsers.

The intuition goes as follows. We could replace

rules **(b)** and **(c)** with the single rule **(m)** in Figure 4 where the right hand side condition $D \in \mathcal{N}$ implies the existence of a discontinuous constituent with label D ,⁸ with left part spanning words i to k and right part spanning words l to j . However, observe that this new rule performs two tests that could be done independently:

1. the right span boundary of the first antecedent must match the left span boundary of the second one;
2. the right span boundary of the second antecedent must match the left span boundary of the third antecedent.

Therefore, we can break the deduction into two sequential deductions, first testing the " k " boundary then the " l " boundary.⁹

To this end, we build a parser based on 4-tuple items $[A, \tau, i, j]$ where $\tau \in \{\top, \perp\}$ indicates whether the item represents a continuous constituent ($\tau = \top$) or an incomplete discontinuous constituent ($\tau = \perp$). More precisely, an item $[A, \perp, i, j]$ represents a partial discontinuous constituent who would be represented as $[A, i, ?, j, ?]$ in the previous formalization. The right boundary of its two spans are unknown: the one of the left span has been "forgotten" and the one on the right span is yet to be determined. The deduction rules of this new parser are listed on Figure 4, with axioms $[A, \tau, i, i + 1]$, $0 \leq i < n$, and goal $[A, \tau, 0, n]$.

We report the running time per sentence length for the $\mathcal{O}(n^4)$ and $\mathcal{O}(n^3)$ parsers in Figure 2. As expected, the running time of the cubic time parser is way lower for long sentences.

4 Experiments

We experiment on the Discontinuous Penn Treebank (Marcus et al., 1993; Evang and Kallmeyer, 2011, DPTB) with standard split, the Tiger treebank (Brants et al., 2002) with the SPMRL 2014 shared task split (Seddah et al., 2014) and the Negra treebank (Skut et al., 1997) with the split proposed by Dubey and Keller (2003).

⁸Without loss of generality, we assume the label D is not null. Although it could be without changing the overall idea, we would just add an extra way to do implicit binarization that can already be handled with rule **(a)**.

⁹This idea of breaking up simultaneous tests in a deduction rule has been previously proposed for improving time complexity of lexicalized grammar parsers (Eisner and Satta, 1999, 2000)

4.1 Data coverage

One important question is whether our parser has a good coverage of the dataset as we can only retrieve constituents of block degree one and two. We report the maximum recall that our parser can achieve in its different variants in Table 1.

First, we observe that our cubic time parser can recover 98% of all constituents in the three treebanks, or around 80% of constituents of block degree of exactly two. Second, the $\mathcal{O}(n^5)$ variant of the parser can recover more than 99% of all treebanks, and, interestingly, there is almost no coverage change when moving to the full deduction system. If we consider the parsers with well-nested restriction, the $\mathcal{O}(n^5)$ and $\mathcal{O}(n^6)$ variants have the same coverage in German datasets and the latter can only recover 2 additional constituents in the English treebanks. If we include ill-nested construction, the difference is either 2 (DPTB and Negra) or 8 (Tiger) constituents. In practice, we observed that both $\mathcal{O}(n^5)$ and $\mathcal{O}(n^6)$ variants predict the same analysis.

4.2 Neural parameterization

We use a neural architecture based on bidirectional LSTMs detailed in Appendix A.

Constituent scores Even with the block degree two restriction, there is a larger number (quartic!) of constituent scores to compute. In early experiments, we observed that weighting such a number of constituents without further decomposition blow up the neural network memory usage and was prohibitively slow. Therefore, we introduce a score decomposition that results in a quadratic number of scores to compute.

We decompose the score of a constituent $[A, i, k, l, j]$ as the sum of a score associated with its outer boundaries (i.e. indices i and j) and one with its gap boundaries (i.e. indices k and l). The score of the constituent is defined as:¹⁰

$$W_{A,i,k,l,j} = \begin{cases} \mathbf{S}_{A,i+1,j}^{(c. \text{ label})} + S_{i+1,j}^{(c. \text{ span})} & \text{if } k = l = -, \\ \mathbf{S}_{A,i+1,j}^{(o. \text{ label})} + S_{i+1,j}^{(o. \text{ span})} & \text{otherwise.} \\ + \mathbf{S}_{A,k+1,l}^{(g. \text{ label})} + S_{k+1,l}^{(g. \text{ span})} & \end{cases}$$

where tensors $\mathbf{S}^{(c. \text{ label})}$, $\mathbf{S}^{(o. \text{ label})}$, $\mathbf{S}^{(g. \text{ label})} \in \mathbb{R}^{|\mathcal{N} \cup \{\emptyset\}| \times n \times n}$ and matrices

¹⁰The +1 in tensor indices result of the fact that we use interstice indices for constituents but that the neural network layers focus on word indices.

		Continuous	$\mathcal{O}(n^3)$	$\mathcal{O}(n^5) / \text{WN}$	$\mathcal{O}(n^5)$	$\mathcal{O}(n^6) / \text{WN}$	$\mathcal{O}(n^6)$
DPTB	All	98.16	99.46	99.81	99.83	99.81	99.83
	BD ≤ 2	98.32	99.63	99.98	99.99	99.98	100.00
	BD = 2	0.00 (0)	78.27 (10713)	99.15 (13572)	99.98 (13685)	99.17 (13574)	100.00 (13687)
TIGER	All	94.51	98.61	99.37	99.49	99.37	99.49
	BD ≤ 2	94.99	99.11	99.88	99.99	99.88	100.00
	BD = 2	0.00 (0)	82.39 (15324)	97.65 (18161)	99.95 (18590)	97.65 (18161)	100.00 (18598)
NEGRA	All	94.37	98.59	99.32	99.46	99.32	99.46
	BD ≤ 2	94.87	99.12	99.85	99.99	99.85	100.00
	BD = 2	0.00 (0)	82.91 (6106)	97.24 (7161)	99.97 (7362)	97.24 (7161)	100.00 (7364)

Table 1: Maximum constituent recall that be can obtained using a continuous constituency parser and all the variants of our parser in three settings: considering all constituents, considering constituents with a block degree less or equal to two and exactly two. For the last case, we also report the number of constituents. We do not remove punctuation. The analysis is done on full treebanks.

	NEGRA		TIGER		DPTB	
	F1	Disc. F1	F1	Disc. F1	F1	Disc. F1
Fully supervised						
Fernández-González and Martins (2015)	77.0				77.3	
Versley (2016)			79.5			
Corro et al. (2017)					89.2	
Coavoux and Crabbé (2017)			79.3			
Coavoux et al. (2019)	83.2	54.6	82.7	55.9	91.0	71.3
Coavoux and Cohen (2019)	83.2	56.3	82.5	55.9	90.9	67.3
Fernández-González and Gómez-Rodríguez (2020)	83.7	54.7	84.6	57.9		
Stanojević and Steedman (2020)	83.6	50.7	83.4	53.5	90.5	67.1
This work, $\mathcal{O}(n^3)$	86.2	54.1	85.5	53.8	92.7	64.2
This work, $\mathcal{O}(n^5)$ and $\mathcal{O}(n^6)$, well-nested	84.9	46.1	84.8	50.4	92.6	62.6
This work, $\mathcal{O}(n^5)$ and $\mathcal{O}(n^6)$	84.9	46.2	84.9	51.0	92.6	62.9
+ gold part-of-speech tags						
Maier (2015)	77.0	19.8	74.7	18.8		
Gebhardt (2018)			75.1			
Coavoux and Crabbé (2017)	82.2	50.0	81.6	49.2		
Corro et al. (2017)			81.6		90.1	
Semi-supervised: pre-trained word embeddings						
Stanojević and Alhama (2017b)			77.0			
Fernández-González and Gómez-Rodríguez (2020), with pred tags	85.4	58.8	85.3	59.1		
Fernández-González and Gómez-Rodríguez (2020), without pred tags	85.7	58.6	85.7	60.4		
This work, $\mathcal{O}(n^3)$	86.3	56.1	85.2	51.2	92.9	64.9
This work, $\mathcal{O}(n^5)$ and $\mathcal{O}(n^6)$, well-nested	85.6	52.9	84.9	50.4	92.6	59.4
This work, $\mathcal{O}(n^5)$ and $\mathcal{O}(n^6)$	85.6	53.0	84.9	51.0	92.6	59.7
+ gold POS tags						
Stanojević and Alhama (2017b)	82.9		81.6			
Fernández-González and Gómez-Rodríguez (2020)	86.1	59.9	86.3	60.7		
Semi-supervised: Bert						
This work, $\mathcal{O}(n^3)$	91.6	66.1	90.0	62.1	94.8	68.9
This work, $\mathcal{O}(n^5)$ and $\mathcal{O}(n^6)$, well-nested	90.5	58.8	89.3	57.8	94.5	64.5
This work, $\mathcal{O}(n^5)$ and $\mathcal{O}(n^6)$	90.6	59.6	89.3	58.7	94.5	64.7

Table 2: Discontinuous constituency parsing results on the three test sets. The $\mathcal{O}(n^5)$ and $\mathcal{O}(n^6)$ variants produced exactly the same results in all settings.

	NEGRA			TIGER			DPTB		
	D. recall	D. prec.	D. F1	D. recall	D. prec.	D. F1	D. recall	D. prec.	D. F1
$\mathcal{O}(n^3)$	42.0	76.0	54.1	40.6	79.8	53.8	49.7	90.8	64.2
$\mathcal{O}(n^5)$ and $\mathcal{O}(n^6)$, wn	47.0	45.2	46.1	46.9	54.3	50.4	63.8	61.4	62.6
$\mathcal{O}(n^5)$ and $\mathcal{O}(n^6)$	47.3	45.2	46.2	47.8	54.8	51.0	64.0	61.8	62.9

Table 3: Detailed discontinuous constituency parsing results for the fully supervised model.

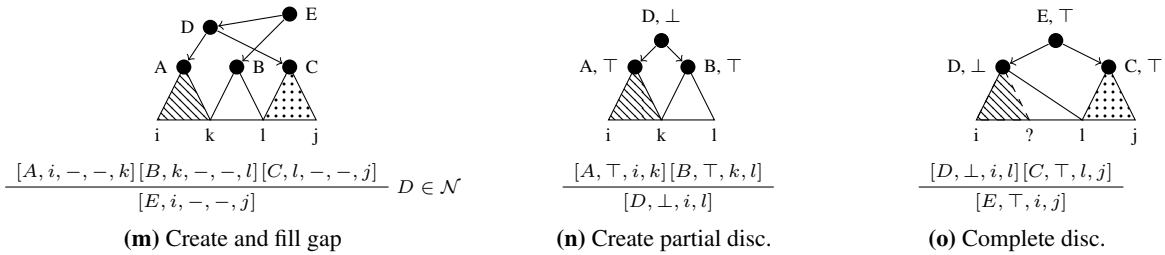


Figure 4: (m) The create gap and fill gap rules can be merged into a single rule if there are no other rule with discontinuous antecedents in the parser. (n)-(o) Rules for the cubic time discontinuous constituency parser. The rule to combine two continuous constituents follows the previous one.

	n. sent.	n. tokens	nn	$\mathcal{O}(n^3)$	$\mathcal{O}(n^4)$	$\mathcal{O}(n^5)$, wn	$\mathcal{O}(n^5)$	$\mathcal{O}(n^6)$, wn	$\mathcal{O}(n^6)$
Negra	1000	6842	1.74	0.35	1.10	3.73	4.48	8.82	22.58
Tiger	5000	38597	7.73	2.81	12.96	98.44	133.22	507.84	1841.95
DPTB	2416	44399	4.67	2.13	6.70	19.35	22.71	43.00	105.98

Table 4: Total time in seconds to parse the full test sets: the nn column corresponds to the time taken by the forward pass of the neural network (max 5000 words per batch on a NVIDIA Tesla V100), each supplementary column is the time taken by each variant of the chart-based algorithm (without any parallelization).

$\mathcal{S}^{(c. \text{span})}$, $\mathcal{S}^{(o. \text{span})}$, $\mathcal{S}^{(g. \text{span})} \in \mathbb{R}^{n \times n}$ are computed using the deep biaffine attention mechanism (Dozat and Manning, 2016). The tensor \mathbf{W} is never explicitly built: during the dynamic program execution we lazily compute requested constituent scores.

Training loss Span-based continuous constituency parsers are usually trained using a decomposable margin-based objective (Stern et al., 2017; Kitaev and Klein, 2018; Kitaev et al., 2019). This approach requires to repeatedly perform loss-augmented inference during training (Taskar et al., 2005), which can be prohibitively slow. A current trend in dependency parsing is to ignore the global structure and rely on negative log-likelihood for head selection independently for each modifier word (Dozat and Manning, 2016; Zhang et al., 2017). We rely on a similar approach and use as training objective the negative log-likelihood loss independently for each span (continuous, outer and gap), adding a null label with a fixed 0 weight as label for spans that do not appear in the gold annotation.

4.3 Evaluation

We evaluate our parser on the test sets of the three treebanks. We report F-measure and discontinuous F-measure as computed using the disco-dop tool¹¹ with standard parameters in Table 2.

First, we observe that the $\mathcal{O}(n^5)$ and $\mathcal{O}(n^6)$ vari-

¹¹<https://github.com/andreasvc/disco-dop>

ants of our parsers produced exactly the same results in all settings. This may be expected as their cover of the original treebanks are almost similar. Second, surprisingly, the $\mathcal{O}(n^3)$ parser produced better results in terms of F-measure than other variants in all cases. We report labeled discontinuous constituent recall and precision measures for the fully supervised model in Table 3. We observe that while the $\mathcal{O}(n^5)$ and $\mathcal{O}(n^6)$ have a better recall than the $\mathcal{O}(n^3)$ parser, their precision is drastically lower. This highlights a benefit of restricting the search space: the parser can retrieve less erroneous constituents leading to an improved overall precision.

Finally, in almost all cases, we achieve a novel state-of-the-art for the task in terms of labeled F-measure. However, we are slightly lower when evaluating discontinuous constituents only. We suspect that this is due to the fact that our best parser is the one with the smallest search space.

4.4 Runtime

The runtime on the test sets of our approach is reported on Table 4. In all cases, the runtime is reasonably fast and we do not need to remove long sentences. Interestingly, the cubic time parser spends most of the time for computing scores with the neural network, even if we use batches to benefit from the GPU architecture while our chart-based algorithm is not parallelized on CPU.

5 Conclusion

We proposed a novel family of algorithms for discontinuous constituency parsing achieving state-of-the-art results. Importantly, we showed that a specific set of discontinuous constituent trees can be parsed in cubic time while covering most of the linguistic structures observed in treebanks. Despite being based on chart-based algorithms, our approach is fast at test time and we can parse all sentences without pruning or filtering long sentences. Future research could explore neural architectures and training losses tailored to our approach.

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