DAPEV2: Process Attention Score as Feature Map for Length Extrapolation

Chuanyang Zheng[1], Yihang Gao[2], Han Shi[3], Jing Xiong[4], Jiankai Sun[1], Jingyao Li[1], Minbin Huang[1], Xiaozhe Ren [3], Michael NG[5], Xin Jiang[3], Zhenguo Li[3], Yu Li[1],
[1]CUHK, [2]NTU, [3]Noah's Ark Lab, [4]HKU, [5]HKBU
Correspondence: cyzhengme@gmail.com

Abstract

The attention mechanism is a fundamental component of the Transformer model, contributing to interactions among distinct tokens. In general, the attention scores are determined simply by the key-query products. However, this work's occasional trial (combining DAPE and NoPE) of including additional MLPs on attention scores without position encoding indicates that the classical key-query multiplication may limit the performance of Transformers. In this work, we conceptualize attention as a feature map and apply the convolution operator (for neighboring attention scores across different heads) to mimic the processing methods in computer vision. Specifically, **the main contribution of this paper is identifying and interpreting the Transformer length extrapolation problem as a result of the limited expressiveness of the naive query and key dot product, and we successfully translate the length extrapolation issue into a well-understood feature map processing problem,** which is called Convolutional Data-Adaptive Position Encoding (CDAPE). The novel insight, which can be adapted to various attention-related models, reveals that the current Transformer architecture has the potential for further evolution. Extensive experiments demonstrate that treating attention as a feature map and applying convolution as a processing method significantly enhances Transformer performance.

1 Introduction

Transformer-based models (Vaswani et al., 2017) have delivered exceptional performances across widespread applications, including language processing (Zhang et al., 2020; Guo et al., 2022; Ainslie et al., 2023), computer vision (Alexey, 2020; Touvron et al., 2021; Liu et al., 2021a; Chen et al., 2024; Peebles and Xie, 2023), quantitative research (Zhou et al., 2024b; Liu et al., 2021b; Wu et al., 2023), and scientific machine learning (Taylor et al., 2022; Geneva and Zabaras, 2022). However, the quadratic cost of the key-query multiplication for processing a sequence raised much concern about the modern architecture of Transformers especially for long context inputs. To address the issue of storage and computation efficiency, recent research delves into developing more efficient architectures, such as sparse structural attention (Xiao et al., 2024c; Zhu et al., 2024), adaptive key selection (Xiao et al., 2024a; Fountas et al., 2024), and hybrid models (Lieber et al., 2024). While these adaptations enhance efficiency, they often involve tradeoffs with model effectiveness.

At the same time, there is another voice advocating for refining the model design for tackling complex tasks, rather than prioritizing efficiency. Positional encoding is one of the key components of the attention mechanism. Although the widely recognized decoder-based Transformer can implicitly incorporate the positional information of tokens, growing evidence both theoretically and empirically shows that the well-designed explicit positional encoding significantly enhances the model performances, especially in long-context tasks (Su et al., 2024b; Press et al., 2021; Zhao et al., 2023). In practice, Transformers depend on positional encoding to explicitly incorporate positional information, enabling the model to make meaningful token predictions. Without these encodings, token generation would lack the necessary contextual order. The well-recognized RoPE (Su et al., 2024b), which is adopted in LLaMA (Touvron et al., 2023), distinguishes the token order by rotating with different angles depending on the token position. However, it demonstrated a notable performance degradation, failing entirely when the input length is double that of the training length (Peng et al., 2023b; Chen et al., 2023a; Ding et al., 2024b). The undesirable performance degradation is also observed for other positional encoding methods, e.g., ALiBi (Press et al., 2021) and Kerple (Chi et al., 2022) . FIRE (Li et al., 2023c) alleviates the long-context extrapo-

Proceedings of the 63rd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers), pages 10628–10666

lation by learnable positional encodings, trying to capture the suitable positional representation by MLPs. Recently, the data-adaptive positional encoding method, namely DAPE (Zheng et al., 2024), which adjusts dynamically with context, enhances the length generalization by incorporating the attention scores and positional information with a more complex mechanism.

In this paper, we propose that precise attention scores are crucial for improving Transformer length extrapolation, and we introduce a new perspective on the attention mechanisms. Traditionally, attention scores are computed through the dot product of the query and key vectors. As illustrated in Figure 1, further processing these attention scores using a neural network—a general case of DAPE (Zheng et al., 2024)—can significantly enhance the length generalization of Transformers, even in the absence of positional encoding (NoPE). Therefore, we suggest treating attention scores as feature maps. By conceptualizing attention as an image feature map (with dimensions $[B, C, W, H]$ for batch size, channel size, width, and height), we can achieve more accurate attention scores by applying techniques used in image processing. In this work, we employ different kernel sizes (such as 1×3) to process attention, finding that the perplexity (ppl) of attention decreases significantly—from over 600 to just above 100—when trained on a sequence length of 128 and evaluated on a length of 8192.

In summary, our contributions are as follows:

1. We highlight that the coarse attention mechanism, which is the direct result of the query and key dot product, limits the Transformer's ability to extrapolate to longer sequences. However, Transformers can achieve good length extrapolation performance with careful processing of attention scores.

2. Besides developing better position encoding (Vaswani et al., 2017) or position interpolation (Chen et al., 2023b) for length extrapolation, we propose the thid direction: by treating attention scores as feature maps and refining them using image processing techniques like convolution, we can enhance the Transformer's extrapolation capabilities.

3. We conducted extensive experiments on language tasks to support our claims and believe that these insights can significantly improve the Transformer's performance in length extrapolation.

2 Related Works

Absolute Positional Encoding. Absolute positional encoding (APE), introduced by (Vaswani et al., 2017), enables Transformers to incorporate positional information. Specifically, at the first layer, each position i is assigned a real-valued encoding $\boldsymbol{e}_i \in \mathbb{R}^d$, which can be either learnable or a fixed sinusoidal encoding (Vaswani et al., 2017; Kiyono et al., 2021; Likhomanenko et al., 2021; Wang et al., 2020; Liu et al., 2020), and this encoding is then added to the input sequence. Although this approach is straightforward, Transformers relying on APE tend to struggle with generalizing to longer sequences (Press et al., 2021).

Relative Positional Encoding. Relative positional encoding (RPE) offers an alternative for embedding positional information (Shaw et al., 2018; Raffel et al., 2020; Press et al., 2021). A widely used RPE method in large language models is rotary positional encoding (RoPE)(Su et al., 2024b; Chowdhery et al., 2023; Touvron et al., 2023). To address length extrapolation challenges(Press et al., 2021; Kazemnejad et al., 2024), positional interpolation (PI) has been introduced (Chen et al., 2023b) to extend the context window. Building on this approach, models like LongLora (Chen et al., 2023c), LongRope (Ding et al., 2024b), YaRN (Peng et al., 2023b), and CLEX (Chen et al., 2023a) have emerged. Another notable direction involves additive positional encoding (ARPE). Different parameterizations of ARPE bias matrix give rise to various RPE variants. Methods supporting arbitrary sequence lengths include T5's RPE (Raffel et al., 2020), ALiBi (Press et al., 2021), Kerple (Chi et al., 2022), Sandwich (Chi et al., 2023a), and FIRE (Li et al., 2023c). Recently, DAPE (Zheng et al., 2024) has been introduced, employing MLPs to dynamically adjust bias values based on the input data.

Data-Adaptive Related Positional Encoding. Transformer-XL (Dai et al., 2019) introduced the use of learnable query and key biases for adaptive positional encodings. Data-Adaptive Positional Encoding (DAPE)(Zheng et al., 2024) extends this idea by leveraging MLPs to adjust positional encodings based on attention over the head dimension for length extrapolation, ensuring different input data receive unique positional encodings. Contex-

tual Positional Encoding(Golovneva et al., 2024) further refines this by conditioning position increments on specific tokens, as determined by the model, allowing positions to adapt based on context."

3 Method

In this section, we first review the previously developed Data-Adaptive Positional Encoding method (DAPE), which incorporates attention scores and positional information through MLPs. As a proof-of-concept, our occasional trial on DAPE without the positional information (as shown in Figure 1) suggests that regarding attention as a feature map and processing it with classical operators (e.g., convolution) can enhance the Transformers' behavior. **The two key differences between DAPE (Zheng et al., 2024) and this work are: 1) Insight**: DAPE attributes length extrapolation performance gains to adaptive position encoding, while this work finds DAPE could still improve performance without position encoding so that we take a broader view, explaining that the Transformer's length extrapolation ability is limited by the expressiveness of the naive query-key dot product, which can be enhanced using image processing techniques; **2) Performance**: As shown in Figure 1, DAPE is designed for additive RPE and may underperform with non-additive RPE (e.g., RoPE), whereas this work suggests that increasing kernel size (e.g., with CDAPE) may improve RoPE's performance. The CDAPE implementation is shown in Appendix Q.

3.1 Additive Relative Positional Encoding

For most additive relative positional encoding (ARPE) methods, the computation of pre-softmax attention logits can be unified under the following formula:

$$\boldsymbol{A}_{\mathrm{ARPE}}(\boldsymbol{X}) = \boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top + \boldsymbol{B}, \quad (1)$$

where the bias matrix $\boldsymbol{B} \in \mathbb{R}^{n\times n}$ is induced by the position encoding function $b : \mathbb{N}^2 \to \mathbb{R}$ and the (i, j)-th entry of $\boldsymbol{B}$ is defined as $b(i, j)$. Various formulations and parameterizations of b give rise to different variants of RPE. Examples of additive RPE include: (1) ALiBi: $b(i, j) = -r|i - j|$, with the scaler $r > 0$ as a hyper-parameter; (2) Kerple: $b(i, j) = -r_1 log(1 + r_2|i - j|)$ with r_1 and r_2 are two learnable parameters; (3) FIRE: $b(i, j) = f_\theta\left(\frac{\psi(i-j)}{\psi(\max\{L,i\})}\right)$, where the positional encoding function f_θ parameterized by θ is learned from data and ψ is a transformation function aimed at assigning more model capacity to local positions.

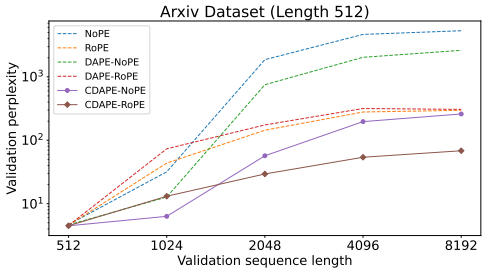

Figure 1: The result of DAPE (Zheng et al., 2024) (equivalent to kernel 1×1 in our explanation) and CDAPE (kernel 1×3 by this work), with baseline NoPE and RoPE. The model is trained with length 512 respectively. The CDAPE denotes that we use $H \times 1 \times 3$ convolutions kernel size on the attention score with shape $[B, H, T, T]$. We find that DAPE can even improve the performance of NoPE (without biased position encoding), suggesting that the explanation in (Zheng et al., 2024), which attributes the improvement to adaptive position encoding, may have a more general underlying cause.

Data-Adaptive Position Encoding (DAPE). The DAPE rewrite the Equation 1 as the following:

$$\boldsymbol{A}_{\mathrm{DAPE}}(\boldsymbol{X}) = \boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top + f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, \boldsymbol{B}). \quad (2)$$

Here, $f : \mathbb{R}^{T\times T} \times \mathbb{R}^{T\times T} \to \mathbb{R}^{T\times T}$ is an element-wise function and T is the sequence length. Another variant of DAPE is with residual, which is the following:

$$\boldsymbol{A}_{\mathrm{DAPE}}(\boldsymbol{X}) = \boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top + \boldsymbol{B} + f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, \boldsymbol{B}). \quad (3)$$

In practice, DAPE (Zheng et al., 2024) utilizes a two-layer *LeakyReLU* MLP with hidden dimension D_{DAPE} (default value is 32) to parameterize $f(\cdot)$ due to its universal approximability (Leshno et al., 1993). All parameters are learned directly from the data during the training process. This architecture allows $f(\cdot)$ to dynamically adjust positional embeddings based on the input sequence data, ensuring that the encoding method is both adaptive and dependent on the input data.

3.2 Special Case of DAPE: Bias is Zero

DAPE was originally designed to dynamically adjust the positional encoding by incorporating input data information. Generally, any additive positional encoding method that includes positional information can be represented as the matrix $\boldsymbol{B}$ in the DAPE model, as outlined in Equation 2. Notably, No Positional Encoding (NoPE) (Kazemnejad et al., 2024) is a special case of additive RPE that assigns zero value to the matrix $\boldsymbol{B}$. The mathematical formulation of DAPE equipped with NoPE

is given by:

$$\boldsymbol{A}_{\mathrm{DAPE}}(\boldsymbol{X}) = \boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top + f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top). \tag{4}$$

The DAPE (Zheng et al., 2024) is designed for additive RPE but not trying NoPE or RoPE, and we present the results of DAPE-NoPE and DAPE-RoPE in the following.

The result of DAPE-NoPE. Compared with the standard Transformer architecture, DAPE-NoPE introduces additional MLPs post the key-query multiplication and prior to the softmax operator. As shown in Figure 1, experimental evidence suggests that DAPE with NoPE significantly outperforms the basic NoPE, prompting a reconsideration of the behaviors of standard Transformers. The additional MLPs (i.e., denoted as $f(\cdot)$ in Equation 4) facilitate information sharing across attention heads and complicate the attention calculation with nonlinear transformation beyond the simple key-query multiplication. This leads to a critical question: *Is the current Transformer architecture, particularly the attention mechanism, sufficiently expressive for real-world language tasks?* Although numerous studies aim to enhance efficiency by reducing computation and storage in standard Transformers, these often come at the cost of effectiveness, potentially hindering the evolution of next-generation Transformer models. Motivated by these insights and observations, we enhance the Transformer's expressiveness and behavior by regarding attention as a feature map and applying convolutional operations, akin to those used in computer vision.

The result of DAPE-RoPE. Building on the hypothesis that DAPE enhances Transformer performance by processing pre-softmax scores with MLPs, we explore its applicability to non-additive positional encoding methods, specifically RoPE (Su et al., 2024b). In the DAPE-RoPE configuration, DAPE-RoPE first computes the classic attention scores of key-query multiplication with RoPE, which are then refined using the MLPs described in Equation 4. The visualized results of the validation perplexity for DAPE-RoPE and other positional encoding methods are presented in Figure 1. The results indicate that DAPE-RoPE may degrade the performance, while CDAPE-RoPE (with kernel size 1×3, proposed by this work) not only improves overall performance but also excels in length extrapolation tasks, particularly at larger sequence lengths. This finding substantiates the effectiveness of CDAPE-RoPE, confirming its superior performance compared to standard RoPE, attributing to the additionally introduced convolution operations to the attention scores.

3.3 CDAPE: Process Attention Scores as Feature Maps

As discussed above, improving Transformer performance necessitates refining the processing of attention score computation beyond the conventional key-query multiplication. We propose regarding the pre-softmax attention scores as feature maps (4-dimensional tensors) and applying convolutional operators, which may could additionally involve position information with zero padding and higher expressiveness (Kayhan and Gemert, 2020) but MLP does not involve additional position information because there is no zero padding. This approach facilitates enhanced communication across neighboring tokens and heads, drawing parallels to popular techniques used in computer vision. This novel method aims to leverage the spatial relationships within tokens, potentially unlocking new aspects of model capabilities.

Rethink the DAPE formulation for CDAPE. In DAPE (Zheng et al., 2024), MLPs are utilized to process and integrate attention and biases. Notably, these MLP operations can be equated to convolution operations with 1×1 kernel (Krizhevsky et al., 2012; Simonyan and Zisserman, 2014; He et al., 2016), a stride of one, and no padding. Consequently, we can reformulate the DAPE in Equation 3 as the following:

$$\boldsymbol{A}_{\mathrm{CDAPE}}(\boldsymbol{X}) = \boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top + \boldsymbol{B} + Conv(tril((\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, \boldsymbol{B})). \tag{5}$$

where $\boldsymbol{X}$ is the input embedding, $\boldsymbol{X}\boldsymbol{W}_Q$ gives the query embedding and the $\boldsymbol{X}\boldsymbol{W}_K$ gives the kery embedding. Under such formulation, DAPE employs convolution operation to process the pre-softmax attention scores of key-query multiplication. The $\texttt{tril}(\cdot)$ returns the lower triangular part of the matrix and the other elements of the result tensor out are set to 0. The resulting attention tensor has a shape of $[B, H, T, T]$, where the four dimensions correspond to the batch size, number of heads, and the context length for both the query and key. This mirrors the structure of an image feature tensor with shape $[B, C, H, W]$, where the dimensions represent the batch size, number of channels, image height, and image width, respectively. This

structural similarity underscores the feasibility of considering attention scores as a tensor of feature mappings, where popular and effective convolution operations can be leveraged for refined processing.

Process attention with more powerful convolution operation. In computer vision, the limitations of 1×1 kernels for processing image features are well-recognized. To improve upon the attention scores processed by these kernels (e.g., DAPE), we introduce $1 \times k$ kernels with a stride of 1 and padding of $k - 1$. This approach allows for wider and deeper convolution across key dimensions and heads without information leakage, as we ensure the attention scores remain lower-triangular. This mechanism is visualized in Appendix P. The use of $1 \times k$ kernels suggests a targeted convolution along the key dimensions across heads. In general, while extending this to include the query dimensions as a standard kernel is theoretically possible, it would significantly increase computational demands. Our forthcoming analysis demonstrates that Transformers modified with $1 \times k$ convolution are adept at associative recall tasks (i.e., the copy task), validating the benefits of integrating convolution in attention calculation. We left as a future work investigating the performances and the soundness of general convolution kernels, such as square sizes. The key contribution of this work is providing a novel insight that suggests applying convolution operations and processing attention as feature maps to improve Transformers' performances.

Realizing associate recall tasks through convolution. As pointed out in some previous works (Arora et al., 2024), the perplexity scores of Transformers mostly result from the performances on associate recall tasks (i.e., the copy tasks). Numerous studies have explored the mechanism of associative recall within Transformers, both from theoretical perspectives and experimental validations (Arora et al., 2024; Bietti et al., 2024; Golovneva et al., 2024). Here, we theoretically prove that the proposed model can realize the associative recall tasks. Notably, this capability is achieved independently of positional encodings, marking a significant advancement in the flexibility and applicability of the proposed architecture. By integrating convolutional operations, we enable the model to handle associative tasks more effectively, leveraging spatial relationships inherent in the data, similar to methods used in image processing. To explain the associative recall mechanism, (Bietti et al., 2024) proved that the first layer of the Transformer is responsible for the previous token mechanism through the positional encoding. More specifically, given a sequence of input tokens $\boldsymbol{X} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_N]$ with corresponding orthogonal positional encoding vectors $[\boldsymbol{p}_1, \boldsymbol{p}_2, \cdots, \boldsymbol{p}_N]$, the first layer primarily facilitates the copying of the previous token to the current token (e.g., $\boldsymbol{x}_i + \boldsymbol{W}_V^1 \boldsymbol{x}_{i-1}$, where $\boldsymbol{W}_V^1$ is the value matrix at the first layer of the Transformer). The input tokens are combined with positional encodings $\boldsymbol{x}_i + \boldsymbol{p}_i$ and the key-query weight matrix is defined as $\boldsymbol{W}_K^{1\top} \boldsymbol{W}_Q^1 = \sum_{i=1}^N \boldsymbol{p}_{i-1} \boldsymbol{p}_i^\top$. The orthogonality of positional encoding vectors and the special choices of the key-query matrix ensure that attention scores predominantly focus on the previous token. In contrast to this implicit mechanism in standard Transformers, our proposed method leverages a convolution operation to explicitly realize associative recall. This approach not only simplifies the process but also enhances its effectiveness by directly manipulating the spatial relationships within tokens and attention scores. Consider a scenario where the word "Hakuna" is consistently followed by "Matata" within a lengthy paragraph. Without the loss of generality, we assume that $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ represent the tokens of "Hakuna" and "Matata" respectively, and $\boldsymbol{x}_N = \boldsymbol{x}_1$ implies that the N-th token in the sequence is "Hakuna". Then we expect that the Transformer can predict and output the next token $\boldsymbol{x}_{N+1}$ as "Matata". For simplicity, we consider a one-head Transformer without positional encoding. We employ a convolution operation with a kernel size of 1×2 and weights $[1, -1]$. Note that the convolution is linear and processing the attention scores along the key dimensions is effectively equivalent to applying convolutions directly to the key vectors themselves. Consequently, the key vector of $\boldsymbol{x}_2$ can be expressed as $\boldsymbol{W}_K^1 (-\boldsymbol{x}_2 + \boldsymbol{x}_1)$ and the query vector for $\boldsymbol{x}_N$ admits $\boldsymbol{W}_Q^1 \boldsymbol{x}_N$. By configuring the matrix $\boldsymbol{W}_K^{1\top} \boldsymbol{W}_Q^1$ to be $\boldsymbol{I}$, the attention mechanism after the convolution predominantly allocates the attention values of $\boldsymbol{x}_N$ to the token $\boldsymbol{x}_2$. This ensures that the token values of $\boldsymbol{x}_2$ are effectively copied to $\boldsymbol{x}_N$, resulting in the model outputting "Matata" following "Hakuna".

Proposition 1. *Transformers incorporating convolution operations can perform associative recall tasks without the need for positional encoding.*

We have discussed the key idea and the sketch of the proof above. For a more detailed compari-

son between CDAPE and the vanilla Transformer, along with a discussion of CDAPE's advantages, we present the main ideas and detailed proof in Appendix A. Additionally, we construct a CDAPE model that successfully performs the simple associative recall task. The results demonstrate that CDAPE, through explicit convolution operations for communication between neighboring tokens, achieves the task more efficiently than the vanilla Transformer, which relies on positional encodings and causal attention to propagate information from neighboring tokens.

Comparisons with hybrid models of convolution and Transformers. Recent developments in hybrid architectures have seen the integration of convolutional and Transformer models to capitalize on the strengths of both. For instance, (Fu et al., 2022) introduced the FlashConv layer, which combines the efficiency of State Space Models (SSMs) with the capabilities of attention-based models. Similarly, (Arora et al., 2024) developed a gated convolution layer, noted for its effectiveness in addressing associative recall tasks. These models typically stack convolution layers directly with standard Transformer layers, resulting in modifications to the token values through convolution. In contrast, our model adopts a distinctive approach by applying convolution along the key dimension during the computation of attention scores. This method preserves the original token values while still leveraging the convolution's benefits for processing attention.

4 Experiment

Baselines. We evaluate the proposed CDAPE against several well-established baselines, including NoPE (Kazemnejad et al., 2024), RoPE (Su et al., 2024b), T5's Bias (Raffel et al., 2020), ALiBi (Press et al., 2021), Kerple (Chi et al., 2022), FIRE (Li et al., 2023c), CoPE (Golovneva et al., 2024), and DAPE (Zheng et al., 2024). As our kernels are applied across all heads, we simplify by omitting the kernel size description at the head dimension. For example, CDAPE indicates the use of a $H \times 1 \times k$ (the k is 3 by default) convolution kernel size on the attention scores, with a shape of $[B, H, T, T]$.

Datasets. Our analysis is based on training language models using the Arxiv and Books3 datasets, commonly employed benchmarks for assessing model performance (Press et al., 2021; Chi et al., 2022; Li et al., 2023c; Ding et al., 2024b). In addition to perplexity, we also leverage downstream datasets with randomized positional encoding (Ruoss et al., 2023) to further assess CDAPE.

Experiment settings. Initially, we compare CDAPE with other baselines at training lengths of 128, 512, and 1024, using 125M decoder-only Transformers (Brown et al., 2020), with model configurations detailed in Appendix L.

4.1 Compare with Baselines

CDAPE-Kerple improves performance within and beyond training length. As shown in Figure 5, when the training length is set to 128 and the evaluation length is extended to 8192, CDAPE-Kerple achieves a perplexity score of 4.60 on the arXiv dataset and 23.52 on the Books3 dataset. These scores are significantly better than those achieved by DAPE-Kerple, which records perplexity scores of 4.97 and 25.01 on the arXiv and Books3 datasets, respectively. Similarly, CoPE performs poorly with perplexity scores of 29.86 on the arXiv dataset and 90.66 on the Books3 dataset under the same conditions. Furthermore, when the training duration is increased to 512, CDAPE-Kerple continues to deliver the best performance, further validating its superior generalization capabilities. These findings highlight the scalability and robustness of CDAPE-Kerple, which is attributed to the introduced convolution operator, making it a promising approach for diverse data scenarios and lengths.

4.2 Performance with Same Training tokens and Different Training Length

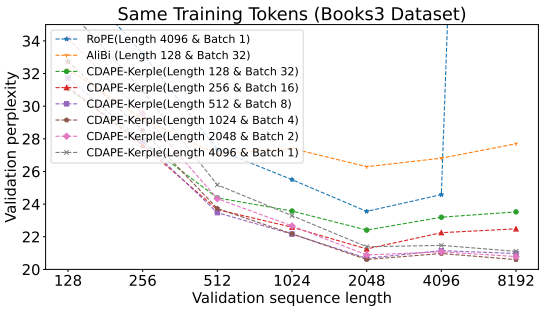

Figure 2: **The performance with same training tokens and different training length**. With the same training tokens, CDAPE with training length 512 could even achieve better performance than RoPE with training length 4096.

Compared to RoPE, with the same training tokens, CDAPE-Kerple with a training length of 128 achieves performance comparable to RoPE

with a training length of 4096, for varying evaluation length. As shown in Figure 2, on the Books3 dataset, CDAPE-Kerple trained with a length of 128 achieves a ppl of 31.07 at an evaluation length of 128 and 23.19 at an evaluation length of 4096, while RoPE trained with a length of 4096 achieves 38.36 and 24.58, respectively. This suggests the superiority of the proposed CDAPE with the introduced convolution operators among heads and neighboring tokens.

With the same training tokens, compared to CDAPE with longer training lengths, CDAPE with shorter training lengths can achieve comparable performance, indicating that CDAPE enhances the model's understanding of text structure. On the arXiv dataset, CDAPE-Kerple with training lengths of 512 demonstrates performance close to that of training with a length of 4096 when the evaluation length is 4096. Moreover, the performance curves for training lengths of 1024, and 2048 are almost identical. This trend is also observed with the Books3 dataset. These results indicate that CDAPE-Kerple effectively helps the model comprehend text structure, enabling it to extend to longer lengths.

4.3 The Effect of Larger Model Size

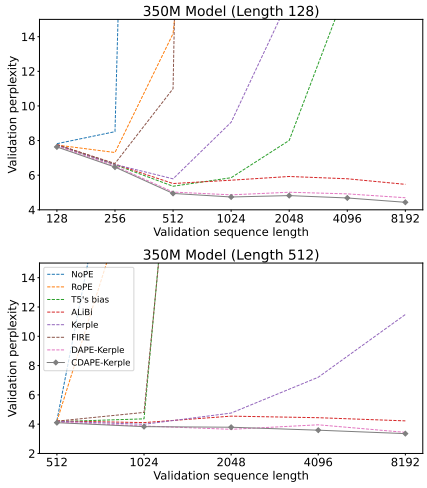

Figure 3: **The Effect of Larger Model Size 350M**. We show the results with training length 128 and training length 512 on Arxiv dataset.

CDAPE performs well with larger model sizes, such as 350M and 2.7B. As illustrated in Figure 3, the proposed CDAPE shows superior performance at varying evaluation lengths with a model size of 350M. For a training length of 128, CDAPE-Kerple achieves a perplexity (ppl) of 7.63 at an evaluation length of 128 and 4.43 at an evaluation length of 8192, compared to DAPE's 7.69 and 4.69, respectively. Similarly, for a training length of 512, CDAPE-Kerple achieves a ppl of 4.10 at an evaluation length of 128 and 3.35 at an evaluation length of 8192, whereas DAPE achieves 4.14 and 3.44, respectively. We also present the 2.7B model size result in Appendix E. Therefore, the proposed CDAPE demonstrates excellent performance with larger model sizes, showing the potential of including the proposed processing techniques in existing large language models.

4.4 The Effect of CDAPE

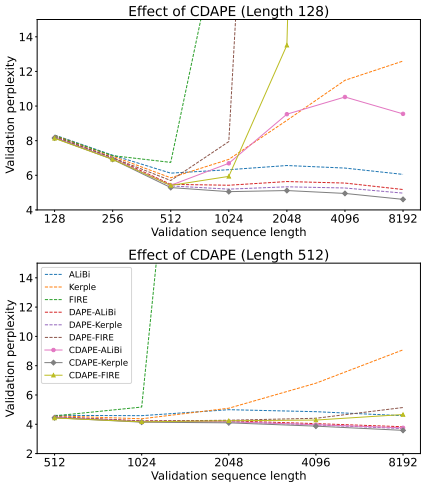

Figure 4: **The effect of CDAPE**. Whatever the baseline is ALiBi, Kerple or FIRE, the proposed CDAPE can all improve their performance. The Figure 1 also proves that the proposed CDAPE is effective for NoPE and RoPE.

CDAPE enhances performance within and beyond the training length, while DAPE does not works well with RoPE. As demonstrated in Figure 4, for varying additive positional encoding such as ALiBi, Kerple, and FIRE, their incorporations with CDAPE (i.e., CDAPE-ALiBi, CDAPE-Kerple, and CDAPE-FIRE) consistently improve performance, while CDAPE-ALiBi may needs longer training length to achieve better performance than DAPE-ALiBi. As illustrated in Figure 1, CDAPE enhances the performance of RoPE, both within and beyond the training length. In contrast, naive DAPE reduces the performance of RoPE, with training lengths of 128 and 512. This indicates that the proposed CDAPE is a versatile and widely applicable method with the potential to be applied to various position encoding techniques on the language modeling task.

4.5 Compare DAPE and CDAPE with Approximate Computational Cost

CDAPE achieves even better performance at a lower computational cost. As shown in Appendix H, when the training length is set to 128, CDAPE-Kerple with D_{DAPE} as 10 achieves a perplexity (ppl) of 8.16 at an evaluation length of 128 and 4.74 at an evaluation length of 8192. This performance is notably better than that of DAPE-Kerple with D_{DAPE} as 64, which achieves perplexities of 8.21 and 4.87, respectively. Moreover, when the training length is extended to 512 and the evaluation length is smaller or equal to 4096, CDAPE-Kerple with D_{DAPE} as 10 continues to surpass the performance of DAPE-Kerple with D_{DAPE} as 64. Also, CDAPE-Kerple with D_{DAPE} as 21 always achieves better performance than DAPE-Kerple with D_{DAPE} as 64. This demonstrates that CDAPE not only maintains its performance advantage across different training lengths but also requires a lower computational cost.

4.6 The Performance with Different Kernel Sizes

Table 1: The performance with different kernel sizes, with training length 128 and evaluation from length 128 to 8192. For different datasets and training length, the optimal kernel size may not always be the largest one, especially when the evaluation length is larger.

Dataset	Method	128	256	512	1024	2048	4096	8192
Arxiv	Kerple	8.30	7.10	5.85	6.91	9.17	11.48	12.59
	DAPE-Kerple (Kernel Size 1x1)	8.21	6.98	5.38	5.20	5.33	5.26	4.97
	CDAPE-Kerple (Kernel Size 1x3)	8.15	6.92	5.29	5.05	5.11	4.95	4.60
	CDAPE-Kerple (Kernel Size 1x5)	8.13	6.91	5.27	5.04	5.10	4.91	4.57
	CDAPE-Kerple (Kernel Size 1x7)	**8.12**	**6.89**	**5.26**	**5.02**	**5.09**	**4.91**	**4.57**
Books3	Kerple	32.10	29.09	28.10	35.75	44.68	56.39	66.23
	DAPE-Kerple (Kernel Size 1x1)	31.49	28.27	24.93	24.31	23.34	24.38	25.01
	CDAPE-Kerple (Kernel Size 1x3)	31.07	27.81	24.38	23.57	22.40	23.19	**23.52**
	CDAPE-Kerple (Kernel Size 1x5)	31.02	27.79	24.36	23.57	22.41	23.32	23.71
	CDAPE-Kerple (Kernel Size 1x7)	**30.98**	**27.76**	**24.31**	**23.47**	**22.30**	**23.00**	23.57

Different experiment settings may have different optimal kernel sizes. Appendix I shows the performance of CDAPE with various kernel sizes, including with kernel size 1 (equivalent to a 1×1 kernel size) to kernel size 7. For the Arxiv dataset, larger kernel sizes consistently achieve better performance, evaluating with training lengths of 128 or 512. However, for the Books3 dataset, CDAPE (with kernel size 3) performs best when the training length is 128 and evaluated at 8192, whereas CDAPE (with kernel size 5) performs best at the same evaluation level when the training length is 512. These results suggest that the optimal kernel size may vary depending on the experimental setting, ranging from 1×1 to larger kernel sizes. Although larger kernel sizes contribute to stronger expressiveness from intuition, we conjecture that the performance degradation for overly large kernel sizes results from optimization challenges.

4.7 The Performance on CHE Benchmark with Accuracy Evaluation Metrics

Different tasks have different optimal kernel sizes, as shown in Appendix J and Appendix I. For example, on MISSING DUPLICATE task, the CDAPE-Kerple improves the 87.57 of DAPE-Kerple to 99.65. However, on the STACK MANIPULATIONtask, the CDAPE-Kerple decreases the 72.04 of DAPE-Kerple to 68.18. Also, as shown in Appendix I, the larger kernel size does not always lead to better performance. Overall, larger kernel size provides a potential way to improve the Transformer length extrapolation performance, and we usually could find a suitable kernel size (ranging from 1×1 to larger kernel sizes) to achieve better performance than without further processing attention score.

The large kernel size performance improvement is related to the baseline bias matrix. As shown in Appendix J, the best performance is usually achieved by further processing attention scores via kernel size 1 or 3. Moreover, on 11 permutation-variant tasks, the CDAPE-Kerple achieves better performance on 8 of 11 tasks compared to Kerple. And the CDAPE-FIRE achieves better performance on 6 of 11 tasks compared to FIRE. This suggests that the large kernel size performance improvement is related to the baseline bias matrix.

4.8 The Time Cost

As the model size increases, the additional computational cost ratio gradually decreases. As shown in Appendix K, when the model size is 350M, the time cost for Kerple is 189.91 ms, while DAPE-Kerple takes 224.22 ms, and CDAPE-Kerple requires 252.84 ms. Compared to CDAPE-Kerple, the time cost ratios for Kerple and DAPE-Kerple are 0.7511 and 0.8868, respectively. As the model size increases from 350M to 2.7B and 6.7B, the time cost ratio for Kerple rises from 0.7511 to 0.8205 and 0.8918, respectively. Similarly, the time cost ratio for DAPE-Kerple increases from 0.8868 to 0.9361 and 0.9677. Therefore, as the model size increases, the time cost ratio also increases,

indicating that the additional computational cost decreases progressively.

5 Conclusion

In this paper, we point out that the key of Transformer length extrapolation is the better and more accurate attention score. Therefore, we develop and analyze CDAPE by processing the attention score as feature maps via convolution operation. Theoretically, we show that the associative recall tasks, which account for the most perplexity scores, can be realized by the proposed Transformer with convolution, in contrast to the vanilla Transformer. We conducted comprehensive experiments on Arxiv, Books3, and CHE to validate the effectiveness of the proposed method, where the proposed method exhibits significant superiority.

Limitations

The proposed method utilizes convolution so that the cost is relatively higher than the previous DAPE. Therefore, there may be additional costs.

References

Muhammad Adnan, Akhil Arunkumar, Gaurav Jain, Prashant J Nair, Ilya Soloveychik, and Purushotham Kamath. 2024. Keyformer: KV cache reduction through key tokens selection for efficient generative inference. *arXiv preprint arXiv:2403.09054*.

Devanshu Agrawal, Shang Gao, and Martin Gajek. 2024. Can't remember details in long documents? you need some r&r. *arXiv preprint arXiv:2403.05004*.

Joshua Ainslie, Tao Lei, Michiel de Jong, Santiago Ontanon, Siddhartha Brahma, Yury Zemlyanskiy, David Uthus, Mandy Guo, James Lee-Thorp, Yi Tay, et al. 2023. CoLT5: Faster long-range transformers with conditional computation. In *The 2023 Conference on Empirical Methods in Natural Language Processing*.

Dosovitskiy Alexey. 2020. An image is worth 16x16 words: Transformers for image recognition at scale. *arXiv preprint arXiv: 2010.11929*.

Chenxin An, Fei Huang, Jun Zhang, Shansan Gong, Xipeng Qiu, Chang Zhou, and Lingpeng Kong. 2024. Training-free long-context scaling of large language models. *arXiv preprint arXiv:2402.17463*.

Simran Arora, Sabri Eyuboglu, Aman Timalsina, Isys Johnson, Michael Poli, James Zou, Atri Rudra, and Christopher Re. 2024. Zoology: Measuring and improving recall in efficient language models. In *The Twelfth International Conference on Learning Representations*.

Dzmitry Bahdanau, Kyunghyun Cho, and Yoshua Bengio. 2015. Neural machine translation by jointly learning to align and translate. *International Conference on Learning Representations*.

Maximilian Beck, Korbinian Pöppel, Markus Spanring, Andreas Auer, Oleksandra Prudnikova, Michael Kopp, Günter Klambauer, Johannes Brandstetter, and Sepp Hochreiter. 2024. xLSTM: Extended long short-term memory. *arXiv preprint arXiv:2405.04517*.

Iz Beltagy, Matthew E Peters, and Arman Cohan. 2020. Longformer: The long-document transformer. *arXiv preprint arXiv:2004.05150*.

Alberto Bietti, Vivien Cabannes, Diane Bouchacourt, Herve Jegou, and Leon Bottou. 2024. Birth of a transformer: A memory viewpoint. *Advances in Neural Information Processing Systems*, 36.

Tom Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared D Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, et al. 2020. Language models are few-shot learners. *Advances in Neural Information Processing Systems*, 33:1877–1901.

Guanzheng Chen, Xin Li, Zaiqiao Meng, Shangsong Liang, and Lidong Bing. 2023a. CLEX: Continuous length extrapolation for large language models. In *International Conference on Learning Representations*.

Junsong Chen, Jincheng YU, Chongjian GE, Lewei Yao, Enze Xie, Zhongdao Wang, James Kwok, Ping Luo, Huchuan Lu, and Zhenguo Li. 2024. Pixart-α: Fast training of diffusion transformer for photorealistic text-to-image synthesis. In *The Twelfth International Conference on Learning Representations*.

Shouyuan Chen, Sherman Wong, Liangjian Chen, and Yuandong Tian. 2023b. Extending context window of large language models via positional interpolation. *arXiv preprint arXiv:2306.15595*.

Yukang Chen, Shengju Qian, Haotian Tang, Xin Lai, Zhijian Liu, Song Han, and Jiaya Jia. 2023c. LongLoRA: Efficient fine-tuning of long-context large language models. *International Conference on Learning Representations*.

Ta-Chung Chi, Ting-Han Fan, Peter J Ramadge, and Alexander Rudnicky. 2022. KERPLE: Kernelized relative positional embedding for length extrapolation. *Advances in Neural Information Processing Systems*, 35:8386–8399.

Ta-Chung Chi, Ting-Han Fan, Alexander Rudnicky, and Peter Ramadge. 2023a. Dissecting transformer length extrapolation via the lens of receptive field analysis. In *Proceedings of the 61st Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 13522–13537.

Ta-Chung Chi, Ting-Han Fan, and Alexander I Rudnicky. 2023b. Attention alignment and flexible positional embeddings improve transformer length extrapolation. *arXiv preprint arXiv:2311.00684*.

Hanseul Cho, Jaeyoung Cha, Pranjal Awasthi, Srinadh Bhojanapalli, Anupam Gupta, and Chulhee Yun. 2024. Position coupling: Leveraging task structure for improved length generalization of transformers. *arXiv preprint arXiv:2405.20671*.

Noam Chomsky. 1956. Three models for the description of language. *IRE Transactions on Information Theory*, 2(3):113–124.

Krzysztof Marcin Choromanski, Valerii Likhosherstov, David Dohan, Xingyou Song, Andreea Gane, Tamas Sarlos, Peter Hawkins, Jared Quincy Davis, Afroz Mohiuddin, Lukasz Kaiser, David Benjamin Belanger, Lucy J Colwell, and Adrian Weller. 2021. Rethinking attention with performers. In *International Conference on Learning Representations*.

Aakanksha Chowdhery, Sharan Narang, Jacob Devlin, Maarten Bosma, Gaurav Mishra, Adam Roberts, Paul Barham, Hyung Won Chung, Charles Sutton, Sebastian Gehrmann, et al. 2023. Palm: Scaling language modeling with pathways. *Journal of Machine Learning Research*, 24(240):1–113.

Zihang Dai, Zhilin Yang, Yiming Yang, Jaime G Carbonell, Quoc Le, and Ruslan Salakhutdinov. 2019. Transformer-XL: Attentive language models beyond a fixed-length context. In *Proceedings of the 57th Annual Meeting of the Association for Computational Linguistics*, pages 2978–2988.

Soham De, Samuel L Smith, Anushan Fernando, Aleksandar Botev, George Cristian-Muraru, Albert Gu, Ruba Haroun, Leonard Berrada, Yutian Chen, Srivatsan Srinivasan, et al. 2024. Griffin: Mixing gated linear recurrences with local attention for efficient language models. *arXiv preprint arXiv:2402.19427*.

Gregoire Deletang, Anian Ruoss, Jordi Grau-Moya, Tim Genewein, Li Kevin Wenliang, Elliot Catt, Chris Cundy, Marcus Hutter, Shane Legg, Joel Veness, et al. 2022. Neural networks and the chomsky hierarchy. In *International Conference on Learning Representations*.

Jacob Devlin, Ming-Wei Chang, Kenton Lee, and Kristina Toutanova. 2019. BERT: Pre-training of deep bidirectional transformers for language understanding. In *Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long and Short Papers)*, pages 4171–4186.

Hantian Ding, Zijian Wang, Giovanni Paolini, Varun Kumar, Anoop Deoras, Dan Roth, and Stefano Soatto. 2024a. Fewer truncations improve language modeling. *arXiv preprint arXiv:2404.10830*.

Yiran Ding, Li Lyna Zhang, Chengruidong Zhang, Yuanyuan Xu, Ning Shang, Jiahang Xu, Fan Yang, and Mao Yang. 2024b. LongRoPE: Extending llm context window beyond 2 million tokens. *arXiv preprint arXiv:2402.13753*.

Zafeirios Fountas, Martin A Benfeghoul, Adnan Oomerjee, Fenia Christopoulou, Gerasimos Lampouras, Haitham Bou-Ammar, and Jun Wang. 2024. Human-like episodic memory for infinite context llms. *arXiv preprint arXiv:2407.09450*.

Daniel Y Fu, Tri Dao, Khaled K Saab, Armin W Thomas, Atri Rudra, and Christopher Ré. 2022. Hungry hungry hippos: Towards language modeling with state space models. *arXiv preprint arXiv:2212.14052*.

Yao Fu, Rameswar Panda, Xinyao Niu, Xiang Yue, Hannaneh Hajishirzi, Yoon Kim, and Hao Peng. 2024. Data engineering for scaling language models to 128k context. *arXiv preprint arXiv:2402.10171*.

Chaochen Gao, Xing Wu, Qi Fu, and Songlin Hu. 2024. Quest: Query-centric data synthesis approach for long-context scaling of large language model. *arXiv preprint arXiv:2405.19846*.

Nicholas Geneva and Nicholas Zabaras. 2022. Transformers for modeling physical systems. *Neural Networks*, 146:272–289.

Olga Golovneva, Tianlu Wang, Jason Weston, and Sainbayar Sukhbaatar. 2024. Contextual position encoding: Learning to count what's important. *arXiv preprint arXiv:2405.18719*.

Albert Gu and Tri Dao. 2023. Mamba: Linear-time sequence modeling with selective state spaces. *arXiv preprint arXiv:2312.00752*.

Mandy Guo, Joshua Ainslie, David Uthus, Santiago Ontanon, Jianmo Ni, Yun-Hsuan Sung, and Yinfei Yang. 2022. LongT5: Efficient text-to-text transformer for long sequences. *Findings of the Association for Computational Linguistics: NAACL*.

Adi Haviv, Ori Ram, Ofir Press, Peter Izsak, and Omer Levy. 2022. Transformer language models without positional encodings still learn positional information. In *Findings of the Association for Computational Linguistics: EMNLP 2022*, pages 1382–1390.

Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. 2016. Deep residual learning for image recognition. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pages 770–778.

Zhenyu He, Guhao Feng, Shengjie Luo, Kai Yang, Di He, Jingjing Xu, Zhi Zhang, Hongxia Yang, and Liwei Wang. 2024. Two stones hit one bird: Bilevel positional encoding for better length extrapolation. *arXiv preprint arXiv:2401.16421*.

Peyman Hosseini, Ignacio Castro, Iacopo Ghinassi, and Matthew Purver. 2024. Efficient solutions for an intriguing failure of llms: Long context window does not mean llms can analyze long sequences flawlessly. *arXiv preprint arXiv:2408.01866*.

Zhiyuan Hu, Yuliang Liu, Jinman Zhao, Suyuchen Wang, Yan Wang, Wei Shen, Qing Gu, Anh Tuan Luu, See-Kiong Ng, Zhiwei Jiang, et al. 2024. Longrecipe: Recipe for efficient long context generalization in large languge models. *arXiv preprint arXiv:2409.00509*.

Hongye Jin, Xiaotian Han, Jingfeng Yang, Zhimeng Jiang, Zirui Liu, Chia-Yuan Chang, Huiyuan Chen, and Xia Hu. 2024. LLM maybe LongLM: Self-extend LLM context window without tuning. *arXiv preprint arXiv:2401.01325*.

Osman Semih Kayhan and Jan C van Gemert. 2020. On translation invariance in cnns: Convolutional layers can exploit absolute spatial location. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 14274–14285.

Amirhossein Kazemnejad, Inkit Padhi, Karthikeyan Natesan Ramamurthy, Payel Das, and Siva Reddy. 2024. The impact of positional encoding on length generalization in transformers. *Advances in Neural Information Processing Systems*, 36.

Guolin Ke, Di He, and Tie-Yan Liu. 2020. Rethinking positional encoding in language pre-training. In *International Conference on Learning Representations*.

Shun Kiyono, Sosuke Kobayashi, Jun Suzuki, and Kentaro Inui. 2021. SHAPE: Shifted absolute position embedding for transformers. In *Proceedings of the 2021 Conference on Empirical Methods in Natural Language Processing*, pages 3309–3321.

Alex Krizhevsky, Ilya Sutskever, and Geoffrey E Hinton. 2012. Imagenet classification with deep convolutional neural networks. *Advances in neural information processing systems*, 25.

Moshe Leshno, Vladimir Ya Lin, Allan Pinkus, and Shimon Schocken. 1993. Multilayer feedforward networks with a nonpolynomial activation function can approximate any function. *Neural Networks*, 6(6):861–867.

Jingyao Li, Pengguang Chen, Zexin He, Shaozuo Yu, Shu Liu, and Jiaya Jia. 2023a. Rethinking out-of-distribution (OOD) detection: Masked image modeling is all you need. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR)*, pages 11578–11589.

Jingyao Li, Pengguang Chen, and Jiaya Jia. 2024a. Motcoder: Elevating large language models with modular of thought for challenging programming tasks. *Preprint*, arXiv:2312.15960.

Jingyao Li, Pengguang Chen, Shengju Qian, and Jiaya Jia. 2023b. Tagclip: Improving discrimination ability of open-vocabulary semantic segmentation. *Preprint*, arXiv:2304.07547.

Jingyao Li, Pengguang Chen, Shaozuo Yu, Shu Liu, and Jiaya Jia. 2024b. Bal: Balancing diversity and novelty for active learning. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 46(5):3653–3664.

Shanda Li, Chong You, Guru Guruganesh, Joshua Ainslie, Santiago Ontanon, Manzil Zaheer, Sumit Sanghai, Yiming Yang, Sanjiv Kumar, and Srinadh Bhojanapalli. 2023c. Functional interpolation for relative positions improves long context transformers. In *International Conference on Learning Representations*.

Zhenyu Li, Yike Zhang, Tengyu Pan, Yutao Sun, Zhichao Duan, Junjie Fang, Rong Han, Zixuan Wang, and Jianyong Wang. 2024c. Focusllm: Scaling llm's context by parallel decoding. *arXiv preprint arXiv:2408.11745*.

Zihan Liao, Jun Wang, Hang Yu, Lingxiao Wei, Jianguo Li, and Wei Zhang. 2024. E2llm: Encoder elongated large language models for long-context understanding and reasoning. *arXiv preprint arXiv:2409.06679*.

Opher Lieber, Barak Lenz, Hofit Bata, Gal Cohen, Jhonathan Osin, Itay Dalmedigos, Erez Safahi, Shaked Meirom, Yonatan Belinkov, Shai Shalev-Shwartz, et al. 2024. Jamba: A hybrid transformer-mamba language model. *arXiv preprint arXiv:2403.19887*.

Tatiana Likhomanenko, Qiantong Xu, Gabriel Synnaeve, Ronan Collobert, and Alex Rogozhnikov. 2021. CAPE: Encoding relative positions with continuous augmented positional embeddings. *Advances in Neural Information Processing Systems*, 34:16079–16092.

Bin Lin, Tao Peng, Chen Zhang, Minmin Sun, Lanbo Li, Hanyu Zhao, Wencong Xiao, Qi Xu, Xiafei Qiu, Shen Li, et al. 2024a. Infinite-LLM: Efficient LLM service for long context with distattention and distributed kvcache. *arXiv preprint arXiv:2401.02669*.

Hongzhan Lin, Ang Lv, Yuhan Chen, Chen Zhu, Yang Song, Hengshu Zhu, and Rui Yan. 2024b. Mixture of in-context experts enhance llms' long context awareness. *arXiv preprint arXiv:2406.19598*.

Jiacheng Liu, Sewon Min, Luke Zettlemoyer, Yejin Choi, and Hannaneh Hajishirzi. 2024a. Infini-gram: Scaling unbounded n-gram language models to a trillion tokens. *arXiv preprint arXiv:2401.17377*.

Jiaheng Liu, Zhiqi Bai, Yuanxing Zhang, Chenchen Zhang, Yu Zhang, Ge Zhang, Jiakai Wang, Haoran Que, Yukang Chen, Wenbo Su, et al. 2024b. Eˆ 2-LLM: Efficient and extreme length extension of large language models. *arXiv preprint arXiv:2401.06951*.

Xiaoran Liu, Hang Yan, Chenxin An, Xipeng Qiu, and Dahua Lin. 2023. Scaling laws of RoPE-based extrapolation. In *International Conference on Learning Representations*.

Xuanqing Liu, Hsiang-Fu Yu, Inderjit Dhillon, and Cho-Jui Hsieh. 2020. Learning to encode position for transformer with continuous dynamical model. In *International Conference on Machine Learning*, pages 6327–6335. PMLR.

Ze Liu, Yutong Lin, Yue Cao, Han Hu, Yixuan Wei, Zheng Zhang, Stephen Lin, and Baining Guo. 2021a. Swin transformer: Hierarchical vision transformer using shifted windows. In *Proceedings of the IEEE/CVF international conference on computer vision*, pages 10012–10022.

Zhuang Liu, Degen Huang, Kaiyu Huang, Zhuang Li, and Jun Zhao. 2021b. Finbert: A pre-trained financial language representation model for financial text mining. In *Proceedings of the Twenty-ninth International Conference on International Joint Conferences on Artificial Intelligence*, pages 4513–4519.

Anton Lozhkov, Loubna Ben Allal, Leandro von Werra, and Thomas Wolf. 2024. Fineweb-edu: the finest collection of educational content.

Shengjie Luo, Shanda Li, Tianle Cai, Di He, Dinglan Peng, Shuxin Zheng, Guolin Ke, Liwei Wang, and Tie-Yan Liu. 2021. Stable, fast and accurate: Kernelized attention with relative positional encoding. *Advances in Neural Information Processing Systems*, 34:22795–22807.

Shengjie Luo, Shanda Li, Shuxin Zheng, Tie-Yan Liu, Liwei Wang, and Di He. 2022. Your transformer may not be as powerful as you expect. *Advances in Neural Information Processing Systems*, 35:4301–4315.

Xindian Ma, Wenyuan Liu, Peng Zhang, and Nan Xu. 2024. 3d-rpe: Enhancing long-context modeling through 3d rotary position encoding. *arXiv preprint arXiv:2406.09897*.

Adam Paszke, Sam Gross, Francisco Massa, Adam Lerer, James Bradbury, Gregory Chanan, Trevor Killeen, Zeming Lin, Natalia Gimelshein, Luca Antiga, et al. 2019. Pytorch: An imperative style, high-performance deep learning library. *Advances in Neural Information Processing Systems*, 32.

William Peebles and Saining Xie. 2023. Scalable diffusion models with transformers. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, pages 4195–4205.

Bo Peng, Eric Alcaide, Quentin Anthony, Alon Albalak, Samuel Arcadinho, Huanqi Cao, Xin Cheng, Michael Chung, Matteo Grella, Kranthi Kiran GV, et al. 2023a. RWKV: Reinventing RNNs for the transformer era. *Findings of the Association for Computational Linguistics: EMNLP*.

Bowen Peng, Jeffrey Quesnelle, Honglu Fan, and Enrico Shippole. 2023b. YaRN: Efficient context window extension of large language models. In *International Conference on Learning Representations*.

Ofir Press, Noah Smith, and Mike Lewis. 2021. Train short, test long: Attention with linear biases enables input length extrapolation. In *International Conference on Learning Representations*.

Zhen Qin, Weigao Sun, Dong Li, Xuyang Shen, Weixuan Sun, and Yiran Zhong. 2024a. Lightning attention-2: A free lunch for handling unlimited sequence lengths in large language models. *arXiv preprint arXiv:2401.04658*.

Zhen Qin, Yiran Zhong, and Hui Deng. 2024b. Exploring transformer extrapolation. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 38, pages 18897–18905.

Alec Radford, Jeffrey Wu, Rewon Child, David Luan, Dario Amodei, Ilya Sutskever, et al. 2019. Language models are unsupervised multitask learners. *OpenAI blog*, 1(8):9.

Colin Raffel, Noam Shazeer, Adam Roberts, Katherine Lee, Sharan Narang, Michael Matena, Yanqi Zhou, Wei Li, and Peter J Liu. 2020. Exploring the limits of transfer learning with a unified text-to-text transformer. *Journal of Machine Learning Research*, 21(140):1–67.

Anian Ruoss, Grégoire Delétang, Tim Genewein, Jordi Grau-Moya, Róbert Csordás, Mehdi Bennani, Shane Legg, and Joel Veness. 2023. Randomized positional encodings boost length generalization of transformers. In *Proceedings of the 61st Annual Meeting of the Association for Computational Linguistics (Volume 2: Short Papers)*, pages 1889–1903.

Mahdi Sabbaghi, George Pappas, Hamed Hassani, and Surbhi Goel. 2024. Explicitly encoding structural symmetry is key to length generalization in arithmetic tasks. *arXiv preprint arXiv:2406.01895*.

Peter Shaw, Jakob Uszkoreit, and Ashish Vaswani. 2018. Self-attention with relative position representations. In *Proceedings of the 2018 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 2 (Short Papers)*, pages 464–468.

Noam Shazeer, Zhenzhong Lan, Youlong Cheng, Nan Ding, and Le Hou. 2020. Talking-heads attention. *arXiv preprint arXiv:2003.02436*.

Weijia Shi, Sewon Min, Maria Lomeli, Chunting Zhou, Margaret Li, Victoria Lin, Noah A Smith, Luke Zettlemoyer, Scott Yih, and Mike Lewis. 2023. In-context pretraining: Language modeling beyond document boundaries. *International Conference on Learning Representations*.

Karen Simonyan and Andrew Zisserman. 2014. Very deep convolutional networks for large-scale image recognition. *arXiv preprint arXiv:1409.1556*.

Matt Stallone, Vaibhav Saxena, Leonid Karlinsky, Bridget McGinn, Tim Bula, Mayank Mishra, Adriana Meza Soria, Gaoyuan Zhang, Aditya Prasad, Yikang Shen, et al. 2024. Scaling granite code models to 128k context. *arXiv preprint arXiv:2407.13739*.

Konrad Staniszewski, Szymon Tworkowski, Sebastian Jaszczur, Henryk Michalewski, Łukasz Kuciński, and Piotr Miłoś. 2023. Structured packing in LLM training improves long context utilization. *arXiv preprint arXiv:2312.17296*.

Jianlin Su, Murtadha Ahmed, Luo Ao, Mingren Zhu, Yunfeng Liu, et al. 2024a. Naive bayes-based context extension for large language models. *arXiv preprint arXiv:2403.17552*.

Jianlin Su, Murtadha Ahmed, Yu Lu, Shengfeng Pan, Wen Bo, and Yunfeng Liu. 2024b. Roformer: Enhanced transformer with rotary position embedding. *Neurocomputing*, 568:127063.

Jiankai Sun, Chuanyang Zheng, Enze Xie, Zhengying Liu, Ruihang Chu, Jianing Qiu, Jiaqi Xu, Mingyu Ding, Hongyang Li, Mengzhe Geng, et al. 2023a. A survey of reasoning with foundation models. *arXiv preprint arXiv:2312.11562*.

Yutao Sun, Li Dong, Barun Patra, Shuming Ma, Shaohan Huang, Alon Benhaim, Vishrav Chaudhary, Xia Song, and Furu Wei. 2023b. A length-extrapolatable transformer. *Proceedings of the 61st Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 14590–14604.

Alon Talmor, Jonathan Herzig, Nicholas Lourie, and Jonathan Berant. 2019. Commonsenseqa: A question answering challenge targeting commonsense knowledge. In *Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long and Short Papers)*, pages 4149–4158.

Yi Tay, Dara Bahri, Liu Yang, Donald Metzler, and Da-Cheng Juan. 2020. Sparse sinkhorn attention. In *International Conference on Machine Learning*, pages 9438–9447. PMLR.

Ross Taylor, Marcin Kardas, Guillem Cucurull, Thomas Scialom, Anthony Hartshorn, Elvis Saravia, Andrew Poulton, Viktor Kerkez, and Robert Stojnic. 2022. Galactica: A large language model for science. *arXiv preprint arXiv:2211.09085*.

Junfeng Tian, Da Zheng, Yang Cheng, Rui Wang, Colin Zhang, and Debing Zhang. 2024. Untie the knots: An efficient data augmentation strategy for long-context pre-training in language models. *arXiv preprint arXiv:2409.04774*.

Hugo Touvron, Matthieu Cord, Matthijs Douze, Francisco Massa, Alexandre Sablayrolles, and Hervé Jégou. 2021. Training data-efficient image transformers & distillation through attention. In *International conference on machine learning*, pages 10347–10357. PMLR.

Hugo Touvron, Thibaut Lavril, Gautier Izacard, Xavier Martinet, Marie-Anne Lachaux, Timothée Lacroix, Baptiste Rozière, Naman Goyal, Eric Hambro, Faisal Azhar, et al. 2023. LLaMA: Open and efficient foundation language models. *arXiv preprint arXiv:2302.13971*.

Szymon Tworkowski, Konrad Staniszewski, Mikołaj Pacek, Yuhuai Wu, Henryk Michalewski, and Piotr Miłoś. 2024. Focused transformer: Contrastive training for context scaling. *Advances in Neural Information Processing Systems*, 36.

Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Łukasz Kaiser, and Illia Polosukhin. 2017. Attention is all you need. *Advances in Neural Information Processing Systems*, 30.

Benyou Wang, Lifeng Shang, Christina Lioma, Xin Jiang, Hao Yang, Qun Liu, and Jakob Grue Simonsen. 2020. On position embeddings in BERT. In *International Conference on Learning Representations*.

Huadong Wang, Xin Shen, Mei Tu, Yimeng Zhuang, and Zhiyuan Liu. 2022. Improved transformer with multi-head dense collaboration. *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, 30:2754–2767.

Jie Wang, Tao Ji, Yuanbin Wu, Hang Yan, Tao Gui, Qi Zhang, Xuanjing Huang, and Xiaoling Wang. 2024a. Length generalization of causal transformers without position encoding. *arXiv preprint arXiv:2404.12224*.

Suyuchen Wang, Ivan Kobyzev, Peng Lu, Mehdi Rezagholizadeh, and Bang Liu. 2024b. Resonance RoPE: Improving context length generalization of large language models. *arXiv preprint arXiv:2403.00071*.

Y Wang, D Ma, and D Cai. 2024c. With greater text comes greater necessity: Inference-time training helps long text generation. *arXiv preprint arXiv:2401.11504*.

BigScience Workshop, Teven Le Scao, Angela Fan, Christopher Akiki, Ellie Pavlick, Suzana Ilić, Daniel Hesslow, Roman Castagné, Alexandra Sasha Luccioni, François Yvon, et al. 2022. Bloom: A 176b-parameter open-access multilingual language model. *arXiv preprint arXiv:2211.05100*.

Shijie Wu, Ozan Irsoy, Steven Lu, Vadim Dabravolski, Mark Dredze, Sebastian Gehrmann, Prabhanjan Kambadur, David Rosenberg, and Gideon Mann. 2023. Bloomberggpt: A large language model for finance. *arXiv preprint arXiv:2303.17564*.

Tong Wu, Yanpeng Zhao, and Zilong Zheng. 2024a. Never miss a beat: An efficient recipe for context window extension of large language models with

consistent" middle" enhancement. *arXiv preprint arXiv:2406.07138*.

Wenhao Wu, Yizhong Wang, Yao Fu, Xiang Yue, Dawei Zhu, and Sujian Li. 2024b. Long context alignment with short instructions and synthesized positions. *arXiv preprint arXiv:2405.03939*.

Chaojun Xiao, Pengle Zhang, Xu Han, Guangxuan Xiao, Yankai Lin, Zhengyan Zhang, Zhiyuan Liu, Song Han, and Maosong Sun. 2024a. InfLLM: Unveiling the intrinsic capacity of LLMs for understanding extremely long sequences with training-free memory. *arXiv preprint arXiv:2402.04617*.

Da Xiao, Qingye Meng, Shengping Li, and Xingyuan Yuan. 2024b. Improving transformers with dynamically composable multi-head attention. *arXiv preprint arXiv:2405.08553*.

Guangxuan Xiao, Yuandong Tian, Beidi Chen, Song Han, and Mike Lewis. 2024c. Efficient streaming language models with attention sinks. In *The Twelfth International Conference on Learning Representations*.

Guangxuan Xiao, Yuandong Tian, Beidi Chen, Song Han, and Mike Lewis. 2024d. Efficient streaming language models with attention sinks. In *International Conference on Learning Representations*.

Wenhan Xiong, Jingyu Liu, Igor Molybog, Hejia Zhang, Prajjwal Bhargava, Rui Hou, Louis Martin, Rashi Rungta, Karthik Abinav Sankararaman, Barlas Oguz, et al. 2023. Effective long-context scaling of foundation models. *arXiv preprint arXiv:2309.16039*.

Peng Xu, Wei Ping, Xianchao Wu, Zihan Liu, Mohammad Shoeybi, and Bryan Catanzaro. 2024. Chatqa 2: Bridging the gap to proprietary llms in long context and rag capabilities. *arXiv preprint arXiv:2407.14482*.

Kai Yang, Jan Ackermann, Zhenyu He, Guhao Feng, Bohang Zhang, Yunzhen Feng, Qiwei Ye, Di He, and Liwei Wang. 2024. Do efficient transformers really save computation? *International Conference on Machine Learning*.

Howard Yen, Tianyu Gao, and Danqi Chen. 2024. Long-context language modeling with parallel context encoding. *arXiv preprint arXiv:2402.16617*.

Jingqing Zhang, Yao Zhao, Mohammad Saleh, and Peter Liu. 2020. PEGASUS: Pre-training with extracted gap-sentences for abstractive summarization. In *International Conference on Machine Learning*, pages 11328–11339. PMLR.

Zhenyu Zhang, Runjin Chen, Shiwei Liu, Zhewei Yao, Olatunji Ruwase, Beidi Chen, Xiaoxia Wu, and Zhangyang Wang. 2024. Found in the middle: How language models use long contexts better via plug-and-play positional encoding. *arXiv preprint arXiv:2403.04797*.

Liang Zhao, Xiaocheng Feng, Xiachong Feng, Bin Qin, and Ting Liu. 2023. Length extrapolation of transformers: A survey from the perspective of position encoding. *arXiv preprint arXiv:2312.17044*.

Liang Zhao, Tianwen Wei, Liang Zeng, Cheng Cheng, Liu Yang, Peng Cheng, Lijie Wang, Chenxia Li, Xuejie Wu, Bo Zhu, et al. 2024. Longskywork: A training recipe for efficiently extending context length in large language models. *arXiv preprint arXiv:2406.00605*.

Chuanyang Zheng, Yihang Gao, Han Shi, Minbin Huang, Jingyao Li, Jing Xiong, Xiaozhe Ren, Michael Ng, Xin Jiang, Zhenguo Li, et al. 2024. Dape: Data-adaptive positional encoding for length extrapolation. *Advances in Neural Information Processing Systems*.

Chuanyang Zheng, Zhengying Liu, Enze Xie, Zhenguo Li, and Yu Li. 2023. Progressive-hint prompting improves reasoning in large language models. *arXiv preprint arXiv:2304.09797*.

Hattie Zhou, Arwen Bradley, Etai Littwin, Noam Razin, Omid Saremi, Joshua M. Susskind, Samy Bengio, and Preetum Nakkiran. 2024a. What algorithms can transformers learn? a study in length generalization. In *International Conference on Learning Representations*.

Jin Peng Zhou, Charles E Staats, Wenda Li, Christian Szegedy, Kilian Q Weinberger, and Yuhuai Wu. 2024b. Don't trust: Verify – grounding LLM quantitative reasoning with autoformalization. In *The Twelfth International Conference on Learning Representations*.

Yongchao Zhou, Uri Alon, Xinyun Chen, Xuezhi Wang, Rishabh Agarwal, and Denny Zhou. 2024c. Transformers can achieve length generalization but not robustly. *arXiv preprint arXiv:2402.09371*.

Dawei Zhu, Nan Yang, Liang Wang, Yifan Song, Wenhao Wu, Furu Wei, and Sujian Li. 2023a. PoSE: Efficient context window extension of llms via positional skip-wise training. In *International Conference on Learning Representations*.

Qianchao Zhu, Jiangfei Duan, Chang Chen, Siran Liu, Xiuhong Li, Guanyu Feng, Xin Lv, Huanqi Cao, Xiao Chuanfu, Xingcheng Zhang, et al. 2024. Near-lossless acceleration of long context llm inference with adaptive structured sparse attention. *arXiv preprint arXiv:2406.15486*.

Shiyi Zhu, Jing Ye, Wei Jiang, Qi Zhang, Yifan Wu, and Jianguo Li. 2023b. CoCA: Fusing position embedding with collinear constrained attention for fine-tuning free context window extending. *arXiv e-prints*, pages arXiv–2309.

A Theoretical Analysis on Associative Recall

As pointed out in some previous works (Arora et al., 2024), the perplexity scores of Transformers mostly result from the performances on associate recall tasks (i.e., the copy tasks). Numerous studies have explored the mechanism of associative recall within Transformers, both from theoretical perspectives and experimental validations (Arora et al., 2024; Bietti et al., 2024; Golovneva et al., 2024). Therefore, theoretical analysis of how the proposed model performs on associative recall tasks is essential and fundamental for understanding its mechanism and superiority over the vanilla Transformer architecture. In this section, we theoretically compare the CDAPE with the vanilla Transformer in solving a simple associative recall task.

Bietti et al. (2024) considers a simple associative recall task in which, given an input sequence of tokens $[\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_N]$, the last token serves as a trigger that has previously appeared in the context. The model is then expected to predict a specific token that should follow this trigger. Without the loss of generality, we assume that the trigger token also appears at position $\boldsymbol{x}_1$ (i.e., $\boldsymbol{x}_N = \boldsymbol{x}_1$) and that token $\boldsymbol{x}_2$ follows $\boldsymbol{x}_1$ in the whole sequence. Therefore, the model is expected to predict $\boldsymbol{x}_{N+1}$ as $\boldsymbol{x}_2$. Bietti et al. (2024) theoretically show that a two-layer vanilla Transformer can solve this task with the help of positional encodings.

Proposition 2. *A two-layer vanilla Transformer model solves the simple associative recall task by making use of positional encodings.*

Proof. Detailed proof can be found in Bietti et al. (2024). To clearly compare the mechanisms and capabilities of the proposed CDAPE model with those of the vanilla Transformer, we summarize and present their key ideas and results below.

After applying additive positional encoding, the input sequence becomes $[\boldsymbol{x}_1 + \boldsymbol{p}_1, \cdots, \boldsymbol{x}_N + \boldsymbol{p}_N]$, where $[\boldsymbol{p}_1, \boldsymbol{p}_2, \cdots, \boldsymbol{p}_N]$ are orthogonal positional encoding vectors. We assume that both the token vectors and positional encodings are nearly orthogonal. For simplicity, we consider a two-layer, single-head vanilla Transformer model without the feedforward layers, focusing merely on the attention mechanism. Let $\{\boldsymbol{W}_K^1, \boldsymbol{W}_Q^1, \boldsymbol{W}_V^1\}$ and $\{\boldsymbol{W}_K^2, \boldsymbol{W}_Q^2, \boldsymbol{W}_V^2\}$denote the key, query, and value projection matrices for the first and second layers, respectively. Let the key and query projection matrices as $\boldsymbol{W}_K^{1\top}\boldsymbol{W}_Q^1 = \sum_{i=1}^N \boldsymbol{p}_{i-1}\boldsymbol{p}_i^\top$. Under this setup, the $(i-1)$-th token representation in the first layer becomes $\boldsymbol{x}_i + \boldsymbol{p}_i + \boldsymbol{W}_V^1\boldsymbol{x}_{i-1}$. The key idea is that the first layer uses causal attention and positional encodings to copy information from the previous token to the current one, enabling associative recall through attention layers.

In the second layer, since token $\boldsymbol{x}_2$ now contains information about the trigger token $\boldsymbol{x}_1$, the attention mechanism enables the trigger token $\boldsymbol{x}_N$ to attend to $\boldsymbol{x}_2$, which effectively embeds the trigger token's context. As a result, the model is able to predict the next token as $\boldsymbol{x}_2$. Specifically, we set $\boldsymbol{W}_K^2\boldsymbol{W}_Q^2 = \boldsymbol{W}_V^1$ so that the attention mechanism guides $\boldsymbol{x}_N$ to attend to $\boldsymbol{x}_2$, using the approximate orthogonality of the token vectors. More concretely, the key vector $\boldsymbol{W}_K^2\boldsymbol{x}_i \approx \boldsymbol{W}_V^1\boldsymbol{x}_{i-1}$ and the query vector $\boldsymbol{W}_Q^2\boldsymbol{x}_i \approx \boldsymbol{W}_V^1\boldsymbol{x}_i$. For the trigger token $\boldsymbol{x}_N$, the query vector becomes $\boldsymbol{W}_V^1\boldsymbol{x}_N$, and the keys for the other tokens are approximately $\boldsymbol{W}_V^1\boldsymbol{x}_{i-1}$. In particular, the key vector corresponding to token $\boldsymbol{x}_2$ is $\boldsymbol{W}_V^1\boldsymbol{x}_1$, which closely matches the query vector of $\boldsymbol{x}_N$ (since $\boldsymbol{x}_N = \boldsymbol{x}_1$). Consequently, after softmax normalization, most of the attention weight is concentrated on token $\boldsymbol{x}_2$. As a result, the representation of $\boldsymbol{x}_N$ in the second layer incorporates the information from $\boldsymbol{x}_2$ through $\boldsymbol{W}_V^2\boldsymbol{x}_2$. In the output layer, the model then predicts the next token as $\boldsymbol{x}_2$, successfully completing the associative recall task.

In summary, the two-layer and single-head vanilla Transformer model realizes the simple associative recall task by making use of the positional encoding and setting parameters as follows:

- $\boldsymbol{W}_V^1$ is randomly generated, and $\boldsymbol{W}_K^{1\top}\boldsymbol{W}_Q^1 = \sum_{i=1}^N \boldsymbol{p}_{i-1}\boldsymbol{p}_i^\top$;
- $\boldsymbol{W}_V^2$ is randomly generated, and $\boldsymbol{W}_K^2\boldsymbol{W}_Q^2 = \boldsymbol{W}_V^1$.

□

We now analyze how the proposed CDAPE model solves the simple associative recall task.

Proposition 3. *A one-layer, single-head CDAPE model is capable of performing the simple associative recall task without relying on positional encodings.*

Proof. Compared to the vanilla Transformer model, CDAPE primarily introduces convolution over attention scores. For simplicity, we similarly

focus on an attention-only version of the model, incorporating linear convolution without any nonlinear activation.

The main modification in CDAPE is the use of convolution, a local operation over neighboring tokens, to copy information from previous tokens, replacing the role of positional encodings in the vanilla Transformer, as discussed before. In this setting, the input token sequence is $[\boldsymbol{x}_1, \boldsymbol{x}2, \cdots, \boldsymbol{x}N]$. We set $\boldsymbol{W}_K^1 = \boldsymbol{I}$ and $\boldsymbol{W}_Q^1 = \boldsymbol{I}$, so that the query vector for x_N is simply $\boldsymbol{x}_N$ itself, and each key vector is equal to the corresponding input token vector, i.e., $\boldsymbol{x}i$. The key vector for each token $\boldsymbol{x}_i$ is also $\boldsymbol{x}_i$. The convolution over attention scores is equivalently implemented by applying convolution directly to the key vectors, if without nonlinear activation. Specifically, we adopt a 2×1 kernel with weights $[1, -1]$, so that the key vector after the convolution for the i-th token becomes $\boldsymbol{x}i - 1 - \boldsymbol{x}_i$. As a result, the key vector for token $\boldsymbol{x}_2$ becomes $\boldsymbol{x}_1 - \boldsymbol{x}_2$, which closely matches the query vector $\boldsymbol{x}_N = \boldsymbol{x}_1$ (recall $\boldsymbol{x}_N$ is the trigger token). Therefore, when computing attention for $\boldsymbol{x}_N$, most of the attention weight is assigned to token $\boldsymbol{x}_2$. After the attention layer, the representation at position N becomes $\boldsymbol{x}_N + \boldsymbol{W}_V^1 \boldsymbol{x}_2$, effectively copying information from $\boldsymbol{x}_2$ to $\boldsymbol{x}_N$. In the output layer, the model then predicts the next token as $\boldsymbol{x}_2$, realizing the simple associative recall task.

We further elaborate on the procedure. If we set the query transformation matrix as $\boldsymbol{W}_Q^1 = \boldsymbol{I}$, then the query vector for token N is simply $\boldsymbol{x}_N$. Similarly, setting the key transformation matrix as $\boldsymbol{W}_K^1 = \boldsymbol{I}$ gives the sequence of key vectors as:

$$[\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_N] .$$

Applying a 2×1 convolutional kernel with weights $[1, -1]$ to the key sequence results in the following transformed key vectors:

$$[-\boldsymbol{x}_1, -\boldsymbol{x}_2 + \boldsymbol{x}_1, \cdots, -\boldsymbol{x}_i + \boldsymbol{x}_{i+1}, \cdots, -\boldsymbol{x}_N + \boldsymbol{x}_{N-1}] .$$

From the key vectors above, we observe that the query vector $\boldsymbol{x}_N = \boldsymbol{x}_1$ closely matches the key vector corresponding to the second token. As a result, after softmax normalization, the attention mechanism assigns the highest weight to the second token when computing the output for the N-th position.

□

Compared to the vanilla Transformer, CDAPE replaces the first layer of neighboring token communication with a convolutional operator. As a result, a one-layer, single-head CDAPE model is capable of solving the simple associative recall task without relying on positional encodings. While the vanilla Transformer implicitly integrates information from neighboring tokens through positional encodings and attention, CDAPE achieves this through explicit convolution, enabling more direct and computationally efficient communication between neighboring tokens.

B Δ Perplexity for Length Extrapolation Evaluation

Table 2: The ΔP on Book dataset with training length 512, compared to baselines.

Method	RoPE	ALiBi	Kerple	DAPE-Kerple	CDAPE-Kerple
$P(M(x_{512}), T_{train})$	19.74	20.04	19.83	19.25	18.95
$P(M(x_{1024}), T_{train})$	261.39	19.74	19.19	18.28	17.92
$P(M(x_{1024}[-T_{train}]:), T_{train})$	19.51	19.79	19.58	19.03	18.74
ΔP_{1024}	-241.88	0.05	0.39	0.75	0.82
$P(M(x_{2048}), T_{train})$	411.23	20.17	20.48	17.20	16.79
$P(M(x_{2048}[-T_{train}]:), T_{train})$	18.74	19.03	19.84	18.28	18.01
ΔP_{2048}	-392.49	-1.14	-0.64	1.08	1.22
$P(M(x_{4096}), T_{train})$	635.80	20.50	28.33	17.58	17.05
$P(M(x_{4096}[-T_{train}]:), T_{train})$	19.11	19.35	19.07	18.59	18.19
ΔP_{4096}	-616.69	-1.15	-9.26	1.01	1.14
$P(M(x_{8192}), T_{train})$	762.86	21.30	40.94	17.85	17.20
$P(M(x_{8192}[-T_{train}]:), T_{train})$	19.78	20.02	19.85	19.38	18.98
ΔP_{8192}	-743.08	-1.28	-21.09	1.53	1.78

In this discussion, we explore how to effectively use perplexity as a metric, incorporating concepts of information gain and entropy. Let $L(\cdot)$ represent the process for calculating loss, and $M(x)$ denote the logit output generated by the model after processing an input sequence x. For evaluating model performance, we define $P(M(x), K)$ as follows:

1. Process the entire sequence x using $M(x)$.
2. Compute the perplexity on the last K tokens of the sequence.

To interpret information gain, we consider the training sequence length T_{train}. Given an input x, we calculate the change in loss/perplexity, ΔP, as:

$$\Delta P = P(M(x[-T_{\text{train}}:]), T_{\text{train}}) - P(M(x), T_{\text{train}}) \quad (6)$$

The term ΔP provides insights into the model's information gain relative to local and global context, allowing us to quantify entropy in terms of model uncertainty reduction. We interpret ΔL as follows:

- When $\Delta P = 0$: The model's information gain from the full sequence is negligible, indicating an entropy level comparable to local attention (e.g., models like ALiBi when the evaluation

length is 1024). This suggests the model does not leverage context beyond a limited range.

- When $\Delta P < 0$: Processing the entire sequence increases entropy, resulting in worse performance than focusing only on the last T_{train} tokens. This implies negative information gain and limited extrapolation capability (e.g. such as RoPE), as the model may overfit to recent tokens without capturing broader context effectively.

- When $\Delta P > 0$: The model benefits from the information within $x[: T_{\text{train}}]$, achieving a reduction in entropy that reflects positive information gain. This suggests the model leverages contextual information beyond the training sequence, indicating extrapolation capability.

- Our suggestion of bias matrix. The Kerple is a good choice for almost all settings, the FIRE may need longer training length/tokens to present its ability, and do not use ALiBi unless necessary . It is easy to train Kerple, as Kerple usually has few trainable parameters compared to FIRE. If you do not know which one to use, directly use Kerple. FIRE may have better performance, but may need longer training length (diverges at 128 but works well at 512, with DAPE). FIRE $b(i,j) = f_\theta\left(\frac{\psi(i-j)}{\psi(\max\{L,i\})}\right)$ so that we may need longer training length or more training tokens to well-train the neural network f_θ. Do not use ALiBi unless necessary. The ALiBi will quickly become local attention as the sequence length increases.

By examining ΔP, we can evaluate the model's ability to reduce entropy and gain information from extended sequences, providing a measure of its extrapolative power.

C Compare with Baselines

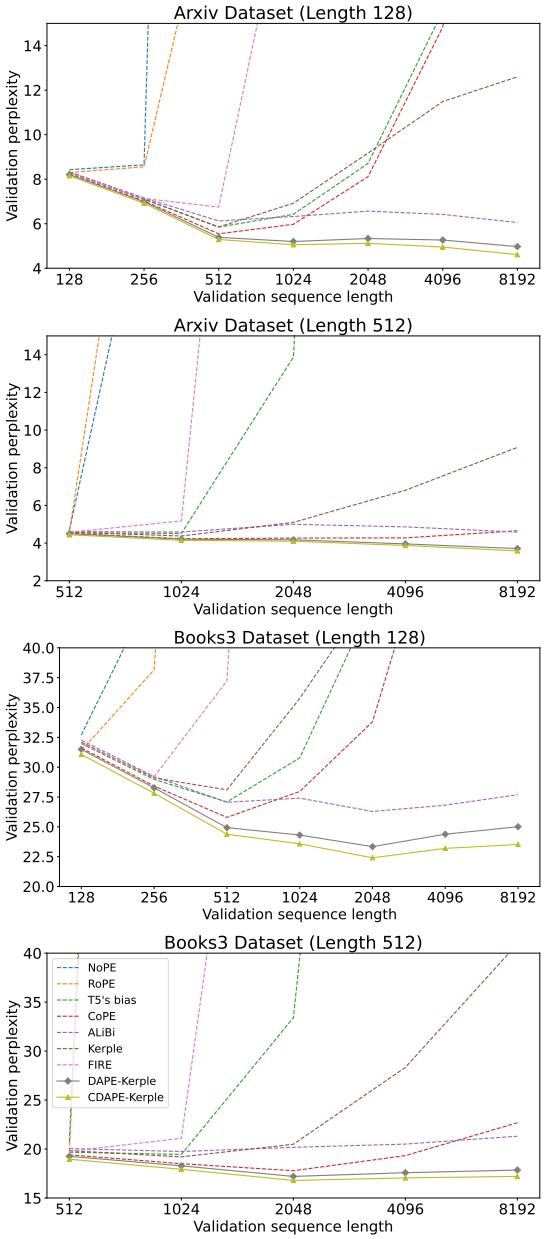

Figure 5: **Comparisons with baselines:** performance with training lengths 128 and 512 on Arxiv and Books3 datasets.

D Compare with Baseline on Arxiv Dataset with Training Length 1024

Table 3: The performance (ppl) on Arxiv dataset with training length 1024, compared to baselines.

Method	1024	2048	4096	8192
NoPE (Kazemnejad et al., 2024)	4.16	42.27	1854.73	17167.32
RoPE (Su et al., 2024b)	4.07	86.20	237.67	256.12
T5's bias (Raffel et al., 2020)	4.03	4.28	13.07	79.55
ALiBi (Press et al., 2021)	4.09	4.53	4.45	4.22
Kerple (Chi et al., 2022)	4.06	4.09	4.68	6.951
FIRE (Li et al., 2023c)	4.06	9.21	236.18	440.60
DAPE-Kerple (Zheng et al., 2024)	3.98	3.91	3.68	3.41
CDAPE-Kerple	3.93	3.86	3.61	3.37

E Large Model Size

Table 4: The performance (ppl) under large model size 2.7B on Books3 dataset.

Method	512	1024	2048	4096
RoPE	21.01	25.00	48.13	160.59
T5's bias	21.10	21.88	23.59	33.23
Kerple	21.14	22.08	23.38	27.21
DAPE-Kerple	20.52	21.01	20.23	19.67
CDAPE-Kerple (kernel size 1x3)	20.16	20.54	19.80	19.02

F The validation on downstream tasks

Table 5: The performance under large model size 125M on FineWeb-Edu (Lozhkov et al., 2024) 50B tokens. The acc_norm is the accuracy norm.

Method	Metric	RoPE	DAPE	CDAPE
Pile_10K	byte_perplexity	2.4967	2.4684	**2.4473**
Pile_10K	word_perplexity	459.0249	425.2719	**397.0870**
ARC_Challenge	acc_norm	25.17	25.43	**26.02**
ARC_Easy	acc_norm	48.02	48.23	**49.62**
Boolq	ACC	56.02	**59.05**	57.40
Hellaswag	acc_norm	33.15	34.35	**34.52**
PIQA	acc_norm	62.30	61.48	**62.35**
SocialIQA	acc	35.93	38.02	**38.59**

G The Performance of CDAPE with Information Leakage

The CDAPE can utilize attention data, which is supported by almost zero loss (perplexity is 1) under information leakage. To prevent the information leakage, we use the $torch.tril$ before CDAPE to make the attention score lower-triangular matrix. For the cheating version, we do not use the `torch.tril`. As shown in Figure 6, whatever CDAPE-ALiBi, CDAPE-Kerple or CDAPE-FIRE, their cheating version can all achieve about zero loss within evaluation length 1024. Furthermore, the CDAPE-Kerple can even aachievezero loss when the evaluation length is extended to 8096. This suggest that the proposed CDAPE can really realize and utilize the information of attention score.

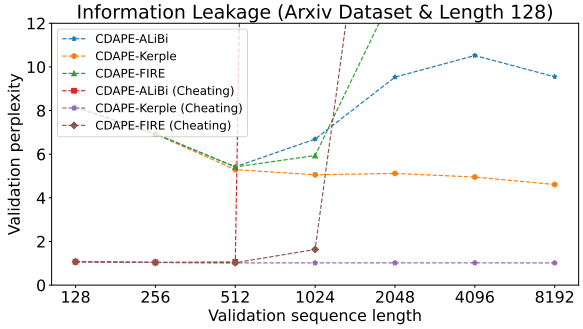

Figure 6: **Result with information leakage.**

H Compare DAPE and CDAPE with Approximate Computational Cost

CDAPE achieves even better performance at a lower computational cost. As shown in Appendix H, when the training length is set to 128, CDAPE-Kerple with D_{DAPE} as 10 achieves a perplexity (ppl) of 8.16 at an evaluation length of 128 and 4.74 at an evaluation length of 8192. This performance is notably better than that of DAPE-Kerple with D_{DAPE} as 64, which achieves perplexities of 8.21 and 4.87, respectively. Moreover, when the training length is extended to 512 and the evaluation length is smaller or equal to 4096, CDAPE-Kerple with D_{DAPE} as 10 continues to surpass the performance of DAPE-Kerple with D_{DAPE} as 64. Also, CDAPE-Kerple with D_{DAPE} as 21 always achieves better performance than DAPE-Kerple with D_{DAPE} as 64. This demonstrates that CDAPE not only maintains its performance advantage across different training lengths but also requires a lower computational cost.

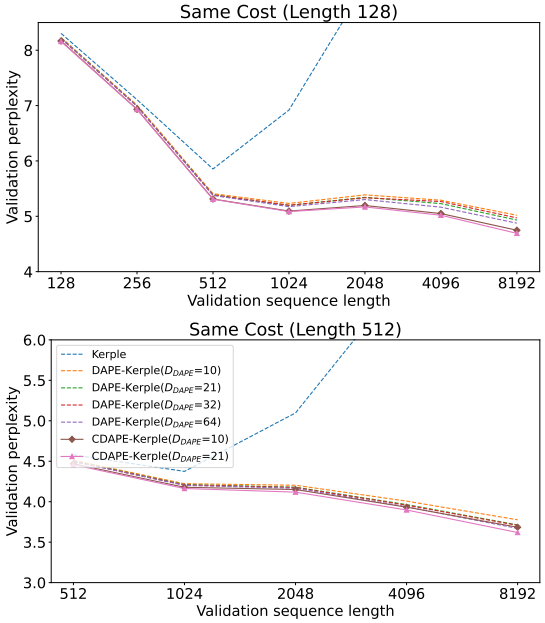

Figure 7: **Compare CDAPE and DAPE with the approximately same cost on Arxiv Dataset.** We compare the CDAPE and DAPE with approximate cost and different D_{DAPE}. As the kernel size of is 1×3, the proposed CDAPE is the triple computation cost of DAPE, with the same D_{DAPE}.

I The Performance with Different Kernel Size

Table 6: The performance with different kernel size, with training length 512 and evaluation from length 512 to 8192. For different datasets and training length, the optimal kernel size may not always be the largest one, especially when the evaluation length is larger.

Dataset	Method	512	1024	2048	4096	8192
Arxiv	Kerple	4.57	4.37	5.09	6.80	9.08
	DAPE-Kerple (Kernel Size 1x1)	4.49	4.20	4.17	3.95	3.70
	CDAPE-Kerple (Kernel Size 1x3)	4.44	4.14	4.09	3.87	3.58
	CDAPE-Kerple (Kernel Size 1x5)	4.44	4.14	4.10	3.85	3.59
	CDAPE-Kerple (Kernel Size 1x7)	**4.43**	**4.13**	**4.08**	**3.85**	**3.57**
Books3	Kerple	19.83	19.19	20.48	28.33	40.94
	DAPE-Kerple (Kernel Size 1x1)	19.25	18.28	17.20	17.58	17.85
	CDAPE-Kerple (Kernel Size 1x3)	18.95	17.92	16.79	17.05	17.20
	CDAPE-Kerple (Kernel Size 1x5)	18.89	17.87	16.76	17.09	**17.10**
	CDAPE-Kerple (Kernel Size 1x7)	**18.86**	**17.82**	**16.70**	**17.01**	17.16

J The Performance of CDAPE on CHE Benchmark

Table 7: Train on length 40 with 200k steps, and test from lengths 41 to 500. The random accuracy is 50%, except for MODULAR ARITHMETIC (SIMPLE), CYCLE NAVIGATION, BUCKET SORT, SOLVE EQUATION and MODULAR ARITHMETIC, where it is 20%. ††† denotes permutation-invariant tasks, which are expected to be solved without positional information. The dataset comes from (Choromanski et al., 2021), with experiment setting from Randomized PE(Ruoss et al., 2023).

		Baseline					CDAPE (Kernel Size 1)			CDAPE (Kernel Size 3)		
Level	**Task**	**RoPE**	**Relative**	**ALiBi**	**Kerple**	**FIRE**	**ALiBi**	**Kerple**	**FIRE**	**ALiBi**	**Kerple**	**FIRE**
R	EVEN PAIRS	99.98	96.60	73.52	57.50	73.86	99.99	99.58	**100**	99.99	**100**	**100**
	MODULAR ARITHMETIC (SIMPLE)	21.35	20.84	20.02	21.79	21.09	23.58	**24.47**	24.46	21.48	23.90	23.43
	PARITY CHECK†††	50.05	50.09	50.09	50.07	50.97	50.30	50.07	50.04	50.13	**52.51**	50.11
	CYCLE NAVIGATION†††	27.63	26.95	24.64	29.47	28.41	22.99	**34.53**	27.54	24.43	24.32	24.34
DCF	STACK MANIPULATION	61.49	64.73	66.42	66.93	69.33	68.18	**72.04**	70.90	58.90	68.18	60.90
	REVERSE STRING	65.23	65.59	71.09	71.54	65.89	73.37	70.74	76.40	56.61	**81.84**	70.11
	MODULAR ARITHMETIC	31.25	31.74	30.56	24.79	30.92	31.34	**32.37**	31.50	29.46	26.13	27.00
	SOLVE EQUATION	21.85	22.93	19.92	21.15	22.06	20.03	22.49	22.42	20.26	**23.95**	23.62
CS	DUPLICATE STRING	64.97	67.66	65.13	66.72	69.03	70.84	**72.95**	72.71	52.96	57.03	66.01
	MISSING DUPLICATE	63.37	72.34	74.21	79.06	79.27	83.41	87.57	89.17	59.33	**99.65**	74.83
	ODDS FIRST	61.00	61.57	59.88	62.59	63.28	63.78	**67.08**	66.34	57.35	56.87	56.57
	BINARY ADDITION	55.59	56.96	54.72	56.35	55.70	59.71	**60.88**	56.62	57.49	55.32	57.86
	COMPUTE SQRT	51.88	51.63	50.63	51.11	50.80	51.64	51.33	**52.46**	52.08	51.76	51.93
	BUCKET SORT†††	98.12	99.31	98.45	99.38	**99.57**	99.38	98.81	99.37	96.61	99.06	98.56

K CDAPE Time Cost

Table 8: The time cost (millisecond) under different testing lengths, with $D_{\mathbf{DAPE}}$ as 32 and default batch size 1, with training length 512.

Method	**350M Total**	**Ratio**	**2.7B Total**	**Ratio**	**6.7B Total**	**Ratio**
RoPE (Su et al., 2024b)	210.01	0.8306	472.63	1.0472	635.57	0.8564
T5's bias (Raffel et al., 2020)	355.16	1.4046	537.62	1.1912	808.85	1.0899
ALiBi (Press et al., 2021)	172.60	0.6826	325.95	0.7222	596.77	0.8041
Kerple (Chi et al., 2022)	189.91	0.7511	370.32	0.8205	661.82	0.8918
FIRE (Li et al., 2023c)	248.13	0.9813	432.63	0.9586	797.68	1.0748
DAPE-Kerple (Zheng et al., 2024)	224.22	0.8868	422.48	0.9361	717.46	0.9667
CDAPE-Kerple	252.84	1.0000	451.29	1.0000	742.10	1.0000

L Model Configuration

All experiments are conducted on 8 GPUs. The 125M and 350M model configuration is the following.

Table 9: **Model Configurations.**

	125M	**350M**
Training sequence length	512	512
Batch size	32×8	32×8
Numer of iterations	50k	50k
Dropout prob.	0.0	0.0
Attention dropout prob.	0.0	0.0
Attention head	12	16
Feature dimension	768	1024
Layer number	12	24
Optimizer	Adam	Adam
Optimizer parameter betas	[0.9, 0.95]	[0.9, 0.95]
Learning rate	$6e-4$	$3e-4$
Precision	float16	float16

M Data-Adaptive Related Position Encoding Performance Comparison

Table 10: The performance comparison between data-related position encoding, with dataset Books3 and training length 128.

Method	128	256	512	1024	2048	4096	8192
Transformer-XL	31.57	28.49	26.07	26.98	27.90	32.76	41.12
CoPE	31.61	28.41	25.79	27.96	33.80	54.08	90.66
DAPE-Kerple (Kernel Size 1x1)	31.49	28.27	24.93	24.31	23.34	24.38	25.01
CDAPE-Kerple (Kernel Size 1x3)	31.07	27.81	24.38	23.57	22.40	23.19	**23.52**

N The Error Bar and Significance Value

Table 11: **The perplexity performances on the Books3 dataset when the training length is 512 and running with three random seeds.**

Method		512	1024	2048	4096	8192
RoPE	mean	19.6805	244.6191	369.1415	528.3689	616.7107
	std	0.1552	36.3003	30.7471	79.5782	107.1226
Alibi	mean	19.9721	19.9142	20.3743	19.9823	20.7975
	std	0.0278	0.0990	0.1246	0.1845	0.2196
Kerple	mean	19.7681	19.2457	21.8713	29.8706	42.2081
	std	0.1687	0.1575	1.6713	4.5761	5.8176
FIRE	mean	19.6919	21.4867	107.1641	312.2332	499.9489
	std	0.1237	0.4015	5.3597	14.1547	20.0708
DAPE-Kerple	mean	19.2322	18.2911	17.5365	17.2084	17.7459
	std	0.1634	0.0315	0.2383	0.4791	0.1845
CDAPE-Kerple	mean	18.9193	17.9277	17.1339	16.6910	17.1153
	std	0.1627	0.0197	0.2431	0.4695	0.2261

According to Table 11, the performances of different methods are evaluated based on perplexity across various validation lengths ranging from 512 to 8192. The results indicate that ALiBi, Kerple and FIRE consistently outperform the RoPE for the length extrapolation, all with p-values less than 0.05. CDAPE-Kerple surpasses DAPE-Kerple, suggesting significant improvements.

O Risk

This work focuses on language modeling so that there is no specific risk. Also, this work use AI assistants for writing.

P CDAPE Visualization

P.1 Visualization on length 512

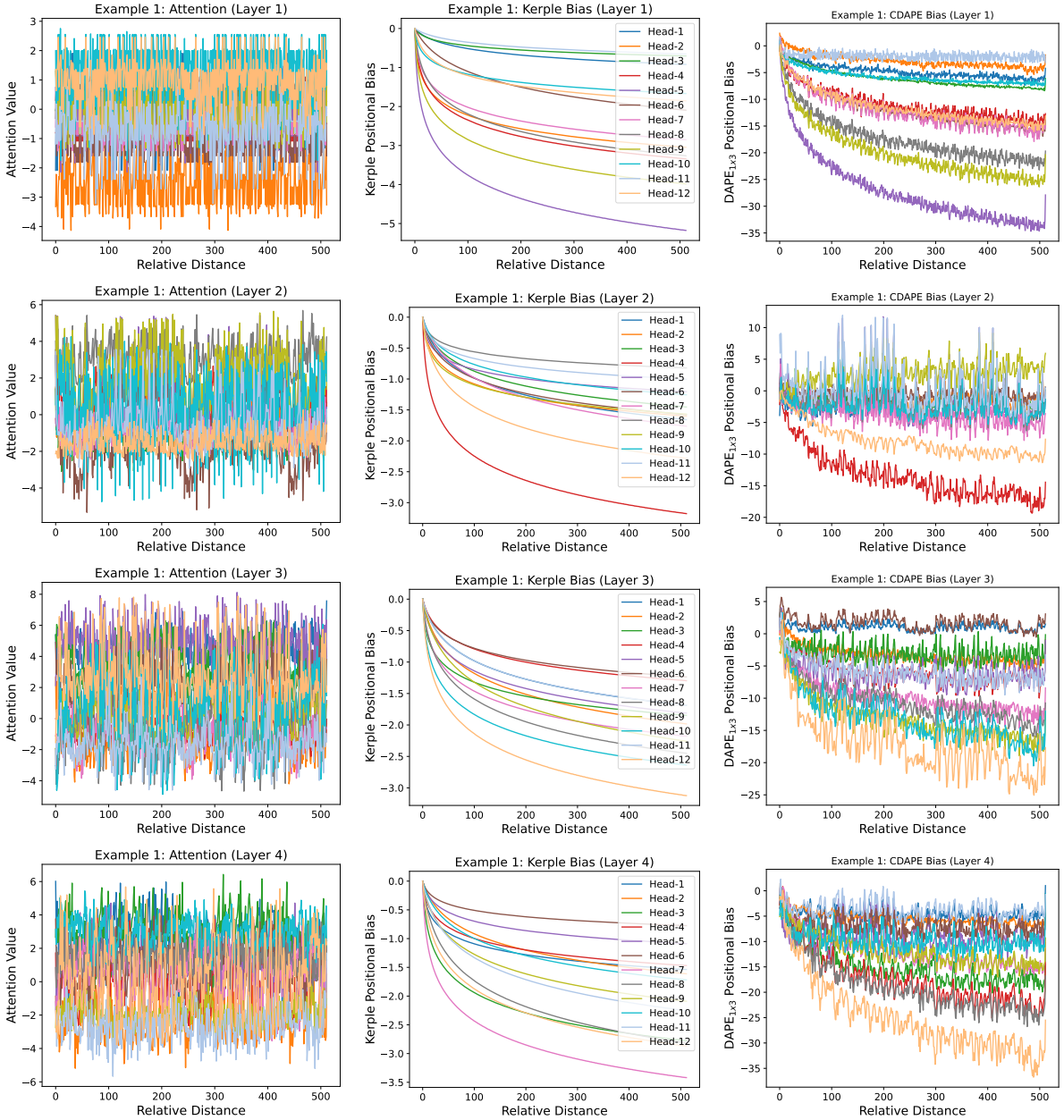

Figure 8: **Evaluation Length 512 Example 1: Part 1. From Left to Right: (1) The Attention is $\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top$; (2) The Kerple bias is $\boldsymbol{B}$; (3) The CDAPE (with Kerple) bias is $f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, \boldsymbol{B})$.**

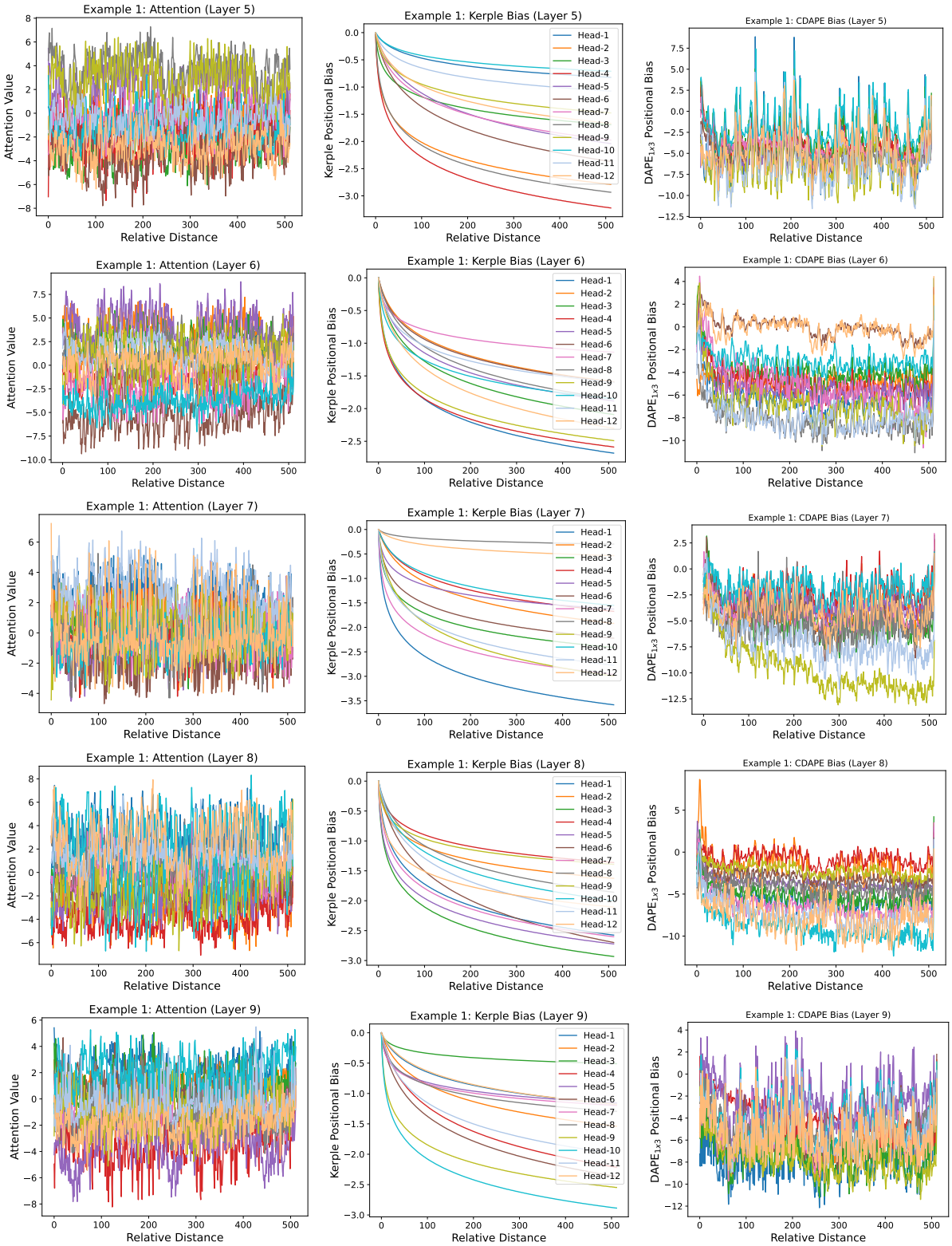

Figure 9: **Evaluation Length 512 Example 1: Part 2. From Left to Right: (1) The Attention is $\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top$; (2) The Kerple bias is $\boldsymbol{B}$; (3) The CDAPE (with Kerple) bias is $f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, \boldsymbol{B})$.**

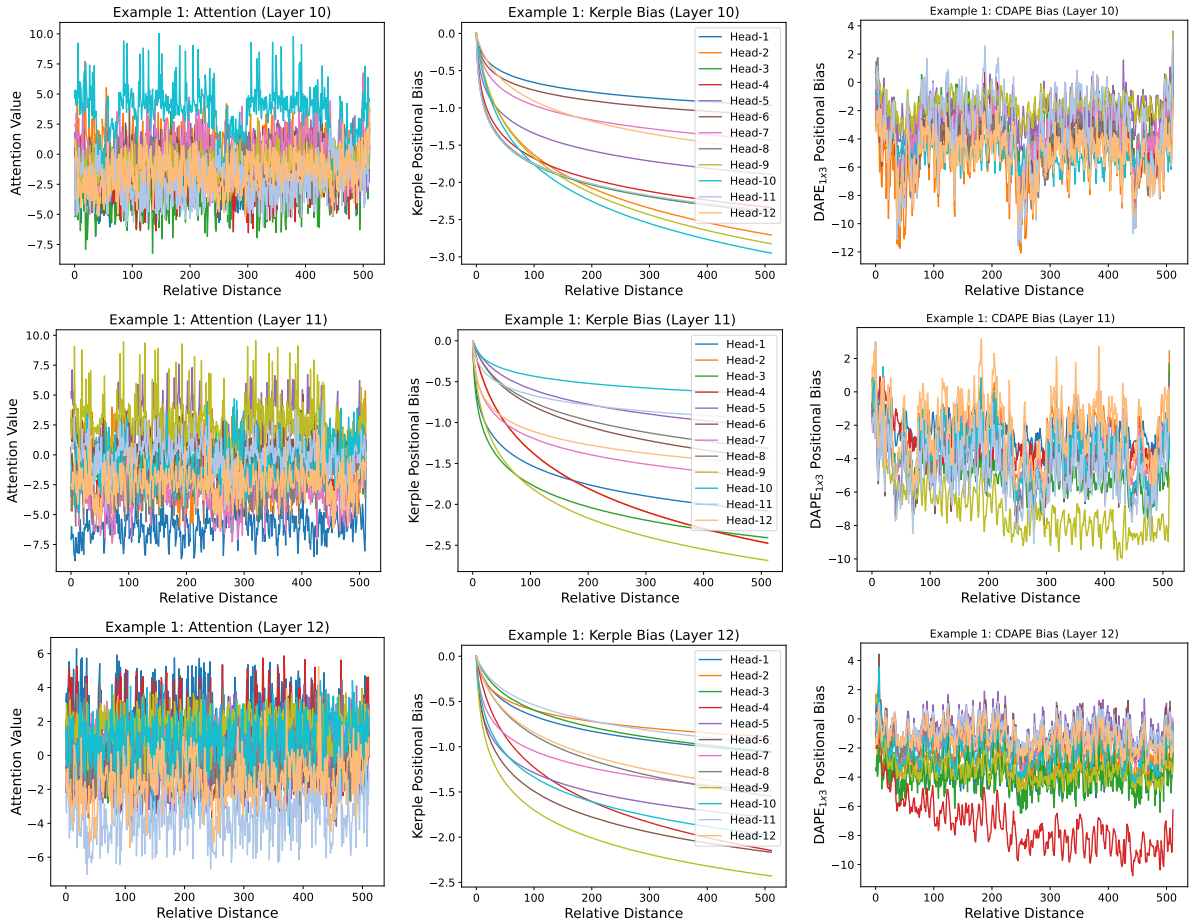

Figure 10: **Evaluation Length 512 Example 1: Part 3. From Left to Right: (1) The Attention is $\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top$; (2) The Kerple bias is $\boldsymbol{B}$; (3) The CDAPE (with Kerple) bias is $f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, \boldsymbol{B})$.**

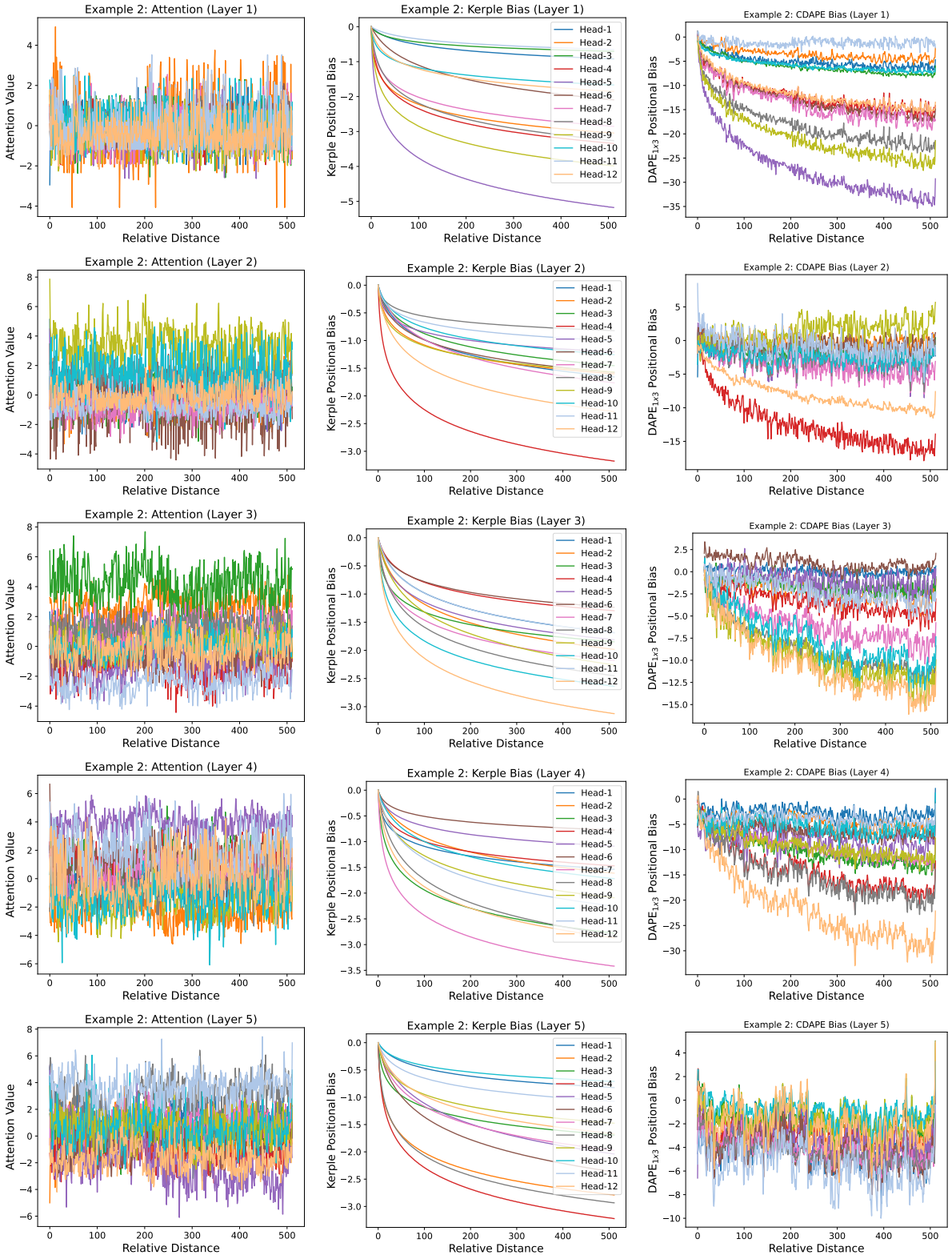

Figure 11: **Evaluation Length 512 Example 2: Part 1. From Left to Right: (1) The Attention is $\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top$; (2) The Kerple bias is $\boldsymbol{B}$; (3) The CDAPE (with Kerple) bias is $f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, \boldsymbol{B})$.**

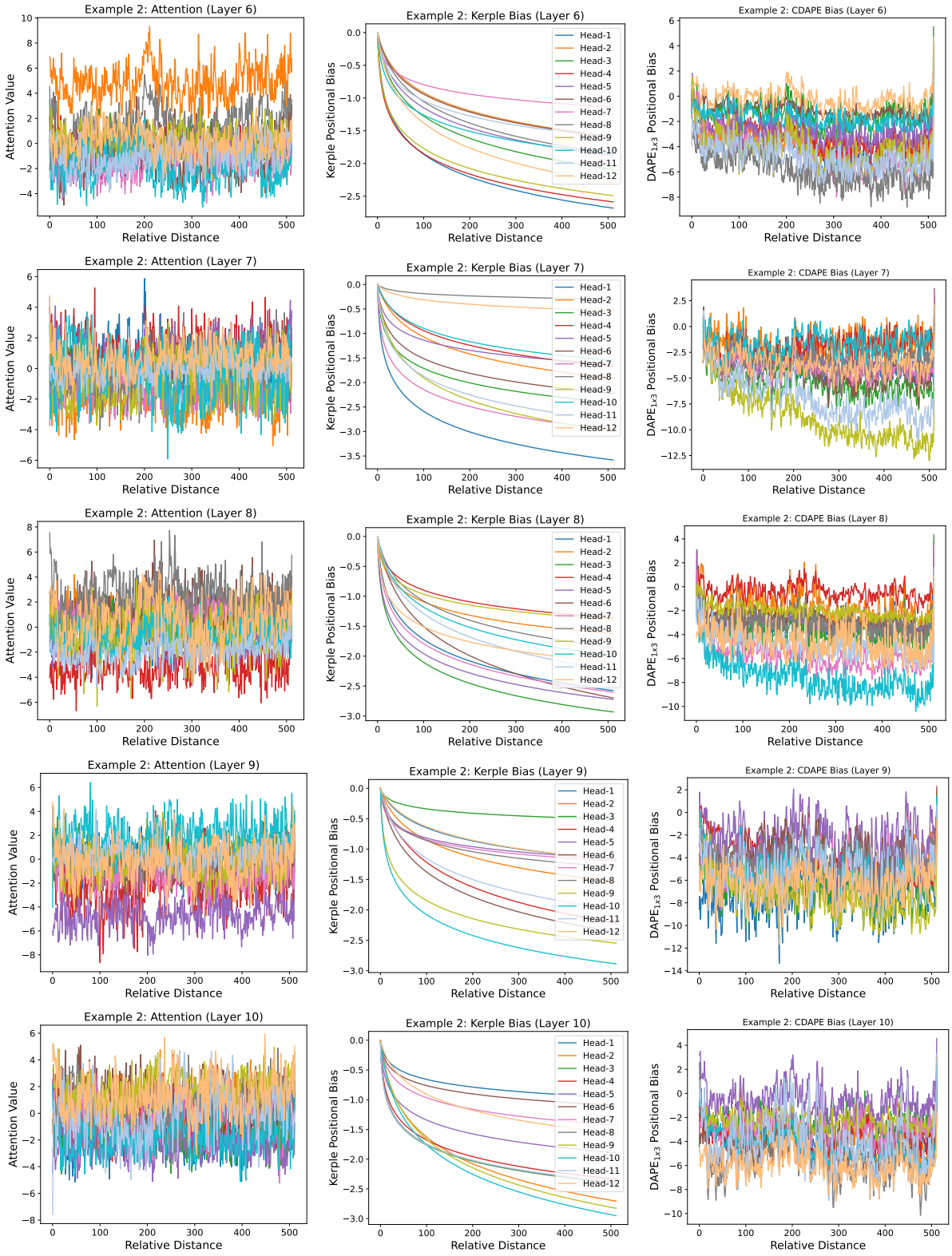

Figure 12: **Evaluation Length 512 Example 2: Part 2. From Left to Right: (1) The Attention is $\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top$; (2) The Kerple bias is $\boldsymbol{B}$; (3) The CDAPE (with Kerple) bias is $f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, \boldsymbol{B})$.**

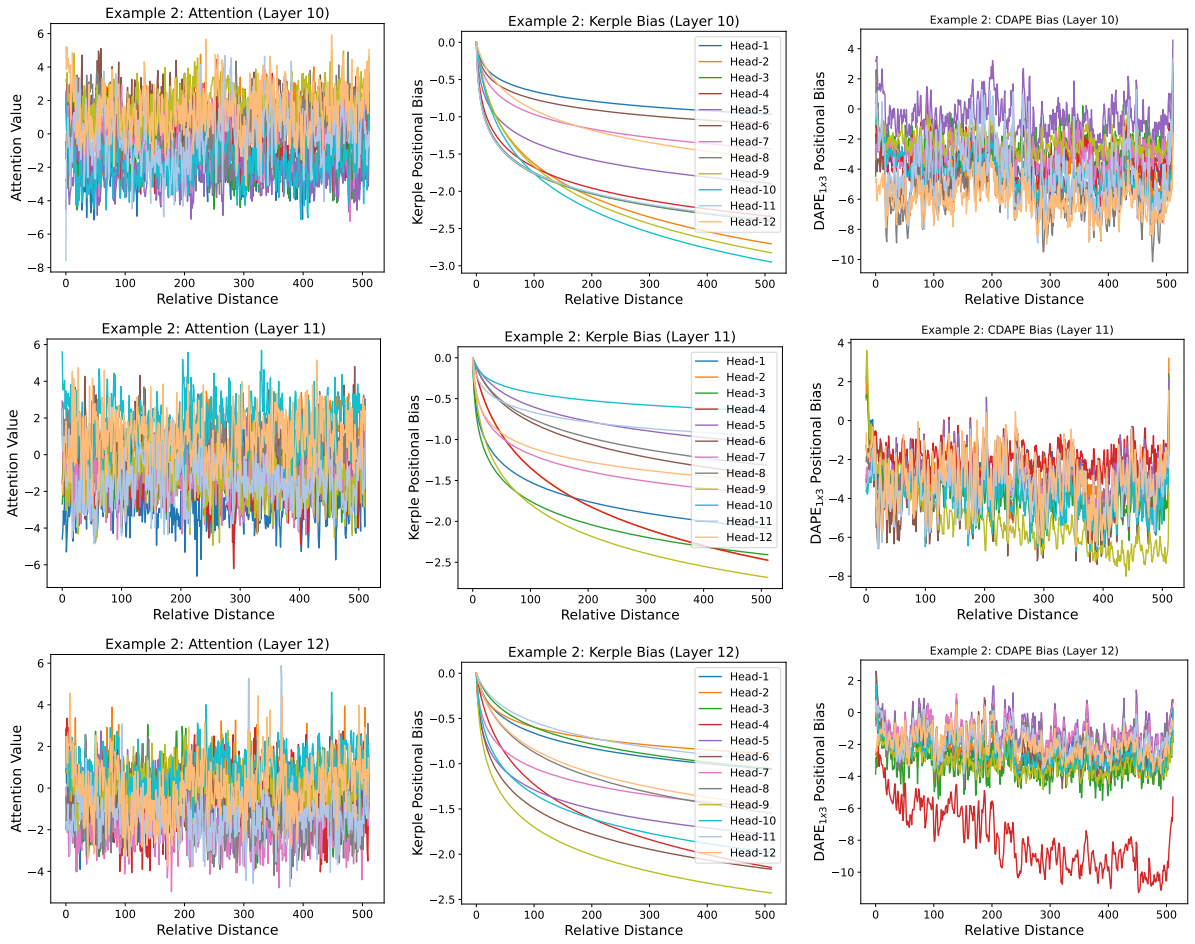

Figure 13: **Evaluation Length 512 Example 2: Part 2. From Left to Right: (1) The Attention is $\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top$; (2) The Kerple bias is $\boldsymbol{B}$; (3) The CDAPE (with Kerple) bias is $f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, \boldsymbol{B})$.**

P.2 Visualization on length 2048

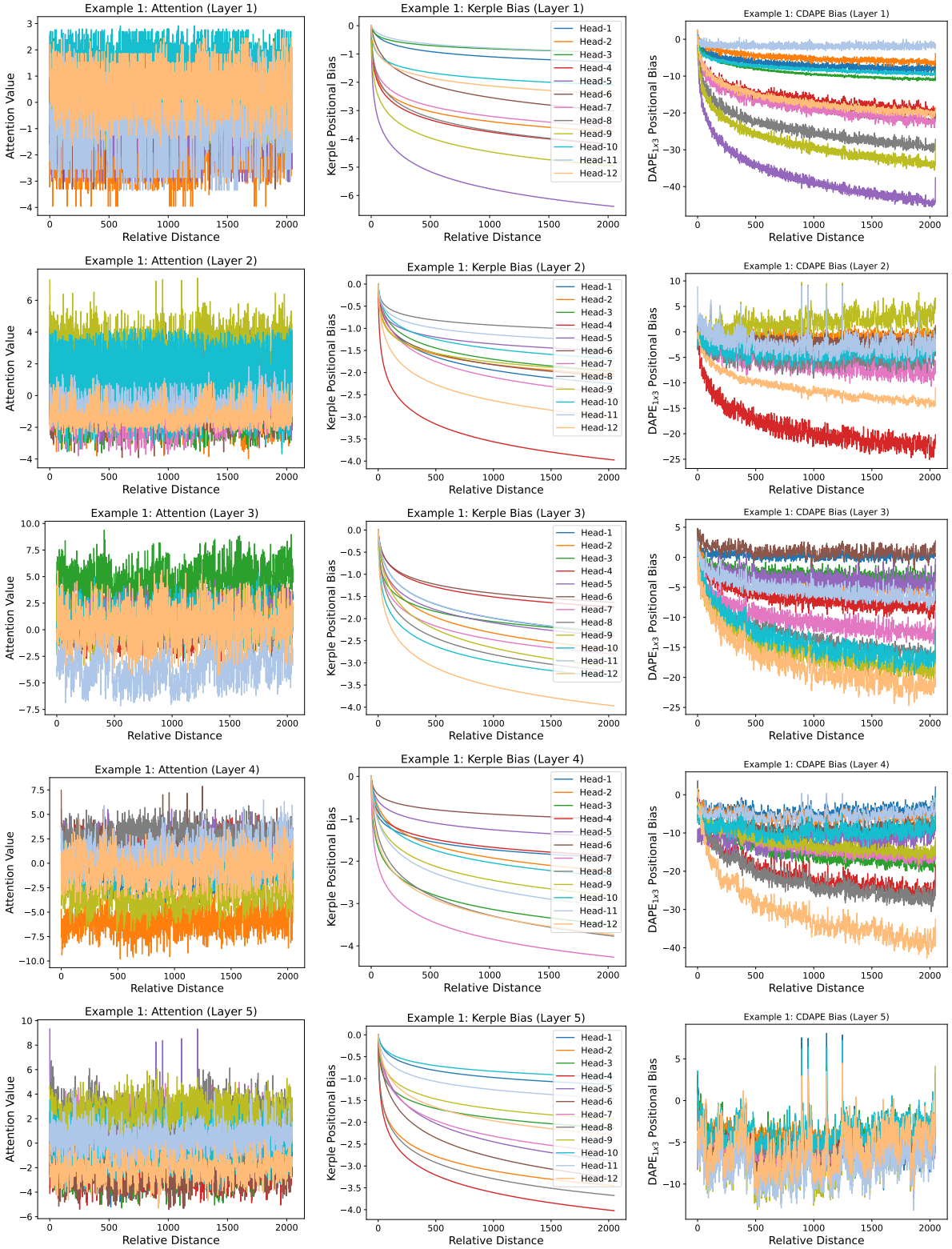

Figure 14: **Evaluation Length 2048 Example 1: Part 1. From Left to Right: (1) The Attention is $\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top$; (2) The Kerple bias is $\boldsymbol{B}$; (3) The CDAPE (with Kerple) bias is $f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, \boldsymbol{B})$.**

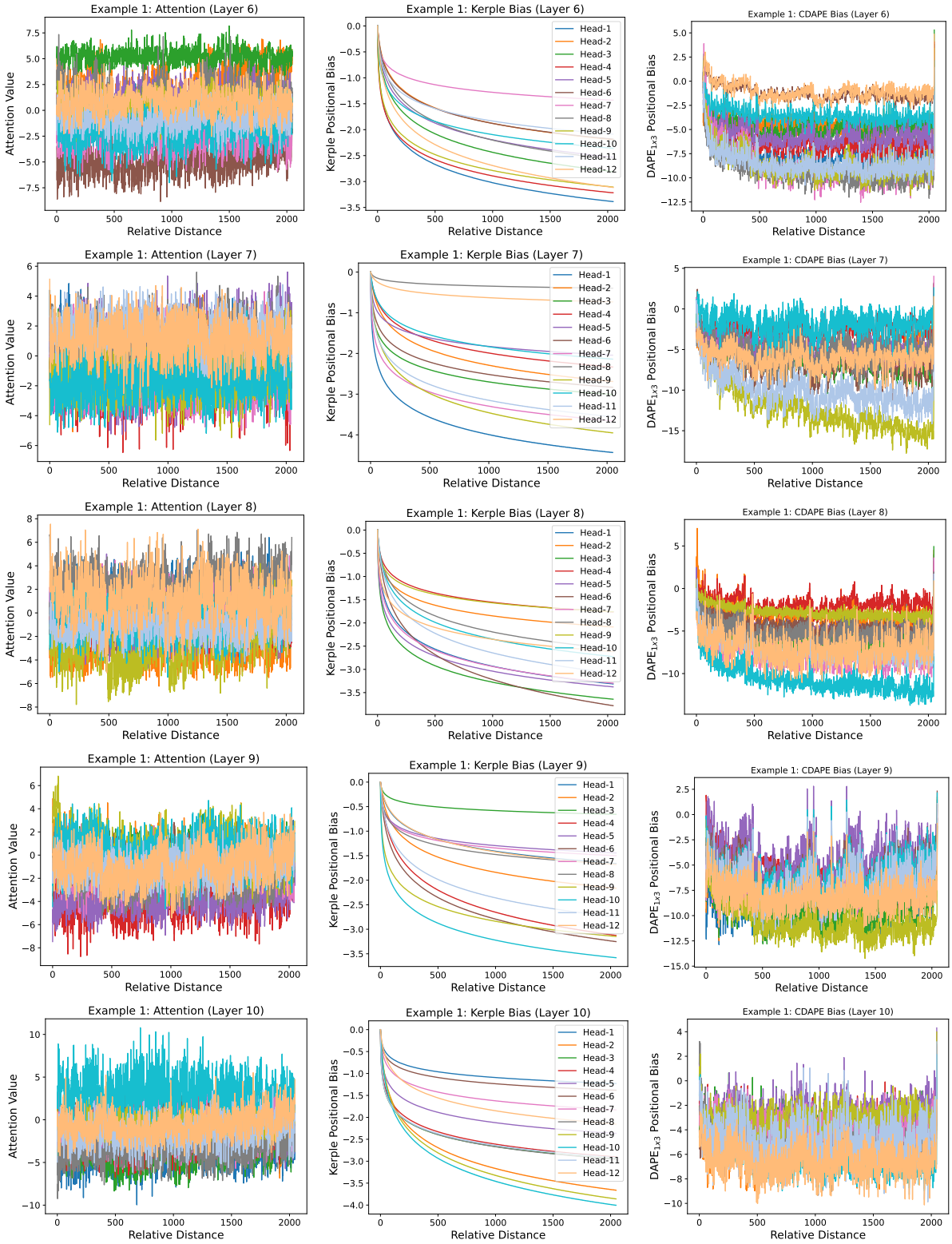

Figure 15: **Evaluation Length 2048 Example 1: Part 2. From Left to Right: (1) The Attention is $\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top$; (2) The Kerple bias is $\boldsymbol{B}$; (3) The CDAPE (with Kerple) bias is $f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, \boldsymbol{B})$.**

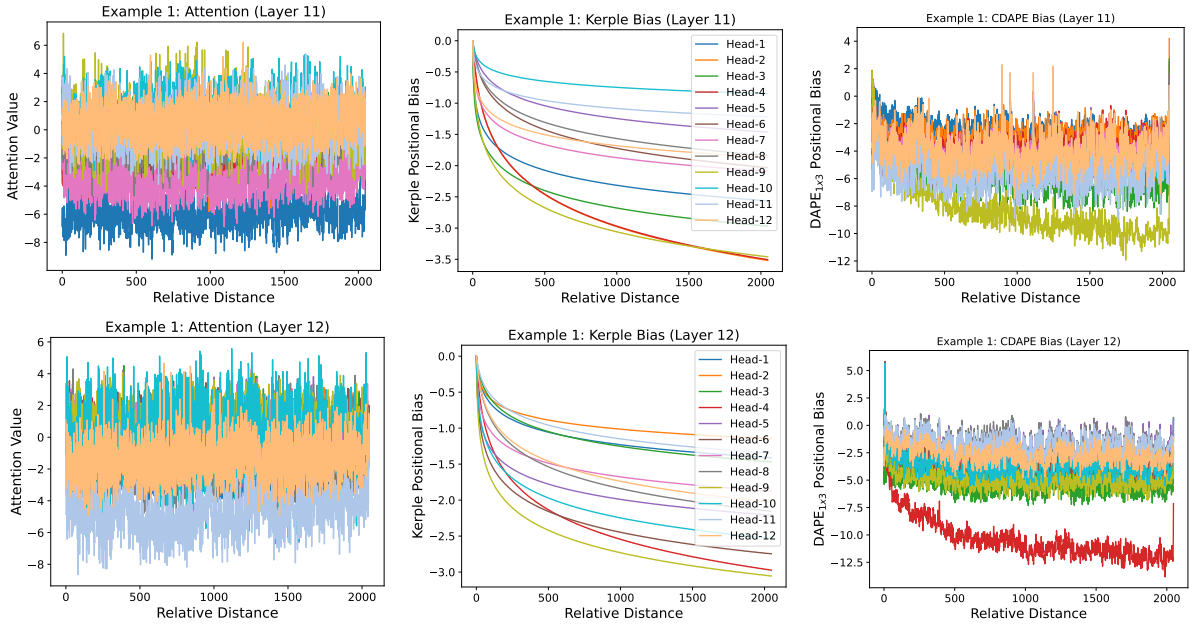

Figure 16: **Evaluation Length 2048 Example 1: Part 3. From Left to Right: (1) The Attention is $\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top$; (2) The Kerple bias is B; (3) The CDAPE (with Kerple) bias is $f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, B)$.**

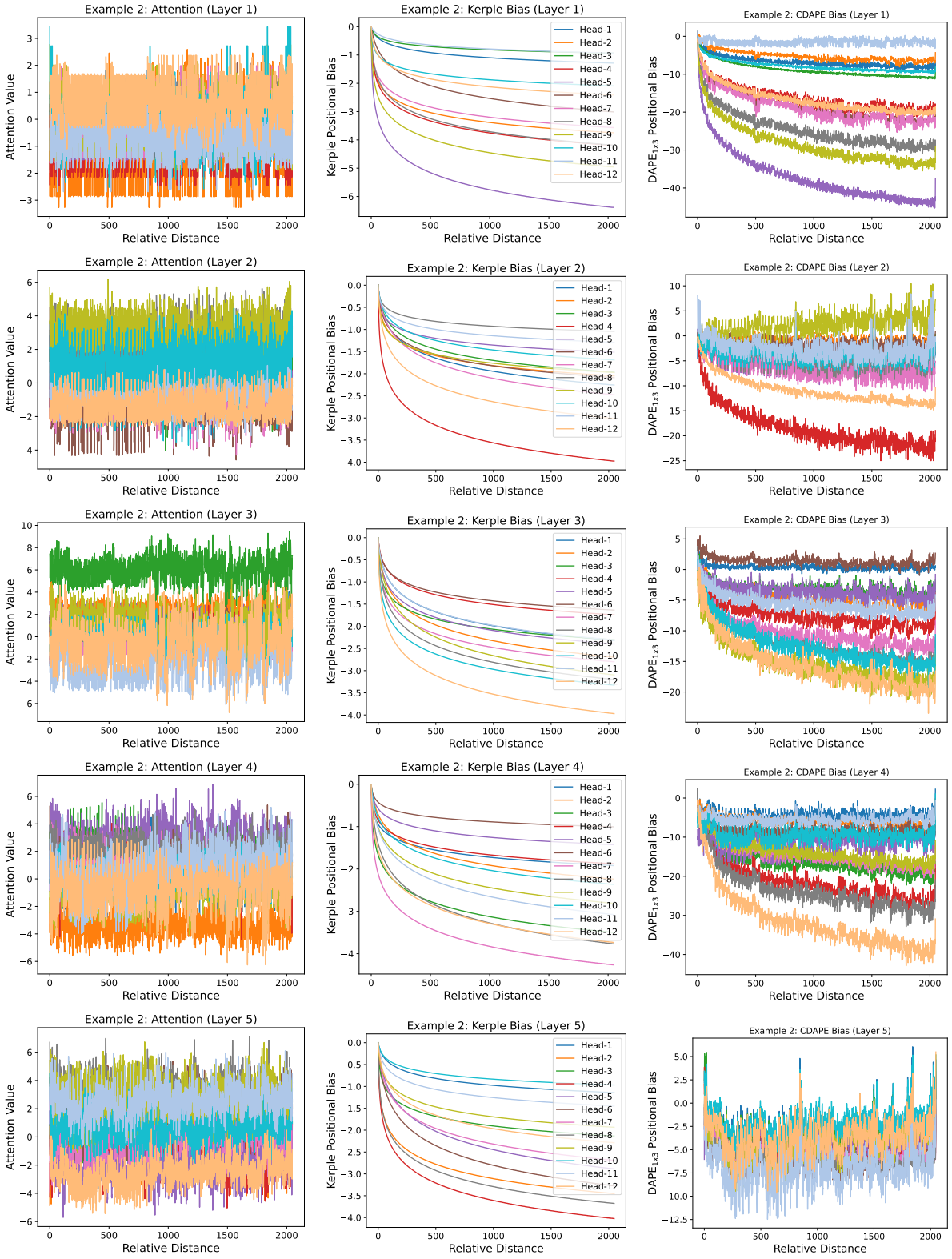

Figure 17: **Evaluation Length 2048 Example 2: Part 1. From Left to Right: (1) The Attention is $\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top$; (2) The Kerple bias is $\boldsymbol{B}$; (3) The CDAPE (with Kerple) bias is $f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, \boldsymbol{B})$.**

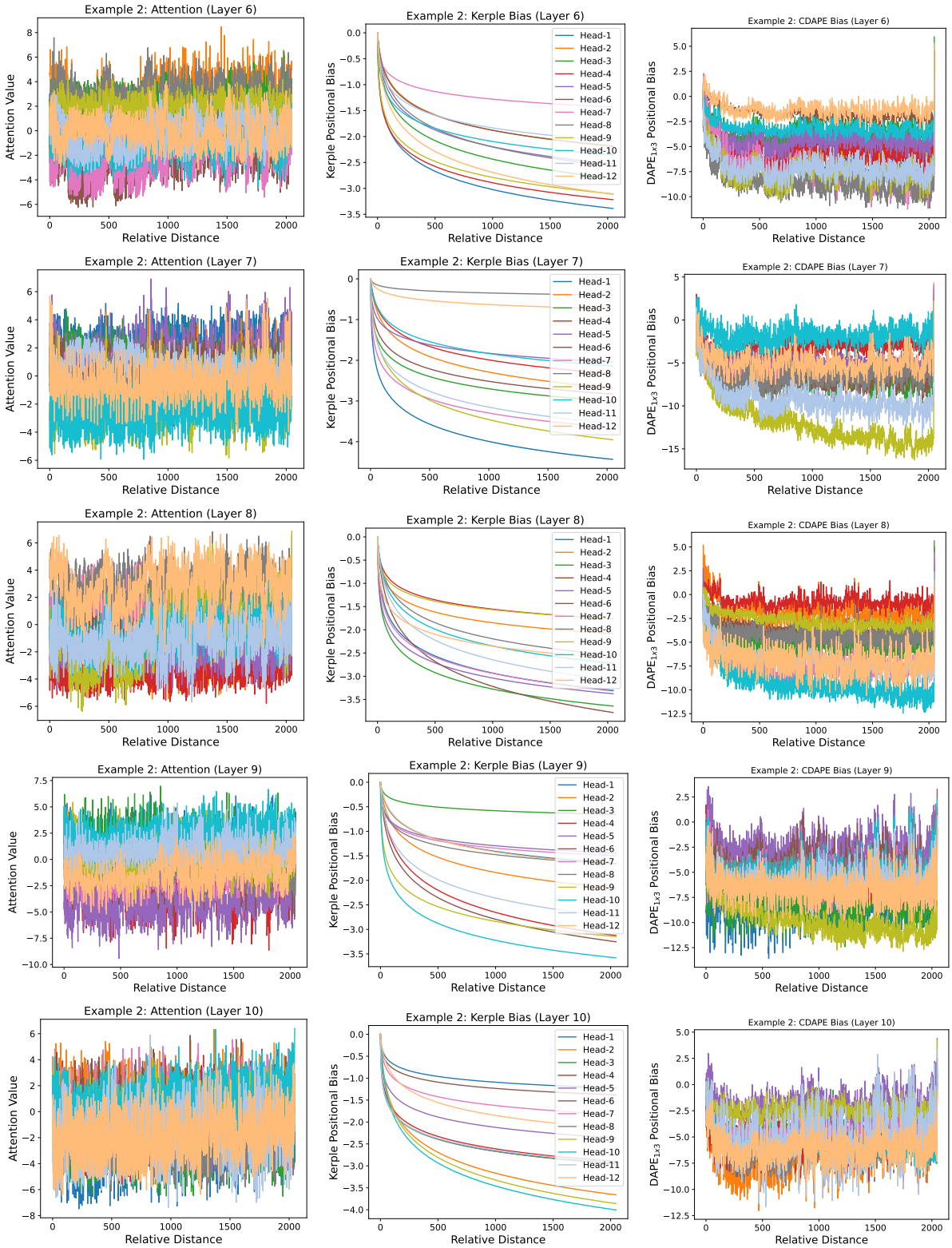

Figure 18: **Evaluation Length 2048 Example 2: Part 2. From Left to Right: (1) The Attention is $\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top$; (2) The Kerple bias is $\boldsymbol{B}$; (3) The CDAPE (with Kerple) bias is $f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, \boldsymbol{B})$.**

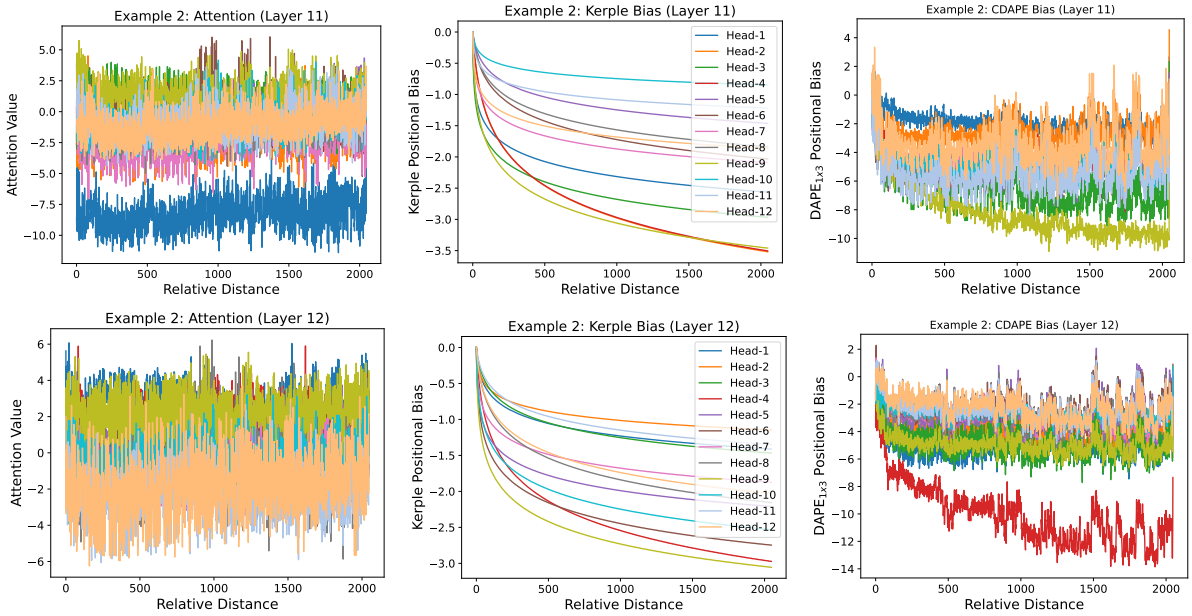

Figure 19: **Evaluation Length 2048 Example 3: Part 2. From Left to Right: (1) The Attention is $\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top$; (2) The Kerple bias is $\boldsymbol{B}$; (3) The CDAPE (with Kerple) bias is $f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, \boldsymbol{B})$.**

P.3 Visualization on length 8192

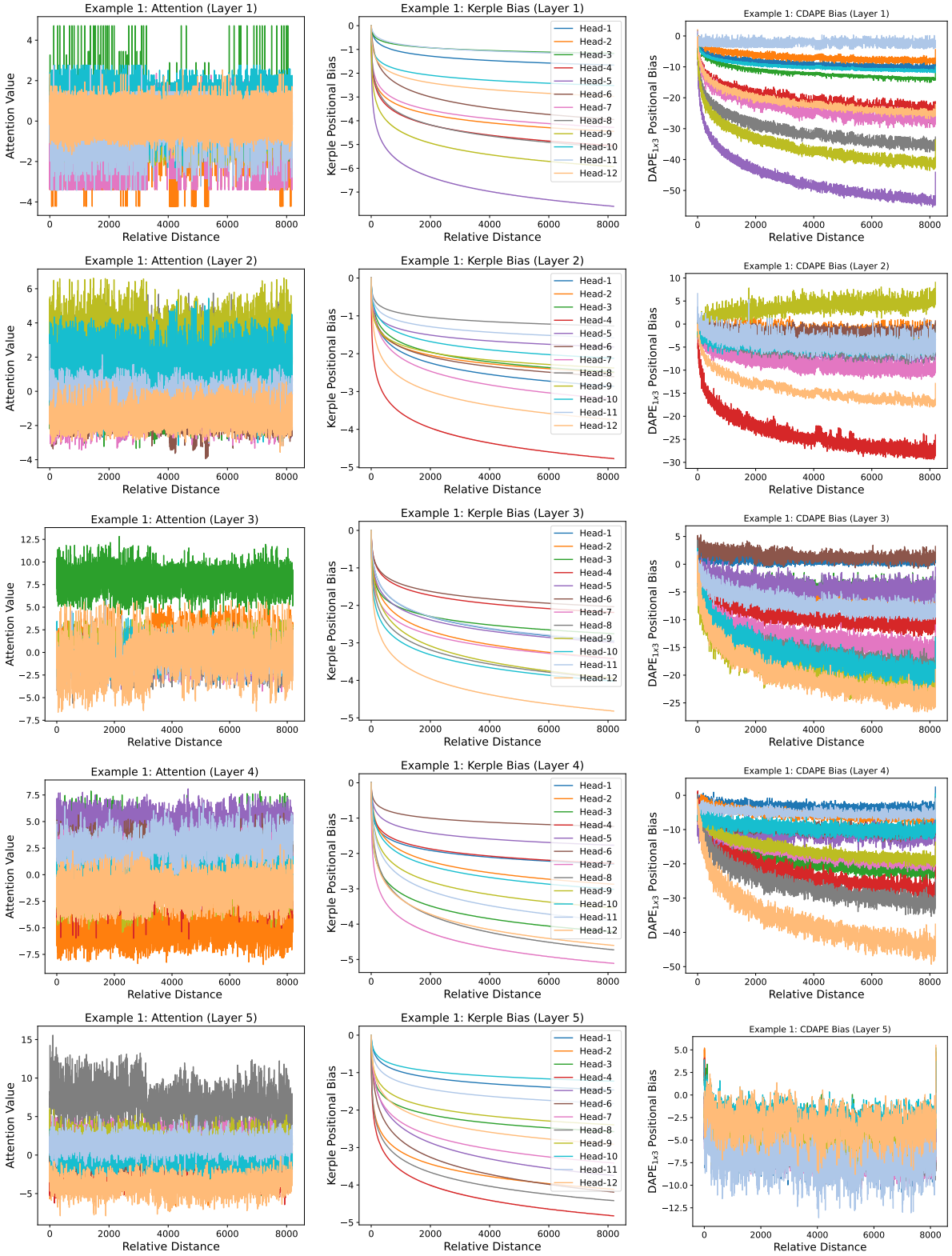

Figure 20: **Evaluation Length 8192 Example 1: Part 1. From Left to Right: (1) The Attention is $\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top$; (2) The Kerple bias is $\boldsymbol{B}$; (3) The CDAPE (with Kerple) bias is $f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, \boldsymbol{B})$.**

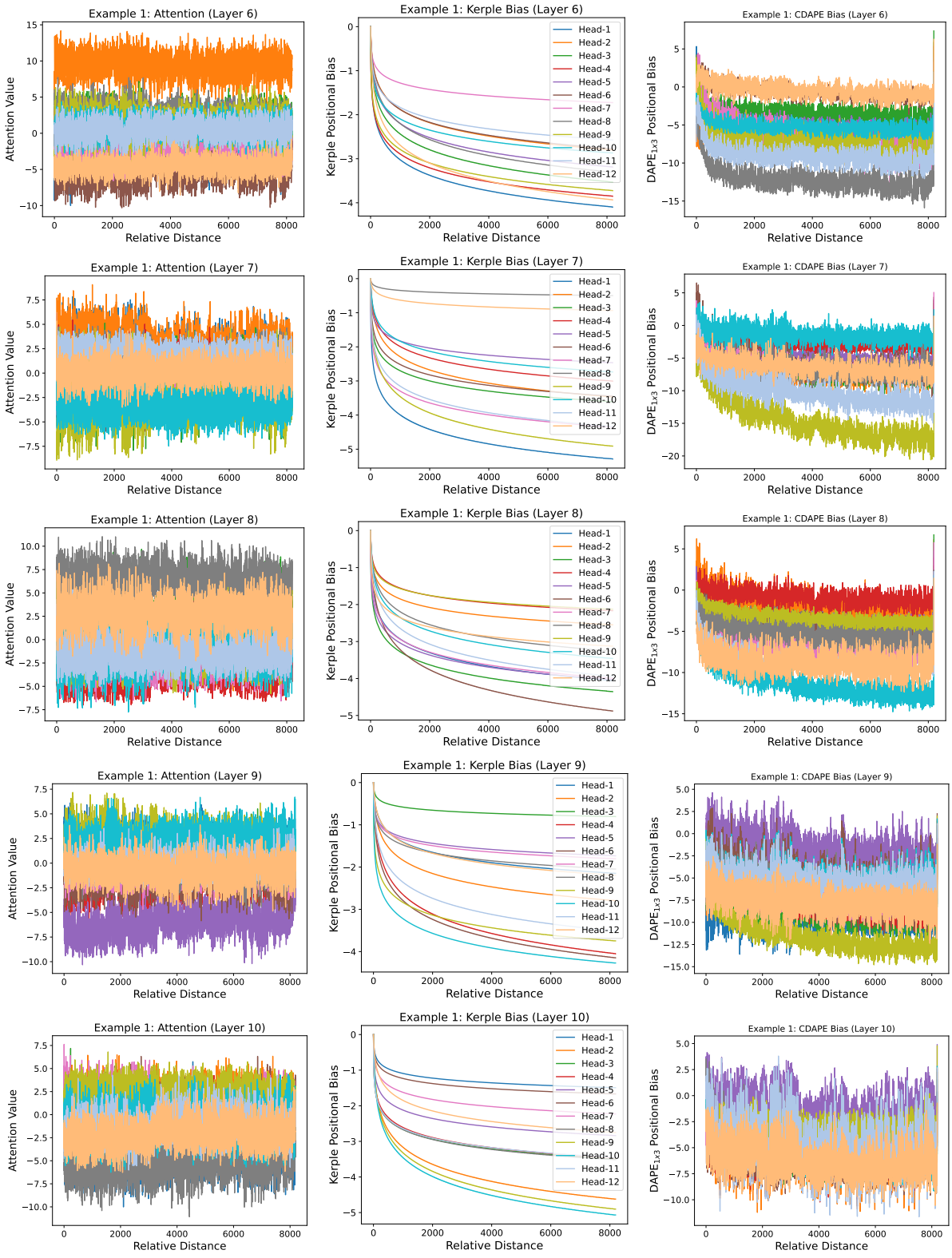

Figure 21: **Evaluation Length 8192 Example 1: Part 2. From Left to Right: (1) The Attention is $\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top$; (2) The Kerple bias is $\boldsymbol{B}$; (3) The CDAPE (with Kerple) bias is $f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, \boldsymbol{B})$.**

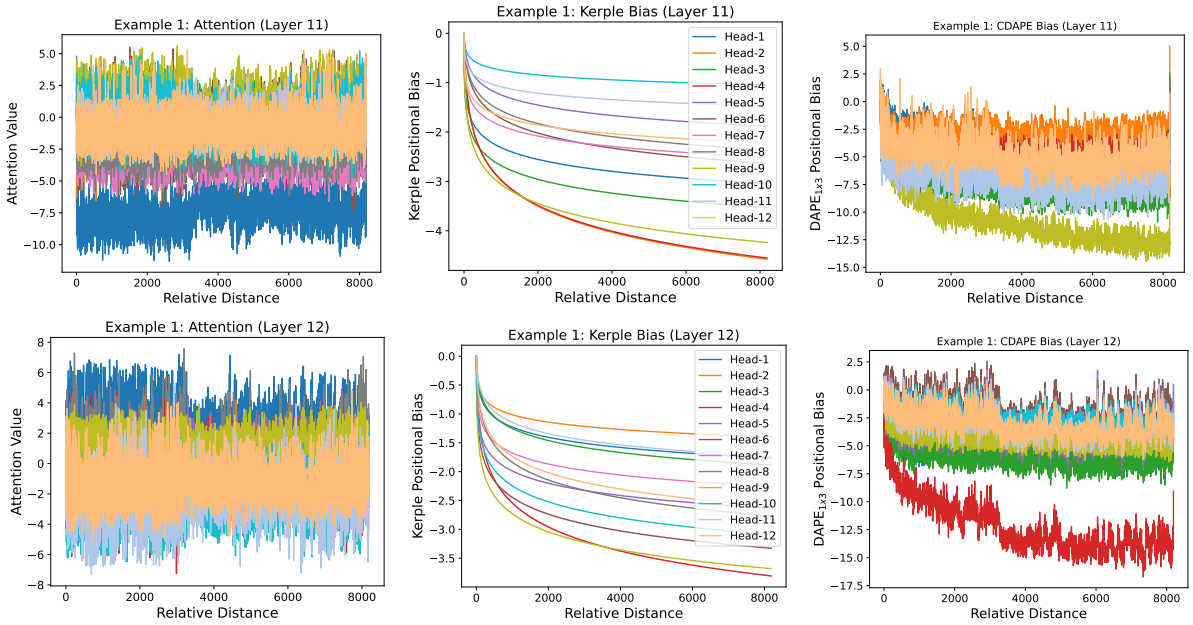

Figure 22: **Evaluation Length 8192 Example 1: Part 3. From Left to Right: (1) The Attention is $\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top$; (2) The Kerple bias is B; (3) The CDAPE (with Kerple) bias is $f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, B)$.**

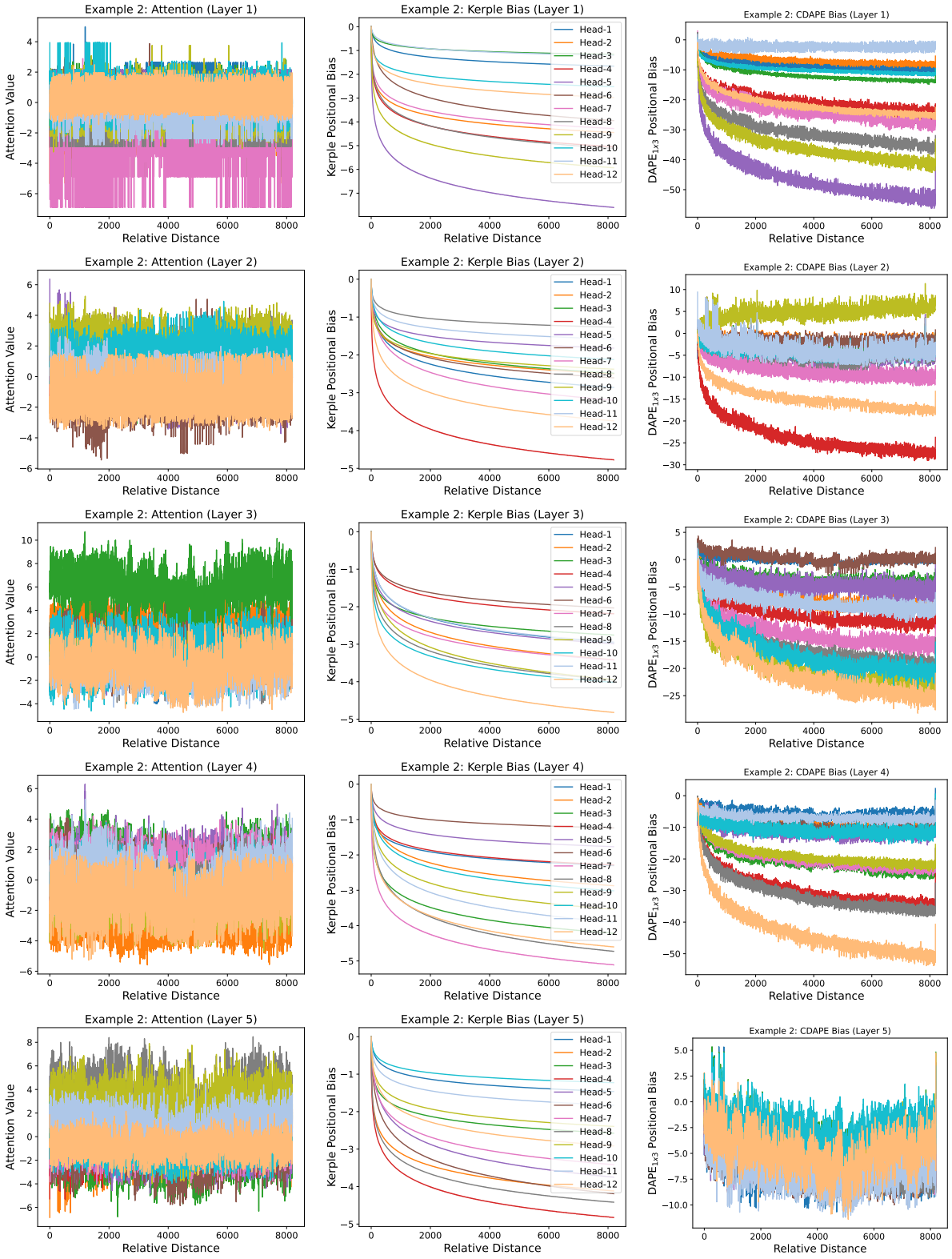

Figure 23: **Evaluation Length 8192 Example 2: Part 1. From Left to Right: (1) The Attention is $\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top$; (2) The Kerple bias is $\boldsymbol{B}$; (3) The CDAPE (with Kerple) bias is $f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, \boldsymbol{B})$.**

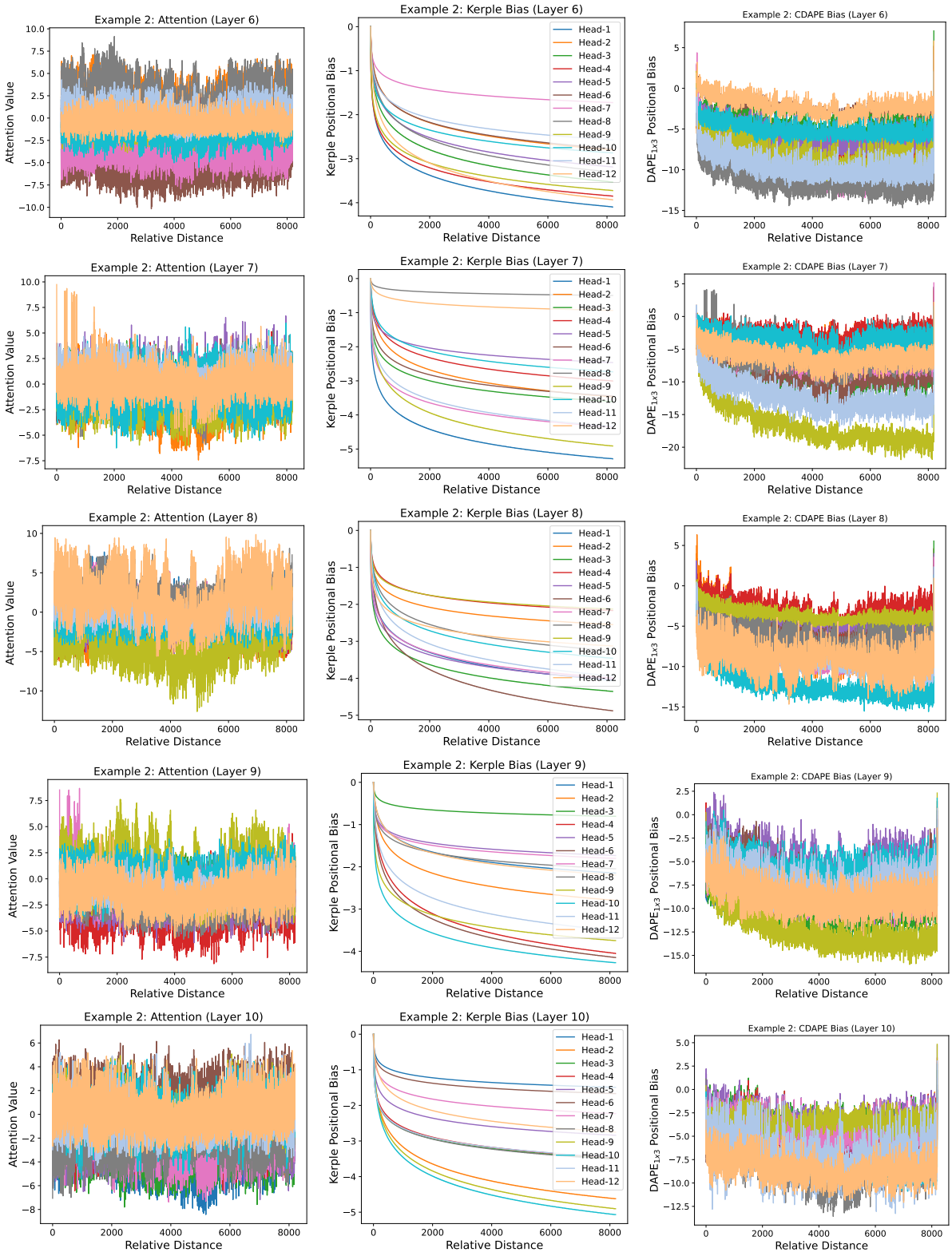

Figure 24: **Evaluation Length 8192 Example 2: Part 2. From Left to Right: (1) The Attention is $\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top$; (2) The Kerple bias is $\boldsymbol{B}$; (3) The CDAPE (with Kerple) bias is $f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, \boldsymbol{B})$.**

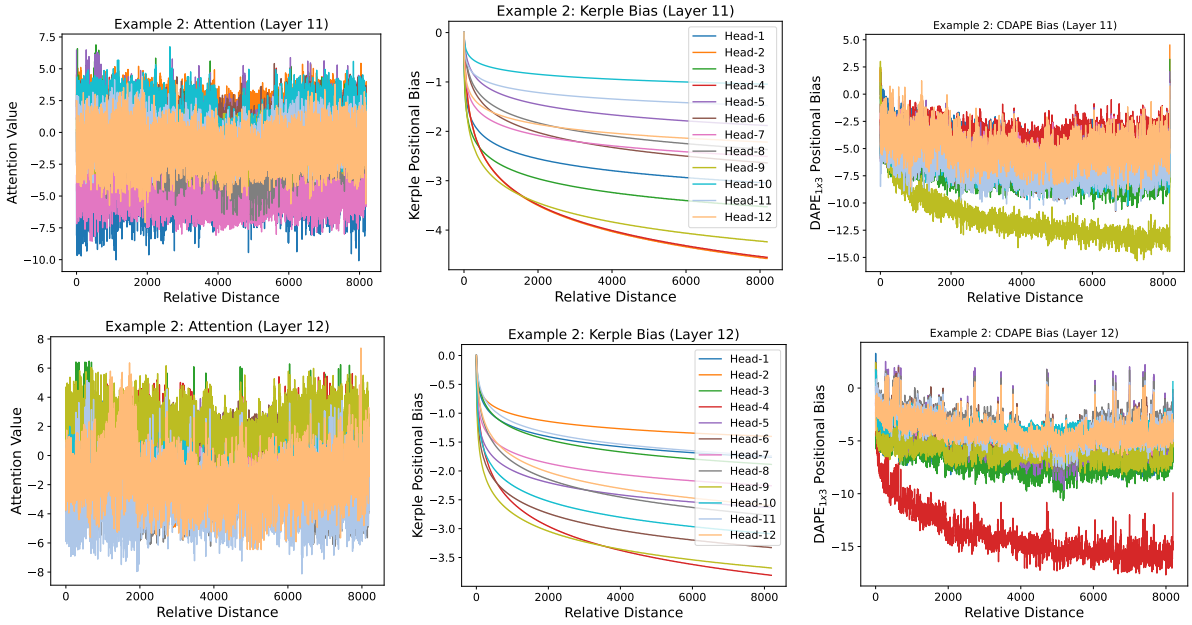

Figure 25: **Evaluation Length 8192 Example 2: Part 2. From Left to Right: (1) The Attention is $\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top$; (2) The Kerple bias is $\boldsymbol{B}$; (3) The CDAPE (with Kerple) bias is $f(\boldsymbol{X}\boldsymbol{W}_Q(\boldsymbol{X}\boldsymbol{W}_K)^\top, \boldsymbol{B})$.**

Q Implementation

In this section, we present the implementation of the proposed CDAPE module in PyTorch for the research purpose, which is consistent with intended use (Paszke et al., 2019).

```
import torch
import torch.nn as nn

class CDAPE(nn.Module):
  def __init__(self, head_number=12, mlp_width=32,kernel_size=3):
    """
    CDAPE attention bias module.

    Args:
      num_heads: number of attention heads.
      mlp_width: Width of MLP.
      kernel_size: convolution kernel size.
    """
    super(CDAPE, self).__init__()


    self.mlp =  nn.Sequential(
            nn.Conv2d(in_channels=head_number*2, out_channels=mlp_width,
                kernel_size=(1,kernel_size),stride=(1,1),padding=(0,kernel_size
                //2),dilation=(1,1)),
            nn.LeakyReLU(),
            nn.Conv2d(in_channels=mlp_width, out_channels=head_number,
                kernel_size=(1,kernel_size),stride=(1,1),padding=(0,kernel_size
                //2),dilation=(1,1)))

  def forward(self, attention: torch.Tensor, bias: torch.Tensor):
    """
    Args:
      attention: input sequence, which is q^T * k,
         shape [bsz, num_heads, seq_len, seq_len]
      bias: bias matrix, which can be generated by ALiBi, Kerple
      FIRE or other additive position encodings
         shape [1,num_heads, seq_len, seq_len]

    Returns:
      attention with DAPEV2,
      shape [bsz, num_heads, seq_len, seq_len]
    """
    bias_tile=torch.tile(fire_bias, (x.shape[0],1,1,1) )
    attention_bias_concat=torch.cat( (attention, bias_tile), dim=1)
    attention_bias_concat=torch.tril(attention_bias_concat)
    attention_bias_concat=self.mlp(attention_bias_concat)


    return attention+bias+attention_bias_concat
```