MODEL THEORETIC SEMANTICS FOR MANY-PURPOSE LANGUAGES AND LANGUAGE HIERARCHIES

H.Andréka*, T.Gergely**, I.Németi*

* Institute of Mathematics of Hungarian Academy of Sciences,Budapest,H-1053,Reáltanoda u.13/15

** Research Institute for Applied Computer Science, Budapest, H-1536, P.O.Box 227.

Summary

Model theoretic semantics (MTS) has a special attitude to describe semantics, to characterize both artificial and natural languages by pure mathematical tools and some of the basic properties of this attitude are disscussed. The arsenal of MTS equipped here with such tools allowing the investigation at such a level of complexity that approximates the real situations. These tools are developed within the frame of category theory.

1. The challange of formal handling of semantics

For long times, natural language has been regarded as some very soft, amorphous, and whimsical phenomenon. Although theoretical considerations showed that this may not be so, the very fact that actual linguistic methodology was quite soft and intuitive seemed to confirm the conviction that language cannot be treated very rigorously. It is clear, however, that the more explicit and transparent framework we use for handling a very complex phenomenon, the more can learn about what its complexity really consists of. It has been the use of more or less mathematical--minded methods improving the situation in recent decades.

A very important first step in the direction of establishing such a framework has been generative grammar. Using the theory of formal languages it gave a fairly abstract picture of *what syntax is*, and it has also proved to be an extremely powerful tool in analysing admittedly very subtle syntactic phenomena and, what is even more, in discovering formerly unnoticed interconnections.

Whatever revealing the results of generative grammar should be with respect to syntax, however, it cannot be regarded as paradigmatic if one is interested in a semantics-oriented model of language. Generative grammarians never put the question of *what semantics is and what role it plays in language* at the same theoretical level they reached with syntax.

It is reasonable to require that any treatment of semantics be adequate to rigorously formalized methods used for syntax. For this we should use formalism not as abbreviation but as basic tool of investigation, e.g. relating exact mathematical objects to components of language. Moreover we aim to characterize language through analysing the corresponding mathematical methods. An appropriate approach can be borrowed from mathematical logic. This results the so called *model theoretic semantics* (MTS). MTS is an attitude to investigate natural language from the point of view of semantics. This attitude provides the investigation of natural language on an abstract level. Namely, it answers the question in the most abstract sense what language is and what its basic components are. The basic properties of the MTS's attitude are analysed in [3].

2. What is MTS?

Language can be analysed only through analysing language carriers. From the different possible functions, the language possesses, we find the cognitive one the most significant and this answers our question above. Considering a language carrying system /whether it be human or a machine or else/ the cognitive function is realized while the language is used to describe objects and events of the environment under cognition. Characterising language we abstract from the cognitive process itself and from the internal organization of the system. Our mere concern is the outcome of the cognitive process, that is, descriptive texts and their relation to the environment which they refer to. MTS attitude demands an ideal external observer (EO) who is to *model* the system (S) and the system's environment world (W). EO forms models of S, of W and of the S-W relation.

In order that EO should be able to form the intended models, he must possess the following kinds of knowledge about the sample situation (and EO being an ideal observer, we assume he really does):

(i) EO knows the aspect and the level at which S may perceive and describe the environment; in other words, EO knows S's sensitivity.

(ii) EO knows those fundamental aspects of W that S may describe.

(i)-(ii) together ensure that EO models W adequately with respect to S.

(iii) EO knows that S is finite whereas W is both infinite and infinitely complex.

(iv) EO knows that S's actual environment is accidental. The knowledge S may obtain at each stage of its cognition is compatible with infinitely many possible worlds. The S-W relation is therefore uncertain: the texts of S always correspond to infinitely many environments, rather than a unique one.

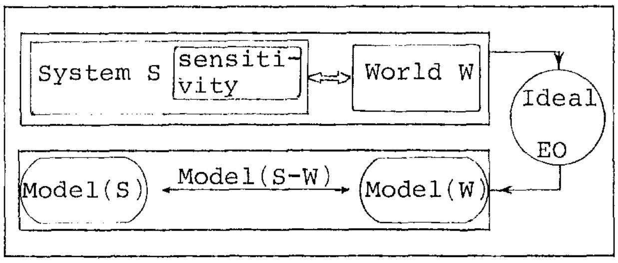
On the basis of (i)-(iv) EO forms the following models: *The model of* S will just be a system producting texts (more precisely, the material bodies of texts, whatever they should be). In case EO happens to be a mathematician, Model (S) will be a formal grammar capable of generating the texts of the language.

The model of W is a metalinguistic description of the world, adequate to S's sensitivity. For purely theoretical purposes, EO only has to take into account that S has some fixed though arbitrary sensitivity, determining the possible character of the objects and phenomena of W S may describe. When modelling some concrete language, S's sensitivity is also fixed though no longer arbitraryly . In case EO happens to be a mathematician, Model (W) will be a mathematical object. Because of the uncertainty of the S-W relation, Model(W) is a class of models of infinitely many possible worlds.

The model of the S-W *relation* is some correspondance between elements of texts and things in the world-models. In case EO happens to be a mathematician, Model(S-W) can be a class of relations or functions.

We have reached the point where we may define language as it appears at this level of abstraction. *By an abstract language* L_A *we mean a triple* <Model (S), Model (W), Model (S-W)>. Furthermore, we call Model (S) the *syntax* of L_A, and Model (W) and Model (S-W) together the *semantics* of L_A. We emphasize that all these models are formed by an ideal external observer and are described in his own language.

The aboves illustrated by the following figure.



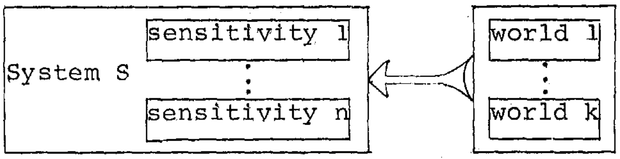
In the case of classical mathematical logic first of all a similarity type t is fixed (t is a function that renders a natural number, the arity, to each relation and function symbols of the basic alphabet, i.e. of the signature). The set F_t of all possible formulas generated from the alphabet with logical connectives in the usual way corresponds to Model(S). The class M_t of all possible t-type relation structures (models) corresponds to Model(W). The so called validity relation $\models \subseteq M_t \times F_t$ corresponds to Model (S-W). Thus a t-type classical first order language L_t is the triple $\langle F_t, M_t, \models \rangle$.

3. MTS in more complex situations

A very simple, we may say, an idealized situation has been considered above. Namely with respect to S it was supposed that its cognition goes on at a fixed level and aspect of analysis, i.e. with a fixed sensitivity. We call this type of cognition *homogeneous* cognition.

However MTS attitude enables us to characterize natural language not only in the above simplicity but in the complexity that approximates more realistic cases.

Indeed a system S can desribe the same objects and events of W from different aspects and at different levels of detailing. Moreover beyond the great spectrum of sensitivity different environment worlds can be the object of cognition. Cognition in this situation is said to be *heterogeneous* cognition. The situation to be described from the point of view of EO is as follows.



The natural language itself virtually seems to enable us to speak about very different kinds of environment at very different levels from very different aspects.

Thus in this light natural language appears as an extremely rich *many-purpose* language.

Beyond the surface natural language consits of such parts which themselves are languages as well (cf. with the subdevision of natural language into a set of dialects or sociolects).

These parts, the sublanguages, are historically formed from others. With the growth of the observable environment the corresponding knowledge also widens. The latter needs new language elements so as to be described. Therefore some words change their meanings, new concepts appear which emerge into new sublanguages.

E.g. the word "tree" has quite a different meaning for a woodman, for a biologist, for a painter, for a child, for a linguist, for a mathematician, etc. The different meanings are connected with different sublanguages which are but different sociolects in this case.

However the sublanguages are not independent. They are in a very complex connection, e.g. one may extract lexical morphological or other kinds of connections on the base of which one or other hierarchy of sublanguages can be sorted out. Such a hierarchy provides a possible "selection" for the natural language. Thus a *hierarchy* of languages consists of the constituent languages together with the relation considered between them.

Note that one can find a detailed survey of different approaches to sublanguages in [6], where another approach has arisen to analyse sublanguages which are called there subsystems of languages.

How natural language as a many purpose one can be investigated with MTS attitude.

First of all a so called *disjunctive* approach can be applied for, according to which EO subdivides the language into such parts each of which can be modelled as a homogeneous one, i.e. as a language that corresponds to a unique and fixed sensitivity.

Now it is supposed that S has several languages rather than a single one. So Model (S) should consist of a conglomerate of sublanguages. However if the sublanguages were independent then EO could model S as a conglomerate of subsystems. But this is not the case because among most of the sublanguages there are some transition possibilities e.g. translation, interpretation.

The MTS attitude possesses tools (developed within the frame of mathematical logic) by the use of which the homogeneous cases can be described. So a conglomerate of languages can also be described by these tools but only as a conglomerate of *independent* languages. What about the connection between two languages? Mathematical logic provides tools only for the case when the languages have the same signature, i.e. when their alphabet is the same. In this case the notion of homomorphism is powerful enough to describe the connection between the languages. But such a case is of not much interest to linguists.

Perhaps it is more interesting to analyse the connection between languages of different type (e.g. between a t_1-type and t_2-type first order classical languages).

Let us see e.g. translation. Having two different languages say, English and Russian, translating a text from one into the other first of all we require not a direct correspondance between the words, but a connection between the corresponding "world conceptions" of the languages and only then is it resonable to establish the connection between the syntactical elements. In MTS this means that for the translation we have to

(i) represent the "world conception" of the languages in question. A "world conception" is but a set of sentences (knowledge) that determines a subclass of Model(W);

(ii) establish the connection between the corresponding subclasses of models, i.e. between the "world conceptions";

(iii) establish the connection among the corresponding syntactical elements.

But up to now MTS has not been in possession of tools to satisfy the above requirements (i)-(iii).

Note that in mathematical logic a set of sentences determines a *theory*. A theory T determines a subclass Mod (T) of models, namely those models where each sentence of T is valid. (Thus a theory T induces a new language $\langle F_t, Mod(T), \models \rangle$.) Thus first of all a connection between the corresponding theories is required for the translation.

However translation between any two languages may not always exist. E.g. let us have two languages physics and biology and we want to establish connection between them. For this we should analyse the connection between the corresponding knowledges. However this analysis, as usual, cannot be established directly. A mediator theory is needed. The mediator is an interdisciplinary theory, e.g. the language of general system theory (see e.g. [2]). By the use of the mediator a new language with a new kind of knowledge arizes from the input languages, namely biophysics.

Our aim is the extension of the MTS attitude to analyse the semantics of many-purpose languages and language hierarchies. We develop such tools (within the frame of mathematical logic) by the use of which EO can model a language carrying system not only in a homogeneous situation, but in a heterogeneous one too, the complexity of which approximates the real cases.

Here we only outline the basic idea providing the basic notions, since the bounds of this paper do not allow us to give a detailed description of the tools This can be found in [1].

Although the first order classical languages do not seem to be adequate for linguistics, it still provides basis for any MTS research. Therefore we introduce the necessary tools of the analysis of the hierarchies of classical first order languages. These tools can be extended for the analysis of different kinds of languages making use of the experience provided by the analysis of the classical case.

4. Basic notions

Definition 1. (*similarity type*)
A similarity type t is a pair $t=\langle H,t'\rangle$ such that t' is a function, t': Dom $t' \to N$ where N is the set of natural numbers and $0 \notin N$, and $H \subseteq$ Dom (t').
Let $\langle r,n\rangle \in t'$ (i.e. let $t'(r) = n$). If $r \in H$ then r is said to be an $n-1$ -ary *function* symbol, if $r \notin H$ then r is said to be an n-ary *relation* symbol. ◉

Let α be an ordinal. F_t^α denotes the set of all t-type formulas containing variable symbols from a set of variables of cardinality α. Thus a t-type first order language is $\langle F_t^\alpha, M_t, \models \rangle$. If $Ax \subseteq F_t^\alpha$ and $\varphi \in F_t^\alpha$ then $Ax \models \varphi$ means that φ is a semantical consequence of Ax.

Definition 2. (*theory*)
A pair $T = \langle Ax, F_t^\alpha \rangle$, where $Ax \subseteq F_t^\alpha$ is said to be a *theory* in α variables. ◉

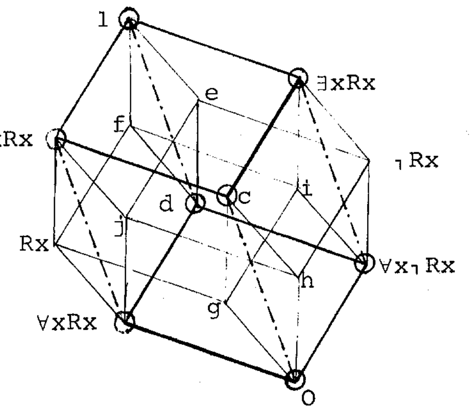
Note that a theory provides a sublanguage of L_t, namely the triple $\langle F_t^\alpha$, Mod $(Ax), \models \rangle$.

Let $T = \langle Ax, F_t^\alpha \rangle$ be a theory, and let $\equiv_T \subseteq F_t^\alpha \times F_t^\alpha$ be the semantical equivalence w.r.t. T defined as follows. For any formulas $\varphi, \psi \in F_t^\alpha$: $\varphi \equiv_T \psi$ iff $Ax \models \varphi \longleftrightarrow \psi$.

Definition 3. (*concept*)
The set of the *concepts* of a theory T is $C_T \overset{d}{=} F_t^\alpha / \equiv_T$. ($F_t^\alpha / \equiv_T$ means the factorization of the set of formulas into such classes any two elements of which are semantically equivalent w.r.t. T.) ◉

Thus in the case of a given theory T C_T contains all the formulas which are compatible with T. Moreover C_T determines what can be described at all about the models by the use of theory T. Note that to C_T a Boole algebra can be corresponded where 0 and 1 correspond to "false" and "true" respectively and the operators correspond to the logical connectives. Let us consider the following

Example
Let $t = \langle \emptyset, \{\langle R,1\rangle\}\rangle$ be the simularity type and $T = \langle \emptyset, F_t^1\rangle$ be a theory. (Note that this theory is axiomless.) We write x instead of x_0, Rx instead of $R(x)$ and φ instead of $\varphi \,/\, \equiv_T$. The concept algebra C_T looks as follows



where we use the following notations:
$c=\exists xRx \wedge \exists x\neg Rx$, $d=\forall xRx \vee \forall x\neg Rx$,
$e=Rx \to \forall xRx$, $f=\neg Rx \to \forall x\neg Rx$,
$g=Rx \wedge \exists x\neg Rx$, $h=\neg Rx \wedge \exists xRx$,
$i=\exists xRx \to (Rx \wedge \exists x\neg Rx)$, $j=\exists x\neg Rx \to (\neg Rx \wedge \exists xRx)$.
The vertexes marker by ⊙ are the fix-points of the operation $\exists x_0$.
The formulas of the above C_T tell all that can be said about the t-type models in the classical first order language of a signature of a single unary relation symbol when the theory is atomless. ◉

Now we define how a theory can be interpreted by an other one.

Definition 4. (interpretation)
Let $T = \langle Ax_1, F_{t_1}^\alpha\rangle$ and $T_2 = \langle Ax_2, F_{t_2}^\alpha\rangle$ be theories in α variables. Let $m: F_{t_1}^\alpha \to F_{t_2}^\alpha$.
The triple $\langle T_1, m, T_2\rangle$ is said to be an *interpretation* going from T_1 into T_2 (or an interpretation of T_1 in T_2) iff the following conditions hold:
a/ $m(x_i = x_j) = (x_i = x_j)$ for every $i,j<\alpha$;
b/ $m(\varphi \wedge \psi) = m(\varphi) \wedge m(\psi)$, $m(\neg\varphi) = \neg m(\varphi)$, $m(\exists x_i \varphi) = \exists x_i m(\varphi)$ for all $\varphi, \psi \in F_{t_1}^\alpha$, $i<\alpha$;
c/ $Ax_2 \models m(\varphi)$ for all $\varphi \in F_{t_1}^\alpha$ such that $Ax_1 \models \varphi$.

We shall often say that m is an interpretation but in these cases we actually mean $\langle T_1, m, T_2\rangle$. ◉

Let m,n be two interpretations of T_1 in T_2.
The interpretations $\langle T_1, m, T_2\rangle$, $\langle T_1, n, T_2\rangle$ are defined to be *semantically equivalent*, in symbols $m \equiv n$, iff the following condition holds:
$Ax_2 \models [m(\varphi) \leftrightarrow n(\varphi)]$ for all $\varphi \in F_{t_1}^\alpha$.

Let $\langle T_1, m, T_2\rangle$ be an interpretation. We define the equivalence class $m/\equiv$ of m (or more precisely $\langle T_1, m, T_2\rangle/\equiv$) to be:
$m/\equiv \stackrel{d}{=} \{\langle T_1, n, T_2\rangle : n \equiv m\}$.
Now we are ready to define the connection between two theories T_1 and T_2.

Definition 5. (theory morphism)
By a *theory morphism* $\mu: T_1 \to T_2$ going from T_1 into T_2 we understand an equivalence class of interpretations of T_1 in T_2, i.e. μ is a theory morphism $\mu: T_1 \to T_2$ iff $\mu = m/\equiv$ for some interpretation $\langle T_1, m, T_2\rangle$. ◉

The following definition provides a tool to represent theory morphisms
Definition 6. (presentation of theory morphisms)
Let $T = \langle Ax_1, F_{t_1}^\alpha\rangle$ and $T_2 = \langle Ax_2, F_{t_2}^\alpha\rangle$ be two theories in α variables.
(i) By a *presentation* of interpretations from T_1 to T_2 we understand a mapping $p: t_1' \to F_{t_2}^\alpha$.
(ii) The interpretation $\langle T_1, m, T_2\rangle$ *satisfies* the presentation $p: t_1' \to F_{t_2}^\alpha$ iff for every $\langle r,n\rangle \in t_1'$ the following conditions hold:
a/ If $r \in H_1$ then $m(r(x_0,\dots,x_{n-2}) = x_{n-1}) = p(r,n)$;
b/ If $r \notin H_1$ then $m(r(x_0,\dots,x_{n-1})) = p(r,n)$.
We define the *theory morphisms* μ to *satisfy the presentation* p if $\langle T_1, m, T_2\rangle$ satisfies p for some $\langle T_1, m, T_2\rangle \in \mu$. ◉

Proposition 1.
Let $T_1 = \langle Ax_1, F_{t_1}^\alpha\rangle$ and $T_2 = \langle Ax_2, F_{t_2}^\alpha\rangle$ be two theories. Let $p: t_1' \to F_{t_2}^\alpha$ be a presentation of interpretations from T_1 to T_2. *Then* there is at most one theory morphism which satisfies p. ◉

Category theory provides the adequate mathematical frame within which theories and theory morphisms can be considered. From now on we use the basic notions of category theory in the usual sense (see e.g. [4] or [5]).

First of all we show how the category interesting for us looks like.

Definition 7.
(i) TH^α is defined to be the pair $TH^\alpha \stackrel{d}{=} \langle \mathrm{Ob}TH^\alpha, \mathrm{Mor}TH^\alpha\rangle$ of classes.
$\mathrm{Ob}TH^\alpha \stackrel{d}{=} \{\langle Ax, F_t^\alpha\rangle$: t is an arbitrary similarity type and $Ax \subseteq F_t^\alpha\}$,
$\mathrm{Mor}TH^\alpha \stackrel{d}{=} \{\langle T_1, \mu, T_2\rangle$: μ is a theory morphism $\mu: T_1 \to T_2, T_1, T_2 \in \mathrm{Ob}TH^\alpha\}$.
(ii) Let $\mu: T_1 \to T_2$ and $\nu: T_2 \to T_3$ be two theory morphisms. We define the *composition* $\nu \circ \mu: T_1 \to T_3$ to be the unique theory morphism for which there exists $m \in \mu$ and $n \in \nu$ such that $\nu \circ \mu = (n \circ m)/\equiv$, where the function $(n \circ m): F_{t_1}^\alpha \to F_{t_2}^\alpha$ is defined

by $(n \circ m)(\varphi)=n(m(\varphi))$ for all $\varphi \in F^{\alpha}_{t_1}$.

(iii) Let $T=\langle Ax, F^{\alpha}_{t}\rangle$ be a theory. The identity function $Id_{F^{\alpha}_{t}}$ is defined to be $Id_{F^{\alpha}_{t}} \overset{d}{=} \{\langle\varphi,\varphi\rangle : \varphi \in F^{\alpha}_{t}\}$.

The *identity morphism* Id_T on T is defined to be $Id_T \overset{d}{=} (Id_{F^{\alpha}_{t}})/\equiv$. ◉

Proposition 2.
TH^{α} is a category with objects $ObTH^{\alpha}$, morphisms $MorTH^{\alpha}$, composition $\mu \circ \nu$ for any $\mu, \nu \in MorTH^{\alpha}$ and identity morphisms Id_T for all $T \in ObTH^{\alpha}$. ◉

5. The main property of TH^{α}

The heterogeneous situation, where the language carrying system uses not only one language to describe the environment world can be described by EO as the category TH^{α}. Note that TH^{α} contains all possible hierarchies, because the connection between any two constituents is but an element of $MorTH^{\alpha}$. The mathematical object TH^{α} provides the usage of the apparatus of category theory to analyse the properties of language hierarchies. Moreover this frame allows us to establish connection between any two theories even if there is not any kind of direct relation between them. In the latter case a "resultant" theory should be constructed which has direct connection with original ones and the power of expression of which joins that of the original ones. This "resultant" theory mediates between the original directly unconnected theories.

Note that the construction of a resultant theory to some given unconnected theories is one of the most important tasks of the General System Theory (see e.g. [2]).

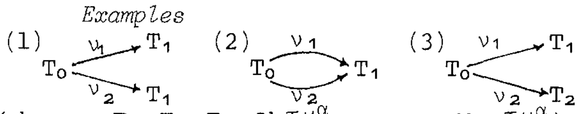
The following theorem claims the completeness of TH^{α} (in the sense of [4] or [5]). This notion corresponds (in category theory) to the above expected property.

Theorem 3.
(i) The category TH^{α} of all theories is complete and cocomplete.
(ii) There is an effective procedure to construct the limits and colimits of the effectively given diagrams in TH^{α}. ◉

Now we enlight the notions used in the above theorem.

A *diagram* D in TH^{α} is a directed graph whose arrows are labelled by morphisms $\mu: T_i \to T_j$ of $MorTH^{\alpha}$ and the nodes by the corresponding objects ($T_i, T_j \in ObTH^{\alpha}$).

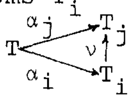
Examples



(where $T_0, T_1, T_2 \in ObTH^{\alpha}$ $\nu_1, \nu_2 \in MorTH^{\alpha}$) are diagrams.

Here the identity morphisms Id_{T_i} (i=0,1,2) are omitted for clarity. We indicate the identity morphisms only if they are needed. ◉

Definition 8.(cone, limit, colimit)
A *cone* over a diagram D is a family $\{\alpha_i : T \to T_i : T_i$ is object of D$\}$ of morphisms from a single object T such that $T \in ObTH^{\alpha}$, for any i $\alpha_i \in MorTH^{\alpha}$ and for any morphisms $T_i \xrightarrow{\nu} T_j$ of D $\alpha_j = \alpha_i \circ \nu$ in TH^{α}

(i.e.  commutes).

The *limit* of a diagram D in TH^{α} is a cone $\{\alpha_i : T \to T_i : T_i$ is object of D$\}$ over D such that for any other cone $\{\beta_i : R \to T_i : T_i$ is object of D$\}$ over D there is a unique morphism $\mu: R \to T$ such that $\beta_i = \mu \circ \alpha_i$.

The *colimit* of D is defined exectly as above but all the arrows are reversed. ◉

Definition 9.(complete, cocomplete)
A category *K* is said to be *complete* and *cocomplete* if for every diagram D in *K* both the limit and the colimit of D exist in *K*. ◉

By aboves we see that Theorem 3 says that *every* diagram in TH^{α} has both limit and colimit in TH^{α}. I.e. in the category TH^{α} of all theories all possible limits and colimits exist (and can be constructed).

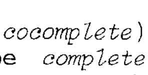
Now let us see some
Examples
Let $T \overset{d}{=} \langle \emptyset, F^{\omega}_{t_0}\rangle$, $T_1 \overset{d}{=} \langle Ax_1, F^{\omega}_{t_1}\rangle$, where $t_0 = \langle \emptyset, \{\langle R,2\rangle\}\rangle$, $t_1 = \langle \{+\}, \{\langle +,3\rangle\}\rangle$ and $Ax_1 \overset{d}{=} \{(x_0+x_0=x_0), ((x_0+x_1)+x_2=x_0+(x_1+x_2)), x_0+x_1=x_1+x_0\}$.

Let $\mu: T_0 \to T_1$ and $\nu: T_0 \to T_1$ be two theory morphisms such that for some $m \in \mu$ and $n \in \nu$ we have

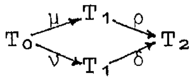
$$m(R(x_0,x_1)) = x_0+x_1=x_1$$

and

$$n(R(x_0,x_1)) = x_0+x_1=x_0 \ .$$

A. Consider the diagram  .

The colimit of this diagram is



where T_2="Lattice theory", i.e. $T_2=\langle Ax_2, F^{\omega}_{t_2}\rangle$, where $t_2=\langle\{+,\cdot\},\{\langle+,3\rangle,\langle\cdot,3\rangle\}\rangle$ and
$Ax_2=\{(x_0+(x_0\cdot x_1)=x_0),(x_0\cdot(x_0+x_1)=x_0)\}\cup$
$\cup\{(x_0\cdot x_0=x_0),((x_0\cdot x_1)\cdot x_2=x_0\cdot(x_1\cdot x_2)),$
$(x_0\cdot x_1=x_1\cdot x_0)\}\cup Ax_1$.
ρ and δ are such that $r(x_0+x_1=x_2)=$
$=x_0+x_1=x_2$ and $d(x_0+x_1=x_2)=x_0\cdot x_1=x_2$
for some $r\in\rho$ and $d\in\delta$.

B. Consider the diagram $T_0 \overset{\mu}{\underset{\nu}{\rightrightarrows}} T_1$.

The colimit of this diagram is

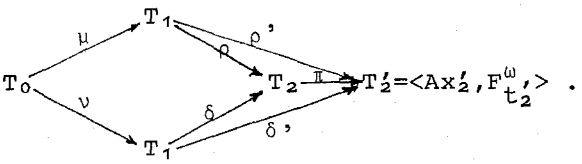
$$T_0 \overset{\mu}{\underset{\nu}{\rightrightarrows}} T_1 \xrightarrow{\rho} T_2$$

where $T_2=\langle Ax_2, F^{\omega}_{t_1}\rangle$ and
$Ax=\{x_0+x_1=x_1+x_0,\ (x_0+x_1)+x_2=x_0+(x_1+x_2),$
$x_0+x_0=x_0,$
$x_0+x_1=x_0 \rightarrow x_1=x_0$

Proof of A

1./ Proof of $\rho\circ\mu=\delta\circ\nu$:
$r(m(R(x_0,x_1)))=r(x_0+x_1=x_1)=x_0+x_1=x_1.$
$d(n(R(x_0,x_1)))=d(x_0+x_1=x_0)=x_0\cdot x_1=x_0.$
We have to prove $r\circ m=d\circ n$, i.e. we have to show $(x_0+x_1=x_1)/\equiv_{T_2}=(x_0\cdot x_1=x_0)/\equiv_{T_2}$,
i.e. that $Ax_2\models(x_0+x_1=x_1\leftrightarrow x_0\cdot x_1=x_0)$.
Suppose $x_0+x_1=x_1$. Then $x_0\cdot x_1=$
$=x_0\cdot(x_0+x_1)=x_0$, by $(x_0\cdot(x_0+x_1)=x_0)\in Ax_2$.
We obtain $Ax_2\models(x_0\cdot x_1=x_0\rightarrow x_0+x_1=x_1)$ similarly.

2./ Suppose $\rho'\circ\mu=\delta'\circ\nu$. We have to show $\pi\circ\rho=\rho'$ and $\pi\circ\delta=\delta'$ for some theory morphism π.



Let $r'\in\rho'$ and $d'\in\delta'$.
$Ax_2'\models(r'(x_0+x_1=x_1)\leftrightarrow d'(x_0+x_1=x_0))$ by $\rho'\circ\mu=\delta'\circ\nu$.
Let $p(x_0+x_1=x_2)\overset{d}{=}r'(x_0+x_1=x_2)$ and
$p(x_0\cdot x_1=x_2)\overset{d}{=}d'(x_0+x_1=x_2)$.
We have to show that p determines a theory morphism $\pi:T_2\rightarrow T_2'$. I.e. we have to show that $(\forall\varphi\in Ax_2)\quad Ax_2'\models p(\varphi)$.
Notation: $r'(+)\overset{d}{=}\oplus$, $d'(+)=\otimes$,
We know that $Ax'\models\{x_0\oplus x_0=x_0,(x_0\oplus x_1)\oplus x_2=$
$=x_0\oplus(x_1\oplus x_2),x_0\oplus x_1=x_1\leftrightarrow x_0\otimes x_1=x_0\}$.
Now $p(x_0\oplus(x_0\cdot x_1)=x_0)=x_0\oplus(x_0\otimes x_1)=x_0$.
We have to show $Ax_2'\models x_0\oplus(x_0\otimes x_1)=x_0$.
$x_0\oplus(x_0\oplus x_1)=(x_0\oplus x_0)\oplus x_1=x_0\oplus x_1$ and therefore $x_0\otimes(x_0\oplus x_1)=x_0$. Similarly for the other elements of Ax_2. ◉

Proof of B:
The proof is based on the fact that
$Th(Ax_2)=Th(Ax_1\cup\{x_0+x_1=x_0\leftrightarrow x_0+x_1=x_1\})$. ◉

Many further interesting features of TH^{α} could be detected had we no limits of our paper.

6. Instead of conclusion

In aboves MTS attitude has been equipped with new tools which might allow the investigation of both natural and artificial languages at such a level of complexity that approximates the real situations. We believe that these open up new perspectives for MTS in the investigation of both computational and theoretical linguistics.

E.g. MTS may provide a description in each case where the connection between two or more sublanguages play a significant role. We think that this is the case in the semantical investigation of certain types of humor as well, where humor might appear by unusual interpretations of texts. This can be described by establishing the connection between the corresponding theories that represent knowledge, i.e. presupositions. The following jokes reflect the afore mentioned type:

1. *"Why didn't you come to the last meeting?"*
   "Had I known it was the last I would have come."
2. Two men were discussing a third.
   "He thinks he is a wit" said one of them.
   "Yes", replied the other, *"but he is only half right"*.

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