# Supplement for Paper "A Multilingual Topic Model for Learning Weighted Topic Links Across Corpora with Low Comparability"

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## A Generative Details of our Multilingual Topic Model

Our MTM is a downstream model with a posterior regularizer. It has two LDA components that generate the documents in languages S and T. Then it generates the posterior regularizer  $\Psi$  which encodes the cross-lingual knowledge. The detailed generative process is as follows.

- 1. For each topic  $k \in \{1, \ldots, K_T\}$  in language T
  - (a) Draw word distribution  $\phi_{T,k} \sim \text{Dirichlet}(\beta_T)$ .
- 2. For each document  $d \in \{1, \ldots, D_T\}$  in language T
  - (a) Draw topic distribution  $\theta_{T,d} \sim \text{Dirichlet}(\alpha_T)$ .
  - (b) For each token  $t_{T,d,n}$  in document d
    - i. Draw a topic  $z_{T,d,n} \sim \text{Multinomial}(\boldsymbol{\theta}_{T,d})$ .
    - ii. Draw a word  $w_{T,d,n} \sim \text{Multinomial}(\phi_{T,z_{T,d,n}})$ .
- 3. For each topic  $k \in \{1, \ldots, K_S\}$  in language S
  - (a) Draw word distribution  $\phi_{S,k} \sim \text{Dirichlet}(\beta_S)$ .
- 4. For each document  $d \in \{1, \ldots, D_S\}$  in language S
  - (a) Draw topic distribution  $\boldsymbol{\theta}_{\boldsymbol{S},\boldsymbol{d}} \sim \text{Dirichlet}(\alpha_S)$ .
  - (b) For each token  $t_{S,d,n}$  in document d
    - i. Draw a topic  $z_{S,d,n} \sim \text{Multinomial}(\boldsymbol{\theta}_{S,d})$ .
    - ii. Draw a word  $w_{S,d,n} \sim \text{Multinomial}(\phi_{S,z_{S,d,n}})$ .

5. Draw the posterior regularizer 
$$\Psi = \left(\prod_{c=1}^{C} \|\mathbf{\Omega}_{S,c} - \boldsymbol{\rho}_{T \to S} \mathbf{\Omega}_{T,c}\|_{2}^{\eta_{c}} \|\boldsymbol{\rho}_{S \to T} \mathbf{\Omega}_{S,c} - \mathbf{\Omega}_{T,c}\|_{2}^{\eta_{c}}\right)^{-1}$$

### A.1 Posterior Inference

As we mentioned in the paper, the posterior inference is based on stochastic EM and consists of an E-step and an M-step [1]. In each iteration, the E-step updates every token's topic assignment using Gibbs sampling, while holding the values in the topic link weight matrices  $\rho$ . The M-step optimizes  $\rho$  while holding the topic assignments.

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#### A.1.1 E-Step: Topic Assignment Sampling

In addition to the usual word and topic dependencies, our MTM encourages topic assignments that maximize the posterior regularizer  $\Psi$ , thus make the related translation pairs' (transformed) topic distributions close. This is reflected in the Gibbs sampling equation to update  $z_{T,d,n}$ , the topic assignment of the *n*-th token of document *d* in language *T*:

$$\Pr(z_{T,d,n} = k \mid \boldsymbol{z}_{-\boldsymbol{T},d,n}, w_{T,d,n} = v, \boldsymbol{w}_{-\boldsymbol{T},d,n}, \boldsymbol{\rho}, \alpha_{T}, \beta_{T}) \propto \underbrace{\left(N_{T,d,k}^{-T,d,n} + \alpha_{T}\right) \frac{N_{T,k,v}^{-T,d,n} + \beta_{T}}{N_{T,k,v}^{-T,d,n} + V_{T}\beta_{T}}}_{\text{LDA Sampling}} \underbrace{\left(\prod_{v' \in \text{Dic}(v)} \left\|\Omega_{\boldsymbol{S},v'} - \boldsymbol{\rho}_{\boldsymbol{T} \to \boldsymbol{S}}\Omega_{\boldsymbol{T},v}\right\|_{2}^{\eta_{v',v}} \left\|\boldsymbol{\rho}_{\boldsymbol{S} \to \boldsymbol{T}}\Omega_{\boldsymbol{S},v'} - \Omega_{\boldsymbol{T},v}\right\|_{2}^{\eta_{v',v}}}_{2}\right)^{-1},$$
(1)

Minimizing the Topic Distribution Distances

where the first two terms are the same as LDA:  $N_{T,d,k}$  denotes the number of tokens in document d assigned to topic k;  $N_{T,k,v}$  denotes the number of times word v is assigned to topic k;  $\cdot$  denotes marginal counts;  ${}^{-T,d,n}$  means the count excludes the token. The final term corresponds to the posterior regularizer: Dic(v) is word v's translation word set in language S; The values of  $\Omega_{T,v}$ , the topic distribution of word v, assume topic k is chosen as follows:

$$\Omega_{T,v,k'} = \frac{N_{T,k',v}^{-T,d,n} + \mathbb{1}\left(k'=k\right)}{N_{T,v}},\tag{2}$$

where  $\mathbb{1}(\cdot)$  is an indicator function.

Symmetrically, the Gibbs sampling equation to update  $z_{S,d,n}$ , the topic assignment of the *n*-th token of document *d* in language *S*, is

$$\Pr(z_{S,d,n} = k \mid \boldsymbol{z}_{-\boldsymbol{S},\boldsymbol{d},\boldsymbol{n}}, w_{S,d,n} = v, \boldsymbol{w}_{-\boldsymbol{S},\boldsymbol{d},\boldsymbol{n}}, \boldsymbol{\rho}, \alpha_{S}, \beta_{S}) \propto \left(N_{S,d,k}^{-S,d,n} + \alpha_{S}\right) \frac{N_{S,k,v}^{-S,d,n} + \beta_{S}}{N_{S,k,\cdot}^{-S,d,n} + V_{S}\beta_{S}} \left(\prod_{v' \in \operatorname{Dic}(v)} \left\|\boldsymbol{\Omega}_{\boldsymbol{S},\boldsymbol{v}} - \boldsymbol{\rho}_{\boldsymbol{T} \to \boldsymbol{S}} \boldsymbol{\Omega}_{\boldsymbol{T},\boldsymbol{v}'}\right\|_{2}^{\eta_{v,v'}} \left\|\boldsymbol{\rho}_{\boldsymbol{S} \to \boldsymbol{T}} \boldsymbol{\Omega}_{\boldsymbol{S},\boldsymbol{v}} - \boldsymbol{\Omega}_{\boldsymbol{T},\boldsymbol{v}'}\right\|_{2}^{\eta_{v,v'}}}\right)^{-1}.$$
(3)

The values of  $\Omega_{S,v}$ , assuming topic k is chosen, are

$$\Omega_{S,v,k'} = \frac{N_{S,k',v}^{-S,d,n} + \mathbb{1}\left(k'=k\right)}{N_{S,v}}.$$
(4)

#### A.1.2 M-Step: Parameter Optimization

Here we optimize the topic link weight matrices  $\rho$  while fixing the topic assignments. As  $\Psi$  is the product over all translation pairs, we modify  $\Psi$  to obtain the objective functions  $J(\rho_{T\to S})$  and  $J(\rho_{S\to T})$  as the weighted logarithmic sum<sup>1</sup>

$$J(\boldsymbol{\rho_{T \to S}}) = \sum_{c=1}^{C} \eta_c \log \|\boldsymbol{\Omega_{S,c}} - \boldsymbol{\rho_{T \to S,i_S}} \boldsymbol{\Omega_{T,c}}\|_2$$
(5)

$$J(\boldsymbol{\rho}_{\boldsymbol{S}\to\boldsymbol{T}}) = \sum_{c=1}^{C} \eta_c \log \|\boldsymbol{\Omega}_{\boldsymbol{T},c} - \boldsymbol{\rho}_{\boldsymbol{S}\to\boldsymbol{T},\boldsymbol{i}_{\boldsymbol{T}}} \boldsymbol{\Omega}_{\boldsymbol{S},c}\|_2.$$
(6)

<sup>&</sup>lt;sup>1</sup>It makes sense to add regularization on  $\rho$ 's to prevent overfitting, but the data already adds a strong constraint on  $\rho$ 's—each word's  $\Omega$  values should add up to one.

The objective functions are then minimized by using L-BFGS [2] and the partial derivatives with respect to  $\rho_{T \to S, k_S, k_T}$  and  $\rho_{S \to T, k_T, k_S}$  as

$$\frac{\partial J(\boldsymbol{\rho_{T \to S}})}{\rho_{T \to S, k_S, k_T}} = -\sum_{c=1}^{C} \frac{\eta_c \Omega_{T, c, k_T} \left(\Omega_{S, c, k_S} - \boldsymbol{\rho_{T \to S, k_S}} \Omega_{T, c}\right)}{\left\|\Omega_{S, c} - \boldsymbol{\rho_{T \to S, i_S}} \Omega_{T, c}\right\|_2^2}$$
(7)

$$\frac{\partial J(\boldsymbol{\rho}_{\boldsymbol{S}\to\boldsymbol{T}})}{\partial \boldsymbol{S}_{\boldsymbol{S}\to\boldsymbol{T},k_{T},k_{S}}} = -\sum_{c=1}^{C} \frac{\eta_{c} \Omega_{S,c,k_{S}} \left(\Omega_{T,c,k_{T}} - \boldsymbol{\rho}_{\boldsymbol{S}\to\boldsymbol{T},\boldsymbol{k}_{T}} \boldsymbol{\Omega}_{\boldsymbol{S},\boldsymbol{c}}\right)}{\|\boldsymbol{\Omega}_{\boldsymbol{T},\boldsymbol{c}} - \boldsymbol{\rho}_{\boldsymbol{S}\to\boldsymbol{T},\boldsymbol{i}_{T}} \boldsymbol{\Omega}_{\boldsymbol{S},\boldsymbol{c}}\|_{2}^{2}}.$$
(8)

### **B** Corpora Statistics

There are four datasets used in our experiments. The first one is collected from Wikipedia for document category classification in English (EN) and Chinese (ZH) [3]. There are six document categories for Wikipedia documents: *film*, *music*, *animals*, *politics*, *religion*, and *food*. The dictionary comes from MDBG, a website for learning Chinese.<sup>2</sup>

The second dataset is about disaster response in English and Sinhalese (SI) [4, 5]. A subset of the documents are annotated with one of eight need types: *evacuation, food supply, search/rescue, utilities, infrastructure, medical assistance, shelter,* and *water supply*. The dictionary is provided by the dataset authors.

The last two are also collected from Wikipedia, one is partially comparable (PACO) and the other one is incomparable (INCO) [6]. Either one contains five bilingual corpora. Among these bilingual corpora, one of the language is always English (EN), while the other language is one of Arabic (AR), Chinese (ZH), Spanish (ES), Farsi (FA), and Russian (RU). The dictionaries are extracted from Wiktionary.<sup>3</sup>

The detailed corpora statistics are in Table 1.

Dataset	Language Pair	Language	#Docs	#Tokens	#Vocabuary	#Translations
Wikipedia EN-ZH		EN	11,043	1,906,142	13,200	6,812
	EN-ZH	ZH	10,135	1,169,056	13,972	
Disaster	EN-SI	EN	1,100	32,714	6,920	6,330
Response		SI	4,790	168,082	31,629	
	EN-AR	EN	1,999	622,955	47,790	4,384
		AR	1,999	107,434	19,900	
	EN-ZH	EN	2,000	405,976	39,847	8,691
		ZH	1,997	86,585	30,481	
DACO	ENES	EN	2,000	238,092	30,278	18,221
PACO EN-ES	EN-ES	ES	2,000	188,469	27,465	
	EN-FA	EN	2,000	513,855	41,685	4,419
	EN-IA	FA	1,814	37,158	9,987	
	EN-RU	EN	1,999	296,148	34,618	2,981
	EN-RU	RU	1,999	101,922	24,341	
		EN	2,000	581,473	45,444	4,380
	EN-AK	AR	1,999	107,434	19,900	
	EN 711	EN	2,000	432,442	38,369	8,766
	EN-ZH	ZH	1,997	86,585	30,481	
	ENEC	EN	1,999	557,602	46,161	20,954
	EN-ES	ES	2,000	188,469	27,465	
	EN-FA	EN	2,000	324,858	34,278	4,280
		FA	1,814	37,158	9,987	
	EN-RU	EN	2,000	547,748	47,167	3,345
		RU	1,999	101,922	24,341	

Table 1: Corpora Statistics.

<sup>&</sup>lt;sup>2</sup>https://www.mdbg.net/chinese/dictionary?page=cc-cedict

<sup>&</sup>lt;sup>3</sup>https://dumps.wikimedia.org/enwiktionary/

## C An Example of Topic Space Transformation with Top-Linked Topics

We give an example to illustrate how we transform topic space with top-linked topics. Suppose that we have a Chinese topic with a probability mass of 0.2 in a document and its topic link weight to English Topics 0–4 are 0.1, 0.4, 0.2, 0.1, 0.2. Given that English Topic 1 has the highest link weight with the Chinese topic, when transforming the document's topic distribution into English, the probability mass of the Chinese topic is transferred to English Topic 1.

## References

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