Reference in Dialogues and Shared Belief Revision

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Abstract

We shall show how to treat shared belief revision and reference management in the process of dialogues by distinguishing three types of discourse referents: (standard) discourse referents, *shared discourse referents*, and the *shared belief referent*, using an extended version of *Discourse Representation Theory*, called *Hyper-DRS*, which can represent shared beliefs semantically in a notation based on *hyper-sets* (sets in non-well-founded set theory).

1 Introduction

Belief sharing in dialogue necessarily involves circular structures, since shared beliefs have circular structures. [1]'s theory of *non-well-founded sets* or *hyper-sets* makes it possible to encode such circular structures in first-order fixed point equations, whose solution is guaranteed by his *solution lemma*. By adopting [1]'s theory, we extend the expressibility of *Discourse Representation Structures*, called *Hyper-DRS*(firstly proposed in [14, 13]), of [9, 8, 10, 2]'s *Discourse Representation Theory*, henceforth *DRT*. (See also [12] for another version of self-referential DRS and its semantics). This extension takes the form of introducing three new types of discourse referents, which we will show reflect our intuitions about dialogue processes. In this paper, we shall show how to treat shared belief revision and reference management in the process of dialogues by distinguishing three types of discourse referents: (standard) discourse referents, *shared discourse referents*, and the *shared belief referent*. Shared discourse referents play a role in preserving the topic of dialogues, and shared belief referents are used to convert circular propositions into first-order equational systems, and we can update and revise a circular proposition by hanging propositions on and taking propositions from shared discourse referents appearing in it. Thus, the three types of discourse referents are basic devices for the management of shared beliefs. In the context of shared belief revision, we can also characterize the three types of discourse referents in terms of the notion of *accessibility* and *life span*. The shared belief referent and standard discourse referents can not be accessed by both conversants. The conversants can access shared discourse referents, and each conversant can access standard discourse referents which are introduced by herself. Shared discourse referents and the shared belief referents can not be deleted and forgotten when shared beliefs are revised or deleted, i.e., the life span of shared belief referents is the longest in revision processes of discourse representation structures, and the life span of shared discourse referents is longer than that of standard discourse referents in their revision processes. Furthermore, we can explain the connection between shared belief revision, reference management, and dialogue process, and give a mothod of evaluation of dialogue in terms of successfulness in information sharing.

As a result, our extended version of DRT with Hyper-DRS's can treat shared beliefs as first-order equational systems, which, being a finite representation of inherently infinite structures, allow us to simulate, and evaluate processes of shared belief revision and reference management in dialogue.

In section 2 we consider a probelm of the type of disourse referents connected to dialogue evaluation. In section 3 we propose Hyper-DRS and their construction procedure from dialogue by distinguishing shared belief referernts from the other types of discourse referents. In section 4 we distinguish and characterize shared discourse referents. In section 5 we propose a procedure for shared belief revion based on Hyper-DRS and the distinction of the three types of discourse referents in dialogue.

2 A Problem of the Type of Discourse Referents concering Dialogue Evaluation

Many scholars ([5],[7]) show that human understanding of definite reference depends on shared belief. However, according to [14], there are some levels of shared belief exploited for definite reference in human dialogues. For example, consider the following exchange where John and Mary are talking about a scene in a park, and they know Bill and Bob.

(1) John: Oh, Bill is running.
Mary: No, *he* is Bob/Bob is dancing.
John: No, Bob has gone to Hongkong.
Mary: Is that so?

And, suppose that it was David who John saw running, who looked like Bill, and it was Martin who Mary saw running, who looked like Bob. That is, they were not seeing the same man, but the two different men, David and Martin. In this situation, John and Mary were wrong by any normal criterion of correctness, but the dialogue was not a failure. The exchanges between John and Mary were successful and rational. As the result, after the dialogue, John and Mary shared the information that it was Bill who they had been seeing. How can we treat such a discourse situation? Moreover, in dialogue (1) Mary changed her belief about her reference, but she didn't abandon her intended referent, i.e., Martin. She didn't become aware of the disagreement between her intended referent and John's intended referent. What did Mary change in her belief?

This problem makes us aware of the existence of a hidden referent in referring in cooperative dialogues: *the referent which conversants believe that they are talking about.* That is, in this case, we can distinguish three types of referents: *referents which they believe that they are talking about and believe to be identical, those which each conversant is perceiving*, and (actual) *individuals.* In (1), the referents which they perceive and intend to refer to are Bill and Bob, the actual individuals are David and Martin, and there is a man they are talking about, who does not exist in the real world. This last referent makes the dialogue successful in spite of the disparity involved in the real situation. We call this kind of referents *shared discourse referents.*

Shared discourse referents can be used even when they bear no value.

(2) John: Who is running over there?
Mary: *It*'s Bill.

In this example, Mary can refer to the referent introduced by John and the dialogue is successful, even when she does not actually share the referent John intends to refer to. In other words, a referent of this kind behaves like a discourse referent in that it is not intended to refer to any particular object ([11]).

And, shared discourse referents are preserved all through a dialogue, and if they are not preserved, then the dialogue fails.

(3) John: Oh, Bill is running.
Mary: Who?/* Who is *it*?
John: Bill.
Mary: Yeah.

This use of 'Who?' is almost equivalent to 'Pardon?'. In this case, we can't use an anaphora. That is, the first part of dialogue (3) failed, and the cause of the failure is the conversants' failure to share discourse referents. For use of an anaphora is impossible.

In (1), at first John and Mary disagreed as to their shared referent but eventually they came to an greement. They changed their shared belief through the conversation. However, they did not change the shared discourse referent: that same thing they were talking about. Shared discourse referents are not changed throughout a dialogue. By contrast, other information may be revised. Therefore, we propose the following principles of dialogue based on this observation.

- Shared discourse referents make dialogues successful, and are not changed throughout a dialogue.

- A dialogue fails and repair question like 'Pardon?' is used in order to establish shared discourse referents.
- The rest of the shared information may be revised through a dialogue.

Thus, we should include shared discourse referents in our formalization of shared information revision in order to treat dialogues properly. In the next section, we will formalize shared discourse referents and revision of shared belief.

3 Discourse Referents as Shared Belief Denoting Devices

In this section, we propose a treatment of shared belief by introducing a discourse referent as shared belief denoting device, which is similar to DRT's treatment of anaphora and *donkey sentences* ([9]), and by translating shared belief into an equational system of propositions, as proposed by [4] and [3]. In Discourse Representation Theory, We can augment information about indefinite objects by introducing discourse referents, which play the role of a peg on which we hang new information. Similarly, we can revise shared belief through Hyper-DRS by regarding shared belief as a kind of discourse referents.

[3, 4, 6] formalize shared belief and shared belief propositions as solutions of equational systems of propositions. Following them, we can formalize John and Mary's shared belief or shared information about a proposition 'Bill is running' as follows:

(4) $Shared(John, Mary, Bill\ is\_running) \Leftrightarrow$
$\exists q[q = (believe(John, Bill\ is\_running \wedge q) \wedge believe(Mary, Bill\ is\_running \wedge q))]$.

Suppose that John and Mary revised their shared belief by a conversation as in (5),

(5) 1. John: Oh, Bill is running.
2. Mary: Yeah.

the shared belief will be what is by (4). How can we formulate this updating process of shared beliefs?

We propose to treat shared belief as a kind of discourse referents. Namely, a shared belief is translated into a discourse referent satisfying an equational system of propositions containing the new discourse referent, as in (6).

(6) $\mathbf{q} = believe(John, \mathbf{q}) \wedge believe(Mary, \mathbf{q}) \wedge$
$believe(John, Bill\ is\_running) \wedge believe(Mary, Bill\ is\_running)$

Revising a shared belief amounts to revising this equational system of propositions. Thus we have only to add to this system a new piece of shared information linearly, as in (6). where $\mathbf{q}$ is a newly introduced discourse referent. More formally, we propose Hyper-DRS which can represent such circular propositions as follows.

Definition 1 *A Hyper-DRS $K(\mathbf{p})$ is a structure $(\mathbf{p}, dom(\mathbf{p}), cond(\mathbf{p}))$, where $\mathbf{p}$ is a discourse referent denoting $K(\mathbf{p})$ itself, $dom(\mathbf{p})$ is a sequence of discourse referents occurring in $cond(\mathbf{p})$, and $cond(\mathbf{p})$ is a sequence of predicates or Hyper-DRS's.* □

A Hyper-DRS $K(\mathbf{p})$ of a dialogue between A and B from A's viewpoint is constructed by the following procedure.

Procedure 1

(i) Initially $dom(\mathbf{p}) = \{\mathbf{A}, \mathbf{B}, \mathbf{u}\}$, $cond(\mathbf{p}) = \{Self(\mathbf{A}), Other(\mathbf{B}), Now(\mathbf{u}), believe(\mathbf{A}, \mathbf{p}), believe(\mathbf{B}, \mathbf{p})\}$.

(ii) If X says φ, then $cond(\mathbf{p}) := cond(\mathbf{p}) \cup \{believe(\mathbf{X}, \mathbf{q}), K(\mathbf{q})\}$, $dom(\mathbf{p}) := dom(\mathbf{p}) \cup \{\mathbf{q}\}$, *where $\mathbf{q}$ is a new discourse referent (See [10] for the detail of the DRS onstruction procedures for English fragments), and $K(\mathbf{q})$ is the Hyper-DRS of φ.*

(iii) If X says 'yeah', then $cond(\mathbf{p}) := cond(\mathbf{p}) \cup \{\mathbf{q} = \mathbf{q}'\}$, $dom(\mathbf{p}) := dom(\mathbf{p}) \cup \{\mathbf{q}\}$, *where $\mathbf{q}$ is a new discourse referent, and $\mathbf{q}'$ is the discourse referent introduced last.*

This method is similar to DRT's treatment of anaphora and *donkey sentences* ([9]). We can augment information about indefinite objects by introducing discourse referents, which play the role of a peg on which we hang new information. Therefore, we can revise shared beliefs through the Hyper-DRS by taking shared beliefs as a kind of discourse referents. We call this type of discourse referents *shared belief referents*.

Thus, we have introduced shared belief referents syntactically. Can we define the semantics for them? According to [4] and [3], in *hyper-universe* introduced by [1], each equational system of propositions has a unique solution by the Solution Lemma. When we define a class of propositions *co-inductively*, we can define a *hyper*-class of propositions containing circular propositions. We can give the semantics of shared belief referents, using the technique of *hyper*-sets, as follows.

Definition 2 *Given a function V from conditions to proposition types, an assignment f verifies a Hyper-DRS $K(\mathbf{p}_i)$ in a model $\mathcal{M}$, written $f \Vdash_{\mathcal{M}} K(\mathbf{p}_i)$ iff for some $g \supseteq f$, there is a unique solution of the equation $g(\mathbf{p}_i) = (\wedge, V_g[cond(\mathbf{p}_i)])$ in $\mathcal{M}(0)$, where $\mathcal{M}$ is a seuqence of hyper-set of proposition types. (See Appendix for more detail)* □

In what follows, we will continue task of differentiating the three types of discourse referents: *(standard) discourse referents*, *shared discourse referents*, and *shared belief referents*, and we can show how to update shared beliefs by a dialogue in terms of Discourse Representation Theory. And, we shall give a solution to the proposed problems in section 2.

4 Shared Discourse Referents

The three types of discourse referents introduced in the previous section shares the property that they play a role in assuring the existence of the denoted object just like *eigen variables*. However, we can further differentiate the three types of discourse referents. So far we have introduced shared discourse referents and shared belief referents. In this section, we shall argue on the distinction between standard discourse referents and shared discourse referents in terms of *accessibility* and *life span*.

In DRT accessibility between discourse referents is defined in terms of the structure of DRS. This concept reflects the use of anaphora in linguistics. We use the term 'accessible' also for the accessibility of anaphora between utterances by conversants. For example, the following dialogue is anomalous.

(7) John: A man is running.
Mary: * *He* is the one we met yesterday.

In DRT 'a man' introduces a new discourse referent into the DRS of (7), but (7) shows that *standard discourse referents introduced by one of the conversants are not accessible from the other of the conversants*, since Mary cannot use 'he' in order to refer to 'a man'.

The following is preferable to (7).

(8) John: A man is running.
Mary: Oh, I know what you are talking about. *He* is the one we met yesterday.

(8) shows that when an unshared discourse referent is shared by an utterance, such as '*I know what you are talking about*', then the other conversant can access discourse referent.

Therefore, (7) and (8) show that conversants distinguish between shared and unshared discourse referents by their accessibility. This accessibility can be encoded by the structure of a Hyper-DRS. Shared discourse referents are registered in the top domain, but unshared discourse referents are in an embedded domain, as follows. Shared discourse referents are bound by the meta-condition '$talking\_about(C, x)$' which represents that the discourse referent x is shared in the dialogue by the group of conversants C.

$$\mathbf{q}: \begin{array}{|l|}\hline \mathbf{A\ B\ u\ p_1\ x\ p_2} \\ \hline \mathit{Self}(\mathbf{A});\ \mathit{Other}(\mathbf{B});\ \mathit{Now}(\mathbf{u}) \\ \mathit{believe}(\mathbf{A},\mathbf{q});\ \mathit{believe}(\mathbf{B},\mathbf{q}) \\ \mathit{believe}(\mathbf{A},\mathbf{p}_1);\ \mathit{talking\_about}(\{\mathbf{A},\mathbf{B}\},\mathbf{x}) \\ \mathbf{p}_1: \begin{array}{|l|}\hline \mathbf{y\ r\ e} \\ \hline \mathbf{y}=\mathbf{x};\ \mathit{Bill}(\mathbf{y});\ \mathbf{u}\mathit{O}\mathbf{r} \\ \mathbf{r}\sqsubseteq\mathbf{e};\ [\mathbf{e}:\mathbf{y}\ \mathit{be\_running}] \\ \hline \end{array} \\ \mathit{believe}(\mathbf{B},\mathbf{p}_2) \\ \mathbf{p}_2=\mathbf{p}_1 \\ \hline \end{array}$$

And, sharing of discourse referents can be carried out by the following procedures on the Hyper-DRS.

Procedure 2 *Given a Hyper-DRS $K(\mathbf{p})$ of a dialogue between A and B from A's viewpoint,*

(i) If X says φ, then $cond(\mathbf{p}) := cond(\mathbf{p}) \cup \{believe(\mathbf{X}, \mathbf{q}), K(\mathbf{q})\}$, $dom(\mathbf{p}) := dom(\mathbf{p}) \cup \{\mathbf{q}\}$, where $\mathbf{q}$ is a new discourse referent, and $K(\mathbf{q})$ is the Hyper-DRS of φ.

(ii) If φ contains a proper name η, then $cond(\mathbf{q}) := \{\varphi(\eta/\mathbf{y}), \eta(\mathbf{y}), \mathbf{y} = \mathbf{x}\}$, $dom(\mathbf{q}) := \{\mathbf{y}\}$, $dom(\mathbf{p}) := dom(\mathbf{p}) \cup \{\mathbf{x}\}$, and $cond(\mathbf{p}) := cond(\mathbf{p}) \cup \{talking\_about(\{\mathbf{A}, \mathbf{B}\}, \mathbf{x})\}$, where $\mathbf{y}$ and $\mathbf{x}$ are new discourse referents, if there is no $\mathbf{x}$ accessible to $\mathbf{y}$ in $dom(\mathbf{p})$, or η is a new proper name introduced in the dialogue. Otherwise, $cond(\mathbf{q}) := \{\varphi(\eta/\mathbf{y}), \eta(\mathbf{y}), \mathbf{y} = \mathbf{x}\}$, $dom(\mathbf{q}) := \{\mathbf{y}\}$, where $\mathbf{y}$ is a new discourse referent and $\mathbf{x}$ is an accessible discourse referent from $\mathbf{y}$ and belongs to $dom(\mathbf{p})$, η is already introduced in the dialogue, and '$talking\_about(\{\mathbf{A}, \mathbf{B}\}, \mathbf{x})$' is in $cond(\mathbf{p})$,

(iii) If φ contains a pronoun η, then $cond(\mathbf{q}) := \{\varphi(\eta/\mathbf{y}), \mathbf{y} = \mathbf{x}\}$, $dom(\mathbf{q}) := \{\mathbf{y}\}$, $dom(\mathbf{p}) := dom(\mathbf{p}) \cup \{\mathbf{x}\}$, and $cond(\mathbf{p}) := cond(\mathbf{p}) \cup \{talking\_about(\{\mathbf{A}, \mathbf{B}\}, \mathbf{x})\}$, where $\mathbf{y}$ and $\mathbf{x}$ are new discourse referents, if there is no $\mathbf{x}$ accessible to $\mathbf{y}$ in $dom(\mathbf{p})$. Otherwise, $cond(\mathbf{q}) := \{\varphi(\eta/\mathbf{y}), \mathbf{y} = \mathbf{x}\}$, $dom(\mathbf{q}) := \{\mathbf{y}\}$, where $\mathbf{y}$ is a new discourse referent and $\mathbf{x}$ is an accessible discourse referent from $\mathbf{y}$ and belongs to $dom(\mathbf{p})$ and '$talking\_about(\{\mathbf{A}, \mathbf{B}\}, \mathbf{x})$' is in $cond(\mathbf{p})$,

(iv) If φ contains an indefinite noun phrase 'a(n) η', then $cond(\mathbf{q}) := \{\varphi(\eta/\mathbf{y}), \eta(\mathbf{y})\}$, $dom(\mathbf{q}) := \{\mathbf{y}\}$, where $\mathbf{y}$ is a new discourse referent.

(v) If X says 'I know what you are talking about', then $dom(\mathbf{p}) := dom(\mathbf{p}) \cup \{\mathbf{z}\}$, and $cond(\mathbf{q}) := dom(\mathbf{q}) \cup \{\mathbf{x} = \mathbf{z}\}$, where $\mathbf{z}$ is a new discourse referent and $\mathbf{x}$ is the last discourse referent in $dom(\mathbf{q})$. □

We can encode this distinction by accessibility of anaphora between conversants into the structure of Hyper-DRS's. In other words, we can define two types of discourse referents in Discourse Representation Theory.

The two types can be distinguished in terms of the *life span* of discourse referents. Suppose that John and Mary disagreed on the referential identity of Bill and Bob.

(9) John: Oh, Bill is running.
Mary: No, he is Bob.

This disagreement is represented by the following Hyper-DRS.

q :
- $\mathbf{A}\ \mathbf{B}\ \mathbf{u}\ \mathbf{p}_1, \mathbf{x}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{x}_2$
- $Self(\mathbf{A})$; $Other(\mathbf{B})$; $Now(\mathbf{u})$
- $believe(\mathbf{A}, \mathbf{q})$; $believe(\mathbf{B}, \mathbf{q})$
- $believe(\mathbf{A}, \mathbf{p}_1)$; $talking\_about(\{\mathbf{A}, \mathbf{B}\}, \mathbf{x}_1)$
- $\mathbf{p}_1$: [$\mathbf{y}_1\ \mathbf{r}_1\ \mathbf{e}_1$ | $\mathbf{y}_1 = \mathbf{x}_1$; $Bill(\mathbf{y}_1)$; $\mathbf{u} O \mathbf{r}_1$; $\mathbf{r}_1 \sqsubseteq \mathbf{e}_1$; $[\mathbf{e}_1 : \mathbf{y}_1\ be\_running]$]
- $believe(\mathbf{B}, \mathbf{p}_2)$
- $\mathbf{p}_2$: [$\mathbf{p}'$ | $\mathbf{p}' = \mathbf{p}_1$; $wrong(\mathbf{p}')$]
- $\mathbf{p}_2$ *because* $\mathbf{p}_3$
- $believe(\mathbf{B}, \mathbf{p}_3)$; $talking\_about(\{\mathbf{A}, \mathbf{B}\}, \mathbf{x}_2)$
- $\mathbf{p}_3$: [$\mathbf{y}_2\ \mathbf{y}_3$ | $\mathbf{y}_2 = \mathbf{x}_1$; $\mathbf{y}_2 = \mathbf{y}_3$; $Bob(\mathbf{y}_3)$; $\mathbf{y}_3 = \mathbf{x}_2$]

This Hyper-DRS represents each conversant as believing she and the other conversant are talking about the same person but their identifications of the person do not coincide with each other. The condition '$\mathbf{p}_2$ *because* $\mathbf{p}_3$' denotes the ground of the refutation. That is, the utterance 'No, p.' is interpreted as meaning 'that is wrong because p.' In dialogue interpretation, we should accomodate this type of relation denoting cohesion.

This Hyper-DRS is constructed by the following procedures.

Procedure 3 *Given a Hyper-DRS $K(\mathbf{p})$ of a dialogue between A and B from A's viewpoint, if X says 'no, P', then $cond(\mathbf{p}) := cond(\mathbf{p}) \cup \{believe(\mathbf{X}, \mathbf{p}_i), K(\mathbf{p}_i), \mathbf{p}_i\ because\ \mathbf{p}_j, believe(\mathbf{X}, \mathbf{p}_j), K(\mathbf{p}_j)\}$,* *dom*($\mathbf{p}$) := *dom*($\mathbf{p}$) $\cup$ {$\mathbf{p}_i, \mathbf{p}_j$}, *where* $\mathbf{p}_i$, $\mathbf{p}_i$ *are new discourse referents, and* $K(\mathbf{p}_i)$ *is the Hyper-DRS of 'no', and* $K(\mathbf{p}_j)$ *is the Hyper-DRS of 'P'.* *cond*($\mathbf{p}_i$) := {*wrong*($\mathbf{p}_k$), $\mathbf{p}_k = \mathbf{p}_h$}, *dom*($\mathbf{p}_i$) := {$\mathbf{p}_k$}, *where* $\mathbf{p}_k$ *is a new discourse referent, and* $\mathbf{p}_h$ *is the last proposition discourse referent in dom*($\mathbf{p}$) *introduced by the other.* □

5 Shared Belief Revision in Dialogues

Furthermore, such a disagreement may be solved in dialogue as follows.

(10) John: Oh, Bill is running.
Mary: No, he is Bob.
John: No, Bob has gone to Hongkong.
Mary: Oh, is that so?

To treat this change from disagreement to agreement, the conversant must revise her belief or at least the shared beliefs. In (10), Mary's utterance 'is that so?' revises their shared beliefs which were at variance and resolves their disagreement. The following procedure can construct the Hyper-DRS representing the agreed states.

Procedure 4 *Given a Hyper-DRS* $K(\mathbf{p})$ *of a dialogue between A and B from A's viewpoint, if X says 'Is that so?' then cond*($\mathbf{p}$) := *cond*($\mathbf{p}$) $\cup$ {*believe*($\mathbf{X}, \mathbf{q}$), $\mathbf{p}_i = \mathbf{p}_h$}, *dom*($\mathbf{p}$) := *dom*($\mathbf{p}$) $\cup$ {$\mathbf{q}, \mathbf{p}_i$}, *where* $\mathbf{q}$, $\mathbf{p}_i$ *are new discourse referents, and* $\mathbf{p}_h$ *is the last proposition discourse referent in dom*($\mathbf{p}$) *introduced by the other.* □

The following Hyper-DRS $K(\mathbf{p})$ is constructed from (10) by procedure 4.

q :

[$\mathbf{A}\ \mathbf{B}\ \mathbf{u}\ \mathbf{p}_1, \mathbf{x}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{x}_2\ \mathbf{p}_4, \mathbf{p}_5, \mathbf{x}_3\ \mathbf{p}_6$]

Self($\mathbf{A}$); *Other*($\mathbf{B}$); *Now*($\mathbf{u}$)
believe($\mathbf{A}, \mathbf{q}$); *believe*($\mathbf{B}, \mathbf{q}$)
believe($\mathbf{A}, \mathbf{p}_1$); *talking_about*({$\mathbf{A}, \mathbf{B}$}, $\mathbf{x}_1$)
$\mathbf{p}_1$: [$\mathbf{y}_1\ \mathbf{r}_1\ \mathbf{e}_1$ | $\mathbf{y}_1 = \mathbf{x}_1$; *Bill*($\mathbf{y}_1$); $\mathbf{u}O\mathbf{r}_1$; $\mathbf{r}_1 \sqsubseteq \mathbf{e}_1$; $\mathbf{e}$: ($\mathbf{y}_1$ *be_running*)]
believe($\mathbf{B}, \mathbf{p}_2$)
$\mathbf{p}_2$: [$\mathbf{p}'$ | $\mathbf{p}' = \mathbf{p}_1$; *wrong*($\mathbf{p}'$)]
$\mathbf{p}_2$ *because* $\mathbf{p}_3$
believe($\mathbf{B}, \mathbf{p}_3$); *talking_about*({$\mathbf{A}, \mathbf{B}$}, $\mathbf{x}_2$)
$\mathbf{p}_3$: [$\mathbf{y}_2\ \mathbf{y}_3$ | $\mathbf{y}_2 = \mathbf{x}_1$; $\mathbf{y}_2 = \mathbf{y}_3$; *Bob*($\mathbf{y}_3$); $\mathbf{y}_3 = \mathbf{x}_2$]
believe($\mathbf{A}, \mathbf{p}_4$)
$\mathbf{p}_4$: [$\mathbf{p}''$ | $\mathbf{p}'' = \mathbf{p}_3$; *wrong*($\mathbf{p}''$)]
$\mathbf{p}_4$ *because* $\mathbf{p}_5$
believe($\mathbf{A}, \mathbf{p}_5$); *talking_about*({$\mathbf{A}, \mathbf{B}$}, $\mathbf{x}_3$)
$\mathbf{p}_5$: [$\mathbf{y}_4\ \mathbf{e}_2\ \mathbf{r}_2\ \mathbf{y}_5$ | $\mathbf{y}_4 = \mathbf{x}_2$; *Bob*($\mathbf{y}_4$); $\mathbf{u}O\mathbf{r}_2$; $\mathbf{r}_2 \sqsubseteq \mathbf{e}_2$; $\mathbf{e}_2$: ($\mathbf{y}_4$ *have_gone_to* $\mathbf{y}_5$); $\mathbf{y}_5 = \mathbf{x}_3$; *Hongkong*($\mathbf{y}_5$)]
believe($\mathbf{B}, \mathbf{p}_6$)
$\mathbf{p}_6 = \mathbf{p}_4$

We must explain how to delete propositions to be abandoned. In (10), Mary revised her belief or shared belief in response to John's previous explanation 'Bob has gone to Hongkong.' This justifies the assertion 'no' against 'he is Bob.' In other words, John's 'no' marks the assertion 'he is Bob' as deletable, and when the ground 'Bob has gone to Hongkong' of John's 'no' has been shared, the marked assertion 'he is Bob' must be deleted. Furthermore, we must delete Mary's *attitude* 'no' against 'Bill is running,' since Mary's 'no' grounds on 'he is Bob,' and this ground is refuted by John's 'no' and its ground 'Bob has gone...' It is to be noted that deletable conditions are not conditions about the content, but conversants' propositional attitudes. *The revised parts are only conversants' attitudes.* This procedure can be more formally defined as follows.

Procedure 5 *Given a Hyper-DRS $K(\mathbf{p})$ of a dialogue between A and B from A's viewpoint, if all of the conversants share $\mathbf{q}$ and the condition '$\mathbf{p}_i$ because $\mathbf{q}$', and $\mathbf{p}_j$ marks $\mathbf{p}_k$ as deletable, i.e., $wrong(\mathbf{p}_k) \in cond(\mathbf{p}_j)$, then delete $believe(X, \mathbf{p}_k)$ from $cond(\mathbf{p})$, and add $believe(X, \mathbf{p}_i)$ to $cond(\mathbf{p})$.*□

As the result, the following Hyper-DRS of (10) is constructed.

$\mathbf{q}$:

$\mathbf{A}\ \mathbf{B}\ \mathbf{u}\ \mathbf{p}_1, \mathbf{x}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{x}_2\ \mathbf{p}_4, \mathbf{p}_5, \mathbf{x}_3\ \mathbf{p}_6$
$Self(\mathbf{A})$; $Other(\mathbf{B})$; $Now(\mathbf{u})$
$believe(\mathbf{A}, \mathbf{q})$; $believe(\mathbf{B}, \mathbf{q})$
$believe(\mathbf{A}, \mathbf{p}_1)$; $talking\_about(\{\mathbf{A}, \mathbf{B}\}, \mathbf{x}_1)$
$\mathbf{p}_1$: [$\mathbf{y}_1\ \mathbf{r}_1\ \mathbf{e}_1$ | $\mathbf{y}_1 = \mathbf{x}_1$; $Bill(\mathbf{y}_1)$; $\mathbf{u} O \mathbf{r}_1$; $\mathbf{r}_1 \sqsubseteq \mathbf{e}_1$; $\mathbf{e}_1 : (\mathbf{y}_1\ be\_running)$]
$\mathbf{p}_2$: [$\mathbf{p}'$ | $\mathbf{p}' = \mathbf{p}_1$; $wrong(\mathbf{p}')$]
$\mathbf{p}_2$ *because* $\mathbf{p}_3$
$talking\_about(\{\mathbf{A}, \mathbf{B}\}, \mathbf{x}_2)$
$\mathbf{p}_3$: [$\mathbf{y}_2\ \mathbf{y}_3$ | $\mathbf{y}_2 = \mathbf{x}_1$; $\mathbf{y}_2 = \mathbf{y}_3$; $Bob(\mathbf{y}_3)$; $\mathbf{y}_3 = \mathbf{x}_2$]
$believe(\mathbf{A}, \mathbf{p}_4)$
$\mathbf{p}_4$: [$\mathbf{p}''$ | $\mathbf{p}'' = \mathbf{p}_3$; $wrong(\mathbf{p}'')$]
$\mathbf{p}_4$ *because* $\mathbf{p}_5$
$believe(\mathbf{A}, \mathbf{p}_5)$; $talking\_about(\{\mathbf{A}, \mathbf{B}\}, \mathbf{x}_3)$
$\mathbf{p}_5$: [$\mathbf{y}_4\ \mathbf{e}_2\ \mathbf{r}_2\ \mathbf{y}_5$ | $\mathbf{y}_4 = \mathbf{x}_2$; $Bob(\mathbf{y}_4)$; $\mathbf{u} O \mathbf{r}_2$; $\mathbf{r}_2 \sqsubseteq \mathbf{e}_2$; $\mathbf{e}_2 : (\mathbf{y}_4\ have\_gone\_to\ \mathbf{y}_5)$; $\mathbf{y}_5 = \mathbf{x}_3$; $Hongkong(\mathbf{y}_5)$]
$believe(\mathbf{B}, \mathbf{p}_6)$
$\mathbf{p}_6 = \mathbf{p}_4$

It should be noted that the shared discourse referent and the shared belief referent can not be deleted and forgotten if the shared beliefs are revised and deleted. Therefore, the life span of shared discourse referents and shared belief referents are the longest in revision processes of Hyper-DRS's.

6 Conclusion

In this paper, we have shown detailed structures of reference in dialogues and shared belief revision from the viewpoint of dialogue as shared belief revision, by extending DRS to Hyper-DRS. We have distinguished and characterized three types of discourse referents: (standard) discourse referents, shared discourse referents, and shared belief referents, and showed how they realize dialogues as processes of shared belief revision. Shared discourse referents play roles in preserving the topic of dialogues, and shared belief discourse referents make circular propositions into first-order equational systems, and we can update and revise the circular propositions. Conversants can access only shared discourse referents except for discourse referents introduced by her own moves. Shared belief referents have the longest life span in the dialogues. Secondly, shared discourse referents live longer than standard discourse referents. They are deleted only if the dialogue fails and a repair question like 'Pardon?' is induced. Shared belief revision is an action affecting only propositional attitudes. So shared belief referents and shared discourse referents are not affected by any shared belief revision in dialogues.

Thus, the three types of discourse referents are foundamental devices for management of shared beliefs.

References

[1] Peter Aczel. *Non-well-founded Sets.* CSLI, Stanford, 1987.

[2] Nicholas Asher. Discourse representation theory and belief dynamics. In A. Fuhrmann and M. Morreau, editors, *The Logic of Theory Change*, pages 282–321. Springer-Verlag, Berlin, 1990.

[3] Jon Barwise. *Situation in Logic.* CSLI, Stanford, 1989.

[4] Jon Barwise and John Etchemendy. *The Liar.* The MIT Press, Cambridge, 1987.

[5] Herbert H. Clark and Catherine R. Marshall. Definite reference and mutual knowledge. In A. K. Joshi, D. Webber, and Ivan Sag, editors, *Elements of Discourse Understanding.* Cambridge University Press, Cambridge, 1981.

[6] Marco Colombetti. Formal semantics for mutual belief. *Artificial Intelligence*, 62:341–353, 1993.

[7] Aravind K. Joshi. Mutual beliefs in question-answer systems. In N. V. Smith, editor, *Mutual Knowledge*, pages 181–197. Academic Press, London, 1982.

[8] Hans Kamp. Context, thought and communication. *The Proceedings of the Aristotelian Society, New Series*, LXXXV(XIII):239–261, 1985.

[9] Hans Kamp. A theory of truth and semantic representation. In Groenendijk et al., editors, *Truth, Interpretation and Information*, pages 1–41. Foris, Dordrecht, 1986.

[10] Hans Kamp and Uwe Reyle. *From Discourse to Logic.* Kluwer Academic Publishers, Dordrecht, 1993.

[11] Lauri Karttunen. Discourse referents. In *Syntax and Semantics*, pages 363–385. Academic Press, New York, 1977.

[12] Hisashi Komatsu. Semantics of Cooperative Dialogues. In Akira Ishikawa and Yoshihiko Nitta eds., *The Proceedings of the 1994 Kyoto Conference: A Festschrift for Professor Akira Ikeya*, Tokyo: The Logico-Linguistics Society of Japan, 1995, 183-192, .

[13] Hisashi Komatsu, Norihiro Ogata, and Akira Ishikawa. Towards a dynamic theory of belief- sharing in cooperative dialogues. In *Proceedings of COLING 94*, 1994.

[14] Norihiro Ogata. Proper Names, Reference, and Information Sharing. (In Japanese) In *Software Bunsho no Nihongo Shori no tame no Kenkyuu*, Tokyo: Information Processing Promotion Agency, 1993, pages 257–310.

[15] Norihiro Ogata. Information sharing models of dialogue and four classes of circularity problems. In Akira Ishikawa and Yoshihiko Nitta eds., *The Proceedings of the 1994 Kyoto Conference: A Festschrift for Professor Akira Ikeya*, Tokyo: The Logico-Linguistics Society of Japan, 1995, 193-202, .

APPENDIX

Modeling Propositions

In [4, 3]'s *Austinian* model of propositions, a proposition is analyzed as a complex object: $(s : \tau)$, where s is a situation which referes to the situation described by a type τ. We define two classes: types ($TYPE$), and propositions ($PROP$) from atoms: individuals $IND = \{a_1, a_2, ...\}$, individual indeterminates $ParIND = \{\mathbf{x}_1, ..., \mathbf{x}_n\}$, time intervals and events, states, processes $i, i' \in TEMP$, relations $REL = \{Believe, ...\}$, polarities $pol \in POL = \{1, 0\}$.

Definition 3 *Assuming ZFC/AFA(See [1]), let $TYPE$, $PROP$, SIT be the largest classes satisfying:*

- $\sigma \in TYPE$ *is of one of the following forms:*
  - $(Believe, x, p; 1)$, *or*

- $(Talking\_about, \{x, y\}, z; 1)$, or
- $(True, p; 1)$, $(True, p; 0)$, or
- $(Because, p, q)$, or
- $(Self, x; 1)$, $(Other, x; 1)$, $(Now, i; 1)$, $(Bill, x; 1)$, $(Bob, x; 1)$ or
- $(Included, i, i'; 1)$, $(Overlap, i, i'; 1)$, $(Included, i, i'; 0)$, $(Overlap, i, i'; 0)$, or
- $(Be\_running, i, x; 1)$, $(Be\_dancing, i, x; 1)$, $(Have\_gone\_to, i, x, y; 1)$, or
- $(\wedge, \Gamma)$, $(\vee, \Gamma)$,

where $p, q \in PROP$, $x, y, z \in IND \cup ParIND$, $\Gamma \subseteq TYPE$.

- $s \in SIT$ iff $s \subseteq TYPE$,
- $p \in PROP$ (an atomic proposition) is a tuple $(:, s, \tau; 1)$, for short $(s : \tau)$, where $s \in SIT$, $\tau \in TYPE$. We also write $\bar{p}$ for the dual of p, and $\bar{\bar{p}} = p$. □

Model Construction

Definition 4 *A model $\mathcal{M}$ is a sequence of subset of $TYPE$, satisfying following conditions: For any natural number n,*

- no proposition and its dual are in $\mathcal{M}(n)$,
- $(Self, x; 1), (Other, y; 1), (Now, i; 1) \in \mathcal{M}(0)$,
- if $(\wedge, \Gamma) \in \mathcal{M}(n)$ then $\Gamma \subseteq \mathcal{M}(n)$,
- if $(Because, p, q; 1), q \in \mathcal{M}(n)$, then $p \in \mathcal{M}(n)$,
- no $(Because, p, q; 1)$ and $(Because, p, \bar{q}; 1)$ are in $\mathcal{M}(n)$,
- no $(Because, p, q; 1)$ and $(Because, \bar{p}, q; 1)$ are in $\mathcal{M}(n)$,
- if $(R, x; 1), (R, y; 1) \in \mathcal{M}(n)$ and $R \in \{Self, Other, Now\}$, then $x = y$,
- if $(Overlap, i, j; 1) \in \mathcal{M}(n)$ then $(Overlap, j, i; 1) \in \mathcal{M}(n)$,
- if $(Included, i, j; 1) \in \mathcal{M}(n)$ then $(Overlap, i, j; 1), \in \mathcal{M}(n)$,
- if $(Overlap, i, j; 0) \in \mathcal{M}(n)$ then $(Included, i, j; 0), \in \mathcal{M}(n)$,
- if $(Overlap, i, j; 1), (Included, j, k; 1) \in \mathcal{M}(n)$ then $(Overlap, i, k; 1) \in \mathcal{M}(n)$,
- if $(Included, i, j; 1), (Included, j, k; 1) \in \mathcal{M}(n)$ then $(Included, i, k; 1) \in \mathcal{M}(n)$,
- if $(Included, i, j; 1), (Included, j, i; 1) \in \mathcal{M}(n)$ then $i = j$,
- if a temporal object k is in $\mathcal{M}(n)$ then $(Included, k, k; 1) \in \mathcal{M}(n)$,
- if $(True, p; 1) \in \mathcal{M}(n)$ then $p \in \mathcal{M}(n)$,
- if $(True, p; 0) \in \mathcal{M}(n)$ then $p \notin \mathcal{M}(n)$.
- if $(Believe, x_m, p; 1) \in \mathcal{M}(n)$ then $p \in \mathcal{M}(m)$. □

Semantics of Hyper-DRS

Definition 5 *Given a Hyper-DRS $K(\mathbf{p}_n)$ of a dialogue between A and B from A's viewpoint, an assignment f from discourse referents to objects in a model $\mathcal{M}$, and a function V from conditions to $TYPE$, f verifies $K(\mathbf{p}_n)$ in $\mathcal{M}$, written $f \Vdash_{\mathcal{M}} K(\mathbf{p}_n))$, iff for some $g \supseteq f$, there is a unique solution of the equation $g(\mathbf{p}_n) = (\wedge, V_g[cond(\mathbf{p}_n)])$ in $\mathcal{M}(0)$, where V_g is satisfying the following conditions:*

- $V_g(Self(\mathbf{x})) = (Self, g(\mathbf{x}); 1)$,
- $V_g(Other(\mathbf{x})) = (Other, g(\mathbf{x}); 1)$,
- $V_g(Now(\mathbf{u})) = (Now, g(\mathbf{u}); 1)$,
- $V_g(Bill(\mathbf{x})) = (Bill, g(\mathbf{x}); 1)$,
- $V_g(Bob(\mathbf{x})) = (Bob, g(\mathbf{x}); 1)$,
- $V_g(Hongkong(\mathbf{x})) = (Hongkong, g(\mathbf{x}); 1)$,
- $V_g(believe(\mathbf{x}, \mathbf{p})) = (Believe, g(\mathbf{x}), g(\mathbf{p}); 1)$,
- $V_g(talking\_about(\{\mathbf{x}, \mathbf{y}\}, \mathbf{z})) = (Talking\_about, \{g(\mathbf{x}), g(\mathbf{y})\}, g(\mathbf{z}); 1)$,
- $V_g(\mathbf{u}O\mathbf{r}) = (Overlap, g(\mathbf{u}), g(\mathbf{r}); 1)$,
- $V_g(\mathbf{u} \sqsubseteq \mathbf{r}) = (Included, g(\mathbf{u}), g(\mathbf{r}); 1)$,
- $V_g([\mathbf{e} : \mathbf{x}\ be\_running]) = (Be\_running, g(\mathbf{e}), g(\mathbf{x}); 1)$,
- $V_g([\mathbf{e} : \mathbf{x}\ be\_dancing]) = (Be\_dancing, g(\mathbf{e}), g(\mathbf{x}); 1)$,
- $V_g([\mathbf{e} : \mathbf{x}\ have\_gone\_to\ \mathbf{y}]) = (Have\_gone\_to, g(\mathbf{e}), g(\mathbf{x}), g(\mathbf{y}); 1)$,
- $V_g(right(\mathbf{p})) = (True, g(\mathbf{p}); 1)$,
- $V_g(wrong(\mathbf{p})) = (True, g(\mathbf{p}); 0)$,
- $V_g(\mathbf{x} = \mathbf{y}) = (g(\mathbf{x}) = g(\mathbf{y}))$,
- $V_g(K(\mathbf{p})) = (g(\mathbf{p}) = V_g[K(\mathbf{p})])$.

$K(\mathbf{p})$ is true in $\mathcal{M}$, written $\Vdash_{\mathcal{M}} K(\mathbf{p})$, iff for some assignment g, $g \Vdash_{\mathcal{M}} K(\mathbf{p})$. □