Articles

Discriminative Reranking for Natural Language Parsing

Michael Collins^{*} Massachusetts Institute of Technology Terry Koo* Massachusetts Institute of Technology

This article considers approaches which rerank the output of an existing probabilistic parser. The base parser produces a set of candidate parses for each input sentence, with associated probabilities that define an initial ranking of these parses. A second model then attempts to improve upon this initial ranking, using additional features of the tree as evidence. The strength of our approach is that it allows a tree to be represented as an arbitrary set of features, without concerns about how these features interact or overlap and without the need to define a derivation or a generative model which takes these features into account. We introduce a new method for the reranking task, based on the boosting approach to ranking problems described in Freund et al. (1998). We apply the boosting method to parsing the Wall Street Journal treebank. The method combined the log-likelihood under a baseline model (that of Collins [1999]) with evidence from an additional 500,000 features over parse trees that were not included in the original model. The new model achieved 89.75% F-measure, a 13% relative decrease in Fmeasure error over the baseline model's score of 88.2%. The article also introduces a new algorithm for the boosting approach which takes advantage of the sparsity of the feature space in the parsing data. Experiments show significant efficiency gains for the new algorithm over the obvious implementation of the boosting approach. We argue that the method is an appealing alternative—in terms of both simplicity and efficiency—to work on feature selection methods within log-linear (maximum-entropy) models. Although the experiments in this article are on natural language parsing (NLP), the approach should be applicable to many other NLP problems which are naturally framed as ranking tasks, for example, speech recognition, machine translation, or natural language generation.

1. Introduction

Machine-learning approaches to natural language parsing have recently shown some success in complex domains such as news wire text. Many of these methods fall into the general category of history-based models, in which a parse tree is represented as a derivation (sequence of decisions) and the probability of the tree is then calculated as a product of decision probabilities. While these approaches have many advantages, it can be awkward to encode some constraints within this framework. In the ideal case, the designer of a statistical parser would be able to easily add features to the model

^{*} MIT Computer Science and Artificial Intelligence Laboratory (CSAIL), the Stata Center, Building 32, 32 Vassar Street, Cambridge, MA 02139. Email: mcollins@csail.mit.edu, maestro@mit.edu.

Submission received: 15th October 2003; Accepted for publication: 29th April 2004

that are believed to be useful in discriminating among candidate trees for a sentence. In practice, however, adding new features to a generative or history-based model can be awkward: The derivation in the model must be altered to take the new features into account, and this can be an intricate task.

This article considers approaches which rerank the output of an existing probabilistic parser. The base parser produces a set of candidate parses for each input sentence, with associated probabilities that define an initial ranking of these parses. A second model then attempts to improve upon this initial ranking, using additional features of the tree as evidence. The strength of our approach is that it allows a tree to be represented as an arbitrary set of features, without concerns about how these features interact or overlap and without the need to define a derivation which takes these features into account.

We introduce a new method for the reranking task, based on the boosting approach to ranking problems described in Freund et al. (1998). The algorithm can be viewed as a feature selection method, optimizing a particular loss function (the exponential loss function) that has been studied in the boosting literature. We applied the boosting method to parsing the Wall Street Journal (WSJ) treebank (Marcus, Santorini, and Marcinkiewicz 1993). The method combines the log-likelihood under a baseline model (that of Collins [1999]) with evidence from an additional 500,000 features over parse trees that were not included in the original model. The baseline model achieved 88.2% *F*-measure on this task. The new model achieves 89.75% *F*-measure, a 13% relative decrease in *F*-measure error.

Although the experiments in this article are on natural language parsing, the approach should be applicable to many other natural language processing (NLP) problems which are naturally framed as ranking tasks, for example, speech recognition, machine translation, or natural language generation. See Collins (2002a) for an application of the boosting approach to named entity recognition, and Walker, Rambow, and Rogati (2001) for the application of boosting techniques for ranking in the context of natural language generation.

The article also introduces a new, more efficient algorithm for the boosting approach which takes advantage of the sparse nature of the feature space in the parsing data. Other NLP tasks are likely to have similar characteristics in terms of sparsity. Experiments show an efficiency gain of a factor of 2,600 for the new algorithm over the obvious implementation of the boosting approach. Efficiency issues are important, because the parsing task is a fairly large problem, involving around one million parse trees and over 500,000 features. The improved algorithm can perform 100,000 rounds of feature selection on our task in a few hours with current processing speeds. The 100,000 rounds of feature selection require computation equivalent to around 40 passes over the entire training set (as opposed to 100,000 passes for the "naive" implementation).

The problems with history-based models and the desire to be able to specify features as arbitrary predicates of the entire tree have been noted before. In particular, previous work (Ratnaparkhi, Roukos, and Ward 1994; Abney 1997; Della Pietra, Della Pietra, and Lafferty 1997; Johnson et al. 1999; Riezler et al. 2002) has investigated the use of Markov random fields (MRFs) or log-linear models as probabilistic models with global features for parsing and other NLP tasks. (Log-linear models are often referred to as maximum-entropy models in the NLP literature.) Similar methods have also been proposed for machine translation (Och and Ney 2002) and language understanding in dialogue systems (Papineni, Roukos, and Ward 1997, 1998). Previous work (Friedman, Hastie, and Tibshirani 1998) has drawn connections between log-linear models and

boosting for classification problems. One contribution of our research is to draw similar connections between the two approaches to ranking problems.

We argue that the efficient boosting algorithm introduced in this article is an attractive alternative to maximum-entropy models, in particular, **feature selection** methods that have been proposed in the literature on maximum-entropy models. The earlier methods for maximum-entropy feature selection methods (Ratnaparkhi, Roukos, and Ward 1994; Berger, Della Pietra, and Della Pietra 1996; Della Pietra, Della Pietra, and Lafferty 1997; Papineni, Roukos, and Ward 1997, 1998) require several full passes over the training set for each round of feature selection, suggesting that at least for the parsing data, the improved boosting algorithm is several orders of magnitude more efficient.¹ In section 6.4 we discuss our approach in comparison to these earlier methods for feature selection, as well as the more recent work of McCallum (2003); Zhou et al. (2003); and Riezler and Vasserman (2004).

The remainder of this article is structured as follows. Section 2 reviews historybased models for NLP and highlights the perceived shortcomings of history-based models which motivate the reranking approaches described in the remainder of the article. Section 3 describes previous work (Friedman, Hastie, and Tibshirani 2000; Duffy and Helmbold 1999; Mason, Bartlett, and Baxter 1999; Lebanon and Lafferty 2001; Collins, Schapire, and Singer 2002) that derives connections between boosting and maximum-entropy models for the simpler case of classification problems; this work forms the basis for the reranking methods. Section 4 describes how these approaches can be generalized to ranking problems. We introduce loss functions for boosting and MRF approaches and discuss optimization methods. We also derive the efficient algorithm for boosting in this section. Section 5 gives experimental results, investigating the performance improvements on parsing, efficiency issues, and the effect of various parameters of the boosting algorithm. Section 6 discusses related work in more detail. Finally, section 7 gives conclusions.

The reranking models in this article were originally introduced in Collins (2000). In this article we give considerably more detail in terms of the algorithms involved, their justification, and their performance in experiments on natural language parsing.

2. History-Based Models

Before discussing the reranking approaches, we describe **history-based models** (Black et al. 1992). They are important for a few reasons. First, several of the best-performing parsers on the WSJ treebank (e.g., Ratnaparkhi 1997; Charniak 1997, 2000; Collins 1997, 1999; Henderson 2003) are cases of history-based models. Many systems applied to part-of-speech tagging, speech recognition, and other language or speech tasks also fall into this class of model. Second, a particular history-based model (that of Collins [1999]) is used as the initial model for our approach. Finally, it is important to describe history-based models—and to explain their limitations—to motivate our departure from them.

Parsing can be framed as a supervised learning task, to induce a function $f : \mathcal{X} \rightarrow \mathcal{Y}$ given training examples (x_i, y_i) , where $x_i \in \mathcal{X}, y_i \in \mathcal{Y}$. We define $\text{GEN}(x) \subset \mathcal{Y}$ to be the set of candidates for a given input x. In the parsing problem x is a sentence, and

¹ Note, however, that log-linear models which employ regularization methods instead of feature selection—see, for example, Johnson et al. (1999) and Lafferty, McCallum, and Pereira (2001)—are likely to be comparable in terms of efficiency to our feature selection approach. See section 6.3 for more discussion.

GEN(x) is a set of candidate trees for that sentence. A particular characteristic of the problem is the complexity of GEN(x): GEN(x) can be very large, and each member of GEN(x) has a rich internal structure. This contrasts with "typical" classification problems in which GEN(x) is a fixed, small set, for example, $\{-1,+1\}$ in binary classification problems.

In probabilistic approaches, a model is defined which assigns a probability P(x, y) to each (x, y) pair.² The most likely parse for each sentence x is then arg $\max_{y \in GEN(x)} P(x, y)$. This leaves the question of how to define P(x, y). In history-based approaches, a one-to-one mapping is defined between each pair (x, y) and a decision sequence $\langle d_1 \dots d_n \rangle$. The sequence $\langle d_1 \dots d_n \rangle$ can be thought of as the sequence of moves that build (x, y) in some canonical order. Given this mapping, the probability of a tree can be written as

$$P(x,y) = \prod_{i=1\dots n} P(d_i | \Phi(d_1 \dots d_{i-1}))$$

Here, $(d_1 \dots d_{i-1})$ is the **history** for the *i*th decision. Φ is a function which groups histories into equivalence classes, thereby making independence assumptions in the model.

Probabilistic context-free grammars (PCFGs) are one example of a history-based model. The decision sequence $\langle d_1 \dots d_n \rangle$ is defined as the sequence of rule expansions in a top-down, leftmost derivation of the tree. The history is equivalent to a partially built tree, and Φ picks out the nonterminal being expanded (i.e., the leftmost nonterminal in the fringe of this tree), making the assumption that $P(d_i|d_1 \dots d_{i-1})$ depends only on the nonterminal being expanded. In the resulting model a tree with rule expansions $\langle A_i \rightarrow \beta_i \rangle$ is assigned a probability $\prod_{i=1}^n P(\beta_i|A_i)$.

Our base model, that of Collins (1999), is also a history-based model. It can be considered to be a type of PCFG, where the rules are lexicalized. An example rule would be

Lexicalization leads to a very large number of rules; to make the number of parameters manageable, the generation of the right-hand side of a rule is broken down into a number of decisions, as follows:

- First the head nonterminal (VBD in the above example) is chosen.
- Next, left and right subcategorization frames are chosen ({} and {NP-C}).
- Nonterminal sequences to the left and right of the VBD are chosen (an empty sequence to the left, (NP-C, NP) to the right).
- Finally, the lexical heads of the modifiers are chosen (her and today).

$$P(y \mid x) = \prod_{i=1\dots n} P(d_i \mid \Phi(d_1 \dots d_{i-1}, x))$$

where $d_1 \dots d_n$ are again the decisions made in building a parse, and Φ is a function that groups histories into equivalence classes. Note that *x* is added to the domain of Φ (the context on which decisions are conditioned). See Ratnaparkhi (1997) for one example of a method using this approach.

² To be more precise, **generative** probabilistic models assign joint probabilities P(x, y) to each (x, y) pair. Similar arguments apply to conditional history-based models, which define conditional probabilities P(y | x) through a definition

Figure 1 illustrates this process. Each of the above decisions has an associated probability conditioned on the left-hand side of the rule (VP(saw)) and other information in some cases.

History-based approaches lead to models in which the log-probability of a parse tree can be written as a linear sum of parameters α_k multiplied by features h_k . Each feature $h_k(x, y)$ is the count of a different "event" or fragment within the tree. As an example, consider a PCFG with rules $\langle A_k \rightarrow \beta_k \rangle$ for $1 \le k \le m$. If $h_k(x, y)$ is the number of times $\langle A_k \rightarrow \beta_k \rangle$ is seen in the tree, and $\alpha_k = \log P(\beta_k | A_k)$ is the parameter associated with that rule, then

$$\log P(x,y) = \sum_{k=1}^{m} \alpha_k h_k(x,y)$$

All models considered in this article take this form, although in the boosting models the score for a parse is not a log-probability. The features h_k define an *m*-dimensional vector of counts which represent the tree. The parameters α_k represent the influence of each feature on the score of a tree.

A drawback of history-based models is that the choice of derivation has a profound influence on the parameterization of the model. (Similar observations have been made in the related cases of belief networks [Pearl 1988], and language models for speech recognition [Rosenfeld 1997].) When designing a model, it would be desirable to have a framework in which features can be easily added to the model. Unfortunately, with history-based models adding new features often requires a modification of the underlying derivations in the model. Modifying the derivation to include a new feature type can be a laborious task. In an ideal situation we would be able to encode arbitrary features h_{kr} without having to worry about formulating a derivation that included these features.

To take a concrete example, consider part-of-speech tagging using a hidden Markov model (HMM). We might have the intuition that almost every sentence has at least one verb and therefore that sequences including at least one verb should have increased scores under the model. Encoding this constraint in a compact way in an HMM takes some ingenuity. The obvious approach—to add to each state the information about whether or not a verb has been generated in the history—doubles



Figure 1

The sequence of decisions involved in generating the right-hand side of a lexical rule.

the number of states (and parameters) in the model. In contrast, it would be trivial to implement a feature $h_k(x, y)$ which is 1 if *y* contains a verb, 0 otherwise.

3. Logistic Regression and Boosting

We now turn to machine-learning methods for the ranking task. In this section we review two methods for binary classification problems: logistic regression (or maximum-entropy) models and boosting. These methods form the basis for the reranking approaches described in later sections of the article. Maximum-entropy models are a very popular method within the computational linguistics community; see, for example, Berger, Della Pietra, and Della Pietra (1996) for an early article which introduces the models and motivates them. Boosting approaches to classification have received considerable attention in the machine-learning community since the introduction of AdaBoost by Freund and Schapire (1997).

Boosting algorithms, and in particular the relationship between boosting algorithms and maximum-entropy models, are perhaps not familiar topics in the NLP literature. However there has recently been much work drawing connections between the two methods (Friedman, Hastie, and Tibshirani 2000; Lafferty 1999; Duffy and Helmbold 1999; Mason, Bartlett, and Baxter 1999; Lebanon and Lafferty 2001; Collins, Schapire, and Singer 2002); in this section we review this work. Much of this work has focused on binary classification problems, and this section is also restricted to problems of this type. Later in the article we show how several of the ideas can be carried across to reranking problems.

3.1 Binary Classification Problems

The general setup for binary classification problems is as follows:

- The "input domain" (set of possible inputs) is X.
- The "output domain" (set of possible labels) is simply a set of two labels, *Y* = {-1, +1}.³
- The training set is an array of *n* labeled examples, $\langle (x_1, y_1), (x_2, y_2), \dots, (x_n, y_n) \rangle$, where each $x_i \in \mathcal{X}, y_i \in \mathcal{Y}$.
- Input examples are represented through *m* "features," which are functions *h_k* : *X*→ ℜ for *k* = 1, . . . , *m*. It is also sometimes convenient to think of an example *x* as being represented by an *m*-dimensional "feature vector" φ(*x*) = ⟨*h*₁(*x*), *h*₂(*x*), . . . , *h_m*(*x*)⟩.
- Finally, there is a parameter vector, $\alpha = \langle \alpha_1, ..., \alpha_m \rangle$, where each $\alpha_k \in \Re$, hence $\bar{\alpha}$ is an *m*-dimensional real-valued vector.

We show that both logistic regression and boosting implement a linear, or hyperplane, classifier. This means that given an input example *x* and parameter values $\bar{\alpha}$, the output from the classifier is

$$\operatorname{sign}\left(F(x,\bar{\alpha})\right) \tag{1}$$

³ It turns out to be convenient to define $\mathcal{Y} = \{-1, +1\}$ rather than $\mathcal{Y} = \{0, +1\}$, for example.

where

$$F(x,\bar{\alpha}) = \sum_{k=1}^{m} \alpha_k h_k(x) = \bar{\alpha} \cdot \phi(x)$$
(2)

Here $\bar{\alpha} \cdot \phi(x)$ is the inner or dot product between the vectors $\bar{\alpha}$ and $\phi(x)$, and sign(z) = 1 if $z \ge 0$, sign(z) = -1 otherwise. Geometrically, the examples x are represented as vectors $\phi(x)$ in some m-dimensional vector space, and the parameters $\bar{\alpha}$ define a hyperplane which passes through the origin⁴ of the space and has $\bar{\alpha}$ as its normal. Points lying on one side of this hyperplane are classified as +1; points on the other side are classified as -1. The central question in learning is how to set the parameters $\bar{\alpha}$, given the training examples $\langle (x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n) \rangle$. Logistic regression and boosting involve different algorithms and criteria for training the parameters $\bar{\alpha}$, but recent work (Friedman, Hastie, and Tibshirani 2000; Lafferty 1999; Duffy and Helmbold 1999; Mason, Bartlett, and Baxter 1999; Lebanon and Lafferty 2001; Collins, Schapire, and Singer 2002) has shown that the methods have strong similarities. The next section describes parameter estimation methods.

3.2 Loss Functions for Logistic Regression and Boosting

A central idea in both logistic regression and boosting is that of a loss function, which drives the parameter estimation methods of the two approaches. This section describes loss functions for binary classification. Later in the article, we introduce loss functions for reranking tasks which are closely related to the loss functions for classification tasks.

First, consider a logistic regression model. The parameters of the model $\bar{\alpha}$ are used to define a conditional probability

$$P(y \mid x, \bar{\alpha}) = \frac{e^{yF(x, \bar{\alpha})}}{1 + e^{yF(x, \bar{\alpha})}}$$
(3)

where $F(x, \bar{\alpha})$ is as defined in equation (2). Some form of maximum-likelihood estimation is often used for parameter estimation. The parameters are chosen to maximize the log-likelihood of the training set; equivalently: we talk (to emphasize the similarities to the boosting approach) about minimizing the negative log-likelihood. The negative log-likelihood, LogLoss($\bar{\alpha}$), is defined as

$$\operatorname{LogLoss}\left(\bar{\alpha}\right) = -\sum_{i=1}^{n} \log P(y_i \mid x_i, \bar{\alpha}) = -\sum_{i=1}^{n} \log\left(\frac{e^{y_i F(x_i, \bar{\alpha})}}{1 + e^{y_i F(x_i, \bar{\alpha})}}\right) = \sum_{i=1}^{n} \log\left(1 + e^{-y_i F(x_i, \bar{\alpha})}\right)$$
(4)

There are many methods in the literature for minimizing $\text{LogLoss}(\bar{\alpha})$ with respect to $\bar{\alpha}$, for example, generalized or improved iterative scaling (Berger, Della Pietra, and

⁴ It might seem to be a restriction to have the hyperplane passing through the origin of the space. However if a constant "bias" feature $h_{m+1}(x) = 1$ for all x is added to the representation, a hyperplane passing through the origin in this new space is equivalent to a hyperplane in general position in the original *m*-dimensional space.

Della Pietra 1996; Della Pietra, Della Pietra, and Lafferty 1997), or conjugate gradient methods (Malouf 2002). In the next section we describe feature selection methods, as described in Berger, Della Pietra, and Della Pietra (1996) and Della Pietra, Della Pietra, and Lafferty (1997).

Once the parameters $\bar{\alpha}$ are estimated on training examples, the output for an example *x* is the most likely label under the model,

$$\arg \max_{y \in \mathcal{Y}} P(y \mid x, \bar{\alpha}) = \arg \max_{y \in \{-1, +1\}} y F(x, \bar{\alpha}) = \operatorname{sign}(F(x, \bar{\alpha}))$$
(5)

where as before, sign (z) = 1 if $z \ge 0$, sign (z) = -1 otherwise. Thus we see that the logistic regression model implements a hyperplane classifier.

In boosting, a different loss function is used, namely, $\text{ExpLoss}(\bar{\alpha}),$ which is defined as

$$\operatorname{ExpLoss}(\bar{\alpha}) = \sum_{i=1}^{n} e^{-y_i F(x_i, \bar{\alpha})}$$
(6)

This loss function is minimized using a feature selection method, which we describe in the next section.

There are strong similarities between LogLoss (equation (4)) and ExpLoss (equation (6)). In making connections between the two functions, it is useful to consider a third function of the parameters and training examples,

$$\operatorname{Error}(\bar{\alpha}) = \sum_{i=1}^{n} \left[y_i F(x_i, \bar{\alpha}) \le 0 \right]$$
(7)

where $[\![\pi]\!]$ is one if π is true, zero otherwise. Error($\bar{\alpha}$) is the number of incorrectly classified training examples under parameter values $\bar{\alpha}$.

Finally, it will be useful to define the **margin** on the *i*th training example, given parameter values $\bar{\alpha}$, as

$$M_i(\bar{\mathbf{a}}) = y_i F(x_i, \bar{\mathbf{a}}) \tag{8}$$

With these definitions, the three loss functions can be written in the following form:

$$\begin{split} &\text{LogLoss}(\bar{\boldsymbol{\alpha}}) = \sum_{i=1}^{n} f(M_{i}(\bar{\boldsymbol{\alpha}})), \text{ where } f(z) = \log(1 + e^{-z}) \\ &\text{ExpLoss}(\bar{\boldsymbol{\alpha}}) = \sum_{i=1}^{n} f(M_{i}(\bar{\boldsymbol{\alpha}})), \text{ where } f(z) = e^{-z} \\ &\text{Error}(\bar{\boldsymbol{\alpha}}) = \sum_{i=1}^{n} f(M_{i}(\bar{\boldsymbol{\alpha}})), \text{ where } f(z) = \llbracket z \leq 0 \rrbracket \end{split}$$

The three loss functions differ only in their choice of an underlying "potential function" of the margins, f(z). This function is $f(z) = \log (1 + e^{-z})$, $f(z) = e^{-z}$, or

 $f(z) = [[z \le 0]]$ for LogLoss, ExpLoss, and Error, respectively. The f(z) functions penalize nonpositive margins on training examples. The simplest function, $f(z) = [[z \le 0]]$, gives a cost of one if a margin is negative (an error is made), zero otherwise. ExpLoss and LogLoss involve definitions for f(z) which quickly tend to zero as $z \to \infty$ but heavily penalize increasingly negative margins.

Figure 2 shows plots for the three definitions of f(z). The functions $f(z) = e^{-z}$ and $f(z) = \log (1 + e^{-z})$ are both upper bounds on the error function, so that minimizing either LogLoss or ExpLoss can be seen as minimizing an upper bound on the number of training errors. (Note that minimizing Error(\bar{a}) itself is known to be at least NP-hard if no parameter settings can achieve zero errors on the training set; see, for example, Hoffgen, van Horn, and Simon [1995].) As $z \to \infty$, the functions $f(z) = e^{-z}$ and $f(z) = \log(1 + e^{-z})$ become increasingly similar, because log $(1 + e^{-z}) \to e^{-z}$ as $e^{-z} \to 0$. For negative *z*, the two functions behave quite differently. $f(z) = e^{-z}$ shows an exponentially growing cost function as $z \to -\infty$. In contrast, as $z \to -\infty$ it can be seen that $\log(1 + e^{-z}) \to \log(e^{-z}) = -z$, so this function shows asymptotically linear growth for negative *z*. As a final remark, note that both $f(z) = e^{-z}$ and $f(z) = \log(1 + e^{-z})$ are convex in *z*, with the result that LogLoss(\bar{a}) and ExpLoss(\bar{a}) are convex in the parameters \bar{a} . This means that there are no problems with local minima when optimizing these two loss functions.

3.3 Feature Selection Methods

In this article we concentrate on feature selection methods: algorithms which aim to make progress in minimizing the loss functions $LogLoss(\bar{\alpha})$ and $ExpLoss(\bar{\alpha})$ while using a small number of features (equivalently, ensuring that most parameter values in



Figure 2

Potential functions underlying ExpLoss, LogLoss, and Error. The graph labeled ExpLoss is a plot of $f(z) = e^{-z}$ for z = [-1.5...1.5]; LogLoss shows a similar plot for $f(z) = \log(1 + e^{-z})$; Error is a plot of $f(z) = [z \le 0]$.

 $\bar{\alpha}$ are zero). Roughly speaking, the motivation for using a small number of features is the hope that this will prevent overfitting in the models.

Feature selection methods have been proposed in the maximum-entropy literature by several authors (Ratnaparkhi, Roukos, and Ward 1994; Berger, Della Pietra, and Della Pietra 1996; Della Pietra, Della Pietra, and Lafferty 1997; Papineni, Roukos, and Ward 1997, 1998; McCallum 2003; Zhou et al. 2003; Riezler and Vasserman 2004). The most basic approach—for example see Ratnaparkhi, Roukos, and Ward (1994) and Berger, Della Pietra, and Della Pietra (1996)—involves selection of a single feature at each iteration, followed by an update to the entire model, as follows:

Step 1: Throughout the algorithm, maintain a set of active features. Initialize this set to be empty.

Step 2: Choose a feature from outside of the set of active features which has the largest estimated impact in terms of reducing the loss function LogLoss, and add this to the active feature set.

Step 3: Minimize $LogLoss(\bar{\alpha})$ with respect to the set of active features; that is, allow only the active features to take nonzero parameter values when minimizing LogLoss. Return to **Step 2**.

Methods in the boosting literature (see, for example, Schapire and Singer [1999]) can be considered to be feature selection methods of the following form:

Step 1: Start with all parameter values set to zero.

Step 2: Choose a feature which has largest estimated impact in terms of reducing the loss function ExpLoss.

Step 3: Update the parameter for the feature chosen at **Step 2** in such a way as to minimize $\text{ExpLoss}(\bar{\alpha})$ with respect to this one parameter. All other parameter values are left fixed. Return to **Step 2**.

The difference with this latter "boosting" approach is that in Step 3, only one parameter value is adjusted, namely, the parameter corresponding to the newly chosen feature. Note that in this framework, the same feature may be chosen at more than one iteration.⁵ The maximum-entropy feature selection method can be quite inefficient, as the entire model is updated at each step. For example, Ratnaparkhi (1998) quotes times of around 30 hours for 500 rounds of feature selection on a prepositional-phrase attachment task. These experiments were performed in 1998, when processors were no doubt considerably slower than those available today. However, the PP attachment task is much smaller than the parsing task that we are addressing: Our task involves around 1,000,000 examples, with perhaps a few hundred features per example, and 100,000 rounds of feature selection; this compares to 20,000 examples, 16 features per example, and 500 rounds of feature selection for the PP attachment task in Ratnaparkhi (1998). As an estimate, assuming that computational complexity scales linearly in these factors,⁶ our task is $\frac{1,000,000}{16} \times \frac{320}{16} \times \frac{100,000}{500} = 200,000$

⁵ That is, the feature may be repeatedly updated, although the same feature will never be chosen in consecutive iterations, because after an update the model is minimized with respect to the selected feature.

⁶ We believe this is a realistic assumption, as each round of feature selection takes O(nf) time, where *n* is the number of training examples, and *f* is the number of active features on each example.

as large as the PP attachment task. These figures suggest that the maximum-entropy feature selection approach may be infeasible for large-scale tasks such as the one in this article.

The fact that the boosting approach does not update the entire model at each round of feature selection may be a disadvantage in terms of the number of features or the test data accuracy of the final model. There is reason for concern that Step 2 will at some iterations mistakenly choose features which are apparently useful in reducing the loss function, but which would have little utility if the entire model had been optimized at the previous iteration of Step 3. However, previous empirical results for boosting have shown that it is a highly effective learning method, suggesting that this is not in fact a problem for the approach. Given the previous strong results for the boosting approach, and for reasons of computational efficiency, we pursue the boosting approach to feature selection in this article.

3.4 Statistical Justification for the Methods

Minimization of LogLoss is most often justified as a parametric, maximum-likelihood (ML) approach to estimation. Thus this approach benefits from the usual guarantees for ML estimation: If the distribution generating examples is within the class of distributions specified by the log-linear form, then in the limit as the sample size goes to infinity, the model will be optimal in the sense of convergence to the true underlying distribution generating examples. As far as we are aware, behavior of the models for finite sample sizes is less well understood. In particular, while feature selection methods have often been proposed for maximum-entropy models, little theoretical justification (in terms of guarantees about generalization) has been given for them. It seems intuitive that a model with a smaller number of parameters will require fewer samples for convergence, but this is not necessarily the case, and at present this intuition lacks a theoretical basis. Feature selection methods can probably be motivated either from a Bayesian perspective (through a prior favoring models with a smaller number of nonzero parameters) or from a frequentist/goodness-of-fit perspective (models with fewer parameters are less likely to fit the data by chance), but this requires additional research.

The statistical justification for boosting approaches is quite different. Boosting algorithms were originally developed within the PAC framework (Valiant 1984) for machine learning, specifically to address questions regarding the equivalence of weak and strong learning. Freund and Schapire (1997) originally introduced AdaBoost and gave a first set of statistical guarantees for the algorithm. Schapire et al. (1998) gave a second set of guarantees based on the analysis of margins on training examples. Both papers assume that a fixed distribution D(x, y) is generating both training and test examples and that the goal is to find a hypothesis with a small number of expected errors with respect to this distribution. The form of the distribution is not assumed to be known, and in this sense the guarantees are nonparametric, or "distribution free." Freund and Schapire (1997) show that if the weak learning assumption holds (i.e., roughly speaking, a feature with error rate better than chance can be found for any distribution over the sample space $\mathcal{X} \times \{-1, +1\}$, then the training error for the ExpLoss method decreases rapidly enough for there to be good generalization to test examples. Schapire et al. (1998) show that under the same assumption, minimization of ExpLoss using the feature selection method ensures that the distribution of margins on training data develops in such a way that good generalization performance on test examples is guaranteed.

3.5 Boosting with Complex Feature Spaces

Thus far in this article we have presented boosting as a feature selection approach. In this section, we note that there is an alternative view of boosting in which it is described as a method for combining multiple models, for example, as a method for forming a linear combination of decision trees. We consider only the simpler, feature selection view of boosting in this article. This section is included for completeness and because the more general view of boosting may be relevant to future work on boosting approaches for parse reranking (note, however, that the discussion in this section is not essential to the rest of the article, so the reader may safely skip this section if she or he wishes to do so).

In feature selection approaches, as described in this article, the set of possible features $h_k(x)$ for k = 1, ..., m is taken to be a fixed set of relatively simple functions. In particular, we have assumed that m is relatively small (for example, small enough for algorithms that require O(m) time or space to be feasible). More generally, however, boosting can be applied in more complex settings. For example, a common use of boosting is to form a linear combination of decision trees. In this case each example x is represented as a number of attribute-value pairs, and each "feature" $h_k(x)$ is a complete decision tree built on predicates over the attribute values in x. In this case the number of features m is huge: There are as many features as there are decision trees over the given set of attributes, thus m grows exponentially quickly with the number of attributes that are used to represent an example x. Boosting may even be applied in situations in which the number of features is infinite. For example, it may be used to form a linear combination of neural networks. In this case each feature $h_k(x)$ corresponds to a different parameter setting within the (infinite) set of possible parameter settings for the neural network.

In more complex settings such as boosting of decision trees or neural networks, it is generally not feasible to perform an exhaustive search (with O(m) time complexity) for the feature which has the greatest impact on the exponential⁷ loss function. Instead, an approximate search is performed. In boosting approaches, this approximate search is achieved through a protocol in which at each round of boosting, a "distribution" over the training examples is maintained. The distribution can be interpreted as assigning an importance weight to each training example, most importantly giving higher weight to examples which are incorrectly classified. At each round of boosting the distribution is passed to an algorithm such as a decision tree or neural network learning method, which attempts to return a feature (a decision tree, or a neural network parameter setting) which has a relatively low error rate with respect to the distribution. The feature that is returned is then incorporated into the linear combination of features. The algorithm which generates a classifier given a distribution over the examples (for example, the decision tree induction method) is usually referred to as "the weak learner." The weak learner generally uses an approximate (for example, greedy) method to find a function with a low error rate with respect to the distribution. Freund and Schapire (1997) show that provided that at each round of boosting the weak learner returns a feature with greater than $(50 + \epsilon)$ % accuracy for some fixed ϵ , the number of training errors falls exponentially quickly with the number of rounds of boosting. This fast drop in training errors translates to statistical bounds on generalization performance (Freund and Schapire 1997).

⁷ Note that it is also possible to apply these methods to the LogLoss function; see, for example, Friedman et al. (2000) and Duffy and Helmbold (1999).

Under this view of boosting, the feature selection methods in this article are a particularly simple case in which the weak learner can afford to exhaustively search through the space of possible features. Future work on reranking approaches might consider other approaches—such as boosting of decision trees—which can effectively consider more complex features.

4. Reranking Approaches

This section describes how the ideas from classification problems can be extended to reranking tasks. A baseline statistical parser is used to generate *N*-best output both for its training set and for test data sentences. Each candidate parse for a sentence is represented as a feature vector which includes the log-likelihood under the baseline model, as well as a large number of additional features. The additional features can in principle be any predicates over sentence/tree pairs. Evidence from the initial log-likelihood and the additional features is combined using a linear model. Parameter estimation becomes a problem of learning how to combine these different sources of information. The boosting algorithm we use is related to the generalization of boosting methods to ranking problems in Freund et al. (1998); we also introduce an approach related to the conditional log-linear models of Ratnaparkhi, Roukos, and Ward (1994), Papineni, Roukos, and Ward (1997, 1998), Johnson et al. (1999), Riezler et al. (2002), and Och and Ney (2002).

Section 4.1 gives a formal definition of the reranking problem. Section 4.2 introduces loss functions for reranking that are analogous to the LogLoss and ExpLoss functions in section 3.2. Section 4.3 describes a general approach to feature selection methods with these loss functions. Section 4.4 describes a first algorithm for the exponential loss (ExpLoss) function; section 4.5 introduces a more efficient algorithm for the case of ExpLoss. Finally, section 4.6 describes issues in feature selection algorithms for the LogLoss function.

4.1 Problem Definition

We use the following notation in the rest of this article:

- *s_i* is the *i*th sentence in the training set. There are *n* sentences in training data, so that 1 ≤ *i* ≤ *n*.
- $x_{i,j}$ is the *j*th parse of the *i*th sentence. There are n_i parses for the *i*th sentence, so that $1 \le i \le n$ and $1 \le j \le n_i$. Each $x_{i,j}$ contains both the tree and the underlying sentence (i.e., each $x_{i,j}$ is a pair $\langle s_i, t_{i,j} \rangle$, where s_i is the *i*th sentence in the training data, and $t_{i,j}$ is the *j*th tree for this sentence). We assume that the parses are distinct, that is, that $x_{i,j} \ne x_{i,j'}$ for $j \ne j'$.
- Score(*x*_{*i*,*j*}) is the "score" for parse *x*_{*i*,*j*}, a measure of the similarity of *x*_{*i*,*j*} to the gold-standard parse. For example, Score(*x*_{*i*,*j*}) might be the *F*-measure accuracy of parse *x*_{*i*,*j*} compared to the gold-standard parse for *s*_{*i*}.
- $Q(x_{i,j})$ is the probability that the base parsing model assigns to parse $x_{i,j}$. $L(x_{i,j}) = \log Q(x_{i,j})$ is the log-probability.

Without loss of generality, we assume x_{i,1} to be the highest-scoring parse for the *i*th sentence.⁸ More precisely, for all *i*, 2 ≤ *j* ≤ n_i, Score(x_{i,1}) > Score(x_{i,j}). Note that x_{i,1} may not be identical to the gold-standard parse; in some cases the parser may fail to propose the correct parse anywhere in its list of candidates.⁹

Thus our training data consist of a set of parses, $\{x_{i,j} : i = 1, ..., n, j = 1, ..., n_i\}$, together with scores $Score(x_{i,j})$ and log-probabilities $L(x_{i,j})$.

We represent candidate parse trees through *m* features, h_k for k = 1, ..., m. Each h_k is an indicator function, for example,

$$h_k(x) = 1$$
 if x contains the rule $\langle S \rightarrow NP \ VP \rangle$
0 otherwise

We show that the restriction to binary-valued features is important for the simplicity and efficiency of the algorithms.¹⁰ We also assume a vector of m + 1 parameters, $\bar{\alpha} = \{\alpha_0, \alpha_1, \ldots, \alpha_m\}$. Each α_i can take any value in the reals. The **ranking function** for a parse tree *x* implied by a parameter vector $\bar{\alpha}$ is defined as

$$F(x,\bar{\alpha}) = \alpha_0 L(x) + \sum_{k=1}^m \alpha_k h_k(x)$$

Given a new test sentence *s*, with parses x_j for j = 1, ..., N, the output of the model is the highest-scoring tree under the ranking function

$$\arg\max_{x\in\{x_1\dots x_N\}}F(x,\bar{\alpha})$$

Thus $F(x, \bar{\alpha})$ can be interpreted as a measure of how plausible a parse x is, with higher scores meaning that x is more plausible. Competing parses for the same sentence are ranked in order of plausibility by this function. We can recover the base ranking function—the log-likelihood L(x)—by setting α_0 to a positive constant and setting all other parameter values to be zero. Our intention is to use the training examples to pick parameter values which improve upon this initial ranking.

We now discuss how to set these parameters. First we discuss loss functions $Loss(\bar{\alpha})$ which can be used to drive the training process. We then go on to describe feature selection methods for the different loss functions.

⁸ In the event that multiple parses get the (same) highest score, the parse with the highest value of log-likelihood *L* under the baseline model is taken as $x_{i_1,1}$. In the event that two parses have the same score and the same log-likelihood—which occurred rarely if ever in our experiments—we make a random choice between the two parses.

⁹ This is not necessarily a significant issue if an application using the output from the parser is sensitive to improvements in evaluation measures such as precision and recall that give credit for partial matches between the parser's output and the correct parse. In this case, it is important only that the precision/ recall for $x_{i,1}$ is significantly higher than that of the baseline parser, that is, that there is some "head room" for the reranking module in terms of precision and recall.

¹⁰ In particular, this restriction allows closed-form parameter updates for the models based on ExpLoss that we consider. Note that features tracking the *counts* of different rules can be simulated through several features which take value one if a rule is seen ≥ 1 time, ≥ 2 times, and so on.

4.2 Loss Functions for Ranking Problems

4.2.1 Ranking Errors and Margins. The loss functions we consider are all related to the number of ranking errors a function *F* makes on the training set. The ranking error rate is the number of times a lower-scoring parse is (incorrectly) ranked above the best parse:

$$\operatorname{Error}\left(\bar{\boldsymbol{\alpha}}\right) = \sum_{i} \sum_{j=2}^{n_{i}} \left[F(x_{i,1}, \bar{\boldsymbol{\alpha}}) \le F(x_{i,j}, \bar{\boldsymbol{\alpha}}) \right] = \sum_{i} \sum_{j=2}^{n_{i}} \left[F(x_{i,1}, \bar{\boldsymbol{\alpha}}) - F(x_{i,j}, \bar{\boldsymbol{\alpha}}) \le 0 \right]$$

where again, $[\pi]$ is one if π is true, zero otherwise. In the ranking problem we define the margin for each example $x_{i,j}$ such that $i = 1, ..., n, j = 2, ..., n_i$, as

$$M_{ij}(\bar{\boldsymbol{\alpha}}) = F(x_{i,1}, \bar{\boldsymbol{\alpha}}) - F(x_{i,j}, \bar{\boldsymbol{\alpha}})$$

Thus $M_{ij}(\bar{\alpha})$ is the difference in ranking score between the correct parse of a sentence and a competing parse $x_{i,j}$. It follows that

$$\operatorname{Error}(ar{m{lpha}}) = \sum_i \sum_{j=2}^{n_i} \llbracket M_{ij}(ar{m{lpha}}) \leq 0
rbrace$$

The ranking error is zero if all margins are positive. The loss functions we discuss all turn out to be direct functions of the margins on training examples.

4.2.2 Log-Likelihood. The first loss function is that suggested by Markov random fields. As suggested by Ratnaparkhi, Roukos, and Ward (1994) and Johnson et al. (1999), the conditional probability of $x_{i,q}$ being the correct parse for the *i*th sentence is defined as

$$P(x_{i,q} \mid s_i, \bar{\alpha}) = \frac{e^{F(x_{i,q}, \bar{\alpha})}}{\sum\limits_{j=1}^{n_i} e^{F(x_{i,j}, \bar{\alpha})}}$$

Given a new test sentence *s*, with parses x_j for j = 1, ..., N, the most likely tree is

$$\arg \max_{x_j} \frac{e^{F(x_j, \bar{\boldsymbol{\alpha}})}}{\sum\limits_{q=1}^{N} e^{F(x_q, \bar{\boldsymbol{\alpha}})}} = \arg \max_{x_j} F(x_j, \bar{\boldsymbol{\alpha}})$$

Hence once the parameters are trained, the ranking function is used to order candidate trees for test examples.

The log-likelihood of the training data is

$$\sum_{i} \log P(x_{i,1} \mid s_i, \bar{\alpha}) = \sum_{i} \log \frac{e^{F(x_{i,1}, \bar{\alpha})}}{\sum_{j=1}^{n_i} e^{F(x_{i,j}, \bar{\alpha})}}$$

Under maximum-likelihood estimation, the parameters $\bar{\alpha}$ would be set to maximize the log-likelihood. Equivalently, we again talk about minimizing the negative

log-likelihood. Some manipulation shows that the negative log-likelihood is a function of the margins on training data:

$$\operatorname{LogLoss}(\bar{\alpha}) = \sum_{i} -\log \frac{e^{F(x_{i,1},\bar{\alpha})}}{\sum_{j=1}^{n_{i}} e^{F(x_{i,j},\bar{\alpha})}} = \sum_{i} -\log \frac{1}{\sum_{j=1}^{n_{i}} e^{-(F(x_{i,1},\bar{\alpha}) - F(x_{i,j},\bar{\alpha}))}} \\
= \sum_{i} \log \left(1 + \sum_{j=2}^{n_{i}} e^{-(F(x_{i,1},\bar{\alpha}) - F(x_{i,j},\bar{\alpha}))} \right) = \sum_{i} \log \left(1 + \sum_{j=2}^{n_{i}} e^{-M_{i,j}(\bar{\alpha})} \right)$$
(9)

Note the similarity of equation (9) to the LogLoss function for classification in equation (4).

4.2.3 Exponential Loss. The next loss function is based on the boosting method described in Schapire and Singer (1999). It is a special case of the general ranking methods described in Freund et al. (1998), with the ranking "feedback" being a simple binary distinction between the highest-scoring parse and the other parses. Again, the loss function is a function of the margins on training data:

$$\operatorname{ExpLoss}(\bar{\alpha}) = \sum_{i} \sum_{j=2}^{n_{i}} e^{-(F(x_{i,1}, \bar{\alpha}) - F(x_{i,j}, \bar{\alpha}))} = \sum_{i} \sum_{j=2}^{n_{i}} e^{-M_{i,j}(\bar{\alpha})}$$
(10)

Note the similarity of equation (10) to the ExpLoss function for classification in equation (6). It can be shown that $\text{ExpLoss}(\bar{\alpha}) \ge \text{Error}(\bar{\alpha})$, so that minimizing $\text{ExpLoss}(\bar{\alpha})$ is closely related to minimizing the number of ranking errors.¹¹ This follows from the fact that for any $x, e^{-x} \ge [x < 0]$, and therefore that

$$\sum_{i} \sum_{j=2}^{n_i} e^{-M_{i,j}(\bar{\boldsymbol{\alpha}})} \geq \sum_{i} \sum_{j=2}^{n_i} \llbracket M_{i,j}(\bar{\boldsymbol{\alpha}}) \leq 0 \rrbracket$$

We generalize the ExpLoss function slightly, by allowing a weight for each example $x_{i,j}$, for $i = 1, ..., n, j = 2, ..., n_i$. We use $S_{i,j}$ to refer to this weight. In particular, in some experiments in this article, we use the following definition:

$$S_{i,i} = \text{Score}(x_{i,1}) - \text{Score}(x_{i,i})$$
(11)

¹¹ Note that LogLoss is not a direct upper bound on the number of ranking errors, although it can be shown that it is a (relatively loose) upper bound on the number of times the correct parse is not the highest-ranked parse on the model. The latter observation follows from the property that the correct parse must be highest ranked if its probability is greater than 0.5.

where, as defined in section 4.1, $Score(x_{i,j})$ is some measure of the "goodness" of a parse, such as the *F*-measure (see section 5 for the exact definition of *Score* used in our experiments). The definition for ExpLoss is modified to be

$$\operatorname{ExpLoss}(\bar{\alpha}) = \sum_{i} \sum_{j=2}^{n_i} S_{i,j} e^{-M_{i,j}(\bar{\alpha})}$$

This definition now takes into account the importance, $S_{i,j}$, of each example. It is an upper bound on the following quantity:

$$\sum_{i}\sum_{j=2}^{n_{i}}S_{i,j}\llbracket M_{i,j}(\bar{\boldsymbol{\alpha}})\leq 0\rrbracket$$

which is the number of errors weighted by the factors $S_{i,j}$. The original definition of ExpLoss in equation (10) can be recovered by setting $S_{i,j} = 1$ for all *i*, *j* (i.e., by giving equal weight to all examples). In our experiments we found that a definition of $S_{i,j}$ such as that in equation (11) gave improved performance on development data, presumably because it takes into account the relative cost of different ranking errors in training-data examples.

4.3 A General Approach to Feature Selection

At this point we have definitions for ExpLoss and LogLoss which are analogous to the definitions in section 3.2 for binary classification tasks. Section 3.3 introduced the idea of feature selection methods; the current section gives a more concrete description of the methods used in our experiments.

The goal of feature selection methods is to find a small subset of the features that contribute most to reducing the loss function. The methods we consider are greedy, at each iteration picking the feature h_k with additive weight δ which has the most impact on the loss function. In general, a separate set of instances is used in cross-validation to choose the stopping point, that is, to decide on the number of features in the model.

At this point we introduce some notation concerning feature selection methods. We define Upd($\bar{\alpha}$, k, δ) to be an updated parameter vector, with the same parameter values as $\bar{\alpha}$ with the exception of α_k , which is incremented by δ :

$$\operatorname{Upd}(\bar{\alpha}, k, \delta) = \{\alpha_0, \alpha_1, \ldots, \alpha_k + \delta, \ldots, \alpha_m\}$$

The δ parameter can potentially take any value in the reals. The loss for the updated model is Loss(Upd($\bar{\alpha}$, k, δ)). Assuming we greedily pick a single feature with some weight to update the model, and given that the current parameter settings are $\bar{\alpha}$, the optimal feature/weight pair (k^* , δ^*) is

$$(k^*, \delta^*) = \arg\min_{k, \delta} \text{Loss}(\text{Upd}(\bar{\alpha}, k, \delta))$$

The feature selection algorithms we consider take the following form ($\bar{\alpha}^t$ is the parameter vector at the *t*th iteration):

Step 1: Initialize $\bar{\alpha}^0$ to some value. (This will generally involve values of zero for $\alpha_1, \ldots, \alpha_m$ and a nonzero value for α_0 , for example, $\bar{\alpha}^0 = \{1, 0, 0, \ldots\}$.)

Step 2: For t = 1 to N (the number of iterations N will be chosen by cross-validation)

a: Find $(k^*, \delta^*) = \arg \min_{k, \delta} \text{Loss}(\text{Upd} (\bar{\alpha}^{t-1}, k, \delta))$ **b:** Set $\bar{\alpha}^t = \text{Upd}(\bar{\alpha}^{t-1}, k^*, \delta^*)$

Note that this is essentially the idea behind the "boosting" approach to feature selection introduced in section 3.3. In contrast, the feature selection method of Berger, Della Pietra, and Della Pietra (1996), also described in section 3.3, would involve updating parameter values for *all* selected features at step 2b.

The main computation for both loss functions involves searching for the optimal feature/weight pair (k^* , δ^*). In both cases we take a two-step approach to solving this problem. In the first step the optimal update for each feature h_k is calculated. We define BestWt(k, $\bar{\alpha}$) as the optimal update for the kth feature (it must be calculated for all features k = 1, ..., m):

$$\mathsf{BestWt}(k,\bar{\alpha}) = \arg\min_{\delta} \mathsf{Loss}(\mathsf{Upd}(\bar{\alpha},k,\delta))$$

The next step is to calculate the Loss for each feature with its optimal update, which we will call

 $\mathsf{BestLoss}(k,\bar{\alpha}) = \min_{\delta} \, \mathsf{Loss}(\mathsf{Upd}(\bar{\alpha},k,\delta)) = \mathsf{Loss}(\mathsf{Upd}(\bar{\alpha},k,\mathsf{BestWt}(k,\bar{\alpha})))$

BestWt and BestLoss for each feature having been computed, the optimal feature/ weight pair can be found:

$$k^* = \arg\min_{k} \text{BestLoss}(k, \bar{\alpha}), \ \delta^* = \text{BestWt}(k^*, \bar{\alpha})$$

The next sections describe how BestWt and BestLoss can be computed for the two loss functions.

4.4 Feature Selection for ExpLoss

At the first iteration, α_0 is set to optimize ExpLoss (recall that $L(x_{i,j})$ is the loglikelihood for parse $x_{i,j}$ under the base parsing model):

$$\alpha_0 = \arg \min_{\alpha} \sum_{i} \sum_{j=2}^{n_i} S_{i,j} e^{-(\alpha [L(x_{i,1}) - L(x_{i,j})])}$$
(12)

In initial experiments we found that this step was crucial to the performance of the method (as opposed to simply setting $\alpha_0 = 1$, for example). It ensures that the

contribution of the log-likelihood feature is well-calibrated with respect to the exponential loss function. In our implementation α_0 was optimized using simple brute-force search. All values of α_0 between 0.001 and 10 at increments of 0.001 were tested, and the value which minimized the function in equation (12) was chosen.¹²

Feature selection then proceeds to search for values of the remaining parameters, $\alpha_1, \ldots, \alpha_m$. (Note that it might be preferable to also allow α_0 to be adjusted as features are added; we leave this to future work.) This requires calculation of the terms BestWt($k, \bar{\alpha}$) and BestLoss($k, \bar{\alpha}$) for each feature. For binary-valued features these values have closed-form solutions, which is computationally very convenient. We now describe the form of these updates. See appendix A for how the updates can be derived (the derivation is essentially the same as that in Schapire and Singer [1999]).

First, we note that for any feature, $[h_k(x_{i,1}) - h_k(x_{i,j})]$ can take on three values: +1, -1, or 0 (this follows from our assumption of binary-valued feature values). For each k we define the following sets:

$$A_k^+ = \{(i,j) : [h_k(x_{i,1}) - h_k(x_{i,j})] = 1\}$$
$$A_k^- = \{(i,j) : [h_k(x_{i,1}) - h_k(x_{i,j})] = -1\}$$

Thus A_k^+ is the set of training examples in which the *k*th feature is seen in the correct parse but not in the competing parse; A_k^- is the set in which the *k*th feature is seen in the incorrect but not the correct parse.

Based on these definitions, we next define W_k^+ and W_k^- as follows:

$$W_{k}^{+} = \sum_{(i,j) \in A_{k}^{+}} S_{i,j} e^{-M_{i,j}(\bar{a})}$$
(13)

$$W_{k}^{-} = \sum_{(i,j) \in A_{k}^{-}} s_{i,j} e^{-M_{i,j}(\tilde{\mathbf{a}})}$$
(14)

Given these definitions, it can be shown (see appendix A) that

$$BestWt(k,\bar{\alpha}) = \frac{1}{2}\log\frac{W_k^+}{W_k^-}$$
(15)

and

BestLoss
$$(k, \bar{\alpha}) = Z - \left(\sqrt{W_k^+} - \sqrt{W_k^-}\right)^2$$
 (16)

¹² A more precise approach, for example, binary search, could also be used to solve this optimization problem. We used the methods that searches through a set of fixed values for simplicity, implicitly assuming that a precision of 0.001 was sufficient for our problem.

where $Z = \sum_{i} \sum_{j=2}^{n_i} S_{i,j} e^{-M_{i,j}(\bar{\alpha})} = \text{ExpLoss}(\bar{\alpha})$ is a constant (for fixed $\bar{\alpha}$) which appears in the BestLoss for all features and therefore does not affect their ranking.

As Schapire and Singer (1999) point out, the updates in equation (15) can be problematic, as they are undefined (infinite) when either W_k^+ or W_k^- is zero. Following Schapire and Singer (1999), we introduce smoothing through a parameter ϵ and the following new definition of BestWt:

BestWt(k,
$$\bar{\alpha}$$
) = $\frac{1}{2} \log \frac{W_k^+ + \epsilon Z}{W_k^- + \epsilon Z}$ (17)

The smoothing parameter ϵ is chosen through optimization on a development set.

See Figure 3 for a direct implementation of the feature selection method for ExpLoss. We use an array of values

$$G_k = \left| \sqrt{W_k^+} - \sqrt{W_k^-} \right|$$

to indicate the *gain* of each feature (i.e., the impact that choosing this feature will have on the ExpLoss function). The features are ranked by this quantity. It can be seen that almost all of the computation involves the calculation of Z and W_k^+ and W_k^- for each feature h_k . Once these values have been computed, the optimal feature and its update can be chosen.

4.5 A New, More Efficient Algorithm for ExpLoss

This section presents a new algorithm which is equivalent to the ExpLoss algorithm in Figure 3, but can be vastly more efficient for problems with sparse feature spaces. In the experimental section of this article we show that it is almost 2,700 times more efficient for our task than the algorithm in Figure 3. The efficiency of the different algorithms is important in the parsing problem. The training data we eventually used contained around 36,000 sentences, with an average of 27 parses per sentence, giving around 1,000,000 parse trees in total. There were over 500,000 different features.

The new algorithm is also applicable, with minor modifications, to boosting approaches for classification problems in which the representation also involves sparse binary features (for example, the text classification problems in Schapire and Singer [2000]). As far as we are aware, the new algorithm has not appeared elsewhere in the boosting literature.

Figure 4 shows the improved boosting algorithm. Inspection of the algorithm in Figure 3 shows that only margins on examples in the sets $A_{k^*}^+$ and $A_{k^*}^-$ are modified when a feature k^* is selected. The feature space in many NLP problems is very sparse (most features only appear on relatively few training examples, or equivalently, most training examples will have only a few nonzero features). It follows that in many cases, the sets $A_{k^*}^+$ and $A_{k^*}^-$ will be much smaller than the overall size of the training set. Therefore when updating the model from $\bar{\alpha}$ to Upd($\bar{\alpha}, k^*, \delta^*$), the values W_k^+ and W_k^- remain unchanged for many features and do not need to be recalculated. In fact, only

Input

- Examples $x_{i,j}$ for $i = 1, ..., n, j = 1, ..., n_i$, drawn from some set \mathcal{X} .
- Weights $S_{i,j}$ representing importance of examples.
- Initial model log-likelihoods $L(x_{i,j})$, for all examples $x_{i,j}$.
- Feature functions $h_k : \mathcal{X} \to \{0, 1\}$ for $k = 1, \ldots, m$.
- \bullet Smoothing parameter ϵ (usually chosen by optimization on development data).
- \bullet Number of rounds N (usually chosen by optimization on development data).

Initialize

- Set $\alpha_0 = \arg \min_{\alpha} \sum_{i} \sum_{j=2}^{n_i} S_{i,j} e^{-\alpha [L(x_{i,1}) L(x_{i,j})]}$.
- Set $\alpha_k = 0$ for $k = 1, \ldots, m$.
- For all $i, 2 \leq j \leq n_i$, set margins $M_{i,j} = \alpha_0 [L(x_{i,1}) L(x_{i,j})].$
- For all $k = 1, \ldots, m$.

• Set
$$A_k^+ = \{(i,j) : [h_k(x_{i,1}) - h_k(x_{i,j})] = 1\}.$$

• Set $A_k^- = \{(i,j) : [h_k(x_{i,1}) - h_k(x_{i,j})] = -1\}$

Repeat for $t = 1, \ldots, N$:

- Calculate $Z = \sum_{i} \sum_{j=2}^{n_i} S_{i,j} e^{-M_{i,j}}$.
- For k = 1, ..., m:
 - Set $W_k^+ = W_k^- = 0$.
 - $$\begin{split} \bullet & \text{For } (i,j) \in A_k^+, \quad W_k^+ = W_k^+ + S_{i,j} e^{-M_{i,j}}. \\ \bullet & \text{For } (i,j) \in A_k^-, \quad W_k^- = W_k^- + S_{i,i} e^{-M_{i,j}}. \end{split}$$

$$\bullet \ G_k = \left| \sqrt{W_k^+} - \sqrt{W_k^-} \right|$$

- Choose $k^* = \arg \max_k G_k$, and $\delta^* = \frac{1}{2} \log \frac{W_{k^*}^+ + \epsilon Z}{W_{k^*}^- + \epsilon Z}$.
- For $(i, j) \in A_{k^*}^+$, $M_{i,j} = M_{i,j} + \delta^*$.
- For $(i, j) \in A_{k^*}^-$, $M_{i,j} = M_{i,j} \delta^*$.
- $\bar{\alpha}^t = \operatorname{Upd}(\bar{\alpha}^{t-1}, k^*, \delta^*).$

Output Final parameter setting $\bar{\alpha}^N$.

Figure 3

A naive algorithm for the boosting loss function.

features which co-occur with k^* on some example must be updated. The algorithm in Figure 4 recalculates the values of A_k^+ and A_k^- only for those features which co-occur with the selected feature k^* .

To achieve this, the algorithm relies on a second pair of indices. For all *i*, $2 \le j \le n_i$, we define

$$B_{i,j}^{+} = \{k : [h_k(x_{i,1}) - h_k(x_{i,j})] = 1\}$$

$$B_{i,j}^{-} = \{k : [h_k(x_{i,1}) - h_k(x_{i,j})] = -1\}$$
(18)

Input

- Examples $x_{i,j}$ for $i = 1, ..., n, j = 1, ..., n_i$, drawn from some set \mathcal{X} .
- Weights $S_{i,j}$ representing importance of examples.
- Initial model log-likelihoods $L(x_{i,j})$, for all examples $x_{i,j}$.
- Feature functions $h_k : \mathcal{X} \to \{0, 1\}$ for $k = 1, \ldots, m$.
- \bullet Smoothing parameter ϵ (usually chosen by optimization on development data).
- \bullet Number of rounds N (usually chosen by optimization on development data).

Initialize

- Set $\alpha_0 = \arg \min_{\alpha} \sum_{i} \sum_{j=2}^{n_i} S_{i,j} e^{-\alpha [L(x_{i,1}) L(x_{i,j})]}.$
- Set $\alpha_k = 0$ for $k = 1, \ldots, m$.
- For all $i, 2 \leq j \leq n_i$, set margins $M_{i,j} = \alpha_0 [L(x_{i,1}) L(x_{i,j})].$
- For all $k = 1, \ldots, m$.

• Set
$$A_k^+ = \{(i,j) : [h_k(x_{i,1}) - h_k(x_{i,j})] = 1\}.$$

• Set $A_k^- = \{(i,j) : [h_k(x_{i,1}) - h_k(x_{i,j})] = -1\}.$

• For all $i, 2 \leq j \leq n_i$,

• Set
$$B_{i,j}^+ = \{k : [h_k(x_{i,1}) - h_k(x_{i,j})] = 1\}.$$

• Set $B_{i,j}^- = \{k : [h_k(x_{i,1}) - h_k(x_{i,j})] = -1\}.$

• Calculate Z and W_k^+ , W_k^- , $G_k = \left| \sqrt{W_k^+} - \sqrt{W_k^-} \right|$ for $k = 1, \ldots, m$ using the algorithm in Figure 3.

Repeat for $t = 1, \ldots, N$

- Choose $k^* = \arg \max_k G_k$ and $\delta^* = \frac{1}{2} \log \frac{W_{k^*}^+ + \epsilon Z}{W_{k^*}^- + \epsilon Z}$.
- For $(i, j) \in A_{k^*}^+$
 - Set $\Delta = S_{i,j} \left(e^{-M_{i,j} \delta^*} e^{-M_{i,j}} \right)$, set $M_{i,j} = M_{i,j} + \delta^*$, and set $Z = Z + \Delta$. • For $k \in B_{i,j}^+$, $W_k^+ = W_k^+ + \Delta$. • For $k \in B_{i,j}^-$, $W_k^- = W_k^- + \Delta$.
- For $(i, j) \in A_{k^*}^-$
 - Set $\Delta = S_{i,j} \left(e^{-M_{i,j} + \delta^*} e^{-M_{i,j}} \right)$, set $M_{i,j} = M_{i,j} \delta^*$, and set $Z = Z + \Delta$. • For $k \in B_{i,j}^+$, $W_k^+ = W_k^+ + \Delta$. • For $k \in B_{i,j}^-$, $W_k^- = W_k^- + \Delta$.
- For features k whose values of W_k^+ and/or W_k^- have changed, update G_k .
- $\bar{\alpha}^t = \operatorname{Upd}(\bar{\alpha}^{t-1}, k^*, \delta^*)$

Output Final parameter setting $\bar{\alpha}^N$.

Figure 4

An improved algorithm for the boosting loss function.

So $B_{i,j}^+$ and $B_{i,j}^-$ are indices from training examples to features. With the algorithm in Figure 4, updating the values of W_k^+ and W_k^- for the features which co-occur with k^* involves the following number of steps:

$$C = \sum_{(i,j) \in A_{k^*}^+} (|B_{i,j}^+| + |B_{i,j}^-|) + \sum_{(i,j) \in A_{k^*}^-} (|B_{i,j}^+| + |B_{i,j}^-|)$$
(19)

In contrast, the naive algorithm requires a pass over the entire training set, which requires the following number of steps:

$$T = \sum_{i=1}^{n} \sum_{j=2}^{n_i} \left(|B_{i,j}^+| + |B_{i,j}^-| \right)$$
(20)

The relative efficiency of the two algorithms depends on the value of C/T at each iteration. In the worst case, when every feature chosen appears on every training example, then C/T = 1, and the two algorithms essentially have the same running time. However in sparse feature spaces there is reason to believe that C/T will be small for most iterations. In section 5.4.3 we show that this is the case for our experiments.

4.6 Feature Selection for LogLoss

We now describe an approach that was implemented for LogLoss. At the first iteration, α_0 is set to one. Feature selection then searches for values of the remaining parameters, $\alpha_1, \ldots, \alpha_m$. We now describe how to calculate the optimal update for a feature *k* with the LogLoss function. First we recap the definition of the probability of a particular parse $x_{i,q}$ given parameter settings $\bar{\alpha}$:

$$P(x_{i,q} \mid s_i, \bar{\boldsymbol{\alpha}}) = \frac{e^{F(x_{i,q}, \bar{\boldsymbol{\alpha}})}}{\sum_{j=1}^{n_i} e^{F(x_{i,j}, \bar{\boldsymbol{\alpha}})}}$$

Recall that the log-loss is

$$\operatorname{LogLoss}(\bar{\alpha}) = \sum_{i} - \log P(x_{i,1} \mid s_i, \bar{\alpha})$$

Unfortunately, unlike the case of ExpLoss, in general an analytic solution for BestWt does not exist. However, we can define an iterative solution using techniques from iterative scaling (Della Pietra, Della Pietra, and Lafferty 1997). We first define \tilde{h}_k , the number of times that feature *k* is seen in the best parse, and $\tilde{p}_k(\bar{\alpha})$, the expected number of times under the model that feature *k* is seen:

$$\tilde{h}_k = \sum_i h_k(x_{i,1}), \quad \tilde{p}_k(\bar{\alpha}) = \sum_i \sum_{j=1}^{n_i} h_k(x_{i,j}) P(x_{i,j} \mid s_i, \bar{\alpha})$$

Iterative scaling then defines the following update $\tilde{\delta}$

$$ilde{\mathbf{\delta}} = \mathrm{log} rac{ ilde{h}_k}{ ilde{p}_k(ar{\mathbf{lpha}})}$$

While in general it is *not* true that $\tilde{\delta} = \text{BestWt}(k, \bar{\alpha})$, it *can* be shown that this update leads to an improvement in the LogLoss (i.e., that LogLoss(Upd($\bar{\alpha}, k, \tilde{\delta}$)) \leq LogLoss $\bar{\alpha}$)), with equality holding only when $\bar{\alpha}_k$ is already at the optimal value, in other words, when arg min_{δ} LogLoss(Upd($\bar{\alpha}, k, \delta$)) = 0. This suggests the following iterative method for finding BestWt($k, \bar{\alpha}$):

Step 1: Initialization: Set $\delta = 0$ and $\bar{\alpha}' = \bar{\alpha}$, calculate \tilde{h}_k .

Step 2: Repeat until convergence of δ :

a: Calculate $\tilde{p}_k(\bar{\alpha}')$. **b**: $\delta \leftarrow \delta + \log \frac{\tilde{h}_k}{\tilde{p}_k(\bar{\alpha}')}$. **c**: $\bar{\alpha}' \leftarrow \text{Upd}(\bar{\alpha}, k, \delta)$.

Step 3: Return BestWt($k, \bar{\alpha}$) = δ .

Given this method for calculating BestWt($k, \bar{\alpha}$), BestLoss($k, \bar{\alpha}$) can be calculated as Loss(k, BestWt($k, \bar{\alpha}$)). Note that this is only one of a number of methods for finding BestWt($k, \bar{\alpha}$): Given that this is a one-parameter, convex optimization problem, it is a fairly simple task, and there are many methods which could be used.

Unfortunately there does not appear to be an efficient algorithm for LogLoss that is analogous to the ExpLoss algorithm in Figure 4 (at least if the feature selection method is required to pick the feature with highest impact on the loss function at each iteration). A similar observation for LogLoss can be made, in that when the model is updated with a feature/weight pair (k^*, δ^*) , many features will have their values for BestWt and BestLoss unchanged. Only those features which co-occur with k^* on some example will need to have their values of BestWt and BestLoss updated. However, this observation does not lead to an efficient algorithm: Updating these values is much more expensive than in the ExpLoss case. The procedure for finding the optimal value BestWt($k, \bar{\alpha}$) must be applied for each feature which co-occurs with the chosen feature k^* . For example, the iterative scaling procedure described above must be applied for a number of features. For each feature, this will involve recalculation of the distribution $\{P(x_{i,1} | s_i), P(x_{i,2} | s_i), \dots, P(x_{i,n_i} | s_i)\}$ for each example *i* on which the feature occurs.¹³ It takes only one feature that is seen on all training examples for the algorithm to involve recalculation of $P(x_{i,i} | s_i)$ for the entire training set. This contrasts with the simple updates in the improved boosting algorithm $(W_k^+ = W_k^+ + \Delta \text{ and } W_k^- = W_k^- + \Delta)$. In fact in the parsing experiments, we were forced to give up on the LogLoss feature selection methods because of their inefficiency (see section 6.4 for more discussion about efficiency).

¹³ This is not a failure of iterative scaling alone: Given that in the general case, closed-form solutions for BestWt and BestLoss do not exist, it is hard to imagine a method that computes these values exactly without some kind of iterative method which requires repeatedly visiting the examples on which a feature is seen.

Note, however, that approximate methods for finding the best feature and updating its weight may lead to efficient algorithms. Appendix B gives a sketch of one such approach, which is based on results from Collins, Schapire, and Singer (2002). We did not test this method; we leave this to future work.

5. Experimental Evaluation

5.1 Generation of Parsing Data Sets

We used the Penn Wall Street Journal treebank (Marcus, Santorini, and Marcinkiewicz 1993) as training and test data. Sections 2–21 inclusive (around 40,000 sentences) were used as training data, section 23 was used as the final test set. Of the 40,000 training sentences, the first 36,000 were used as the main training set. The remaining 4,000 sentences were used as development data and to cross-validate the number of rounds (features) in the model. Model 2 of Collins (1999) was used to parse both the training and test data, producing multiple hypotheses for each sentence. We achieved this by disabling dynamic programming in the parser and choosing a relatively narrow beam width of 1,000. The resulting parser returns all parses that fall within the beam. The number of such parses varies sentence by sentence.

In order to gain a representative set of training data, the 36,000 training sentences were parsed in 2,000 sentence chunks, each chunk being parsed with a model trained on the remaining 34,000 sentences (this prevented the initial model from being unrealistically "good" on the training sentences). The 4,000 development sentences were parsed with a model trained on the 36,000 training sentences. Section 23 was parsed with a model trained on all 40,000 sentences.

In the experiments we used the following definition for the Score of the parse:

$$Score(x_{i,j}) = \frac{F-measure(x_{i,j})}{100} \times Size(x_{i,j})$$

where *F*-measure($x_{i,j}$) is the F_1 score¹⁴ of the parse when compared to the goldstandard parse (a value between 0 and 100), and Size($x_{i,j}$) is the number of constituents in the gold-standard parse for the *i*th sentence. Hence the Score function is sensitive to both the accuracy of the parse, and also the number of constituents in the goldstandard parse.

5.2 Features

The following types of features were included in the model. We will use the rule VP \rightarrow PP VBD NP NP SBAR with head VBD as an example. Note that the output of our baseline parser produces syntactic trees with headword annotations (see Collins [1999]) for a description of the rules used to find headwords).

¹⁴ Note that in the rare cases in which the baseline parser produces no constituents, the precision is undefined; in these cases we defined the *F*-measure to be 0.

Rules. These include all context-free rules in the tree, for example, $VP \rightarrow PP VBD NP NP SBAR$.

Bigrams. These are adjacent pairs of nonterminals to the left and right of the head. As shown, the example rule would contribute the bigrams (Right,VP,NP,NP), (Right,VP,NP,SBAR), (Right,VP,SBAR,STOP) to the right of the head and (Left,VP,PP,STOP) to the left of the head.

Grandparent rules. Same as **Rules**, but also including the nonterminal above the rule.

Grandparent bigrams. Same as **Bigrams**, but also including the nonterminal above the bigrams.



Lexical bigrams. Same as **Bigrams**, but with the lexical heads of the two nonterminals also included.

Two-level rules. Same as **Rules**, but also including the entire rule above the rule.

Two-level bigrams. Same as **Bigrams**, but also including the entire rule above the rule.

Trigrams. All trigrams within the rule. The example rule would contribute the trigrams (VP, STOP, PP, VBD!), (VP, PP, VBD!, NP), (VP, VBD!, NP, NP), (VP, NP, NP, SBAR), and (VP,NP, SBAR, STOP) (! is used to mark the head of the rule).



50









S







Head Modifiers. All head-modifier pairs, with the grandparent nonterminal also included. An adj flag is also included, which is one if the modifier is adjacent to the head, zero otherwise. As an example, say the nonterminal dominating the example rule is S. The example rule would contribute (Left, S, VP, VBD, PP, adj = 1), (Right, S, VP, VBD, NP, adj = 1), (Right, S, VP, VBD, NP, adj = 0), and (Right, S, VP, VBD, SBAR, adj = 0).

PPs. Lexical trigrams involving the heads of arguments of prepositional phrases. The example shown at right would contribute the trigram (NP, NP, PP, NP, president, of, U.S.), in addition to the relation (NP, NP, PP, NP, of, U.S.), which ignores the headword of the constituent being modified by the PP. The three nonterminals (for example, NP, NP, PP) identify the parent of the entire phrase, the nonterminal of the head of the phrase, and the nonterminal label for the PP.



Distance head modifiers. Features involving the distance between headwords. For example, assume *dist* is the number of words between the headwords of the VBD and SBAR in the (VP, VBD, SBAR) head-modifier relation in the above rule. This relation would then generate features (VP, VBD, SBAR, = *dist*), and (VP, VBD, SBAR, $\leq x$) for all *dist* $\leq x \leq 9$ and (VP, VBD, SBAR, $\geq x$) for all $1 \leq x \leq dist$.

Further lexicalization. In order to generate more features, a second pass was made in which all nonterminals were augmented with their lexical heads when these headwords were closed-class words. All features apart from **head modifiers**, **PPs**, and **distance head modifiers** were then generated with these augmented nonterminals.

All of these features were initially generated, but only features seen on at least one parse for at least five different sentences were included in the final model (this count cutoff was implemented to keep the number of features down to a tractable number).

5.3 Applying the Reranking Methods

The ExpLoss method was trained with several values for the smoothing parameter ϵ : {0.0001, 0.00025, 0.0005, 0.00075, 0.001, 0.0025, 0.005, 0.0075}. For each value of ϵ , the method was run for 100,000 rounds on the training data. The implementation was such that the feature updates for all 100,000 rounds for each training run were recorded in a file. This made it simple to test the model on development data for all values of *N* between 0 and 100,000.

The different values of ϵ and *N* were compared on development data through the following criterion:

$$\sum_{i} \text{Score}(z_i) \tag{21}$$

where Score is as defined above, and z_i is the output of the model on the *i*th development set example. The ϵ , *N* values which maximized this quantity were used to define the final model applied to the test data (section 23 of the treebank). The optimal values were $\epsilon = 0.0025$ and N = 90,386, at which point 11,673 features had nonzero values (note that the feature selection techniques may result in a given feature being updated more than once). The computation took roughly 3–4 hours on a machine with a 1.6 GHz pentium processor and around 2 GB of memory.

Table 1 shows results for the method. The model of Collins (1999) was the base model; the ExpLoss model gave a 1.5% absolute improvement over this method. The method gives very similar accuracy to the model of Charniak (2000), which also uses a rich set of initial features in addition to Charniak's (1997) original model.

The LogLoss method was too inefficient to run on the full data set. Instead we made some tests on a smaller subset of the data (5,934 sentences, giving 200,000 parse trees) and 52,294 features.¹⁵ On an older machine (an order of magnitude or more slower than the machine used for the final tests) the boosting method took 40 minutes for 10,000 rounds on this data set. The LogLoss method took 20 hours to complete 3,500 rounds (a factor of about 85 times slower). This was in spite of various heuristics that were implemented in an attempt to speed up LogLoss: for example, selecting multiple features at each round or recalculating the statistics for only the best *K* features for some small *K* at the previous round of feature selection. In initial experiments we found ExpLoss to give similar, perhaps slightly better, accuracy than LogLoss.

5.4 Further Experiments

This section describes further experiments investigating various aspects of the boosting algorithm: the effect of the ϵ and *N* parameters, learning curves, the choice of the *S*_{*i*,*j*} weights, and efficiency issues.

5.4.1 The Effect of the ϵ and *N* **Parameters.** Figure 5 shows the learning curve on development data for the optimal value of ϵ (0.0025). The accuracy shown is the performance relative to the baseline method of using the probability from the generative model alone in ranking parses, where the measure in equation (21) is used to measure performance. For example, a score of 101.5 indicates a 1.5% increase in this score. The learning curve is initially steep, eventually flattening off, but reaching its peak value after a large number (90,386) of rounds of feature selection.

Table 2 indicates how the peak performance varies with the smoothing parameter ϵ . Figure 6 shows learning curves for various values of ϵ . It can be seen that values other than $\epsilon = 0.0025$ can lead to undertraining or overtraining of the model.

¹⁵ All features described above except distance head modifiers and further lexicalization were included.

Table 1

Results on section 23 of the WSJ Treebank. "LR" is labeled recall; "LP" is labeled precision; "CBs" is the average number of crossing brackets per sentence; "0 CBs" is the percentage of sentences with 0 crossing brackets; "2 CBs" is the percentage of sentences with two or more crossing brackets. All the results in this table are for models trained and tested on the same data, using the same evaluation metric. Note that the ExpLoss results are very slightly different from the original results published in Collins (2000). We recently reimplemented the boosting code and reran the experiments, and minor differences in the code and ϵ values tested on development data led to minor improvements in the results.

Model	\leq 40 Words (2,245 sentences)					
	LR	LP	CBs	0 CBs	2 CBs	
Charniak 1997 Collins 1999 Charniak 2000 ExpLoss	87.5% 88.5% 90.1% 90.2%	87.4% 88.7% 90.1% 90.4%	1.00 0.92 0.74 0.73	62.1% 66.7% 70.1% 71.2%	86.1% 87.1% 89.6% 90.2%	
	\leq 100 Words (2,416 sentences)					
Model	$\leq 100 \text{ V}$	Nords (2	,416 se	ntences)		
Model	$\frac{\leq 100 \text{ V}}{\text{LR}}$	Words (2 LP	,416 ser CBs	ntences) 0 CBs	2 CBs	



Figure 5

Learning curve on development data for the optimal value for ϵ (0.0025). The *y*-axis is the level of accuracy (100 is the baseline score), and the *x*-axis is the number of rounds of boosting.

Table 2

Peak performance achieved for various values of ϵ . "Best *N*" refers to the number of rounds at which peak development set accuracy was reached. "Best score" indicates the relative performance, compared to the baseline method, at the optimal value for *N*.

e	Best N	Best score
0.0001	29,471	101.743
0.00025	22,468	101.849
0.0005	48,795	101.845
0.00075	43,386	101.809
0.001	43,975	101.849
0.0025	90,386	101.864
0.005	66,378	101.824
0.0075	80,746	101.722

5.4.2 The Effect of the $S_{i,j}$ **Weights on Examples.** In section 4.2.3 we introduced the idea of weights $S_{i,j}$ representing the importance of examples. Thus far, in the experiments in this article, we have used the definition

$$S_{i,i} = \text{Score}(x_{i,1}) - \text{Score}(x_{i,i})$$
(22)

thereby weighting examples in proportion to their difference in score from the correct parse for the sentence in question. In this section we compare this approach to a default definition of $S_{i,j}$, namely,

$$S_{i,j} = 1 \tag{23}$$

Using this definition, we trained the ExpLoss method on the same training set for several values of the smoothing parameter ϵ and evaluated the performance on development data. Table 3 compares the peak performance achieved under the two definitions of $S_{i,j}$ on the development set. It can be seen that the definition in equation (22) outperforms the simpler method in equation (23). Figure 7 shows the learning curves for the optimal values of ϵ for the two methods. It can be seen that the learning curve for the definition of $S_{i,j}$ in equation (22) consistently dominates the curve for the simpler definition.

5.4.3 Efficiency Gains. Section 4.5 introduced an efficient algorithm for optimizing ExpLoss. In this section we explore the empirical gains in efficiency seen on the parsing data sets in this article.

We first define the quantity *T* as follows:

$$T = \sum_{i} \sum_{j=2}^{n_i} (|B_{i,j}^+| + |B_{i,j}^-|)$$



Figure 6

Learning curves on development data for various values of ϵ . In each case the *y*-axis is the level of accuracy (100 is the baseline score), and the *x*-axis is the number of rounds of boosting. The three graphs compare the curve for $\epsilon = 0.0025$ (the optimal value) to (from top to bottom) $\epsilon = 0.0001$, $\epsilon = 0.0075$, and $\epsilon = 0.001$. The top graph shows that $\epsilon = 0.0001$ leads to undersmoothing (overtraining). Initially the graph is higher than that for $\epsilon = 0.0025$, but on later rounds the performance starts to decrease. The middle graph shows that $\epsilon = 0.0075$ leads to oversmoothing (undertraining). The graph shows consistently lower performance than that for $\epsilon = 0.0025$. The bottom graph shows that there is little difference in performance for $\epsilon = 0.001$ versus $\epsilon = 0.0025$.

Table 3

Peak performance achieved for various values of ϵ for $S_{i,j} = \text{Score}(x_{i,1}) - \text{Score}(x_{i,j})$ (column labeled "weighted") and $S_{i,j} = 1$ (column labeled "unweighted").

e	Best score (weighted)	Best score (unweighted)
0.0001	101.743	101.744
0.00025	101.849	101.754
0.0005	101.845	101.778
0.00075	101.809	101.762
0.001	101.849	101.778
0.0025	101.864	101.699
0.005	101.824	101.610
0.0075	101.722	101.604

This is a measure of the number of updates to the W_k^+ and W_k^- variables required in making a pass over the entire training set. Thus it is a measure of the amount of computation that the naive algorithm for ExpLoss, presented in Figure 3, requires for each round of feature selection.

Next, say the improved algorithm in Figure 4 selects feature k^* on the *t* th round of feature selection. Then we define the following quantity:

$$C_t = \sum_{(i,j)\in A_{k^*}^+} (|B_{i,j}^+| + |B_{i,j}^-|) + \sum_{(i,j)\in A_{k^*}^-} (|B_{i,j}^+| + |B_{i,j}^-|)$$

This is a measure of the number of summations required by the improved algorithm in Figure 4 at the t th round of feature selection.



Figure 7

Performance versus number of rounds of boosting for $S_{i,j} = \text{Score}(x_{i,1}) - \text{Score}(x_{i,j})$ (curve labeled "Weighted") and $S_{i,j} = 1$ (curve labeled "Not Weighted").

Collins and Koo

We are now in a position to compare the running times of the two algorithms. We define the following quantities:

$$Work(n) = \sum_{t=1}^{n} \frac{C_t}{T}$$
(24)

$$Savings(n) = \frac{nT}{\sum_{t=1}^{n} C_t}$$
(25)

$$Savings(a, b) = \frac{(1+b-a)T}{\sum_{t=a}^{b} C_t}$$
(26)

Here, Work(n) is the computation required for *n* rounds of feature selection, where a single unit of computation corresponds to a pass over the entire training set. Savings(n) tracks the relative efficiency of the two algorithms as a function of the number of features, *n*. For example, if Savings(100) = 1,200, this signifies that for the first 100 rounds of feature selection, the improved algorithm is 1,200 times as efficient as the naive algorithm. Finally, Savings(a, b) indicates the relative efficiency between rounds *a* and *b*, inclusive, of feature selection. For example, Savings(11, 100) = 83 signifies that between rounds 11 and 100 inclusive of the algorithm, the improved algorithm was 83 times as efficient.

Figures 8 and 9 show graphs of Work(n) and Savings(n) versus *n*. The savings from the improved algorithm are dramatic. In 100,000 rounds of feature selection, the improved algorithm requires total computation that is equivalent to a mere 37.1 passes over the training set. This is a saving of a factor of 2,692 over the naive algorithm.

Table 4 shows the value of Savings(a,b) for various values of (a,b). It can be seen that the performance gains are significantly larger in later rounds of feature selection, presumably because in later stages relatively infrequent features are being selected. Even so, there are still savings of a factor of almost 50 in the early stages of the method.

6. Related Work

6.1 History-Based Models with Complex Features

Charniak (2000) describes a parser which incorporates additional features into a previously developed parser, that of Charniak (1997). The method gives substantial improvements over the original parser and results which are very close to the results of the boosting method we have described in this article (see section 5 for experimental results comparing the two methods). Our features are in many ways similar to those of Charniak (2000). The model in Charniak (2000) is quite different, however. The additional features are incorporated using a method inspired by maximum-entropy models (e.g., the model of Ratnaparkhi [1997]).

Ratnaparkhi (1997) describes the use of maximum-entropy techniques applied to parsing. Log-linear models are used to estimate the conditional probabilities $P(d_i | \Phi(d_1, ..., d_{i-1}))$ in a history-based parser. As a result the model can take into account quite a rich set of features in the history.



Figure 8 Work(n)(y-axis) versus n (x-axis).



values of Savings (<i>a</i> , <i>b</i>) for various values of <i>a</i> , <i>b</i> .				
а—b	Savings (a, b)			
$\begin{array}{r} 1-100,000\\ 1-10\\ 11-100\\ 101-1,000\\ 1,001-10,000\\ 10,001-50,000\\ 50,001-100,000\end{array}$	2,692.7 48.6 83.5 280.0 1,263.9 2,920.2 4,229.8			

Table 4 Values of Savings (*a*, *b*) for various values of *a*, *b*.

Both approaches still rely on decomposing a parse tree into a sequence of decisions, and we would argue that the techniques described in this article have more flexibility in terms of the features that can be included in the model.

6.2 Joint Log-Linear Models

Abney (1997) describes the application of log-linear models to stochastic headdriven phrase structure grammars (HPSGs). Della Pietra, Della Pietra, and Lafferty (1997) describe feature selection methods for log-linear models, and Rosenfeld (1997) describes application of these methods to language modeling for speech recognition. These methods all emphasize models which define a joint probability over the space of all parse trees (or structures in question): For this reason we describe these approaches as "Joint log-linear models." The probability of a tree $x_{i,j}$ is

$$P(x_{i,j}) = \frac{e^{F(x_{i,j})}}{\sum_{x \in Z} e^{F(x)}}$$
(27)

Here Z is the (infinite) set of possible trees, and the denominator cannot be calculated explicitly. This is a problem for parameter estimation, in which an estimate of the denominator is required, and Monte Carlo methods have been proposed (Della Pietra, Della Pietra, and Lafferty 1997; Abney 1997; Rosenfeld 1997) as a technique for estimation of this value. Our sense is that these methods can be computationally expensive. Notice that the joint likelihood in equation (27) is not a direct function of the margins on training examples, and its relation to error rate is therefore not so clear as in the discriminative approaches described in this article.

6.3 Conditional Log-Linear Models

Ratnaparkhi, Roukos, and Ward (1994), Johnson et al. (1999), and Riezler et al. (2002) suggest training log-linear models (i.e., the LogLoss function in equation (9)) for parsing problems. Ratnaparkhi, Roukos, and Ward (1994) use feature selection techniques for the task. Johnson et al. (1999) and Riezler et al. (2002) do not use a feature selection technique, employing instead an objective function which includes a

Gaussian prior on the parameter values, thereby penalizing parameter values which become too large:

$$\bar{\alpha}^* = \arg \min_{\alpha} \left(\text{LogLoss}(\bar{\alpha}) + \sum_{k=0...m} \frac{\alpha_k^2}{\phi_k^2} \right)$$
(28)

Closed-form updates under iterative scaling are not possible with this objective function; instead, optimization algorithms such as gradient descent or conjugate gradient methods are used to estimate parameter values.

In more recent work, Lafferty, McCallum, and Pereira (2001) describe the use of conditional Markov random fields (CRFs) for tagging tasks such as named entity recognition or part-of-speech tagging (hidden Markov models are a common method applied to these tasks). CRFs employ the objective function in equation (28). A key insight of Lafferty, McCallum, and Pereira (2001) is that when features are of a significantly local nature, the gradient of the function in equation (28) can be calculated efficiently using dynamic programming, even in cases in which the set of candidates involves all possible tagged sequences and is therefore exponential in size. See also Sha and Pereira (2003) for more recent work on CRFs.

Optimizing a log-linear model with a Gaussian prior (i.e., choosing parameter values which achieve the global minimum of the objective function in equation (28)) is a plausible alternative to the feature selection approaches described in the current article or to the feature selection methods previously applied to log-linear models. The Gaussian prior (i.e., the $\sum_k \alpha_k^2/\varphi_k^2$ penalty) has been found in practice to be very effective in combating overfitting of the parameters to the training data (Chen and Rosenfeld 1999; Johnson et al. 1999; Lafferty, McCallum, and Pereira 2001; Riezler et al. 2002). The function in equation (28) can be optimized using variants of gradient descent, which in practice require tens or at most hundreds of passes over the training data (see, e.g., Sha and Pereira 2003). Thus log-linear models with a Gaussian prior are likely to be comparable in terms of efficiency to the feature selection approach described in this article (in the experimental section, we showed that for the parse-reranking task, the efficient boosting algorithm requires computation that is equivalent to around 40 passes over the training data).

Note, however, that the two methods will differ considerably in terms of the sparsity of the resulting reranker. Whereas the feature selection approach leads to around 11,000 (2%) of the features in our model having nonzero parameter values, log-linear models with Gaussian priors typically have very few nonzero parameters (see, e.g., Riezler and Vasserman 2004). This may be important in some domains, for example, those in which there are a very large number of features and this large number leads to difficulties in terms of memory requirements or computation time.

6.4 Feature Selection Methods

A number of previous papers (Berger, Della Pietra, and Della Pietra 1996; Ratnaparkhi 1998; Della Pietra, Della Pietra, and Lafferty 1997; McCallum 2003; Zhou et al. 2003; Riezler and Vasserman 2004) describe feature selection approaches for log-linear models applied to NLP problems. Earlier work (Berger, Della Pietra, and Della Pietra 1996; Ratnaparkhi 1998; Della Pietra, Della Pietra, and Lafferty 1997) suggested methods that added a feature at a time to the model and updated all parameters in the current model at each step (for more detail, see section 3.3).

Assuming that selection of a feature takes one pass over the training set and that fitting a model takes p passes over the training set, these methods require $f \times (p + 1)$ passes over the training set, where f is the number of features selected. In our experiments, $f \approx 10,000$. It is difficult to estimate the value for p, but assuming (very conservatively) that p = 2, selecting 10,000 features would require 30,000 passes over the training set. This is around 1,000 times as much computation as that required for the efficient boosting algorithm applied to our data, suggesting that the feature selection methods in Berger, Della Pietra, and Della Pietra (1996), Ratnaparkhi (1998), and Della Pietra, Della Pietra, and Lafferty (1997) are not sufficiently efficient for the parsing task.

More recent work (McCallum 2003; Zhou et al. 2003; Riezler and Vasserman 2004) has considered methods for speeding up the feature selection methods described in Berger, Della Pietra, and Della Pietra (1996), Ratnaparkhi (1998), and Della Pietra, Della Pietra, and Lafferty (1997). McCallum (2003) and Riezler and Vasserman (2004) describe approaches that add k features at each step, where k is some constant greater than one. The running time for these methods is therefore $O(f \times (p + 1)/k)$. Riezler and Vasserman (2004) test a variety of values for k, finding that k = 100 gives optimal performance. McCallum (2003) uses a value of k = 1,000. Zhou et al. (2003) use a different heuristic that avoids having to recompute the gain for every feature at every iteration.

We would argue that the alternative feature selection methods in the current article may be preferable on the grounds of both efficiency and simplicity. Even with large values of k in the approach of McCallum (2003) and Riezler and Vasserman (2004) (e.g., k = 1,000), the approach we describe is likely to be at least as efficient as these alternative approaches. In terms of simplicity, the methods in McCallum (2003) and Riezler and Vasserman (2004) require selection of a number of free parameters governing the behavior of the algorithm: the value for k, the value for a regularizer constant (used in both McCallum [2003] and Riezler and Vasserman [2004]), and the precision with which the model is optimized at each stage of feature selection (McCallum [2003] describes using "just a few BFGS iterations" at each stage). In contrast, our method requires a single parameter to be chosen (the value for the ϵ smoothing parameter) and makes a single approximation (that only a single feature is updated at each round of feature selection). The latter approximation is particularly important, as it leads to the efficient algorithm in Figure 4, which avoids a pass over the training set at each iteration of feature selection (note that in sparse feature spaces, *f* rounds of feature selection in our approach can take considerably fewer than *f* passes over the training set, in contrast to other work on feature selection within log-linear models).

Note that there are other important differences among the approaches. Both Della Pietra, Della Pietra, and Lafferty (1997) and McCallum (2003) describe methods that induce conjunctions of "base" features, in a way similar to decision tree learners. Thus a relatively small number of base features can lead to a very large number of possible conjoined features. In future work it might be interesting to consider these kinds of approaches for the parsing problem. Another difference is that both McCallum, and Riezler and Vasserman, describe approaches that use a regularizer in addition to feature selection: McCallum uses a two-norm regularizer; Riezler and Vasserman use a one-norm regularizer.

Finally, note that other feature selection methods have been proposed within the machine-learning community: for example, "filter" methods, in which feature selection is performed as a preprocessing step before applying a learning method,

and backward selection methods (Koller and Sahami 1996), in which initially all features are added to the model and features are then incrementally removed from the model.

6.5 Boosting, Perceptron, and Support Vector Machine Approaches for Ranking Problems

Freund et al. (1998) introduced a formulation of boosting for ranking problems. The problem we have considered is a special case of the problem in Freund et al. (1998), in that we have considered a binary distinction between candidates (i.e., the best parse vs. other parses), whereas Freund et al. consider learning full or partial orderings over candidates. The improved algorithm that we introduced in Figure 4 is, however, a new algorithm that could perhaps be generalized to the full problem of Freund et al. (1998); we leave this to future research.

Altun, Hofmann, and Johnson (2003) and Altun, Johnson, and Hofmann (2003) describe experiments on tagging tasks using the ExpLoss function, in contrast to the LogLoss function used in Lafferty, McCallum, and Pereira (2001). Altun, Hofmann, and Johnson (2003) describe how dynamic programming methods can be used to calculate gradients of the ExpLoss function even in cases in which the set of candidates again includes all possible tagged sequences, a set which grows exponentially in size with the length of the sentence being tagged. Results in Altun, Johnson, and Hofmann (2003) suggest that the choice of ExpLoss versus LogLoss does not have a major impact on accuracy for the tagging task in question.

Perceptron-based algorithms, or the voted perceptron approach of Freund and Schapire (1999), are another alternative to boosting and LogLoss methods. See Collins (2002a, 2002b) and Collins and Duffy (2001, 2002) for applications of the perceptron algorithm. Collins (2002b) gives convergence proofs for the methods; Collins (2002a) directly compares the boosting and perceptron approaches on a named entity task; and Collins and Duffy (2001, 2002) use a reranking approach with kernels, which allow representations of parse trees or labeled sequences in very-high-dimensional spaces.

Shen, Sarkar, and Joshi (2003) describe support vector machine approaches to ranking problems and apply support vector machines (SVMs) using tree-adjoining grammar (Joshi, Levy, and Takahashi 1975) features to the parsing data sets we have described in this article, with good empirical results.

See Collins (2004) for a discussion of many of these methods, including an overview of statistical bounds for the boosting, perceptron, and SVM methods, as well as a discussion of the computational issues involved in the different algorithms.

7. Conclusions

This article has introduced a new algorithm, based on boosting approaches in machine learning, for ranking problems in natural language processing. The approach gives a 13% relative reduction in error on parsing Wall Street Journal data. While in this article the experimental focus has been on parsing, many other problems in natural language processing or speech recognition can also be framed as reranking problems, so the methods described should be quite broadly applicable. The boosting approach to ranking has been applied to named entity segmentation (Collins 2002a) and natural language generation (Walker, Rambow, and Rogati 2001). The key characteristics of the approach are the use of global features and of a training criterion (optimization

problem) that is discriminative and closely related to the task at hand (i.e., parse accuracy).

In addition, the article introduced a new algorithm for the boosting approach which takes advantage of the sparse nature of the feature space in the parsing data that we use. Other NLP tasks are likely to have similar characteristics in terms of sparsity. Experiments show an efficiency gain of a factor of over 2,600 on the parsing data for the new algorithm over the obvious implementation of the boosting approach. We would argue that the improved boosting algorithm is a natural alternative to maximum-entropy or (conditional) log-linear models. The article has drawn connections between boosting and maximum-entropy models in terms of the optimization problems that they involve, the algorithms used, their relative efficiency, and their performance in empirical tests.

Appendix A: Derivation of Updates for ExpLoss

This appendix gives a derivation of the optimal updates for ExpLoss. The derivation is very close to that in Schapire and Singer (1999). Recall that for parameter values $\bar{\alpha}$, we need to compute BestWt($k, \bar{\alpha}$) and BestLoss($k, \bar{\alpha}$) for k = 1, ..., m, where

$$BestWt(k,\bar{\alpha}) = \arg\min_{\delta} ExpLoss(Upd(\bar{\alpha},k,\delta))$$

and

$$BestLoss(k, \bar{\alpha}) = ExpLoss(Upd(\bar{\alpha}, k, BestWt(k, \bar{\alpha})))$$

The first thing to note is that an update in parameters from $\bar{\alpha}$ to Upd($\bar{\alpha}$, k, δ)) results in a simple additive update to the ranking function *F*:

$$F(x_{i,i}, \operatorname{Upd}(\bar{\alpha}, k, \delta)) = F(x_{i,i}, \alpha) + \delta h_k(x_{i,i})$$

It follows that the margin on example (i, j) also has a simple update:

$$M_{i,j}(\operatorname{Upd}(\bar{\mathbf{a}}, k, \delta)) = F(x_{i,1}, \operatorname{Upd}(\bar{\mathbf{a}}, k, \delta)) - F(x_{i,j}, \operatorname{Upd}(\bar{\mathbf{a}}, k, \delta))$$
$$= F(x_{i,1}, \bar{\mathbf{a}}) - F(x_{i,j}, \bar{\mathbf{a}}) + \delta[h_k(x_{i,1}) - h_k(x_{i,j})]$$
$$= M_{i,j}(\bar{\mathbf{a}}) + \delta[h_k(x_{i,1}) - h_k(x_{i,j})]$$

The updated ExpLoss function can then be written as

$$\begin{aligned} \operatorname{ExpLoss}\left(\operatorname{Upd}(\bar{\boldsymbol{\alpha}},k,\delta)\right) &= \sum_{i} \sum_{j=2}^{n_{i}} S_{i,j} e^{-M_{i,j}\left(\operatorname{Upd}(\bar{\boldsymbol{\alpha}},k,\delta)\right)} \\ &= \sum_{i} \sum_{j=2}^{n_{i}} S_{i,j} e^{-M_{i,j}\left(\alpha\right) - \delta\left[h_{k}\left(x_{i,1}\right) - h_{k}\left(x_{i,j}\right)\right]} \end{aligned}$$

Next, we note that $[h_k(x_{i,1}) - h_k(x_{i,j})]$ can take on three values: +1, -1, or 0. We split the training sample into three sets depending on this value:

$$A_k^+ = \{(i,j) : [h_k(x_{i,1}) - h_k(x_{i,j})] = 1\}$$

$$A_k^- = \{(i,j) : [h_k(x_{i,1}) - h_k(x_{i,j})] = -1\}$$
$$A_k^0 = \{(i,j) : [h_k(x_{i,1}) - h_k(x_{i,j})] = 0\}$$

Given these definitions, we define W_k^+ , W_k^- , and W_k^0 as

$$\begin{split} W_k^+ &= \sum_{(i,j)\in A_k^+} S_{i,j} e^{-M_{i,j}(\tilde{\mathbf{\alpha}})} \\ W_k^- &= \sum_{(i,j)\in A_k^-} S_{i,j} e^{-M_{i,j}(\tilde{\mathbf{\alpha}})} \\ W_k^0 &= \sum_{(i,j)\in A_k^0} S_{i,j} e^{-M_{i,j}(\tilde{\mathbf{\alpha}})} \end{split}$$

ExpLoss is now rewritten in terms of these quantities:

$$\begin{aligned} \operatorname{ExpLoss}(\operatorname{Upd}\left(\bar{\boldsymbol{\alpha}},k,\delta\right)) &= \sum_{(i,j)\in A_{k}^{+}} S_{i,j}e^{-M_{i,j}\left(\bar{\boldsymbol{\alpha}}\right)-\delta} + \sum_{(i,j)\in A_{k}^{-}} S_{i,j}e^{-M_{i,j}\left(\bar{\boldsymbol{\alpha}}\right)+\delta} + \sum_{(i,j)\in A_{k}^{0}} S_{i,j}e^{-M_{i,j}\left(\bar{\boldsymbol{\alpha}}\right)} \\ &= e^{-\delta}W_{k}^{+} + e^{\delta}W_{k}^{-} + W_{k}^{0} \end{aligned}$$
(A.1)

To find the value of δ that minimizes this loss, we set the derivative of (A.1) with respect to δ to zero, giving the following solution:

BestWt(
$$k, \bar{\alpha}$$
) = $\frac{1}{2} \log \frac{W_k^+}{W_k^-}$

Plugging this value of δ back into (A.1) gives the best loss:

BestLoss
$$(k, \bar{\alpha}) = 2\sqrt{W_k^+ W_k^-} + W_k^0$$

= $2\sqrt{W_k^+ W_k^-} + Z - W_k^+ - W_k^-$
= $Z - \left(\sqrt{W_k^+} - \sqrt{W_k^-}\right)^2$ (A.2)

where $Z = \text{ExpLoss}(\bar{\alpha}) = \sum_{i} \sum_{j=2}^{n_i} S_{i,j} e^{-M_{i,j}(\bar{\alpha})}$ is a constant (for constant $\bar{\alpha}$) which appears in the BestLoss for all features and therefore does not affect their ranking.

Appendix B: An Alternative Method for LogLoss

In this appendix we sketch an alternative approach for feature selection in LogLoss that is potentially an efficient method, at the cost of introducing an approximation

in the feature selection method. Until now, we have defined $\text{BestLoss}(k, \bar{\alpha})$ to be the minimum of the loss given that the *k*th feature is updated an optimal amount:

$$BestLoss(k, \bar{\alpha}) = \min_{\delta} LogLoss(Upd(\bar{\alpha}, k, \delta))$$

In this section we sketch a different approach, based on results from Collins, Schapire, and Singer (2002), which leads to an algorithm very similar to that for ExpLoss in Figures 3 and 4. Take the following definitions (note the similarity to the definitions in equations (13), (14), (15), and (16), with only the definitions for W_k^+ and W_k^- being altered):

$$W_{k}^{+} = \sum_{(i,j)\in A_{k}^{+}} q_{i,j}, \quad W_{k}^{-} = \sum_{(i,j)\in A_{k}^{-}} q_{i,j}, \quad \text{where} \quad q_{i,j} = \frac{e^{-M_{i,j}(\tilde{\mathbf{a}})}}{1 + \sum_{q=2}^{n_{i}} e^{-M_{i,q}(\tilde{\mathbf{a}})}}$$
(B.1)

BestWt(k,
$$\bar{\alpha}$$
) = $\frac{1}{2} \log \frac{W_k^+}{W_k^-}$ (B.2)

BestLoss
$$(k, \bar{\alpha}) = \text{LogLoss}(\bar{\alpha}) - \left(\sqrt{W_k^+} - \sqrt{W_k^-}\right)^2$$
 (B.3)

Note that the ExpLoss computations can be recovered by replacing $q_{i,j}$ in equation (B.1) with $q_{i,j} = e^{-M_{i,j}(\bar{\alpha})}$. This is the only essential difference between the new algorithm and the ExpLoss method.

Results from Collins, Schapire and Singer (2002) show that under these definitions the following guarantee holds:

$$LogLoss(Upd(\bar{\alpha},k,BestWt(k,\bar{\alpha}))) \leq BestLoss(k,\bar{\alpha})$$

So it can be seen that the update from $\bar{\alpha}$ to Upd($\bar{\alpha}$, *k*, BestWt(k, $\bar{\alpha}$)) is guaranteed to decrease LogLoss by at least $(\sqrt{w_k^+} - \sqrt{w_k^-})^2$. From these results, the algorithms in Figures 3 and 4 could be altered to take the revised definitions of W_k^+ and W_k^- into account. Selecting the feature with the minimum value of BestLoss(k, $\bar{\alpha}$) at each iteration leads to the largest guaranteed decrease in LogLoss. Note that this is now an approximation, in that BestLoss(k, $\bar{\alpha}$) is an upper bound on the log-likelihood which may or may not be tight. There are convergence guarantees for the method, however, in that as the number of rounds of feature selection goes to infinity, the LogLoss approaches its minimum value.

The algorithms in Figures 3 and 4 could be modified to take the alternative definitions of W_k^+ and W_k^- into account, thereby being modified to optimize LogLoss instead of ExpLoss. The denominator terms in the $q_{i,j}$ definitions in equation (B.1) may complicate the algorithms somewhat, but it should still be possible to derive relatively efficient algorithms using the technique.

For a full derivation of the modified updates and for quite technical convergence proofs, see Collins, Schapire and Singer (2002). We give a sketch of the argument here. First, we show that

$$\operatorname{LogLoss}(\operatorname{Upd}(\bar{\alpha},k,\delta)) \le \operatorname{LogLoss}(\bar{\alpha} - W_k^+ - W_k^- + W_k^+ e^{-\delta} + W_k^- e^{\delta})$$
(B.4)

This can be derived as follows (in this derivation we use $g_k(x_{i,j}) = h_k(x_{i,1}) - h_k(x_{i,j})$):

$$\begin{aligned} \text{LogLoss}(\text{Upd}(\bar{\alpha},k,\delta)) &= \text{LogLoss}(\bar{\alpha}) + \text{LogLoss}(\text{Upd}(\bar{\alpha},k,\delta)) - \text{LogLoss}(\bar{\alpha}) \\ &= \text{LogLoss}(\bar{\alpha}) + \sum_{i} \log \left(\frac{1 + \sum_{j=2}^{n_i} e^{-M_{i,j}(\bar{\alpha}) - \delta g_k(x_{i,j})}}{1 + \sum_{j=2}^{n_i} e^{-M_{i,j}(\bar{\alpha})}} \right) \\ &= \text{LogLoss}(\bar{\alpha}) + \sum_{i} \log \left(\frac{1}{1 + \sum_{j=2}^{n_i} e^{-M_{i,j}(\bar{\alpha})}} + \frac{\sum_{j=2}^{n_i} e^{-M_{i,j}(\bar{\alpha}) - \delta g_k(x_{i,j})}}{1 + \sum_{j=2}^{n_i} e^{-M_{i,j}(\bar{\alpha})}} \right) \\ &= \text{LogLoss}(\bar{\alpha}) + \sum_{i} \log \left(1 - \sum_{j=2}^{n_i} q_{i,j} + \sum_{j=2}^{n_i} q_{i,j} e^{-\delta g_k(x_{i,j})} \right) \end{aligned}$$
(B.5)

$$\leq \text{LogLoss}(\bar{\alpha}) - \sum_{i} \sum_{j=2}^{n_{i}} q_{i,j} + \sum_{i} \sum_{j=2}^{n_{i}} q_{i,j}e^{-\delta g_{k}(x_{i,j})}$$

$$= \text{LogLoss}(\bar{\alpha}) - (W_{k}^{0} + W_{k}^{+} + W_{k}^{-}) + W_{k}^{0} + W_{k}^{+}e^{-\delta} + W_{k}^{-}e^{\delta}$$

$$= \text{LogLoss}(\bar{\alpha}) - W_{k}^{+} - W_{k}^{-} + W_{k}^{+}e^{-\delta} + W_{k}^{-}e^{\delta}$$
(B.6)

Equation (B.6) can be derived from equation (B.5) through the bound $log(1 + x) \le x$ for all *x*.

The second step is to minimize the right-hand side of the bound in equation (B.4) with respect to δ . It can be verified that the minimum is found at

$$\delta = \frac{1}{2} \log \frac{W_k^+}{W_k^-}$$

at which value the right-hand side of equation (B.4) is equal to

$$\text{LogLoss}(\delta) - \left(\sqrt{W_k^+} - \sqrt{W_k^-}\right)^2$$

Acknowledgments

Thanks to Rob Schapire and Yoram Singer for useful discussions on boosting algorithms and to Mark Johnson for useful discussions about linear models for parse ranking. Steve Abney and Fernando Pereira gave useful feedback on earlier drafts of this work. Finally, thanks to the anonymous reviewers for several useful comments.

References

Abney, Steven. 1997. Stochastic attribute-value grammars. Computational Linguistics, 23(4):597–618.

- Altun, Yasemin, Thomas Hofmann, and Mark Johnson. 2003. Discriminative learning for label sequences via boosting. In *Advances in Neural Information Processing Systems (NIPS 15)*, Vancouver.
- Altun, Yasemin, Mark Johnson, and Thomas Hofmann. 2003. Loss functions and optimization methods for discriminative learning of label sequences. In *Proceedings of Empirical Methods in Natural Language Processing (EMNLP 2003),* Sapporo, Japan.
- Berger, Adam L., Stephen A. Della Pietra, and Vincent J. Della Pietra. 1996. A maximum entropy approach to natural language

processing. *Computational Linguistics*, 22(1):39–71.

Black, Ezra, Frederick Jelinek, John Lafferty, David Magerman, Robert Mercer, and Salim Roukos. 1992. Towards history-based grammars: Using richer models for probabilistic parsing. In *Proceedings of the Fifth DARPA Speech and Natural Language Workshop*, Harriman, NY.

Charniak, Eugene. 1997. Statistical parsing with a context-free grammar and word statistics. *Proceedings of the 14th National Conference on Artificial Intelligence*, Menlo Park, CA. AAAI Press/MIT Press.

Charniak, Eugene. 2000. A maximum-entropy-inspired parser. In *Proceedings of NAACL-2000*, Seattle.

Chen, Stanley F., and Ronald Rosenfeld. 1999. A gaussian prior for smoothing maximum entropy models. Technical Report CMU-CS-99-108, Computer Science Department, Carnegie Mellon University.

Collins, Michael. 1997. Three generative, lexicalised models for statistical parsing. In Proceedings of the 35th Annual Meeting of the Association for Computational Linguistics and Eighth Conference of the European Chapter of the Association for Computational Linguistics, pages 16–23, Madrid.

Collins, Michael. 1999. Head-Driven Statistical Models for Natural Language Parsing. Ph.D. thesis, University of Pennsylvania, Philadelphia.

Collins, Michael. 2000. Discriminative reranking for natural language parsing. In Proceedings of the 17th International Conference on Machine Learning (ICML 2000), Stanford, CA. Morgan Kaufmann, San Francisco.

Collins, Michael. 2002a. Ranking algorithms for named-entity extraction: Boosting and the voted perceptron. In ACL 2002: Proceedings of the 40th Annual Meeting of the Association for Computational Linguistics, Philadelphia.

Collins, Michael. 2002b. Discriminative training methods for hidden Markov models: Theory and experiments with the perceptron algorithm. In *Proceedings of EMNLP 2002*, Philadelphia.

Collins, Michael. 2004. Parameter estimation for statistical parsing models: Theory and practice of distribution-free methods. In Harry Bunt, John Carroll, and Giorgio Satta, editors, *New Developments in Parsing Technology*. Kluwer.

Collins, Michael and Nigel Duffy. 2001. Convolution kernels for natural language. In Advances in Neural Information Processing Systems (NIPS 14), Vancouver.

Collins, Michael, and Nigel Duffy. 2002. New ranking algorithms for parsing and tagging: Kernels over discrete structures, and the voted perceptron. In ACL 2002: Proceedings of the 40th Annual Meeting of the Association for Computational Linguistics, Philadelphia.

Collins, Michael, Robert E. Schapire, and Yoram Singer. 2002. Logistic regression, AdaBoost and Bregman distances. *Machine Learning*, 48(1/2/3):253–285.

Della Pietra, Stephen, Vincent Della Pietra, and John Lafferty. 1997. Inducing features of random fields. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 19(4):380–393.

Duffy, Nigel and David Helmbold. 1999. Potential boosters? In Advances in Neural Information Processing Systems (NIPS 12), Denver.

Freund, Yoav, Raj Iyer, Robert E. Schapire, and Yoram Singer. 1998. An efficient boosting algorithm for combining preferences. In *Machine Learning: Proceedings of the 15th International Conference*, Madison, WI.

Freund, Yoav and Robert E. Schapire. 1997. A decision-theoretic generalization of on-line learning and an application to boosting. *Journal of Computer and System Sciences*, 55(1):119–139.

Freund, Yoav and Robert E. Schapire. 1999. Large margin classification using the perceptron algorithm. *Machine Learning*, 37(3):277–296.

Friedman, Jerome H., Trevor Hastie, and Robert Tibshirani. 2000. Additive logistic regression: A statistical view of boosting. *Annals of Statistics*, 38(2):337–374.

Henderson, James. 2003. Inducing history representations for broad coverage statistical parsing. In *Proceedings of the Joint Meeting of the North American Chapter of the Association for Computational Linguistics and the Human Language Technology Conference (HLT-NAACL 2003)*, pages 103–110, Edmonton, Alberta, Canada.

Hoffgen, Klauss U., Kevin S. van Horn, and Hans U. Simon. 1995. Robust trainability of single neurons. *Journal of Computer and System Sciences*, 50:114–125.

Johnson, Mark, Stuart Geman, Stephen Canon, Zhiyi Chi, and Stefan Riezler. 1999. Estimators for stochastic "unification-based" grammars. In *Proceedings of ACL 1999*, College Park, MD.

- Joshi, Aravind K., Leon S. Levy, and Masako Takahashi. 1975. Tree adjunct grammars. *Journal of Computer and System Science* 10, no. 1:136–163.
- Koller, Daphne, and Mehran Sahami. 1996. Toward optimal feature selection. In Proceedings of the 13th International Conference on Machine Learning (ICML), pages 284–292, Bari, Italy, July.
- Lafferty, John. 1999. Additive models, boosting, and inference for generalized divergences. In *Proceedings of the 12th Annual Conference on Computational Learning Theory (COLT'99)*, Santa Cruz, CA.

Lafferty, John, Andrew McCallum, and Fernando Pereira. 2001. Conditional random fields: Probabilistic models for segmenting and labeling sequence data. In *Proceedings of ICML 2001*, Williamstown, MA.

- Lebanon, Guy and John Lafferty. 2001. Boosting and maximum likelihood for exponential models. In *Advances in Neural Information Processing Systems* (*NIPS 14*), Vancouver.
- Malouf, Robert. 2002. A comparison of algorithms for maximum entropy parameter estimation. In *Proceedings of the Sixth Conference on Natural Language Learning (CoNNL-2002)*, Taipei, Taiwan.
- Marcus, Mitchell, Beatrice Santorini, and Mary Ann Marcinkiewicz. 1993. Building a large annotated corpus of English: The Penn Treebank. *Computational Linguistics*, 19(2):313–330.
- Mason, Llew, Peter L. Bartlett, and Jonathan Baxter. 1999. Direct optimization of margins improves generalization in combined classifiers. In *Advances in Neural Information Processing Systems (NIPS 12)*, Denver.
- McCallum, Andrew. 2003. Efficiently inducing features of conditional random fields. In *Proceedings of the Conference on Uncertainty in Artificial Intelligence (UAI* 2003), Acapulco.
- Och, Franz Josef, and Hermann Ney. 2002. Discriminative training and maximum entropy models for statistical machine translation. In ACL 2002: Proceedings of the 40th Annual Meeting of the Association for Computational Linguistics, pages 295–302, Philadelphia.

Papineni, Kishore A., Salim Roukos, and R. T. Ward. 1997. Feature-based language understanding. In *Proceedings of EuroSpeech'97*, vol. 3, pages 1435–1438, Rhodes, Greece.

Papineni, Kishore A., Salim Roukos, and R. T. Ward. 1998. Maximum likelihood and discriminative training of direct translation models. In *Proceedings of the* 1998 IEEE International Conference on Acoustics, Speech and Signal Processing, vol. 1, pages 189–192, Seattle.

- Pearl, Judea. 1988. Probabilistic Reasoning in Intelligent Systems. Morgan Kaufmann, San Mateo, CA.
- Ratnaparkhi, Adwait. 1997. A linear observed time statistical parser based on maximum entropy models. In *Proceedings of the Second Conference on Empirical Methods in Natural Language Processing*, Brown University, Providence, RI.
- Ratnaparkhi, Adwait. 1998. Maximum Entropy Models for Natural Language Ambiguity Resolution. Ph.D. thesis, University of Pennsylvania, Philadelphia.
- Ratnaparkhi, Adwait, Salim Roukos, and R. T. Ward. 1994. A maximum entropy model for parsing. In *Proceedings of the International Conference on Spoken Language Processing*, pages 803–806, Yokohama, Japan.
- Riezler, Stefan, Tracy H. King, Ronald M. Kaplan, Richard Crouch, John T. Maxwell III, and Mark Johnson. 2002. Parsing the Wall Street Journal using a lexical-functional grammar and discriminative estimation techniques. In ACL 2002: Proceedings of the 40th Annual Meeting of the Association for Computational Linguistics, Philadelphia.
- Riezler, Stefan and Alexander Vasserman. 2004. Incremental feature selection and *l*₁ regularization for relaxed maximum-entropy modeling. In *Proceedings* of the 2004 Conference on Empirical Methods in Natural Language Processing (EMNLP'04), Barcelona, Spain.
- Rosenfeld, Ronald. 1997. A whole sentence maximum entropy language model. In *Proceedings of the IEEE Workshop on Speech Recognition and Understanding,* Santa Barbara, CA, December.
- Schapire, Robert E., Yoav Freund, Peter Bartlett, and W. S. Lee. 1998. Boosting the margin: A new explanation for the effectiveness of voting methods. *Annals of Statistics*, 26(5):1651–1686.
- Schapire, Robert E. and Yoram Singer. 1999. Improved boosting algorithms using confidence-rated predictions. *Machine Learning*, 37(3):297–336.
- Schapire, Robert E. and Yoram Singer. 2000. BoosTexter: A boosting-based system for text categorization. *Machine Learning*, 39(2/3):135–168.

- Sha, Fei and Fernando Pereira. 2003. Shallow parsing with conditional random fields. In *Proceedings of HLT-NAACL 2003*, Edmonton, Alberta, Canada.
- Shen, Libin, Anoop Sarkar, and Aravind K. Joshi. 2003. Using LTAG based features in parse reranking. In *Proceedings of the 2003 Conference on Empirical Methods in Natural Language Processing (EMNLP 2003)*, Sapporo, Japan.
- Valiant, Leslie G. 1984. A theory of the learnable. *Communications of the ACM*, 27(11):1134–1142.
- Walker, Marilyn, Owen Rambow, and Monica Rogati. 2001. SPoT: A trainable sentence planner. In *Proceedings of the Second Meeting of the North American Chapter of the Association for Computational Linguistics (NAACL 2001)*, Pittsburgh.
- Zhou, Yaqian, Fuliang Weng, Lide Wu, and Hauke Schmidt. 2003. A fast algorithm for feature selection in conditional maximum entropy modeling. In *Proceedings of the 2003 Conference on Empirical Methods in Natural Language Processing (EMNLP 2003)*, Sapporo, Japan.