The Entailment Relationship Between Transparent Perceptual Reports and Opaque Infinitival Complements: An Approach Without Possible Worlds

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Abstract

This paper proposes a solution to the problem that remains in Tomita (2019). I propose an approach that resembles Hobbs (1985), updated in Hobbs (2003b). His idea is that all predicates take arguments in the *Platonic (or Meinongian) universe*, which consists of everything (every object and eventuality) that possibly exists, regardless of whether it exists in the actual world. This study aims to refine their approach by constructing a semantic system without possible worlds. Then, I discuss the remaining problems and truth-makers.

1 Introduction

This paper proposes a way to solve a problem in the entailment relationship between embedded and matrix clauses, discussing a combination of perceptual reports and infinitival complements as crucial examples. The example (1a) is called a perceptual report and is not problematic in event semantics (see, e.g., Higginbotham, 1983). When (1a) is true, it is entailed that *Mary left*. In general cases of infinitival complements, however, the sentence usually does not necessarily entail the content in its infinitival complement, as shown in (1b).

(1) a. John saw Mary leave. $\hookrightarrow$ Mary left.
    b. John forbade Mary to leave. $\not\hookrightarrow$ Mary left.

First, I will review two approaches: (i) the non-existing eventuality approach (Hobbs, 1985; Parsons, 1991), where an infinitival complement serves as an argument to perception verbs, and (ii) quantificational event semantics (Champollion, 2015), where a sentence has a GQ type over events, and in an opaque context, entailment relations wreak havoc. However, since both approaches in (i) are non-compositional, the event-quantification problem was not considered.

Here, I use the term *truth-makers* covering such objects for truth-making. In other words, I define truth-makers as something in semantics that verifies (or falsifies) the truth of statements. According to Fine (2017), a major class of these truth-making objects can be divided with different properties. One is *worldly*, and the others are *stately*. The former, a set of possible worlds, and the latter, a set of eventualities (events and states), work almost similarly.

The remaining roadmap of the paper consists of the following sections. In Section 2, I review possible worlds from the perspective of *Truth-Maker semantics* (Fine, 2017) and compare them with the event(ualitie)s that Hobbs (1985, 2003b) proposed, considering the entailment relationships between perceptual reports and infinitival complements. I argue that possible worlds are unnecessary for analyzing infinitival complements. In Section 3, I introduce a formalism called Outer Quantificational Event semantics (Tomita, 2019), modifying some notations. Then, I will highlight the problem in Tomita (2019) in Section 4 and propose a modification by embedding an additional restriction in perceptual verbs in Section 5. This predicts different existential entailments between the matrix and embedded clauses. In the final section, I discuss the limitations of the proposed analysis and future work on other truth-makers.

2 Backgrounds

Here, I define truth-makers as something in semantics that verifies (or falsifies) the truth of statements, following Fine (2017, p.557).

> On the objectual approach[...], the *truth-conditions are objects*, rather than clauses, which stand in a relation of truth-making to the statements they make true.

According to Fine (2017), various kinds of *truth makers* inhabit this field. The summary of these truth-makers is presented below:

(2) a. *worldly truth-making objects* (= possible worlds)

b. *stately truth-making objects* (verifiers)

i. exact verifier (events and states): wholly relevant to the statement

ii. inexact verifier (situations): partially relevant to the statement

iii. loose verifier: no requirement of relevance

From this perspective, I regard worlds, events, and situations as instances of truth-makers: their coverages overlap. Note that Fine (2017) calls exact truth-making objects *states*. Still, it seems to have similarities to *situations* in linguistics, while Hobbs (2003a) calls all stately truth-making objects *events*. Here, I tentatively use the term *eventualities* to cover all stately exact verifiers in (2b), focusing on the difference between **possible worlds** (2a) and **eventualities**, and making no distinction among stately truth-making objects (in Fine's sense).

2.1 Possible world semantics

These properties characterize the possible worlds:

(3) a. **Completeness:** If one possible world is given, it can settle (verify or falsify) any propositions.

b. **Accessibility relation:** Some possible world is reachable from another one close enough to it.

Here, I describe the possible worlds as a class of objectual verifiers, each of which completely verifies any proposition and is connected with some accessibility relationship. I think of possible worlds as semantic (and mathematical) tools of truth-making. It is widely accepted that they are necessary for analyzing modals, conditionals, and *de re/ de dicto* distinction. In possible-world semantics, propositions denote a set of possible worlds or, equivalently, characteristic functions of worlds to truth values, as shown in (4).

(4) a. $[\![\text{Brutus stabbed Caesar}]\!] = \{w \mid \text{Brutus stabbed Caesar in } w\}$

b. $[\![\text{Brutus stabbed Caesar}]\!](w) = 1 \text{ iff Brutus stabbed Caesar in } w$

2.2 Difference between Eventualities and Possible Worlds

Here, I compare some properties of possible worlds and eventualities. Fine (2017) pointed out that the possible worlds can, unlike other truth-makers, settle the truth value of any propositions; if one possible world is given, it can verify that any statement is true or false. In other words, the possible world can settle any proposition and play the roles of both a verifier and a falsifier. In contrast, eventualities do not always settle all propositions.

In addition to Fine's claim on completeness, possible worlds are usually characterized by a directed graph structure representing the accessibility relationships among them. Compared with a partial order, typically common in events and situations, the graph structure is stronger since any partial order can be written as a directed graph, but not vice versa. There is a dilemma of cardinality. If you assume an infinite number of possible worlds in natural language semantics, you should also consider an infinite number of accessibility relations among them, strengthening my argument. However, if you try to reduce the cardinality of possible worlds, Fine's argument will become more serious. See also Tomita (2019); Tancredi and Sharvit (2022) for critical remarks on the possible-world approach.

In summary, even if possible worlds are descriptively adequate, they still carry inevitable shortcomings: (i) they are too strong, and (ii) they sometimes fail to describe the meaning. Therefore, possible worlds should be considered *too massive truth-makers*.

2.3 Opaque infinitival complements

Opaque object readings such as one in *John seeks a unicorn* provide empirical support in favor of possible-world semantics (Montague, 1970). However, in recent years, few semantic studies such as Moltmann (2020) and Tancredi and Sharvit (2022) analyzed attitude verbs and their complements without possible worlds. Each of them appealed to other truth-makers, such as the *attitudinal objects* and the *judge parameter*, respectively. Here, I will focus on the difference between the transparent and opaque reading of infinitival clauses.

In addition, Tomita (2019) proposed Outer-Quantificational Event Semantics, which is based on Champollion's (2015) quantificational event semantics, arguing that an entailment problem in infinitival complements can be solved without possible worlds. He embedded $\mathbf{E!}(x)$ into the thematic-relation head [**theme**] that corresponds to the verb *see* and always entails the actual existence of the object for the seeing event. In the next section, I will lay out some formalism along the lines of Tomita (2019), modifying some notational conventions.

3 Formalism

Here, I describe the ideas in Hobbs (1985, 2003b) that accepted the Platonic universe as the quantification domain. To allow quantification over the Platonic universe as its domain, a non-standard formalism called *first-order free logic* is introduced, which adds the special predicate over the Platonic universe $\mathbf{E!}(x)$ that is true if and only if x actually exists.

3.1 Mathematical tools and notational conventions

Here, I use λ-calculus, adding some constants for natural language semantics. Only e (entity), v (eventuality), and t (truth value) are atomic types. The Greek letters ρ, τ, σ are type variables. Due to space limitations, I define logical constants of type $\langle \sigma t, \langle \sigma t, \sigma t\rangle\rangle$ over one-place predicates A and B of type σt. Instead of some standard binary operators, namely, conjunction (&) and implication ($\rightarrow$) symbols, $[_e A \wedge B]$ and $[_e A \supset B]$ are used:

$$[_e A \wedge B] \equiv (\lambda e.[A(e) \& B(e)])$$
$$[_e A \supset B] \equiv (\lambda e.[A(e) \rightarrow B(e)])$$

Since the conjunction is associative, I write $(\lambda e.[A_0(e) \& \ldots \& A_n(e)])$ as $[_e A_0 \wedge \ldots \wedge A_n]$.[1]

3.2 Outer and inner quantifiers

In our formalism, quantifiers are divided into two pairs: One's domain covers the Platonic universe, which contains all entities and eventualities. In contrast, the other's domain covers a subset that only consists of something that exists actually or some eventuality that occurs actually. I define these quantifiers as higher-order lambda terms of type $\langle \sigma t, t\rangle$ (on the typed lambda calculus, see, e.g., Unger (2010)) in (5);

(5) Outer quantifiers:
  a. Existential (particular) quantifier: $\Sigma(\lambda x.M)$
  b. Universal quantifier: $\Pi(\lambda x.M)$

[1]Though these notations are similar to those in Icard and Moss (2023), I additionally use brackets with a subscript variable $[_e \ldots]$ for the sake of readability.

where M is of type t and σ is a type variable ranging over entities and eventualities. In (5), each quantifier binds to every occurrence of the free variable x in M. In particular, $\Sigma(\lambda e.[A_0(e) \& \ldots \& A_n(e)]) \equiv \Sigma[_e A_0 \wedge \ldots \wedge A_n]$.

Inner quantifiers such as $\exists x.M (= \exists(\lambda x.M))$ and $\forall x.M (= \forall(\lambda x.M))$ are defined with respect to the outer ones.

(6) Inner quantifiers
  a. Existential (particular) quantifier: $\exists(\lambda x.M) := \Sigma[_x \mathbf{E!} \wedge (\lambda x.M)]$
  b. Universal quantifier: $\forall(\lambda x.M) := \Pi[_x \mathbf{E!} \supset (\lambda x.M)]$

As shown in (6), each of the inner quantifiers is defined with each of the outer counterparts and **E!**.

3.3 Outer-quantificational event semantics

Our steps proceed in almost the same way as Coppock and Champollion (2019). In their framework, verbal denotations lexically contain the existential quantification of events. Thus, they are predicates of type $\langle vt, t\rangle$.

Along the line proposed by Tomita (2019), I incorporate first-order free logic into Champollion's Quantificational Event Semantics, as in (7).

(7) Type $\langle vt, t\rangle$ expressions (to be revised):
  a. $[\![\text{(to) leave}]\!] \rightsquigarrow \lambda f.\Sigma[_e \textbf{leaving} \wedge f)]$
  b. $[\![\text{forbid}]\!] \rightsquigarrow \lambda f.\Sigma[_e \textbf{forbidding} \wedge f]$
  c. $[\![\text{(to) see}]\!] \rightsquigarrow \lambda f.\Sigma[_e \textbf{seeing} \wedge f]$

where f stands for a variable of type vt. Thematic relations such as [agent] and [theme] are separated from verbal denotations, as shown in (8).

(8) Type $\langle e \cup v, \langle\langle vt, t\rangle, \langle vt, t\rangle\rangle\rangle$ expressions:

$$[\![\text{[agent]}]\!] \rightsquigarrow \lambda x \lambda N \lambda f.N[_e x : \textbf{ag} \wedge f]$$
$$[\![\text{[theme]}]\!] \rightsquigarrow \lambda x \lambda N \lambda f.N[_e x : \textbf{th} \wedge f]$$
$$[\![\text{[experiencer]}]\!] \rightsquigarrow \lambda x \lambda N \lambda f.N[_e x : \textbf{ex} \wedge f]$$

where $x : \mathbf{r}$ is abbreviation for $\lambda e.(x = \mathbf{r}(e))$ of type vt, and $\mathbf{r}$ stands for some thematic-role function, such as **ag(ent)**, **th(eme)**, **ex(periencer)**. They are the functions of the set of eventualities to the set of everything, namely, the Platonic universe. In other words, these functions assign each eventuality to something in the Platonic universe. Here is an example calculation for the sentence *Brutus stabbed Caesar*.

(9) a. $[\![\text{[agent]}]\!]([\![\text{Brutus}]\!])$
  $\Rightarrow \lambda N \lambda f.N[_e \mathbf{b} : \textbf{ag} \wedge f]$

b. $[\![\text{theme}]\!]([\![\text{Caesar}]\!])$
$\Rightarrow \lambda N \lambda f.N[_e\mathbf{c}:\mathbf{th} \wedge f]$

c. (9b)($[\![\text{stab}]\!]$)
$\Rightarrow \lambda f.\Sigma[_e\mathbf{stabbing} \wedge \mathbf{c}:\mathbf{th} \wedge f]$

d. (9a)((9c))
$\Rightarrow \lambda f.\Sigma[_e\mathbf{stabbing} \wedge \mathbf{c}:\mathbf{th} \wedge \mathbf{b}:\mathbf{ag} \wedge f]$

The first two steps (9a) and (9b) show that each verbal argument is combined with a thematic-relation head. Then, in the subsequent two steps, they each take the verbal denotation as their "arguments in λ-calculus". Instead of the original sentential closure in Champollion (2015), Tomita (2019) adopt **E!**. Figure 1 shows the derivation steps in the sentence. With the sentential closure **E!** of type vt, the corresponding neo-Davidsonian logical form $\exists e.[\mathbf{stabbing}(e) \wedge \mathbf{th}(e) = \mathbf{c} \wedge \mathbf{ag}(e) = \mathbf{b}]$ is obtained.

4 Problem in Tomita (2019)

Tomita (2019) proposed that **E!** is applied to the eventuality in the matrix clause but not to the one in the embedded clauses because the embedded infinitival clause is treated as an argument of the matrix verb.

Because the sentence (1b) implies $\Sigma[_{e'}\mathbf{leaving} \wedge \mathbf{m}:\mathbf{ag}]$, which does not commit to the actual existence of any leaving eventuality, this is compatible with any situation whether Mary left or not. However, according to my old proposal, the denotation for (1a) does not entail *Mary left*. Therefore, in Tomita (2019), I argued that complements of perceptual verbs denote the following expression.

(10) $[\![\text{XP} + [\mathbf{theme}]]\!] \rightsquigarrow \lambda N \lambda f.[[\![\text{XP}]\!](\lambda x.[N(\lambda e.\mathbf{th}(e) = x \wedge \mathbf{E!}(x) \wedge f(e))])]$

Then, the sentence (1a) commits to the existence of the event *Mary left* since **E!** applies to the embedded event.

However, some problems remain against his proposal. First, when a thematic-relation head contains the existence predicate **E!**, it can combine non-perceptual verbs such as *forbade* in (1b) that have a different entailment relationship. Second, since (11) entails neither *John saw Mary leave* nor *Mary left*, his proposal cannot predict the correct entailment.

(11) Paul forbade John to see Mary leave.

In the next section, I propose an alternative to this problem.

5 Proposal: Percolation of the Existence Property

Here, I consider the problem of entailment in infinitival complements. Intuitively, if an event of seeing occurs, it implies that something or some eventuality seen in that event also exists or occurs. I describe this intuition as a lambda term of type vt, such that $\lambda e.[\mathbf{E!}(e) \rightarrow \mathbf{E!}(\mathbf{th}(e))] \equiv [_e\mathbf{E!} \supset (\mathbf{E!} \circ \mathbf{th})]$. As mentioned in the previous sections, **th** is a function of a set of eventualities to something in the Platonic universe, and thus $\mathbf{E!} \circ \mathbf{th} \equiv \lambda e.\mathbf{E!}(\mathbf{th}(e))$. The proposed denotation of the verb *see* is now revised as (12), which implies that if some event of seeing exists in the actual world, its theme (internal argument) also exists.

(12) Type $\langle vt, t\rangle$ expression (perceptual verbs):
$[\![\text{(to) see}]\!] \rightsquigarrow \lambda f.\Sigma[_e\mathbf{seeing} \wedge (\mathbf{E!} \supset (\mathbf{E!} \circ \mathbf{th})) \wedge f]$

Unlike my old proposal in (10), thematic-relation heads do not contain such existential predicates as shown in (13).

(13) Type $\langle v, \langle\langle vt, t\rangle, \langle vt, t\rangle\rangle\rangle$ expression:
$[\![\text{[theme]}]\!] \rightsquigarrow \lambda e' \lambda N \lambda f.N[_e e':\mathbf{th} \wedge f]$

In Coppock and Champollion (2019), Hendriks' (1993) raising rule is applied to some thematic-relation heads due to the type mismatch between them and quantifiers of type $\langle et, t\rangle$. For example, (13) is shifted to the following expression:

(14) Type $\langle\langle vt, t\rangle, \langle\langle vt, t\rangle, \langle vt, t\rangle\rangle\rangle$ expression:
$[\![\text{[theme]}]\!] \rightsquigarrow \lambda M \lambda N \lambda f.M(\lambda e'.N[_e e':\mathbf{th} \wedge f])$

where $\vec{a}$ is a null sequence, $\tau = \sigma = t$, and $\langle \vec{c}, \sigma\rangle = \langle\langle vt, t\rangle, \langle vt, \sigma\rangle\rangle = \langle\langle vt, t\rangle, \langle vt, t\rangle\rangle$. The details of this rule are shown in Coppock and Champollion (2019), Hendriks (1993), and the Appendix in this paper. (14) takes a GQ-type eventuality for the verb *see*. The logical form corresponds to *(to) see Mary leave* is calculated in (15). The infinitival complements are of the GQ-type over events $\langle vt, t\rangle$.

(15) (Infinitival) VP: to see Mary leave

a. (14)($[\![\text{Mary leave}]\!]$) $\Rightarrow$
$\lambda N \lambda f.\Sigma[_{e'}\mathbf{leaving} \wedge \mathbf{m}:\mathbf{ag} \wedge (\lambda e'.N[_e e':\mathbf{th} \wedge f])]$

b. (15a)(12) $\Rightarrow \lambda f.\Sigma[_{e'}\mathbf{leaving} \wedge \mathbf{m}:\mathbf{ag} \wedge (\lambda e'.\Sigma[_e\mathbf{seeing} \wedge (\mathbf{E!} \supset (\mathbf{E!} \circ \mathbf{th})) \wedge e':\mathbf{th} \wedge f])]$

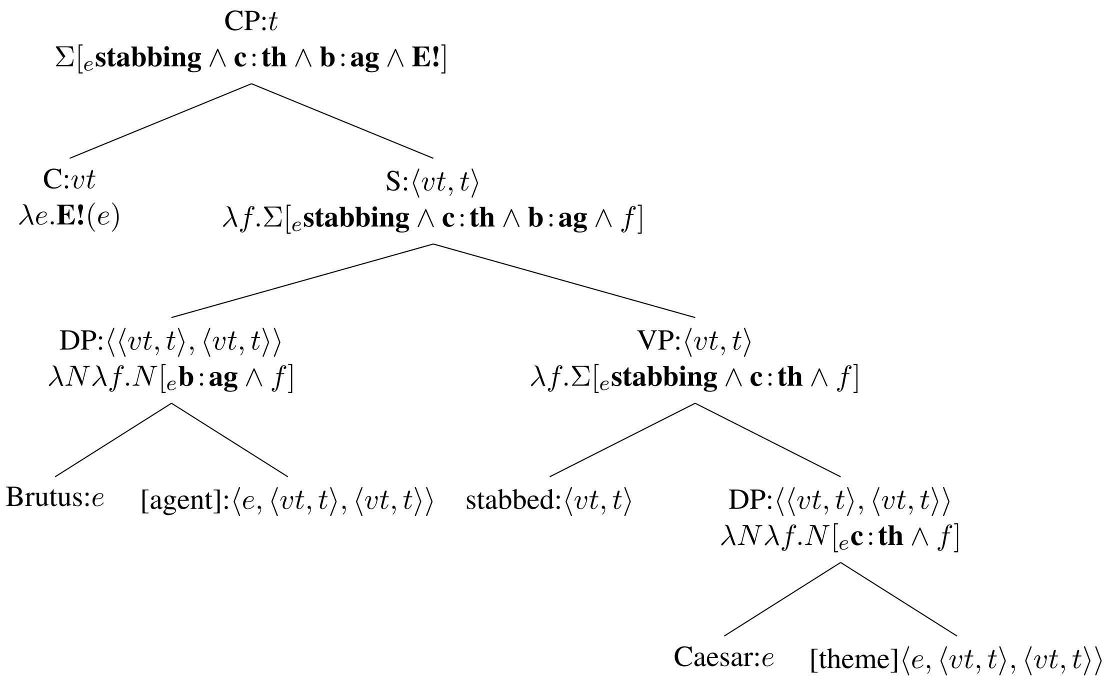

Figure 1: Derivation tree of *Brutus stabbed Caesar*

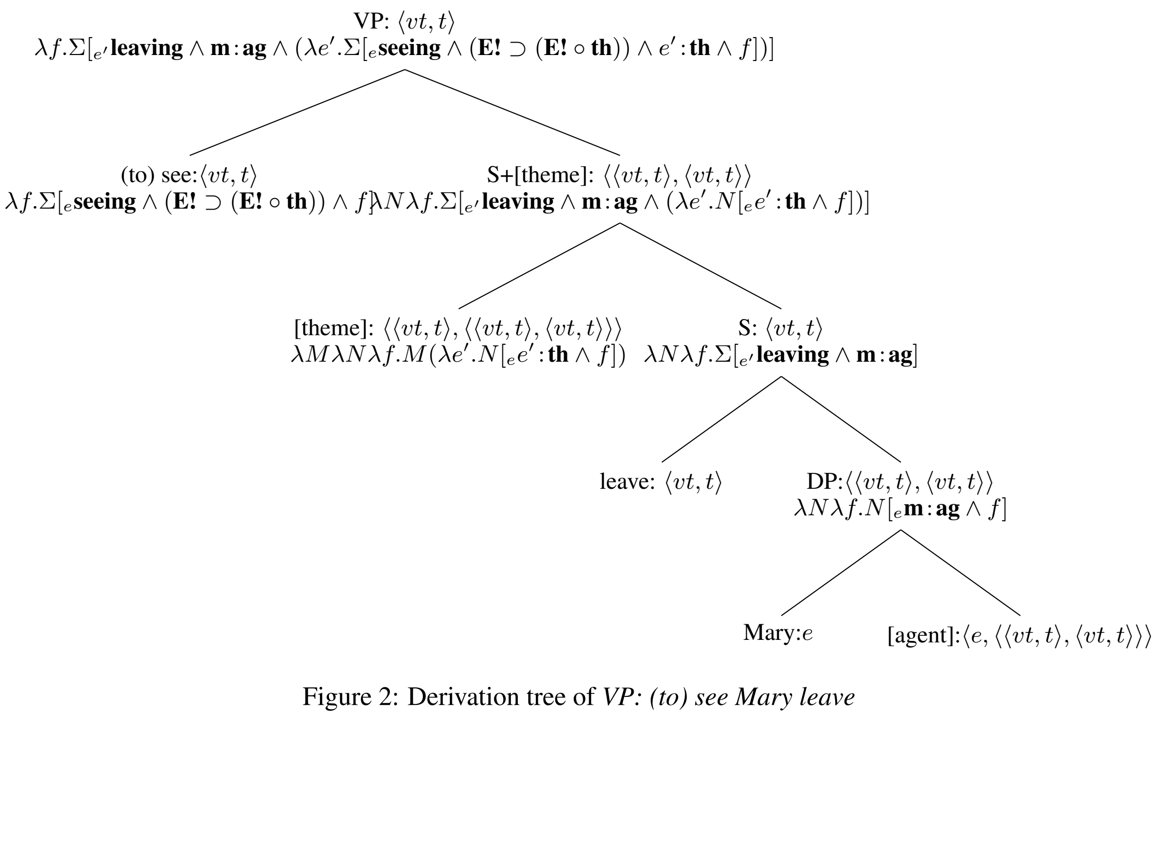

Figure 2: Derivation tree of *VP: (to) see Mary leave*

Figure 2 shows the corresponding derivation tree.

Unlike the previous proposal in (10), this does not entail *Mary left* unless the existential closure is given. If (15b) is given as a VP of the perceptual report *John saw Mary leave*, then the logical form contains $\mathbf{E!}(e) \supset (\mathbf{E!}(\mathbf{th}(e))$ and $\mathbf{E!}(e)$, which indicates that $\mathbf{th}(e) = e'$ also actually exists ($\mathbf{E!}(e')$), which results in a transparent reading. In contrast, when (15b) is given as a VP of infinitival complements in *Paul forbade John to see Mary leave*, this does not entail *Mary left* since the existence of the event in the matrix clause does not percolate to the events in the embedded clause, resulting in an opaque infinitival reading.

6 Discussion

Until this section, I argued that there is no need to use possible worlds to analyze some infinitival complements. However, there are some limitations and necessary future work. I will now discuss them and some related work.

6.1 Limitations

I further investigate which types of truth-makers are necessary for the other problems shown below.

Here are both transparent and opaque examples.

(16) a. Ralph saw Ortcutt to be a spy
b. Ralph saw ORTCUTT not to be a spy

In the above example, ORTCUTT (semantically focused proper name) in (16b) is a bit different from the non-focused one (16a) in that ORTCUTT in (16b) can designate a different person other than Ortcutt in (16a); see, e.g., Schwarzschild (1999). The same problems can be found in opaque infinitivals.

(17) a. Ralph considered the man in the brown hat to be a spy.
b. Ralph considered the man seen on the beach not to be a spy.

In both sentences in (17), *the man* can denote the same entity in context. However, both sentences can have different values (see, e.g., Quine, 1956). As like Tancredi and Sharvit (2022), the judge-parameter should be taken into consideration, which typically varies between an evaluator and a speaker according to the evaluator; none of which is introduced as a kind of stately truth-makers in Fine (2017). I do not address whether such truth-makers can be classified as different or be reduced to more general truth-makers. My current conjecture is that such truth-making tools are still necessary for semantics and can be incorporated into my proposal since these truth-makers play a completely different role from truth-making objects.

6.2 Related work

In the proposed framework, the existential quantifier in an infinitival complement always takes scope over the matrix clause, regardless of its syntactic position. From the syntactic point of view, the infinitival is extraposed and takes scope over some constituents in an upper clause. See also Higginbotham (1983, Sec. 2) for the related discussion. Similar discussions can be found in recent work on finite relative clauses; see, e.g., Koval (2019) and papers cited therein. As stated in the first two sections of this paper, possible worlds are required to analyze these embedded clauses. However, as shown in this proposal, when useful (but unusual) tools are admitted, it may become possible to reveal semantic phenomena without possible worlds.

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Appnedix: Generalized Raising Rule

Here, I modify Hendriks' raising rule to allow thematic-relation heads to take an argument of type $\langle et, \sigma\rangle$. If an expression denotes M of type $\langle \vec{a}, \langle b, \langle \vec{c}, \tau\rangle\rangle\rangle$ where $\vec{a}$ and $\vec{c}$ are possibly null sequences of any types, then this also denotes the following term:

(18) Type $\langle \vec{a}, \langle\langle b\tau, \sigma\rangle, \langle \vec{c}, \sigma\rangle\rangle\rangle$ expression:
$\lambda \vec{x} \lambda v \lambda \vec{y}.[v(\lambda z.[M(\vec{x})(z)(\vec{y})])]$

If this rule is applied to the thematic-relation head, it can take any argument of type $\langle et, \sigma\rangle$, such as the wh-phrase in the framework proposed in Unger (2010).