# Make Prompt-based Black-Box Tuning Colorful: Boosting Model Generalization from Three Orthogonal Perspectives

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#### Abstract

Large language models (LLMs) have shown increasing power on various natural language processing (NLP) tasks. However, tuning these models for downstream tasks usually needs exorbitant costs or is unavailable due to commercial considerations. Recently, black-box tuning has been proposed to address this problem by optimizing task-specific prompts without accessing the gradients and hidden representations. However, most existing works have yet fully exploited the potential of gradient-free optimization under the scenario of few-shot learning. In this paper, we describe BBT-RGB, a suite of straightforward and complementary techniques for enhancing the efficiency and performance of black-box optimization. Specifically, our method includes three plug-and-play components: (1) Two-stage derivative-free optimization strategy that facilitates fast convergence and mitigates overfitting; (2) Automatic verbalizer construction with its novel usage under few-shot settings; (3) Better prompt initialization policy based on instruction search and auto-selected demonstration. Extensive experiments across various tasks on natural language understanding and inference demonstrate the effectiveness of our method. Our codes and data are available at https://github.com/QiushiSun/BBT-RGB.

Keywords: Black-box Language Models, Derivative-free Optimization, Parameter-Efficient Tuning

### 1. Introduction

Transformer-based Language models (Vaswani et al., 2017) have achieved remarkable improvements among various NLP tasks (Qiu et al., 2020; Lin et al., 2022) in recent years. These models are mainly first pre-trained on a large-scale unsupervised corpus and then fine-tuned on a specific downstream task. However, this paradigm of pre-train and fine-tune face challenges in the era of Large Language Models (LLMs) (Brown et al., 2020; Ouyang et al., 2022; Chowdhery et al., 2022; Zhang et al., 2022; Scao et al., 2022; Touvron et al., 2023, inter alia). The ever-growing model size leads to a non-stop increase in the cost of tuning, and deploying separate copies of LLMs in real applications becomes exorbitantly expensive. Though recent research on Parameter-Efficient Tuning (Li and Liang, 2021; Lester et al., 2021, inter alia) alleviates the problem by tuning a small percentage of parameters while keeping the backbone frozen, the second problem arises: most LLMs are released as a service, and users can only access them through black-box APIs. This implies that the aforementioned tuning strategies become less viable owing to the inaccessibility of parameters and gradients, thereby causing a dilemma for downstream applications. Sun et al. (2022c) describe this scenario as Language Model-as-a-Service (LMaaS): Users are unable to tune the model parameters but can accomplish the tasks of interest by finding appropriate prompts with limited examples. Then, Black-Box Tuning (BBT) is proposed as a framework for derivative-free optimization under few-shot settings. Subsequently, BBTv2 (Sun et al., 2022a) was introduced as an improved version that prepends prompts to the hidden states of models, rather than merely injecting prompt tokens at the input layer. Recent works (Chai et al., 2022; Diao et al., 2023; Hou et al., 2023) have also revealed that black-box tuning based on this paradigm is promising.

However, the potential of black-box optimization is still not fully exploited. Previous tuning methods are prone to overfit / fall into local optimum under the scenario of few-shot learning. This phenomenon is triggered by both the characteristics of the Derivative-Free Optimization (DFO) algorithm and the unavailability of pre-trained prompts under few-shot settings. Moreover, they do not fully utilize the information returned by the black-box.

In this paper, we present BBT-RGB, a suite of practical, complementary, and pluggable techniques that further explore the possibility of blackbox tuning. We take one step forward in black-box tuning from the following three aspects 1) Employ-

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ing a two-stage DFO strategy for the attenuation of overfitting. 2) Utilizing multiple auto-selected verbalizers to exploit the context further. 3) Combining manual prompt with new search approach for task instructions improvement. Extensive experiments across various NLP downstream tasks demonstrate the superiority of our method. Besides, BBT-RGB can significantly outperform current gradient-based Parameter-Efficient tuning methods (Houlsby et al., 2019; Ben Zaken et al., 2022; Hu et al., 2022; Liu et al., 2022) under few-shot learning. Our main contributions can be summarized as follows:

- We propose a two-stage derivative-free optimization strategy that enables stable convergence of training tunable prompts while effectively mitigating the issue of overfitting.
- To further exploit the LLM's output, we propose a verbalizer selection process to derive multiple appropriate candidates. Moreover, instruction with judiciously selected demonstration is adopted for prompt initialization.
- A wide range of NLP tasks is covered to verify the effectiveness of our approach. By employing our method, optimization<sup>1</sup> under the derivative-free framework can reach comparative performance to full fine-tuning.

### 2. Preliminaries

In this section, we briefly introduce the basics of LLMs, prompt-based learning, and DFO.

Large Language Models and APIs Large language models (LLMs) (Devlin et al., 2019; Liu et al., 2019; Brown et al., 2020) have revolutionized the NLP landscape in the past few years. Given some examples of tasks as input, LLMs can be "prompted" to conduct a wide range of NLP tasks. These huge models are usually released as a service (Brown et al., 2020; Chen et al., 2021; Ouyang et al., 2022), which allows users to interact with the models deployed on the cloud servers through APIs. Unlike some popular open-source LMs (Devlin et al., 2019; Liu et al., 2019) that can be directly utilized by researchers, access to the parameters and gradients of LLMs is restricted due to commercial, ethical, and security concerns.

**Prompt-based Learning** Prompt-based learning (Liu et al., 2023) transforms an NLP downstream task into a masked language modeling (MLM) task and narrows the discrepancy between pre-training and fine-tuning. Based on the prompt

format, prompt-based learning can be categorized into discrete prompts and continuous prompts. Discrete prompts can be designed manually (Brown et al., 2020; Schick et al., 2020) or generated automatically (Gao et al., 2021). Continuous prompts are designed as a sequence of vectors (Qin and Eisner, 2021; Lester et al., 2021) that are usually prepended to the input and optimized by gradients. Recently, Sun et al. (2022c) propose BBT for optimizing prompts under gradient-free settings, as is shown in section 3. We mainly focus on the optimization of continuous prompts under the black-box settings in this paper.

Derivative-free Optimization Derivative-free optimization (DFO) algorithms are capable of solving complex problems without the back-propagation process. DFO generally employs a sampling-andupdating framework (Rios and Sahinidis, 2013; Wierstra et al., 2014; Qian et al., 2016) to improve the solution iteratively. For instance, Covariance Matrix Adaptation Evolution Strategy (Hansen and Ostermeier, 2001; Hansen et al., 2003), namely CMA-ES, is a widely adopted evolutionary algorithm for non-linear non-convex continuous optimization. At each iteration, the algorithm samples new potential solutions from a parameterized distribution model (e.g., multivariate normal distribution). Besides, we have COBYLA algorithm (Constrained Optimization BY Linear Approximation) (Powell, 1994, 1998) that builds a linear approximation model of the objective function and constraints within a trust region, iteratively updating the model based on the progress made in minimizing the objective function.

### 3. Black-Box Tuning

Given a batch of samples (X, Y) converted with prompt templates and label words, the original derivative-free prompt learning, as introduced by Sun et al. (2022a) first use a set of prompt embeddings p to concatenate the input tokens, creating the prompted input for LLMs with frozen backbones. The prompt  $p = p_0 + p_\theta$  consists of the initial prompt  $p_0 \in \mathbb{R}^D$ , which is manually/randomly selected and a tunable prompt  $p_{\theta} \in \mathbb{R}^{D}$  that is progressively optimized through a DFO algorithm like CMA-ES (Hansen et al., 2003). DFOs suffer slow convergence on high-dimensional problems, but fortunately, Aghajanyan et al. (2021) discover that PLMs exhibit low-dimensional reparameterization that is as effective for fine-tuning as the full parameter space. This finding indicates that the search space of  $p_{\theta}$  can be condensed into an intrinsic dimensionality  $z \in \mathbb{R}^d$  ( $d \ll D$ ) by using a (frozen) random projection matrix  $\Pi \in \mathbb{R}^{D \times d}$ , such that  $p_{\theta} = \Pi \cdot z$  will significantly decrease the cost

<sup>&</sup>lt;sup>1</sup>We follow bbtv2 (Sun et al., 2022a) to use random projection matrices to transform prompt parameters into low-dimensional subspaces.



Figure 1: An illustration of BBT-RGB. Given a backbone model with L layers. The target is to optimize continuous prompts  $z^l, l \in [1, L]$ . We use **Red**, **G**reen and **B**lue to indicate three distinct aspects of our strategy, which inspired the naming of our method. M<sup>2</sup> Verbalizers (Multi-Mixed Verbalizers) further utilize the information provided by the LLMs. In<sup>2</sup> Initialization (Instruction learning + In-context learning) improves prompt tuning by integrating both instruction and demonstration, noted as  $p_l$ . And Two-Stage DFOs exploit the advantages of different optimization methods. B represents the combination of DFOs.

of optimization. Subsequently, the task-specific inference of model *f* through API Call is performed to determine the fitness of candidate prompts using an objective function  $\mathcal{L}(f([P; X]), Y)$ , where  $\mathcal{L}$  is a loss function such as cross-entropy. Finally, the DFO algorithm iteratively refines the prompt for seeking  $p^* = \arg \min_p \mathcal{L}(f([P; X]), Y)$ .

In the era of LLMs, black-box optimization is a promising research target that can drive models for few-shot learning without access to gradients. Sun et al. (2022c) first propose BBT that focuses on optimizing continuous prompt by only accessing inference APIs and then present BBTv2 (Sun et al., 2022a) as an improved version. While some recent works focus on optimizing discrete prompts concurrent with our work. Diao et al. (2023) present blackbox discrete prompt learning with gradient estimation as their key feature. Hou et al. (2023) first use gradient-free methods to sample sub-optimal discrete prompts and then ensemble them by boosting algorithm. And Chai et al. (2022) acquire informative feedback to enhance derivative-free optimization through using frozen subnetworks as critics. Recently, Han et al. (2023) ingeniously leverage knowledge distillation to combine gradient descent and gradient-free optimization.

## 4. BBT-RGB

As is shown in Figure 1, we introduce our method: BBT-RGB, which contains three orthogonal optimization perspectives of derivative-free learning.

#### 4.1. Two-Stage DFOs

Previous works of black-box tuning mainly use CMA-ES to optimize the intrinsic dimensionality (Aghajanyan et al., 2021) of LLMs. Nonetheless, in the early training stage, the evolutionary algorithm (EA) exhibits a considerably faster convergence rate compared to the search-based algorithm (SA), which potentially causes fast overfitting. Then, the following steps would be futile. Thus, we design a novel two-stage DFO algorithm for black-box tuning, as is shown in algorithm 1.

We leverage the advantages of two different kinds of DFOs respectively. In stage I, we use EA to perform coarse-grained population-level optimization, which has a specific budget (Number of API Calls) to move toward the target swiftly. And the SA will use the remaining budgets in stage II for approximating the solution by dimension-level fine-grained search.

# 4.2. M<sup>2</sup> Verbalizers

Verbalizers, defined as words that can serve as labels, play a crucial role in prompt learning (Schick et al., 2020; Schick and Schütze, 2021a). However, most prior works employ a single verbalizer for gradient-free optimization, which cannot fully use the information, *i.e.*, logits returned by the black box model. To address this problem, we propose **M**ulti-**M**ixed verbalizers, which are constructed through the following methods: 1) manual verbalizer selection<sup>2</sup>. 2) search-based verbalizer construction

<sup>&</sup>lt;sup>2</sup>Specifically, we use synonyms in practice.

#### Algorithm 1 Two-Stage DFOs

- german i me en ge - ee
<b>Input:</b> popsize: $\lambda$ , intrinsic dimension: $d$
<b>Input:</b> budget1: <i>b</i> 1, budget2: <i>b</i> 2, backbone: <i>f</i> <sub>model</sub>
<b>Input:</b> $m, \delta, C, D //$ initialize state variables
<b>Output:</b> hidden variable: <i>z</i>
1: function Two-Stage DFO
// CMA-ES
2: repeat
3: <b>for</b> each hidden layer <b>do</b>
4: <b>for</b> <i>i</i> in 1 to $\lambda$ <b>do</b>
5: Sample $z_i$ from $\mathcal{N}(m,\delta^2 C)$
$6: \qquad f_i = f_{model}(z_i)$
7: end for
8: Update $m, \delta, C$ with $f$
9: end for
10: Update $z$ to min(f)
11: <b>until</b> $b1$ times $f_{model}$ call
// COBYLA
12: <b>for</b> each hidden layer <b>do</b>
13: repeat
14: <b>for</b> each search direction i in D <b>do</b>
15: Update <i>z</i> to min(f) along i
16: <b>end for</b>
17: Select a new set of D
18: <b>until</b> $b2//d$ times $f_{model}$ call
19: <b>end for</b>
20: end function

based on importance estimation by TF-IDF. 3) auto verbalizer generation based on neural nets (Gao et al., 2021). After the aforementioned approaches select verbalizers, the confidence of each category is represented by the average prediction probability of multiple verbalizers. Compared with the previous approach,  $M^2$  verbalizers take one step forward to exploit the information provided by the black box. Additionally, this approach can prevent the negative impact on model performance caused by a single unsuitable label word.

# 4.3. In<sup>2</sup> Initialization

An appropriate initialization has proven to play an essential role in effective prompt-based tuning while existing BBT methods mainly employ arbitrary selections for instructions and demonstrations (*e.g.*, randomly selecting some tokens from the vocabulary). Inspired by previous efforts (An et al., 2022; Prasad et al., 2022), we propose a modelagnostic strategy named as  $\ln^2$  initialization. The first component of our approach is a task-specific manual **In**struction. For the second part, we iterate through the training set and take each sample as a demonstration (Min et al., 2022), which is assessed on the validation set together with the pre-selected instruction. After that, the sample with the best performance is selected for **In**-context learning.

# 5. Experiments

## 5.1. Experimental Settings

**Backbones** We use  $RoBERTa_{LARGE}$  (Liu et al., 2019) as backbone throughout the main experiments. To verify the versatility, we also evaluate on other models including GPT-2<sub>LARGE</sub>, T5<sub>LARGE</sub> (Raffel et al., 2020) and BART<sub>LARGE</sub> (Lewis et al., 2020).

**Datasets.** To evaluate our proposed methods, we choose a series of tasks from GLUE (Wang et al., 2018). Specifically, we employ SST-2 (Socher et al., 2013) and Yelp (Zhang et al., 2015) for sentiment analysis, AGNews and DBPedia (Zhang et al., 2015) for topic classification, SNLI (Bowman et al., 2015) and RTE (Dagan et al., 2005) for natural language inference, and MRPC (Dolan and Brockett, 2005) for semantic paraphrasing.

**Methods and Hyperparameters.** For all the experiments, we adhered to the same settings as Sun et al. (2022a). For the optimization, the API call limit for each DFO algorithm is set to 8000. Regarding the baselines, we employ the results reported by Sun et al. (2022a) for comparison.

## 5.2. Main Results

As is demonstrated in table 1, we compare BBT-RGB with both gradient-based and gradient-free tuning methods. We observed different levels of improvement on various NLP tasks.

**Sentiment Analysis.** On both the SST-2 and Yelp datasets, our method surpasses all prior white-box methods, consistently demonstrating superior performance compared to the established baselines.

**Topic Classification.** Compared with the previous gradient-free method, BBT-RGB has a significant advancement in the evaluation based on DB-Pedia and AGNews but still needs to catch up to full model tuning. We hold the view that this is caused by a relatively large number of classes (categories), and it is difficult for the model to learn enough knowledge under few-shot settings.

**Entailment and Inference.** BBT-RGB benefits entailment and natural language inference tasks significantly; both experiments on SNLI and MRPC indicate surpassing full fine-tuning performance. In addition, we can observe a leap in the accuracy of RTE compared with previous baselines.

# 5.3. Ablation Studies and Analysis

**Ablation Studies.** We conduct ablation studies to verify the effectiveness of the three proposed techniques that formed the core of this paper, as demonstrated in Table 2.

Method	Tunable Params	SST-2 acc	Yelp P. acc	AG's News acc	DBPedia acc	MRPC F1	SNLI acc	RTE acc	Avg.
			Gradie	ent-Based Metho	ods				
Model Tuning	355M	85.39 ±2.84	91.82 ±0.79	86.36 ±1.85	97.98 ±0.14	77.35 ±5.70	54.64 ±5.29	58.60 ±6.21	78.88
Prompt Tuning	50K	68.23 ±3.78	61.02 ±6.65	84.81 ±0.66	87.75 ±1.48	51.61 ±8.67	36.13 ±1.51	54.69 ±3.79	63.46
P-Tuning v2	1.2M	64.33 ±3.05	92.63 ±1.39	83.46 ±1.01	97.05 ±0.41	68.14 ±3.89	36.89 ±0.79	50.78 ±2.28	70.47
Adapter	2.4M	83.91 ±2.90	90.99 ±2.86	86.01 ±2.18	97.99 ±0.07	69.20 ±3.58	57.46 ±6.63	48.62 ±4.74	76.31
LoRA	786K	88.49 ±2.90	90.21 ±4.00	87.09 ±0.85	97.86 ±0.17	72.14 ±2.23	61.03 ±8.55	49.22 ±5.12	78.01
BitFit	172K	81.19 ±6.08	88.63 ±6.69	86.83 ±0.62	$94.42 \pm 0.94$	66.26 ±6.81	$53.42 \pm 10.63$	52.59 ±5.31	74.76
	Gradient-Free Methods								
Manual Prompt	0	79.82	89.65	76.96	41.33	67.40	31.11	51.62	62.56
In-Context Learning	0	79.79 ±3.06	85.38 ±3.92	62.21 ±13.46	34.83 ±7.59	45.81 ±6.67	47.11 ±0.63	60.36 ±1.56	59.36
BBT	500	89.56 ±0.25	91.50 ±0.16	81.51 ±0.79	79.99 ±2.95	61.56 ±4.34	46.58 ±1.33	52.59 ±2.21	71.90
BBTv2	12K	90.33 ±1.73	92.86 ±0.62	85.28 ±0.49	93.64 ±0.68	77.01 ±4.73	57.27 ±2.27	56.68 ±3.32	79.01
BBT-RGB (ours)	12K	$92.89 \pm 0.26$	$\textbf{94.20} \pm 0.48$	85.60 ±0.41	$94.41 \pm 0.73$	<b>79.49</b> ±1.84	$60.71 \pm 0.66$	61.82 ±1.20	81.30

Table 1: Overall comparison between BBT-RGB and other methods (both gradient-based and gradient-free). The results are obtained with the RoBERTa<sub>LARGE</sub> backbone in 16-shot (per class) setting.

Method	SST-2	AG's News	RTE
BBT-RGB	91.00	85.65	61.80
w/o. M <sup>2</sup> Verb	91.00	85.59	61.33
w/o. Two-Stage	90.39	85.57	61.17
w/o. In <sup>2</sup> Init	90.77	84.31	60.17
w/o. Two-Stage & M <sup>2</sup> Verb	90.83	85.56	59.77
w/o. Two-Stage & In <sup>2</sup> Init	90.28	83.79	59.30
w/o. In <sup>2</sup> Init & M <sup>2</sup> Verb	90.66	83.75	59.47

Table 2: Ablation studies of BBT-RGB on SST-2, AG's News, and RTE

**Analysis.** We select two cases<sup>3</sup> to analyze the effectiveness of two-stage DFOs on Yelp. In Figure 2, the training loss (orange curve) converges to zero for both methods. While the oscillation of validation loss observed in pure CMA-ES case is mainly attributed to the nature of the adaptive algorithm.



Figure 2: Comparison of original CMA-ES and Twostage DFOs on Yelp dataset.

In stage II of our proposed two-stage DFOs method, a relatively gentle decrease in validation loss can be observed, demonstrating that dimension-level updates by COBYLA make the overall learning process smoother, which helps us curb the problem of fast overfitting. **BBT-RGB Across Models.** As shown in Table 3, to demonstrate versatility, we conducted additional experiments on both Decoder-only and Encoder-Decoder architecture models. It is evident that beyond encoder-only models, BBT-RGB also exhibits superior performance on other model architectures.

LM	Method	SST-2	AG's News	DBPedia			
	Decoder-Only Models						
	BBT	75.53 ±1.98	77.63 ±1.89	77.46 ±0.69			
GPT-2	BBTv2	83.72 ±3.05	79.96 ±0.75	91.36 ±0.73			
	BBT-RGB	$86.32 \pm 0.97$	82.01 ±0.81	<b>93.52</b> ±1.13			
	Encoder-Decoder Models						
	BBT	89.15 ±2.01	83.98 ±1.87	92.76 ±0.83			
T5	BBTv2	91.40 ±1.17	85.11 ±1.11	93.36 ±0.80			
	BBT-RGB	<b>92.91</b> ±0.97	85.50 ±1.32	<b>93.74</b> ±0.56			
	BBT	77.87 ±2.57	77.70 ±2.46	79.64 ±1.55			
BART	BBTv2	89.53 ±2.02	81.30 ±2.58	87.10 ±2.01			
	BBT-RGB	<b>92.63</b> ±1.43	82.76 ±1.74	88.26 ±1.06			

Table 3: Comparison of BBT-RGB and baselines on the large versions of GPT-2, BART, and T5.

#### 6. Conclusion

This paper proposes BBT-RGB, a set of practical techniques to drive more powerful derivative-free prompt-based learning. We make improvements from three independent aspects: (1) Two-stage derivative-free optimization algorithms for attenuating overfitting; (2) Versatile verbalizer construction with a robust selection process; (3) Using Instruction learning and demonstrations to exploit in-context information. All the modules are "plugand-play", and empirical studies across a series of tasks verify the effectiveness of our method.

## Limitations and Ethical Consideration

Limitations. Our limitations are threefold:

 Following previous works (Sun et al., 2022c,a), our proposed method lays much emphasis on

<sup>&</sup>lt;sup>3</sup>We choose CMA-ES (8,000 budgets) and COBYLA (12,000 budgets) for the two-stage DFOs in illustrations.

the optimization of continuous prompts. It can be applied to a majority of open-source Large Language Models (LLMs), but for some commercial models that do not provide loss, logits, or perplexity, the optimization is constrained to remain in the discrete form at the initial layer of the model.

- Since the algorithm is unable to achieve linear convergence, some of the tasks require more API calls, which may lead to extra costs when running on commercial models.
- Given that In<sup>2</sup>Init and M<sup>2</sup>Verb involve the search for verbalizers and demonstrations, our method takes a longer execution time compared to BBTv2, requiring approximately 25% additional runtime.

However, the essence of our contributions could be extended to broader scenarios under gradientfree settings, and we leave them as future research.

**Ethical Considerations.** Our method: BBT-RGB, aims to exploit the potential of black-box tuning further, and the contribution in this paper is fully methodological. Therefore, this contribution has no direct negative social or ethical impacts. Moreover, given that our approach requires significantly less computational resources compared to full-fine tuning, it is poised to contribute positively to the sustainable development of the AI community.

# Acknowledgements

This work is supported by Shanghai "Science and Technology Innovation Action Plan" Project (No.23511100700). Our method is also derived from a prize-winning solution of the *First International Algorithm Case Competition: PLM Tuning Track, Guangdong-Hong Kong-Macao Greater Bay Area.* Finally, we thank our anonymous reviewers for their insightful comments and suggestions.

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### A. Experimental Details

#### A.1. Implementation

Most of our experiments<sup>4</sup> are conducted with a single NVIDIA GTX 3090 GPU.

#### A.2. BBT-RGB Settings

The details of using BBT-RGB across seven NLP tasks are listed in Table 4. For each task, we report the average performance and standard deviation across three random seeds (42, 50, 66).

#### A.3. Hyperparameters Settings

The experimental settings in our paper are listed in Table 5. Sigma1 and Sigma2 are the hyperparameters for CMA-ES. Alpha refers to a constant scalar for stretching the distribution of random projection matrices, as is shown in equation 1.

$$\sigma_A = \frac{\alpha \hat{\sigma}}{\sqrt{d}\sigma_z},\tag{1}$$

 $\hat{\sigma}$  is the standard deviation of word embeddings from RoBERTa-Large, and  $\sigma_z$  is the standard deviation of the normal distribution maintained by the CMA-ES algorithm. The random projection matrices are frozen during the whole optimization process.

#### A.4. Templates and Verbalizers

The templates and verbalizers we employed are listed in Table 6 and Table 7 respectively.

### B. Clarifications on Two-Stage DFOs

Here we make some clarification about the choice of DFOs mentioned in Section 4.1, mainly about the advantages of using a search-based algorithm along with evolutionary algorithms for optimization:

- Search Precision: Search-based algorithm (SA) excels in fine-tuned searches due to better local adaptation, unlike evolutionary algorithms (EA) which may falter in detailed adjustments near optima.
- Better Convergence: SA demonstrates superior convergence, especially for local optima, by effectively leveraging minor variations in the search space.
- Parameters: SA offers greater flexibility in parameter tuning, enabling more precise adjustments when approaching the optimal solution.
- Robustness: SA are more resistant to evaluation noise, crucial for stability in later optimization stages, compared to the noise sensitivity of evolutionary algorithms.

<sup>&</sup>lt;sup>4</sup>Due to the memory requirements, experiments on MRPC and DBPedia datasets are conducted with NVIDIA V100 GPUs.

	SST-2	Yelp	AGNews	DBPedia	SNLI	RTE	MRPC
Two-Stage DFO	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	×
M <sup>2</sup> Verbalizers	×	$\checkmark$	×	$\checkmark$	×	$\checkmark$	$\checkmark$
In <sup>2</sup> Initialization	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	×

Table 4: The details of employing BBT-RGB,  $\checkmark$  refers to use the given technique on this task, and > vice versa

Task	Budget1 (CMA-ES)	Budget2 (COBYLA)	Alpha	Sigma1	Sigma2
SST-2	7,000	6,000	0.5	0.7	0.7
Yelp	8,000	6,000	0.9	0.4	0.2
ĀĠNews	8,000	6,000	0.1	<u>0</u> .ē	0.2
DBPedia	8,000	6,000	0.3	0.2	0.2
- SNLI	8,000	6,000	0.5	0.45	0.2
RTE	8,000	6,000	0.5	1	0.2
MRPC	8,000	0	0.3	0.3	0.2

Table 5: Hyperparameter Settings for BBT-RGB in different tasks.

Dataset	Template
SST-2	$\langle P \rangle \langle S \rangle$ . It was [MASK]
Yelp P.	$\langle P \rangle \langle S \rangle$ . It was [MASK]
AGNews	$\langle P \rangle$ [mask] News: $\langle S \rangle$
DBPedia	$\langle P \rangle$ [Category: [MASK]] $\langle S \rangle$
MRPC	$\left< P \right> \left< S_1 \right>$ ? [MASK], $\left< S_2 \right>$
RTE	$\left< P \right> \left< S_1 \right>$ ? [MASK], $\left< S_2 \right>$
SNLI	$\left< P \right> \left< S_1 \right>$ ? [MASK], $\left< S_2 \right>$

Table 6: Prompt templates used in this paper.  $\langle P \rangle$  is a sequence of continuous prompt tokens.  $\langle S \rangle$  is the original input text.

Dataset	M <sup>2</sup> Verbalizers
SST 2	Positive: exciting, all, indeed,
SST-2	Negative: ridiculous, worse, stupid, ···
Yelp P.	Positive: addictive, sensational, classic, ···
Telp F.	Negative: boring, worse, ugly, …
	World: South, China, Africa,
AG's News	Sports: Athletics, SPORTS, Sporting, ···
AG 3 News	Business: Banking, Manufacturing, Trade, …
	Tech: Digital, Internet, Tech,
	Company: Business, Products, ···
	Educational/Institution: Education, Schools, ···
	Artist: Artists, ···
	Athlete: Profile, ···
	Office Holder: Politics, ···
	Mean Of Transportation: Vehicles, ···
DBPedia	Building: Architecture, ···
221 00.00	Natural Place: Lakes, ···
	Village: Rural, ···
	Animal: Animals, Birds, ···
	Plant: Plants, plants, Flowers, ···
	Album: Album, Records, ···
	Film: Movies, Films, ···
	Written Work: Books, Fiction, ···
MRPC	Equivalent: Finally, Notably, Next,
RTE	Not Equivalent: Instead, Although, That,
	Yes: Indeed, So, Wordwide,
	No: Also, Now, meanwhile, ···
SNLI	Yes: Whatever, YES, Regardless,
SINLI	Maybe: Imagine, Usually, Typically, ···
	No: Besides, Unfortunately, Surprisingly, ···

Table 7: Examples of the M<sup>2</sup> Verbalizers used in practice.