Scaling Synthetic Logical Reasoning Datasets with Context-Sensitive Declarative Grammars

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Abstract

Logical reasoning remains a challenge for natural language processing, but it can be improved by training language models to mimic theorem provers on procedurally generated problems. Previous work used domain-specific proof generation algorithms, which biases reasoning toward specific proof traces and limits auditability and extensibility. We present a simpler and more general declarative framework with flexible context-sensitive rules binding multiple languages (specifically, simplified English and the TPTP theorem-proving language). We construct first-order logic problems by selecting up to 32 premises and one hypothesis. We demonstrate that using semantic constraints during generation and careful English verbalization of predicates enhances logical reasoning without hurting natural English tasks. We use relatively small DeBERTa-v3 models to achieve state-ofthe-art accuracy on the FOLIO human-authored logic dataset, surpassing GPT-4 in accuracy with or without an external solver by 12%.

1 Introduction

Language models trained only on natural language show lackluster capabilities at logical reasoning (McCoy et al., 2023; Mahowald et al., 2024). As a countermeasure, we can train neural models to match the output of symbolic reasoning systems (e.g., logic theorem provers, or other algorithms) on procedurally generated problems, to sharpen their reasoning capabilities. This process improves accuracy on some human-authored problems (Wu et al., 2021; Clark et al., 2020; Wu et al., 2022; Liu et al., 2023).

Previous work on synthetic first-order logic (FOL) reasoning datasets, RuleTaker (Clark et al., 2020), LogicNLI (Goodwin et al., 2020a) and FLD (Morishita et al., 2023), write dedicated code reimplementing the FOL axioms from scratch to generate proofs, and translate the generated problems to natural language. We propose Unigram, a framework for synthetic reasoning data generation, specifically designed to generate problems jointly into multiple languages. We represent grammars with concise and expressive rules binding two languages, and constraints to prune unwanted generations.

We write the most extensive grammar of FOL semantic fragments to our knowledge. We structure the generated expressions into PREMISE, HYPOTHESIS pairs, and annotate their logical relationship (entailment/contradiction/neutral) with a FOL solver, following the natural language inference (NLI) framework (Goodwin et al., 2020b). A simplistic FOL NLI problem is: PREMISE: *Everyone who is happy is rich. Mary is rich.* HYPOTHESIS: *Mary is happy* LABEL: *Neutral.*

We fine-tune DeBERTa NLI models (He et al., 2021) on Unigram-FOL and compare it with previous similar datasets. The 184M parameters (basesize) beats GPT-4 augmented or not with external theorem provers, on the FOLIO (Han et al., 2022) dataset. Our contributions are as follows: (i) A dataset of reasoning problems expressed in English and TPTP (a language that can be interfaced with numerous theorem provers) alongside Vampire proof annotations, covering FOL with equality and both finite and open domains, improved compositionality, and more extensive quantifiers. (ii) Ablations measuring the effect of constraining material conditionals usage, of using realistic English predicates, and of reimplementing LogicNLI with declarative generation instead of proof tree generation, highlighting that declarative can work better but that a richer logical modeling drives most of the improvement. (iii) A general reasoning problem grammar-based generation framework relying on solvers. The generation library, grammars, models, and generated dataset are publicly available¹.

¹[code:GitHub **?**][data:HF-datasets ²]

2 Related work

Synthetic datasets for reasoning Numerous works investigate the logical capabilities of NLP models using textual datasets and symbolic reasoning (Helwe et al., 2022). We focus on the grammarderived synthetic datasets. RuleTaker (Clark et al., 2020) explores this area with a subset of first-order logic. LogicNLI addresses a broader FOL subset (Tian et al., 2021). FLD explores full FOL (Morishita et al., 2023) and increased compositionality. Richardson et al. (2020); Richardson and Sabharwal (2022) use a solver to study the satisfiability in natural language using the Z3 solver and dedicated generation logic on constrained problems. PrOntoQA (Saparov and He, 2023) generates proofs from ontologies and then derives questions from the proofs to analyze chains of thoughts in language models. Other work explore non-standard logic with synthetic dataset, notably probabilistic (Sileo and Moens, 2023), paraconsistant (Kazemi et al., 2024), epistemic (Sileo and Lernould, 2023) logics.

Generation frameworks Multiple frameworks already implement generation from handwritten grammars. NLTK (Bird and Loper, 2004) has a context-free grammar tool, but cannot natively handle multiple languages or large-scale generation. Grammatical Framework (Ranta, 2004) is the closest tool to ours. It enables generation from abstract grammars and linearization into concrete grammars (e.g. French and English) but it is translationoriented and not context-sensitive. GLIF (Schaefer and Kohlhase, 2020) extends Grammatical Framework to parse English into logical formulas but is not suited for generation either.

3 Scalable dataset generation without forward inference

3.1 Forward inference

Previous NLI-style FOL reasoning datasets (Rule-Taker, LogicNLI, FLD) generate examples using proof generators that are based on the axioms of FOL. This requires domain-specific generation code and introduces unwanted complexity. Elimination and Introduction rules can cancel each other and create an illusion of reasoning depth. We found that some examples in the Proofwriter dataset (Tafjord et al., 2021) directly contain the premise in the hypothesis despite having a reasoning depth of 5. When constructing NLI pairs, generating neutral examples requires special strategies introducing a sampling bias, and it can be the same for contradiction generation. Proof generation techniques enable high reasoning depth but at the cost of breadth (linguistic variety and reasoning variety).

3.2 Declarative generation

We fully rely on an existing FOL solver and we propose Unigram, a simpler, more generic method to generate problems with multilingual grammars where rules bind multiple surface form realization templates. A Unigram Rule declaration specifies a type signature, and two surface form realizers, and optional validity constraints: **R**(output_type, input_types, realizers, constraints) The signature specifies the type of the rule output and the type of the arguments. The realizers take the arguments as input and map them to a string. We can have a realizer for a logical representation and a realizer for English. Using functions allows more expressivity than context-free grammars (Hunter, 2021), but for most cases with can treat template strings as functions using Python string.format. Constraints and realizers can access the state of the current generation as an anytree tree. Constraints are binary functions checking construction validity. One useful constraint is distinctness, e.g. (arguments of the same type should have a different realization), to avoid repetitions or statements like Mary likes Mary. We enable this constraint by default.

Generation algorithm We use a depth-first algorithm that recursively fills in the leftmost nonterminal leaf with random type-matching rule sampling until constraints are satisfied. This enables left-to-right generation, allowing realizers and constraints to access the current context. We recursively call realizers to construct surface forms (e.g. English text).

4 Application to first-order logic (FOL)

We use Unigram to enrich FOL problem generation while also avoiding ambiguity, starting as a superset of LogicNLI (grammar in Appendix B). To create a problem, we uniformly sample 1 to 32 sentences as premises and 1 sentence per hypothesis ensuring that all symbols are present in the premise. We exclude non-satisfiable formulas (paradoxes) in premise groups and hypotheses. We label pairs as ENTAILMENT if (premise $\land \neg$ hypothesis) is unsatisfiable, as CONTRADICTION if $(premise \land hypothesis)$ is unsatisfiable, and as NEUTRAL otherwise. Following Ruletaker and LogicNLI, we create problems with predicates over named individuals (e.g. *Mary is young*). We generate genderbalanced English surnames with CensusName. We now present new logical modeling features absent from the previous comparable datasets:

Explicit finite and open domains We explicitly mention the domain when using the quantifiers. We introduce two locations, *anywhere*, and a *room* with occupants e.g. *Mary, and Paul are the only persons in the room*. which logically means $\forall x, room(x) \rightarrow (x = \text{Mary} \lor x = \text{Paul})$. We can then quantify over the room (*everyone in the room*) or anywhere (*everyone anywhere*). By doing this, we can generate induction problems (checking that everyone in the room is happy if Mary and Paul are happy) and test reasoning with both finite and open domains. This requires handling FOL with equality which was not implemented in previous work.

Quantifiers and logical relationships We extend previous work with more complete quantifiers *not all, nobody, not everyone.* We leverage context-sensitivity to create a rule for polysyllogisms (predicate chains of the form all A are B, all B are C, all C are D. We also introduce *only if, unless, otherwise* as conditionals and allow sentence-level negation.

Constraining material conditionals Like previous work, we use material conditional to express conditional statements: *if p then q* is formalized as $p \rightarrow q$ i.e. $\neg p \lor q$. This means that the implication is true if *p* is false, and that negating $p \rightarrow q$ entails *q* both $\neg p$ which can be counter-intuitive. We use a constraint to eliminate all conditionals within the scope of negations and of other conditionals.

Improving predicate verbalization RuleTaker and LogicNLI use adjectives as logical predicates but do not handle their semantic interference. Rule-Taker do not consider being both *blue* and being *green* as contradictory. LogicNLI uses 379 adjectives treated as independent, including *ugly* and *ugliest*. FLD uses pseudo language like *the lard does hurtle pushup*. We prompted GPT-4 (May version) to *Generate 150 predicates where each predicate does not contradict nor entail any other predicate. Two examples: "enjoys wildlife photography" and "owns a smart tv"*. We remove errors and provide manual negations. We also use relationships (*like, is a sibling of*, modeling symmetry axioms when relevant, and adjectives.

Logical representation language Previous LogicNLI, RuleTaker, FLD, and FOLIO all use their own logical format, representing formulas as lists or strings. We use the TPTP (Sutcliffe, 2010) FOF language which is a standard syntax for theorem provers evaluation and is compatible with many theorem provers, notably Vampire (Reger et al., 2022), Z3 (De Moura and Bjørner, 2008) or Prover9 (Mc-Cune, 2005). We select the Vampire (Reger et al., 2022) theorem prover which provides short and readable proofs and details all the premises used during a derivation.

Complexity control Methods based on forward inference can theoretically control the proof depth using hyperparameters. Here, to avoid mostly sampling shallow problems, we limit the number of non-neutral examples where the proof to the number of examples using 5 inputs, for each number of inputs. Neutrals are still a majority by an order of magnitude. To sample hard neutral examples, we use a Gradient Boosting classifier with 100 trees (and scikit-learn 1.5.0 (Pedregosa et al., 2011) default parameters otherwise) to predict the labels based on unigram counts of the logical operators in the premise and hypothesis. We train on 1k examples, discard these, and then discard the most confident neutral predictions to achieve balanced labels.

5 Experiments

5.1 Methodology

We fine-tune a pre-trained NLI model on multiple synthetic FOL datasets: LogicNLI, FLD, Rule-Taker, and on Unigram-FOL. We then evaluate the direct effect on other three-way entailment downstream tasks, and on further fine-tuning on the training data of evaluation tasks (Phang et al., 2018).

We use the DeBERTa-v3 (He et al., 2021) NLI models trained on the tasksource collection (Sileo, 2024)². We use a learning rate of 1e-5 for DeBERTa-large and 2e-5 (Mosbach et al., 2021) for DeBERTa-base, 1 or 3 epochs (based on intrinsic validation accuracy) and Huggingface Transformers (Wolf et al., 2019) version 4.41 default Trainer arguments otherwise.

²hf.co/deberta-v3-base-tasksource-nli

Model size	Auxilary training	FOLIO	+ft	WANLI	+ft	CTRL	+ft	Fragments
D-base	-	49.5	74.3	65.2	77.4	46.2	56.7	63.6
D-base	RuleTaker	55.1	71.3	60.9	73.8	36.0	53.0	48.7
D-base	LogicNLI	50.5	69.3	61.1	72.4	38.4	54.4	56.3
D-base	FLD	<u>59.9</u>	72.3	60.0	73.6	38.2	55.8	56.8
D-base	Unigram-FOL	64.4	78.2	<u>63.6</u>	<u>75.6</u>	<u>42.8</u>	<u>56.6</u>	65.4
D-base	- Constrained_Conditionals	63.4	81.2	62.2	71.8	40.6	55.4	59.8
D-base	 Realistic_Predicates 	62.4	76.2	65.8	74.4	41.8	53.2	68.2
D-base	Unigram-LogicNLI	57.4	71.3	61.6	76.4	38.6	55.6	57.8
D-large	-	49.5	70.0	66.2	77.0	49.6	62.0	67.6
D-large	RuleTaker	58.1	77.2	68.5	<u>77.9</u>	43.1	60.7	61.7
D-large	LogicNLI	58.7	73.3	<u>68.5</u>	77.4	45.4	60.9	64.4
D-large	FLD	<u>60.9</u>	78.2	68.0	77.6	44.0	59.8	61.7
D-large	Unigram-FOL	63.4	82.2	75.4	81.6	<u>48.2</u>	62.2	73.2
D-large	Unigram-FOL+FLD	78.2	88.6	65.2	78.4	42.2	57.9	75.4

Figure 1: Comparison of auxiliary synthetic training datasets effect on the evaluation tasks. We report the average accuracy of two runs. D column refer to zero-shot D test accuracy after synthetic auxiliary training, and +ft refers to the test accuracy after auxiliary training then further fine-tuning D training set (in the previous column).

We generate 100k examples with a 80/10/10 train/dev/test split. but we only use 40k training training examples to match FLD. We use the FLD* version of FLD. We use the ProofWriter (Tafjord et al., 2021) open-world-assumption version of RuleTaker. We exclude LogicNLI examples labeled as paradoxes and we map all labels to NLI labels.

5.2 Evaluation datasets

We evaluate on two pure reasoning datasets, FO-LIO and Fragments, and on two more general datasets: FOLIO (Han et al., 2022) contains human-written FOL problems. We evaluate on the validation set to compare to Olausson et al. (2023) results who report 72.5% accuracy using a GPT-4 with a solver and 75.3% with chain-ofthoughts. We construct another validation set from 10% of train and map labels to NLI labels. (Wei et al., 2022) WANLI (Liu et al., 2022) is a NLI dataset with diverse and challenging reasoning patterns. ConTRoL (Liu et al., 2021) is a NLI dataset requiring multiple premises to derive the correct label, measuring contextual reasoning. Fragments (Richardson et al., 2020) is based on formal semantics templates and evaluate reasoning with quantifiers; this dataset is mostly suited to evaluation, as training quickly leads to almost perfect test accuracy.

Comparison with previous synthetic datasets Table 1 shows the accuracy of multiple auxiliary training datasets on the evaluation dataset. Unigram-FOL outperforms RuleTaker, LogicNLI, and FLD on all tasks with a comfortable margin, and leads to lesser degradation on the datasets that are not only focused on logic (WANLI, ConTRoL). The last line of the table combines Unigram-FOL (with the full 100k examples) with FLD and shows that combining generation methods can further push the state of the art on FOLIO.

We conduct ablations to better understand the source of this improvement, presented in the middle of Table 1.

Unigram-LogicNLI We use our declarative generation method on the base LogicNLI grammar to disentangle the effect of the generation technique from the grammar itself. This outperforms the original LogicNLI but not Unigram-FOL which highlights the value of our additional constructions.

Replacing Realistic Predicates We replace our generated predicates with the original LogicNLI adjectives (containing semantic interferences); this degrades FOLIO accuracy but does not strongly impact other NLI tasks, notably Fragments which mainly use adjectives as predicates.

Removing Conditionals Constraints Unrestricting usage of material conditionals harms the zeroshot transfer on FOLIO and the capabilities at more general reasoning, which confirms that removing counter-intuitive constructs can help transferability.

6 Conclusion

We showed that simple declarative grammars paired with solvers can outperform complex proof tree generators for reasoning dataset generations and released a new FOL reasoning dataset, models, and ablations. Our framework can help future reasoning research, notably on explanation since fully aligned TPTP code can be leveraged to model necessity and sufficiency. We plan to extend Unigram to planning, constraint satisfaction and modal logic.

Limitations

Reasoning methods based on neural networks do not provide formal guarantees and can introduce biases in real applications. They can be used as a complement to externalization methods (Olausson et al., 2023). Automatically formalizing a problem is difficult and can lead to mistakes (Olausson et al., 2023) which could be detected by internalizationbased methods. Our dataset could be used to automate formalization but we did not try such experiments. In addition, our work is only conducted with English language and encoder models, mainly used for verification and not generation. We only used one model architecture, DeBERTa, while other architectures like Albert (Lan et al., 2020) or other recursive architectures could be more suited to reasoning.

Ethical considerations

Our models are derived from language models which inherit bias from their training corpus. We did not conduct any human annotations, relying on already annotated datasets to validate our methodology. We use encoder models which have lower energy consumption than decoders (Luccioni et al., 2024) and performed experiments with less than 20 total days on a Nvidia A100 GPU.

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A FOL-nli example

PREMISE :

Christopher, Donald, Gene are the only persons in the room. Everyone in the room who collects antique jewelry plays the drums. Someone in the room designs and sews custom cosplay costumes for conventions. Christopher collects classic novels. Everyone in the room who enjoys deep-sea diving and exploring underwater caves enjoys kayaking or is a night owl or both. Christopher enjoys kayaking. Everyone in the room enjoys kayaking only if they collects antique jewelry. **HYPOTHESIS** : Christopher collects antique jewelry. LABEL entailment PREMISE (TPTP): room(c) & room(d) & room(g) & (![X]:(room(X) => (X='c' | X='d' | X='g'))) & (![X]:(room(X) => ((collects_jewelry(X)) => (plays_drums(X))))) & (?[X]:(room(X) & (designs_cosplay(X)))) & collects_novels(c) & (![X]:(room(X) => ((enjoys_diving(X)) => (enjoys_kayaking(X) | is_night_owl(X)))) & enjoys_kayaking(c) & (![X]:(room(X) => (enjoys_kayaking(X) <= collects_jewelry(X))))</pre>

B Unigram LogicNLI reimplementation

```
from unigram import Rule as R
ADJECTIVES = ['rich', 'quiet', 'old', 'tall', 'kind', 'brave', 'wise',
'happy', 'strong','curious','patient','funny','generous','humble']
# (We selected adjectives with no clear semantic interference)
NAMES = ['mary', 'paul', 'fred', 'alice', 'john', 'susan', 'lucy']
R.init(['tptp', 'eng'], "fof")
R('premise(' + ','.join(['rule']*16')'+','+'.'join(['fact]*8)+')',
  '&\n'.join([f'({i})' for i in range(24)]),
   '\n'.join([f'{i}' for i in range(24)]))
R('hypothesis(person,a)','1(0)','0 is 1')
for a in ADJECTIVES:
    R('adj', a), R('adj', f'~{a}', f'not {a}', weight=0.2)
R('property(adj,adj)', '(0(?)&1(?))', 'both 0 and 1')
R('property(adj,adj)', '(0(?)|1(?))', '0 or 1')
R('property(adj,adj)', '(0(?) <~>1(?))', 'either 0 or 1', weight=0.5)
R('property(adj)', '0(?)', '0')
R('rule(property, property)', '![X]:(0[?+X]=>1[?+X])',
'everyone who is 0 is 1')
R('rule(property, property)', '![X]:(0[?+X]<=>1[?+X])',
'everyone who is 0 is 1 and vice versa')
for p in NAMES:
    R('person', p)
R('fact(person, property)', '1[?+0]', '0 is 1')
R('fact(property)', '?[X]:(0[?+X])', 'someone is 0', weight=0.2)
R('rule(fact, fact)', '(0) => (1)', 'if 0 then 1')
R('rule(fact, fact)', '(0) \leq >(1)', 'if 0 then 1 and vice versa')
```