Assessing Resource-Performance Trade-off of Natural Language Models using Data Envelopment Analysis

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Abstract

Natural language models are often summarized through a high-dimensional set of descriptive metrics including training corpus size, training time, the number of trainable parameters, inference times, and evaluation statistics that assess performance across tasks. The high dimensional nature of these metrics yields challenges with regard to objectively comparing models; in particular it is challenging to assess the trade-off models make between performance and resources (compute time, memory, etc.).

We apply Data Envelopment Analysis (DEA) to this problem of assessing the resourceperformance trade-off. DEA is a nonparametric method that measures productive efficiency of abstract units that consume one or more inputs and yield at least one output. We recast natural language models as units suitable for DEA, and we show that DEA can be used to create an effective framework for quantifying model performance and efficiency. A central feature of DEA is that it identifies a subset of models that live on an efficient frontier of performance. DEA is also scalable, having been applied to problems with thousands of units. We report empirical results of DEA applied to 14 different language models that have a variety of architectures, and we show that DEA can be used to identify a subset of models that effectively balance resource demands against performance.

1 Introduction

A standard task in the machine learning lifecycle is to compare performance of many models; typically this process involves analyzing high-dimensional sets of summary statistics (hyperparameters, evaluation metrics, etc.). A common use case is quantifying the trade-off between performance and resource constraints; the goal being to achieve the best possible performance using minimal resources.

Meanwhile, multitask performance benchmarks (e.g., GLUE) have found widespread adoption in

the natural language processing (NLP) community, with transformer-based models often leading in evaluation performance (Vaswani et al., 2017). While the performance of transformer-based language models is impressive, they are notoriously resource-intensive, and often smaller models can more efficiently leverage a limited resource budget. However, it is nontrivial to demonstrate this fact by formulating a rational and fair comparison among models of different sizes and architectures.

In this paper, we apply data envelopment analysis (DEA) to this challenge of assessing model resource-performance trade-off (Charnes et al., 1978; Banker et al., 1984). DEA is a technique that originated in the operations research community, and it has been applied to a wide range of settings over many decades. It is traditionally concerned with rigorously defining *decision making* efficiency for teams, departments, companies, and other types of people-oriented organizations. But DEA is a generic technique that is based on solving a series of linear programs that are constructed to analyze the relative efficiency of *decision making* units (DMUs). A DMU is an abstract object that converts a set of inputs or resources into a set of outputs or benefits.

Our adaptation of DEA to the NLP context begins by treating each model as a DMU. Example inputs for the analysis include training time, training corpus size, the number of trainable parameters, and total monetary cost to train. Typical outputs would be performance evaluation metrics, evaluation throughput, etc.

Our main contribution in this paper is that we apply DEA to the problem of assessing the resourceperformance trade-off of machine learning models with an emphasis on evaluating the efficiency of language models. To our knowledge, this application of DEA to machine learning has not appeared in prior work. We do not assume familiarity with DEA, so in Section 4 we provide sufficient detail



Figure 1: Two simple examples of DEA. (*Left*) Each DMU, represented by a blue dot, has two outputs and a single input. Efficient DMUs generally lie far from the origin, which corresponds to DMUs that have high output per unit of input. (*Right*) Each DMU ingests two inputs and yields a single output. The set of DMUs that lie close to either axis are more efficient, which has the interpretation of low unit input per unit of output. The Pareto fronts are indicated with the heavy black line segments. Note that the dashed lines segments are not considered part of the front. Suboptimal DMUs are said to be "enveloped" by efficient DMUs.

to interpret our empirical results, which apply DEA to a variety of models, in Section 5.

2 Background

DEA was developed to enable performance assessment of teams of people and organizations such as not-for-profits, governmental organizations, departments within larger organizations and metaanalyses of industries. Traditional inputs include organizational staff salary, operational costs, and time. Traditional outputs include revenue, sales volume, and other organizational goals. The Pareto front of DEA in this context is also known as the *best practice frontier*, with the name derived from the observation that if a decision making unit (DMU) is on this frontier, it is objectively more efficient at transforming its inputs into outputs.

DEA analysis is applied to the inputs and outputs of a population of DMUs, and it assigns a scalar value between 0 and 1 to each DMU which expresses its efficiency. A DMU that is more effective at transforming inputs into outputs is considered more efficient.

Figure 1 illustrates two simple scenarios where DEA-type efficiency can be endowed with an intuitive representation. On the left, several DMUs are represented as blue dots, and each DMU ingests a single input and yields two outputs. A process that generates more output for a given input is considered to have greater efficiency. In this scenario, processes that lie on the Pareto front are far from the origin. At right, a different set of DMUs is shown. In this example, each DMU ingests two inputs and performance is assessed through a single output. A DMU with low input or large output will live close to one of the axes, and DMUs close to the origin are more efficient.

We now illustrate how DEA quantifies model efficiency by briefly describing a hypothetical, and simple, example. Consider a language model trained on a small amount of data with high accuracy for some task. DEA classifies this model as *more efficient* than (1) a model that achieves the same accuracy with more training data or (2) a model that achieves a lower accuracy with the same amount of training data.

We describe the formal definition of DEA below in Section 4, but for now it suffices to understand that DEA is the result of solving a sequence of linear programs. In particular, global solutions are guaranteed to be found rapidly and with high numerical precision.

A DEA-based approach to model comparison has several advantages. Since it is based on linear programming, the DEA framework lends itself to detailed theoretical analysis, which extends to interpreting solutions and modifying the programs in a controlled way. Furthermore, DEA is extensible both in the number of models that one can consider as well as the metrics that are used to represent each model's inputs and outputs. Finally, DEA is a scalable technique, since it allows one to analyze model performance of tens of thousands of models.

DEA can be applied to almost arbitrary data that satisfies a small number of weak conditions, but this flexibility comes with some cost. In order to derive meaning from DEA, one must carefully choose the set of inputs and outputs. This process of selection is necessarily subjective. The specifics of our implementation are not meant to be universal prescriptions but rather a demonstration of the concept and useful starting point.

3 Related work

DEA was introduced to the operations research community as a tool to help organizations quantify efficiency, and to objectively identify suborganizations that perform especially well (Charnes et al., 1978). Since its introduction, DEA has been applied to a vast array of fields, including international banking, cloud computing operations, economic sustainability, police department operations, hospital operations, and logistical applications (Charnes et al., 1995; Emrouznejad et al., 2016; Sun, 2002; Thanassoulis, 1995; Tsaples et al., 2022). Recent work has applied DEA to the machine learning context to optimize generalization error of models (Guerrero et al., 2022); we are unaware of prior work that applies DEA to the purpose of assessing model efficiency as we do here.

The theory of DEA continues to be an active field of research, and there have been many developments over the years in an attempt to address perceived shortcomings. In addition to the relaxation of constant returns to scale, "cross-efficiency" was introduced to generate unique efficiency rankings, and "stochastic data envelopment analysis" was developed to account for noise and uncertainty in the measurements that are used to inform DEA (Banker et al., 1984; Doyle and Green, 1994; Olesen and Petersen, 2016).

DEA is parallelizable, and it has been applied to problems with tens of thousands of DMUs (Phillips et al., 1990; Khezrimotlagh et al., 2019). Reducing the required computation time of DEA has also been explored (Ali, 1990, 1993).

Assessment of natural language understanding requires models to execute a range of linguistic tasks across different domains. Recognizing this, the GLUE benchmarks were introduced (Wang et al., 2018). The GLUE benchmarks consist of nine English sentence understanding tasks, so the performance of a single model on the GLUE benchmark yields a nine-dimensional vector. Typically, this vector is summarized through an average and reported as a single score.

A challenge of modern transformer-based machine learning is the large number of different architectures that can be tested. A fairly comprehensive overview of recent performance results related to language models is reported in (Narang et al., 2021). The primary method of reporting the results is in tabular form (see, for example Tables 1 and 2 of that reference), and comparative analysis is challenging. Others propose rigorous scientific methods and experiment design to help manage these challenges (Ulmer et al., 2022), (Dror et al., 2019), (Dror et al., 2017); we believe DEA is another tool that can be leveraged for these analyses.

Multidimensional descriptions of models are an inescapable feature of machine learning, and scalarization of such descriptions are equally common. Several metrics commonly used to describe models include precision, recall, accuracy, model size, and a variety measures of performance including BLEU and the family of ROUGE scores (Lin, 2004; Papineni et al., 2002). Aside from DEA, other wellknown examples of scalarization techniques used within the machine learning community include the F1 score, the Matthews correlation coefficient, and the Fowlkes–Mallows index, which summarizes the confusion matrix (Matthews, 1975; Yule, 1912; Fowlkes and Mallows, 1983).

4 Mathematical background

In this section, we provide sufficient background for one who is unfamiliar with DEA to interpret the results of Section 5.

When DEA was originally introduced, a technical requirement of the processes being assessed was that they exhibit *constant returns to scale*; for example this means that doubling the value of each input (e.g., sales staff) should cause the doubling of the value of all the outputs (e.g., monthly sales). DEA found widespread adoption despite this assumption almost never holding in practice. To address this perceived deficiency in DEA, the original formulation of DEA was modified to relax the constant returns to scale assumption. Several other extensions of DEA are now in common use, and we provide an overview below. More details can be found in (Cooper et al., 2007).

We introduce the setup and notation as follows. There are n DMUs, each of which consumes minputs and produces s outputs. Concretely, DMU_i consumes $x_{ij} \ge 0$ units of input and produces $y_{rj} \geq 0$ units of output, where $1 \leq i \leq m$, $1 \leq r \leq s$, and $1 \leq j \leq n$. The measurement units of the different inputs and outputs need not be congruent. For shorthand, we can express the data corresponding to DMU_i with the pair $(x_j, y_j) \in \mathbb{R}^{m+s}$ where $x_j = (x_{ij})_{i=1}^m$ and $y_i = (y_{rj})_{r=1}^s$. We call the pair (x_i, y_j) an activity. We can additionally arrange the input data in an $m \times n$ matrix $X = (x_{ij})$ and the output data in an $s \times n$ matrix $Y = (y_{rj})$. All the vectors x_i and y_i are assumed to be *semipositive*, meaning their entries are nonnegative and at least one entry is strictly positive. Equivalently, this means DMU_i consumes a positive amount of some input and produces a positive amount of some output.

4.1 CCR efficiency

We first introduce the (input-oriented) CCR model (Charnes et al., 1978), named as such after its creators Charnes, Cooper, and Rhodes. The CCR model is widely regarded as the first DEA model, and assumes constant returns to scale.

For each o, where $1 \le o \le n$, we evaluate DMU_o against its peers. Let $v = (v_i)_{i=1}^m \in \mathbb{R}^m_+$ and $u = (u_r)_{r=1}^s \in \mathbb{R}^s_+$ denote the weights that are applied to all the inputs and all the outputs of DMU_o , respectively. For an arbitrary activity $(x, y) \in \mathbb{R}^{m+s}_+$, the ratio $u^\top y / v^\top x$ measures efficiency by reducing the multiple inputs (resp. outputs) to a single "virtual" input (resp. "virtual" output), then returning the ratio of virtual output to virtual input. The CCR model aims to solve the following fractional program, indexed by o, where $1 \le o \le n$:

$$\underset{v,u}{\text{maximize}} \quad \theta \coloneqq \theta_o = \frac{u^\top y_o}{v^\top x_o} \tag{1a}$$

subject to $\frac{u^{\top}y_j}{v^{\top}x_j} \le 1$ for $j = 1, \dots, n$ (1b)

$$v \in \mathbb{R}^m_+, \ u \in \mathbb{R}^s_+.$$
 (1c)

The constraints (1b) bound the efficiency ratio of each DMU above by 1. The objective (1a) aims to find multipliers v, u that maximize the efficiency ratio of target DMU_o; due to the constraints (1b), clearly the optimal value θ^* is at most 1. It can be shown that Eq. (1) is equivalent to the following linear program, called the CCR multiplier form:

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$$\underset{v.u}{\text{maximize}} \quad \theta = u^{\top} y_o \tag{2a}$$

ubject to
$$v^{\top} x_o = 1$$
 (2b)

$$-v^{\top}X + u^{\top}Y \le 0^{\top} \qquad (2c)$$

$$v \in \mathbb{R}^m_+, \ u \in \mathbb{R}^s_+.$$
 (2d)

Equivalence of (1) and (2) can be verified through a simple exercise (Cooper et al., 2007). We call DMU_o CCR-efficient if $\theta^* = 1$ and there exists an optimal (v^*, u^*) with $v^* > 0$ and $u^* > 0$. Otherwise we call DMU_o CCR-inefficient.

It is possible for DMU_o to achieve the maximal value $\theta^* = 1$ and still be CCR-inefficient; this occurs when some $DMU_i \neq DMU_o$ consumes no more input than DMU_o, produces at least as much output as DMU_o, and either consumes strictly less of some input or produces strictly more of some output than DMU_{o} . In the literature, such CCRinefficient DMUs are occasionally referred to as weakly efficient, whereas DMUs satisfying both $\theta^* = 1$ and $(v^*, u^*) > 0$ are called strongly efficient (Cooper et al., 2004). In Figure 1, the weakly inefficient points are the endpoints of the dashed line segments that are parallel to the axes, and they are labeled "suboptimal." For the most part, we will not use this terminology and simply refer to DMUs satisfying $\theta^* = 1$ and not $(v^*, u^*) > 0$ as inefficient.

Computationally, one typically does not work with the CCR multiplier form directly, but rather with its dual. The dual of (2) is referred to as the *CCR envelopment form*:

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$$\underset{\theta,\lambda}{\text{minimize}} \quad \theta \tag{3a}$$

subject to $\theta x_o - X\lambda \ge 0$ (3b)

$$\lambda \ge y_o \tag{3c}$$

We now describe the connection between the CCR model and the assumption of constant returns to scale with an alternative interpretation of the envelopment form. Recall that an arbitrary pair of vectors $(x, y) \in \mathbb{R}^{m+s}_+$ is called an *activity*. The CCR model assumes there is a set of feasible activities, called the *production possibility set*, denoted P_{CCR} , which is defined as the polytope

$$P_{CCR} \coloneqq \{ (x, y) \in \mathbb{R}^{m+s}_+ : \\ x \ge X\lambda, \ y \le Y\lambda, \ \lambda \in \mathbb{R}^n_+ \},$$

and which has the following properties:

- 1. We assume the observed activities $\{(x_j, y_j)\}_{j=1}^n$ are contained in P_{CCR} .
- If (x, y) ∈ P_{CCR}, then (x̄, ȳ) ∈ P_{CCR} for any x̄ ≥ x, ȳ ≥ y. (In the economics literature, this is known as *free disposability* (Carter and Koopmans, 1952).)
- 3. Conic combinations of activities in P_{CCR} belong to P_{CCR} .

The last property implies constant returns to scale, as $(x, y) \in P_{CCR}$ implies $(tx, ty) \in P_{CCR}$ for any t > 0.

Eq. (3) can be viewed as finding the minimum θ such that $(\theta x_o, y_o) \in P_{CCR}$. More intuitively, Eq. (3) aims to synthesize a new activity using conic combinations of the observed activities $\{(x_j, y_j)\}_{j=1}^n$, i.e., $(X\lambda, Y\lambda)$ where $\lambda \in \mathbb{R}^n_+$. Eq. (3) tries to scale the inputs x_o as small as possible by the factor θ while ensuring that the synthesized activity $(X\lambda, Y\lambda)$ consumes no more inputs than θx_o and maintains output levels at least as high as y_o .

The envelopment form allows for an alternative characterization of CCR-efficiency: DMU_o is CCR-efficient if for any optimal solution (θ^*, λ^*) to (3), $\theta^* = 1$ and the solution has zero slack, i.e., the constraints (3b) and (3c) hold at equality; DMU_o is CCR-inefficient otherwise. If DMU_o is CCR-inefficient, then there exists $\lambda \in \mathbb{R}^n_+$ such that $x_o \ge X\lambda$, $Y\lambda \ge y_o$ and at least one inequality in the system holds strictly; the synthesized activity $(X\lambda, Y\lambda)$ is thus strictly better than (x_o, y_o) , and so DMU_o is said to be *enveloped* by the observed activities $\{(x_j, y_j)\}_{i=1}^n$.

Solving (3) alone is not enough to determine whether DMU_o is CCR-efficient; to determine whether every optimal solution to (3) has zero slack, one additionally solves the following linear program:

$$\underset{\lambda, s^-, s^+}{\text{maximize}} \quad \mathbf{1}^\top s^- + \mathbf{1}^\top s^+ \tag{4a}$$

subject to $s^- = \theta^* x_o - X\lambda$ (4b)

$$s^+ = Y\lambda - y_o \tag{4c}$$

$$\lambda \in \mathbb{R}^n_+, \ s^- \in \mathbb{R}^m_+, \ s^+ \in \mathbb{R}^s_+, \ (4d)$$

where θ^* in (4) is the optimal value of (3). If DMU_o is CCR-inefficient, we can additionally find its *reference set*, the set of CCR-efficient DMUs that envelop DMU_o thus making it CCR-inefficient. The

reference set is defined based on the max-slack solution $(\theta^*, \lambda^*, s^{-*}, s^{+*})$ of (3) and (4) to be

$$E_o^{CCR} = \{j \in \{1, \dots, n\} : \lambda_j^* > 0\}.$$

4.2 BCC efficiency

The constant returns to scale assumption of the CCR model can be problematic when comparing language models, e.g., one typically expects diminishing returns from increased training time. Fortunately, this can be relaxed with a very simple modification to the CCR formulation (Banker et al., 1984). The so-called BCC model, named after its creators Banker, Charnes, and Cooper, addresses this shortcoming and allows for variable returns to scale by adding a single additional constraint, namely $1^{T}\lambda = 1$, on the production possibility set. The BCC envelopment form, which is almost identical to Eq. (3), is as follows:

$$\underset{\theta,\lambda}{\text{minimize}} \quad \theta \tag{5a}$$

subject to
$$\theta x_o - X\lambda \ge 0$$
 (5b)

$$Y\lambda \ge y_o$$
 (5c)

$$\mathbf{1}^{\top} \lambda = 1 \tag{5d}$$

$$\theta \in \mathbb{R}, \ \lambda \in \mathbb{R}^n_+.$$
 (5e)

The dual of (5) is the BCC multiplier form:

 $\underset{v,u,u_0}{\text{maximize}} \quad u^{\top} y_o - u_0 \tag{6a}$

ubject to
$$v^{\top}x_o = 1$$
 (6b)

$$-v'X + u'Y - u_01' \le 0'$$
 (6c)

 $v \in \mathbb{R}^m_+, \ u \in \mathbb{R}^s_+, \ u_0 \in \mathbb{R}.$ (6d)

The production possibility set P_{BCC} of the BCC model is defined as

$$P_{BCC} = \{ (x, y) \in \mathbb{R}^{m+s}_+ : \\ x \ge X\lambda, \ y \le Y\lambda, \ 1^\top \lambda = 1, \ \lambda \in \mathbb{R}^n_+ \}.$$

The envelopment form (5) can be viewed as finding the minimum θ such that $(\theta x_o, y_o) \in P_{BCC}$. We call $DMU_o BCC$ -efficient if for any optimal solution (θ^*, λ^*) to (5), $\theta^* = 1$ and the solution has zero slack, i.e., the constraints (5b) and (5c) hold at equality; DMU_o is *BCC-inefficient* otherwise. As in the case of the CCR model, one not only solves

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(5) but also the following:

$$\underset{\lambda, s^-, s^+}{\text{maximize}} \quad \mathbf{1}^\top s^- + \mathbf{1}^\top s^+ \tag{7a}$$

subject to $s^- = \theta^* x_o - X\lambda$ (7b)

$$s^+ = Y\lambda - y_o \tag{7c}$$

$$1^{\top}\lambda = 1 \tag{7d}$$

$$\lambda \in \mathbb{R}^n_+, \ s^- \in \mathbb{R}^m_+, \ s^+ \in \mathbb{R}^s_+, \ (7e)$$

where θ^* in (7) is the optimal value of (5).

If DMU_o is BCC-inefficient, we are interested in finding its *reference set*, the set of BCC-efficient DMUs that envelop DMU_o thus making it BCCinefficient. The reference set is defined based on the max-slack solution (θ^* , λ^* , s^{-*} , s^{+*}) of (5) and (7) to be

$$E_o^{BCC} = \{ j \in \{1, \dots, n\} : \lambda_j^* > 0 \}.$$

If DMU_o is BCC-efficient, we can additionally determine returns to scale as follows:

- 1. Increasing returns to scale prevails at (x_o, y_o) iff $u_0^* < 0$ for all optimal solutions to (6).
- 2. Decreasing returns to scale prevails at (x_o, y_o) iff $u_0^* > 0$ for all optimal solutions to (6).
- 3. Constant returns to scale prevails at (x_o, y_o) iff $u_0^* = 0$ for some optimal solution to (6).

Suppose we solve (6) and obtain $u_0^* < 0$. We then solve the following modified program:

$$\max_{\substack{v,u,u_0\\v,u,u_0}} u_0 \tag{8a}$$

subject to $v^{\top} x_o = 1$ (8b)

$$u^{\top} y_o - u_0 = 1 \tag{8c}$$
$$- v^{\top} X + u^{\top} Y - u_0 1^{\top} \leq 0^{\top}$$

$$v \in \mathbb{R}^m_+, \ u \in \mathbb{R}^s_+, \ u_0 \le 0.$$
 (8e)

If (8) yields an optimal solution with $u_0^* = 0$, then constant returns to scale prevails at (x_o, y_o) , otherwise increasing returns to scale prevails. If on the other hand we solve (6) and obtain $u_0^* > 0$, (8) can be modified by replacing the constraint $u_0 \le 0$ with $u_0 \ge 0$ and switching the optimization sense to minimize u_0 .

Since the BCC envelopment form differs from the CCR envelopment form only in the addition of the convexity constraint $1^{\top}\lambda = 1$, if DMU_o is CCR-efficient, it is also BCC-efficient, and constant returns to scale prevail at DMU_o. The CCR score θ^*_{CCR} is called the (global) technical efficiency (TE) as the CCR model ignores the effects of scaling. The BCC score θ^*_{BCC} is called the (local) pure technical efficiency (PTE) as the BCC model accounts for variable returns to scale. The scale efficiency (SE) is defined as

$$SE = \frac{TE}{PTE} = \frac{\theta^*_{CCR}}{\theta^*_{BCC}}.$$
 (9)

Note that $0 \le SE \le 1$. Eq. (9) implies a decomposition of technical efficiency into pure technical efficiency and scale efficiency; if technical efficiency TE is low, it is either because of inefficient operation (low PTE) or poor scaling of resources (low SE).

We remark that all of the CCR and BCC models we consider are *input-oriented*, as they attempt to reduce input consumption while maintaining the same if not higher level of output production. We do not consider *output-oriented* models which consider the opposite situation where output production is increased while maintaining the same or lower level of input consumption.

5 Results and analysis

In this section, we describe the results of applying DEA to compare a variety of NLP models. The input features and the output features were selected to incorporate aspects of training, evaluation and task performance. Since training is one part of our analysis, several identical versions of the same models are represented in the set of models considered but with different learning rates selected. We also incorporate several simpler models as baselines including TF-IDF and GloVe embeddings with linear classifiers (Pennington et al., 2014).

The transformer models are pretrained models that are sourced from the Hugging Face Model Hub (Hugging Face, 2022). Each transformer model appears three times: once for each of the learning rates 10^{-3} , 10^{-4} , and 10^{-5} . The base models are bert-base-uncased, bert-large-uncased (Devlin et al., 2019), roberta-base (Liu et al., 2019), and their distilled versions: distilbert-base-uncased, distilroberta-base (Sanh et al., 2019). The GloVe embeddings used are all trained on the Wikipedia 2014 and Gigaword 5 6B corpuses and vary in embedding dimension between 50, 100, 200 and 300 (Pennington et al., 2014). As our simplest baseline we use scikit-learn's implementation of TF-IDF which varies in vocabulary size

between 100, 500, 1000, 5000, 10000 and 15000 (Pedregosa et al., 2011).

The number of trainable parameters for the transformer and other deep network models is determined by the model architecture and is typically in the millions. For the simpler embedding-based models, the number of trainable parameters is determined by the embedding dimension or vocabulary size. The GLUE benchmarks were coalesced in the standard manner by applying an average of all the scores. This score was treated as an output.

We ran each model through the standard GLUE benchmark by training them on the train split of the dataset and evaluating them on the eval split; in doing so we generated several dozen metrics for each model. These metrics include standard metrics that capture model throughput, running time and performance; a condensed representative summary is presented in Table 1.

In practical applications of DEA, if the analysis uses far more inputs and outputs than the number of DMUs, then the typical outcome classifies all DMUs as Pareto efficient. There can still be value in analyses where this happens, this is atypical and we wish to avoid it. A rule of thumb advises that the number of DMUs should be at least twice the number of inputs and outputs considered (Cook et al., 2014; Golany and Roll, 1989). Following this advice, we run an analysis with just two inputs and two outputs. The inputs we use are log (# trainable params) and total train runtime. The outputs were average score across all GLUE tasks and average eval throughput (samples/second).

We nonlinearly transform the number of trainable parameters by applying log to it for two reasons. First, there is a large disparity between the number of trainable parameters that the simple models have, and the number of trainable parameters that the transformer models have. The result of this gap is that the feature effectively becomes a binary indicator of whether the model is a transformer or not, and this is not what we would like the feature to convey. The second reason is based on empirical observations about performance. Informally, we expect performance to be a sublinear function of model size. That is, model performance should improve as a function of model size, but with decreasing returns.

We ran our experiments via Google Cloud Platform's Vertex AI Pipelines. Transformer models were trained on n1-highmem-8 instances (8 vC-PUs, 52 GiB memory) and one NVIDIA T4 GPU with CUDA toolkit version 11.2. Non-transformer models were trained on e2-standard-4 instances (4 vCPUs, 16 GiB memory). All experiments used Python 3.8 and, at the time of writing, the latest versions of major libraries¹. Our experiment script was a modified version of the run_glue.py script included with Hugging Face's examples ². Runtimes for all tasks varied from minutes to hours depending on the task and model but all experiments were completed within 24 hours.

After generating the model metrics, we constructed the relevant linear programs described in Section 4, and we solved them using Gurobi version 9.5.2.

The results shown in Table 2 use the following definitions. The column headed "CCR score" reports the optimal objective value of the program in Eq. (3), and the "BCC score" reports the optimal objective value of the program in Eq. (5). The column headed "scale efficiency" reports the ratio of the two optimal values, and is defined explicitly in Eq. (9). The column "CCR eff." indicates whether the optimal solution to (3) has zero slack, and reports the result of solving Eq. (4). The column headed "BCC eff." indicates whether the optimal solution to Eq. (5) has zero slack and requires solving Eq. (7) to make the determination. Note that several models exhibit a BCC score of 1.000 while not being BCC-efficient, i.e., they are weakly efficient. Finally, the "ret. to scale" column containing either CRS or DRS (IRS does not occur here) reports the results of Eq. (6) and Eq. (8). CRS indicates constant returns to scale, which corresponds to item 3 in the list appearing just prior to Eq. (8). DRS indicates decreasing returns to scale. In this case, DRS corresponds to item 2 in that same list.

As a result, of this table, it is clear that the models glove-50-linear, tfidf-1000-linear, and roberta-base with lr=1e-4 perform well overall. It is also clear that the BCC equations provide a view of model performance that benefits the more complex models. This confirms the general intuition that large changes in model size, complexity and

¹The libraries and their versions are: (torch, 1.11.0), (transformers, 4.20.1), and (scikit-learn, 1.1.1).

²https://github.com/huggingface/ transformers/blob/v4.20.1/examples/ pytorch/text-classification/run_glue.py

other inputs yield incremental improvements in performance. Additionally, it shows that bert-large-uncased models are suboptimal, requiring a lot of time and space in exchange for performance that is similar to that of other models.

6 Conclusions and Future Work

We have applied Data Envelopment Analysis to the challenge of quantifying the trade-off that exists between model performance and resource demands. We base this analysis on standard high-dimensional summary statistics that describe each model. We apply DEA to the analysis of 14 natural language models, and from this analysis we identify both simple and transformer-based models that effectively balance the competing objectives.

We demonstrate that the method is feasible and scales well. Future work can refine the approach presented above in several directions. First, specifics of our analysis can be modified by selecting different sets of inputs and outputs, or by selecting different ways of normalizing the inputs and outputs. Although DEA is a quantitative framework, there is much subjectivity in how the analysis is set up and interpreted. Second, it would be interesting to consider a more principled approach to the normalization of inputs and output attributes used in the analysis. We take the \log of the number of trainable parameters to amplify the difference between models where the number of parameters is small, as well as to capture diminishing resource cost once models are sufficiently large. For future work, one may apply exp to achieve the opposite effect. In addition, for attributes that take on negative values, since DEA assumes semipositive data, one may consider splitting the attribute into its positive and negative parts. Third, we have only considered input-oriented models, and so inherent in our approach is the goal of minimizing input consumption while maintaining best-in-class performance. An output-oriented approach is conversely interested in holding input resources constant while producing superior results. We leave investigation of these types of models to future work. Finally, it seems possible that DEA might be integrated into the training process, where the analysis is used to direct training time, parameter size, performance criteria. Due to the high-dimensional nature of language model descriptions, we believe that DEA is well-suited for language model assessment.

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Metric	Percentiles: 25	50	75
eval_steps_per_second	0.258	0.295	0.561
train_samples_per_second	70.223	101.606	143.307
num_trainable_params	82119169	109483009	124646401
eval_samples_per_second	129.03	147.476	280.338
eval_runtime	5.3507	10.1712	11.6252
train_runtime	4011.6	5658.1	8186.7
train_steps_per_second	0.828	1.148	1.674
eval_combined_score	0.867	0.880	0.894
eval_spearmanr	0.866	0.878	0.893
eval_pearson	0.868	0.881	0.895

Table 1: Representative data for the stsb tests. The five models tested are: bert-base-uncased, bert-large-uncased, distilbert-base-uncased, distilbert-base, and roberta-base with three different learning rates 10^{-5} , 10^{-4} and 10^{-3} . In addition to stsb, the other tests are mrpc, qqp, wnli, rte, mnli, cola, sst2, and qnli. For each distinct model, each test, and each learning rate, similar metrics are generated, for a total of over 100 different metrics.

	θ^*_{CCR}	θ^*_{BCC}	SE	CCR eff.	BCC eff.	RTS	GLUE score
glove-50-linear	1.000	1.000	1.000	+	+	\rightarrow	0.408
tfidf-1000-linear	1.000	1.000	1.000	+	+	\rightarrow	0.591
roberta-base, lr=1e-4	0.501	1.000	0.501		+	\downarrow	0.830
distilroberta-base, lr=1e-5	0.499	1.000	0.499		+	\downarrow	0.815
tfidf-10000-linear	0.999	1.000	0.999				0.591
tfidf-5000-linear	0.999	1.000	0.999				0.591
tfidf-500-linear	0.999	1.000	0.999				0.610
tfidf-15000-linear	0.999	1.000	0.999				0.591
glove-100-linear	0.952	0.961	0.990				0.455
roberta-base, lr=1e-5	0.460	0.919	0.500				0.807
bert-base-uncased, lr=1e-4	0.486	0.913	0.533				0.799
distilroberta-base, lr=1e-4	0.479	0.863	0.555				0.782
glove-200-linear	0.841	0.845	0.996				0.446
distilbert-base-uncased, lr=1e-5	0.460	0.842	0.546				0.579
bert-base-uncased, lr=1e-5	0.437	0.841	0.519				0.789
bert-large-uncased, lr=1e-5	0.385	0.835	0.461				0.800
glove-300-linear	0.827	0.834	0.991				0.460
distilbert-base-uncased, lr=1e-3	0.466	0.819	0.569				0.740
distilbert-base-uncased, lr=1e-4	0.473	0.803	0.588				0.769
bert-large-uncased, lr=1e-4	0.411	0.741	0.554				0.772
distilroberta-base, lr=1e-3	0.452	0.679	0.665				0.729
roberta-base, lr=1e-3	0.436	0.654	0.666				0.729
bert-base-uncased, lr=1e-3	0.410	0.543	0.756				0.703
bert-large-uncased, lr=1e-3	0.225	0.312	0.721				0.378

Table 2: Efficiency scores, returns to scale characterizations of BCC-efficient models, and GLUE scores (average performance across tasks). Models are ranked first by their BCC score, then by their CCR score. Returns to scale characteristics (increasing = \uparrow , decreasing = \downarrow , constant = \rightarrow) indicated only for BCC-efficient models.

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