Learning Morphological Productivity as Meaning-Form Mappings

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Abstract

Child language acquisition is famously accurate despite the sparsity of linguistic input. In this paper, we introduce a cognitively motivated method for morphological acquisition with a special focus on verbal inflections. Using UniMorph annotations as an approximation of children's semantic representation of verbal inflection, we use the Tolerance Principle to explicitly identify the formal processes of segmentation and mutation that productively encode the semantic relations (e.g., past tense) between stems and inflected forms. Using a child-directed corpus of verbal inflection forms, our model acquires the verbal inflection morphemes of Spanish and English as a list of explicit and linguistically interpretable rules of suffixation and stem change corresponding to sets of semantic features.

1 Introduction

One of the greatest challenges a child faces as they acquire their native language is to combat the sparsity of the linguistic input. Paradigm saturation, the proportion of possible inflectional categories that a lemma realizes in a corpus, is a measure of morphological sparsity. An analysis of childdirected Spanish, for example, shows that each verb lemma only appears in 7.9% of the possible inflectional categories on average; even the most saturated lemma (i.e., decir 'to say') only appears in 46 of the 55 inflectional categories that are available in a child-directed corpus (Lignos and Yang, 2016). Similarly, some inflectional categories are used far more frequently than others: the most common, 3rd-person singular present indicative, which is often taken as the default form in linguistic analysis, appears tens of thousands times whereas some (e.g., 2nd-person plural imperfect subjunctive) appear only once, if at all (Lignos and Yang, 2016). We provide an illustration of the sparsity of our

own Spanish training data (\S 3.2) in Figures 1 and 2. In order to acquire the verbal morphology of a language, it is necessary for the child to generalize well beyond the input.

The uneveness of morphological distributions entails that children learn morphology in a piecemeal fashion. Brown's (1973) study, for instance, establishes that English-learning children generally acquire the progressive -ing before the noun plural -s, followed by irregular past tense and then the regular past tense -ed. The order of acquisition is related to the input frequency of these morphemes as well as their regularity (Yang 2016). However, children recognize morphological regularity very early and apparently categorically: productive rules are often extended to exceptions whereas analogical errors on the basis of similarity are virtually unattested. For example, as is well known, past tense over-regularization errors (e.g., go-goed) are fairly common after -ed becomes productive, typically before age 3 (Marcus et al., 1992). But children almost never over-irregularize by analogizing an irregular pattern incorrectly (e.g., **wipe-wope* from write-wrote, *beep-bept from sleep-slept; Xu and Pinker 1995). The regular vs. irregular distinction is also clear in Spanish acquisition. Irregularity in Spanish inflection can be seen in both the stem (e.g., diphthongization) and the suffix (e.g., quis-e instead of *quis-i). In a corpus of about 5,700 verb inflections produced by young children (Clahsen et al., 2002), only 3% (168) are errors: of these only 2 are over-irregularization while all others are overregularization. Not a single misuse of diphthongization is found in a corpus of almost 2,000 tokens for which irregularization could have taken place (Mayol, 2007). The asymmetry between the generalization of productive rules and the lexicalization of non-productive forms has been observed in the cross-linguistic study of child morphology (Demuth, 2003; Deen, 2005; Clahsen et al., 1992;

Proceedings of the Society for Computation in Linguistics (SCiL) 2021, pages 177-187. Held on-line February 14-19, 2021 Caprin and Guasti, 2009), including polysynthetic languages such as Inuktitut (Allen, 1996).

The child acquisition results may seem at odds with gradient claims of morphological productivity (Baayen, 1992; Albright and Hayes, 2003; Seidenberg and Plaut, 2014). But such gradient measures of productivity are definitional, in the form of a ratio between two quantitative measures of the corpus statistics. Furthermore, the psychological evidence adduced for gradience generally involves inherently gradient tasks such as rating which encourage gradient responses even for uncontroverisally discrete categories (e.g., even numbers; Armstrong et al. 1983). Finally, gradient responses to morphological productivity are almost always obtained from adults rather than children. In the original Wug study (Berko, 1958), nonce verbs such as gling often lead adults to produce analogical irregularized forms (glang, glung), which are nearly completely absent in children's responses. Still, adults rate the regularized forms above irregularized forms when both forms are available (Ambridge, 2010). It is worth noting that English has added three novel verbs, Bing, bling and gling, in the past few decades. These verbs belong to the category of irregular verbs most favored for analogy in (adult) experimental studies (Albright and Hayes, 2003; Bybee and Moder, 1983), but both are regular in actual language use.

Taken together, these results suggest that (a) children form morphological generalizations on a relatively small amount of data, (b) children grasp the semantic features of morphological processes as indicated by the rarity of misapplications, and (c) children – and probably adults too, when experimental confounds are eliminated – draw a nearcategorical distinction between productive and unproductive processes, with the former occasionally over-used but almost never the latter. The last part, in particular, has been a challenging problem for computational morphology (Gorman et al., 2019). Such developmental findings may serve as design specifications for a psychological theory of morphological acquisition.

1.1 Related Work

Many systems for automatic morpheme segmentation have been proposed in the NLP community: see Tzoukermann and Liberman (1990); Klenk and Langer (1989); Méndez-Cruz et al. (2016) for Spanish, Cotterell et al. (2015); Sirts and Goldwater (2013); Ruokolainen et al. (2014) for English, and Hammarström and Borin (2011) for a review. Of particular relevance here are the Morpho-Challenge tasks (Kurimo et al., 2010) and resulting unsupervised morphological learning models (e.g., Creutz and Lagus 2005, 2007; Monson et al. 2007; Lignos 2010). More recently, Xu et al. (2020) provide a method of unsupervised morphological analysis exploiting a universal framework for morphological typology. The Morpho-Challenge-inspired models rely only on a set of word pairs as their input, making them better-suited for low-resource languages and more cognitively plausible than models that rely on larger or more-saturated data. However, these models focus exclusively on segmentation.

To the best of our knowledge, our model is the first to acquire morphological mappings in a manner consistent with the developmental constraints from child language acquisition reviewed earlier. While there is a large literature on the modeling of child morphological learning stemming from the so-called Past Tense Debate, these models only learn a single category (e.g., English past tense). Our model, by contrast, is designed to handle complex processes that realize multiple semantic features in a single inflected form.

1.2 Outline

In this paper, we present an unsupervised learning model that identifies form-meaning mappings in Spanish and English inflectional morphology and is intended to model the process of child language acquisition. The input to the model is a sample of words from child-directed speech in these languages (MacWhinney, 2000) comparable in size to those acquired during the first years of morphological learning. Children's accurate understanding of the semantic features of morphology is approximated by supplying the words in the input with annotations from the UniMorph project (Sylak-Glassman et al., 2015; Kirov et al., 2018).

The goal of our model is to find the morphological processes – affixation and stem changes – that provide systematic mappings between the semantic features of the stem (e.g., *walk*) and their inflection forms (e.g., *walked*). The most critical aim of our model is to acquire the distinction between productive and unproductive morphological processes, which young children almost never fail to recognize. Similarly, our model is able to learn both the productive processes, which can be applied to novel forms beyond the training data, as well as the unproductive processes (e.g., the stem change in *think-thought*, *sweep-swept*, and *singsang*), which will be restricted to the attested items in the training data and memorized accordingly. Productive patterns may be "broad," widely applicable and possibly default, or "narrow" in that they supersede a broader pattern under specific conditions. In sum, our model acquires broad defaults, narrowly-conditioned productive processes, and unproductive exceptions that must be memorized.

A central component of our learning model is the Tolerance Principle (TP; Yang 2016), a simple mathematical model that specifies the threshold for (morphological) productivity, one which been extensively used in the empirical and experimental studies of child language acquisition (e.g. Schuler et al. 2016). As in our model, the TP makes a categorical distinction between productive and unproductive rules and supports a notion of broadly applicable and conditioned productive processes.

Our model is presented in § 2, with the algorithm laid out in the Appendix. In a nutshell, our model extracts the morphological processes that map the semantic features of the stem to those of the inflected form (e.g., Spanish ama-rá-n 'love-FUT-3.PL'). These processes are then subjected to the TP's productivity test. If a process is deemed productive, it will be recorded as such while lexically listing its exceptions. If a process fails the TP test, the model subdivides the words into subclasses delimited by finer-grained semantic features and applies the TP test recursively to find narrower productive rules within. The experiments reported in § 3 show that our model successfully acquires almost all of the inflectional rules of English and Spanish available in the training data along with their exceptions, thereby providing a reasonable psychological account of young children's morphological acquisition. § 4 discusses related work and considers directions for future research.

2 Model Description

2.1 Linguistic background

In the present study, each input item is a tuple that consists of the orthographic form of a verb stem, an inflected form of the stem, and its UniMorph annotation (although other conventions adequately describing the semantic aspects of inflectional morphology could also be used). Table 1 provides some example annotations in both languages. We also as-

Lemma	Form	Features				
English						
find	find	2 SG PRS				
fall	fallen	PTCP PST				
call	called	3 pl pst				
Spanish						
poder	podría	cond 3 sg				
imaginar	imaginar	NFIN				
quedar	quedaron	ind pst 3 pl pfv				
mirar	mirad	pos imp 2 pl				
caer	caigo	IND PRS 1 SG				

Table 1: English and Spanish UniMorph annotations

sume that the model, like a child language learner, has knowledge of feature *categories*: person (1, 2, 3), number (SG, PL), tense (e.g., present PRS, past PST), aspect (e.g., participle PTCP), mood (e.g., indicative IND, subjunctive SBJV), and so on. These semantic aspects of morphology are well under control for children acquiring English and Spanish before the age of 3 (Brown, 1973; Kvaal et al., 1988). Note that although Spanish and English are typically considered to be fusional languages, Spanish inflection is also agglutinative, realizing person/number after tense/aspect (again, *ama-rá-n* 'love-FUT-3.PL').

A morphological process is defined as the mapping that manipulates the stem into the inflected form. For example, an input item may be (walk, {*walking*, {PTCP, PRS}}), and the morphological process -ing, which must be learned, is regarded as the realization of the features {PTCP, PRS}. In some cases, the morphological process may be conditioned on the stem. For example, the morphological process defined over the pair (sing, {sang, $\{1, SG, PST\}\}$ will be $i \rightarrow a$, which realizes the feature {1, SG, PST}. Such stem-conditioned processes are all unproductive in English, as indicated by the virtual absence of over-irregularized forms in child productions (Xu and Pinker, 1995). However, some of the productive morphological processes in Spanish are conditioned on the stem (i.e., the conjugation classes). Our model uses simple string edit methods to extract these morphological processes, which roughly correspond to the familiar suffixation and stem changes in the Indo-European languages. We emphasize that the extraction of the morphological processes and the evaluation of these processes, which is the core component of our learning model, are completely independent. Should we encounter a language with other morphological properties (e.g., prefixation, harmony, reduplication), or indeed from the derivational domain, the evaluation component of our model can apply without modification.

2.2 The Tolerance Principle

The central component of our learning model is the Tolerance Principle (TP; Yang 2016), which asserts that a rule is productive if and only if:

$$e \le \theta_N = \frac{N}{\ln N}$$

where N is the number of words a rule can apply to and e is the number of words which do not follow the rule.¹ The value θ_N is thus the threshold for generalization. For example, say we have 100 first person singular present forms, 90 of which are realized in Spanish with the process \circ . Because $100/\ln 100 \approx 21.7$ which is greater than the 10 exceptions, \circ is deemed as the productive mapping for {1, SG, PRS}. It is worth stressing that the TP operates on type frequencies. The token frequency of words does not enter into the TP calculation or any other component of our learning model; its only role is that a high frequency word is more likely to be sampled in the training data, thereby contributing as a single type count.

A crucial feature of the TP is its recursive application. In a case like the English past tense, a semantic feature (PST) may be realized by a single productive process (-ed), as long as the number of exceptions (i.e., the irregular verbs) does not exceed the threshold. In other cases, however, a semantic feature may not be productively realized by a single process. For example, the semantic feature (PRS) is realized as -s for 3rd person singular (3 SG PRS), -ing for the participle (PRS PTCP), and null for all other feature sets that contain (PRS). None of the three options can survive as the productive process for (PRS) as the other (two) competitors would collectively exceed the threshold in any reasonable sample. Thus, the learner will subdivide the verbs into more-specific feature sets and search for productive rules within each. In this case, this leads to a set of complementary mappings at varying levels of specification: $\{PRS\} = \emptyset$, $\{3 \text{ SG PRS}\} = s$, and {PTCP PRS} = ing, where the first mapping may be considered a broad mapping and the second two narrow mappings. Note that, when considered

in order of decreasing specificity, these mappings function identically to the disjoint mappings given by: $\{PRS \land \overline{PTCP} \land \overline{3} SG\} = \emptyset$, $\{3 SG PRS\} = s$, and $\{PTCP PRS\} = ing$. The search for productivity over a feature set terminates when the productive mapping is found.

2.3 The Search for Productive Mappings

For each instance of (lemma l, inflected form i, feature-set F) in its input data, our learner applies the TP to the mappings given by:

 $\mathcal{P}(F) \times \sigma(i)$

where $\mathcal{P}(F)$ is the power-set of the feature-set and $\sigma(i)$ is the set of all substrings at the end of the inflected form. Here we assume $\sigma(i)$ contains nonempty substrings of at most 4 characters unless i = l, in which case $\sigma(i) = \{\emptyset\}$. For example, the substrings considered for $(l=walk, \{i=walk, F=\{1, SG, PRS\}\})$ are just $\sigma(i)=\{\emptyset\}$ since l = i, while for $(l=walk, \{i=walking, F=\{PTCP, PRS\}\})$, they are $\sigma(i)=\{king, ing, ng, g\}$. Because we may generate multiple possible suffixes from each inflected form, we define e – the number of exceptions in TP calculation – to be the number of strings that are not equal to, but are also not a sub- or super-string, of the suffix in question.

The model calculates the frequencies of each of the feature categories {person, number, tense, mood, aspect} on its input, and iterates through them in order of decreasing frequency, following the well-attested frequency effects in language acquisition (Brown, 1973; Yang, 2016). At each pass, it attempts a mapping from the feature space to the suffix space using the Tolerance Principle, constraining the feature space to the features currently under consideration. If this mapping is empty, additional feature-categories are added in order of decreasing frequency until a mapping constrained to these categories is non-empty. That is, we apply the TP at each pass to:

$$\bigcup_{(l,i,F)\in \text{input},\ F\cap C\neq \emptyset} \mathcal{P}(F\cap C)\times \sigma(i),$$

where C is the set of feature categories under consideration at this pass, so $F \cap C$ constrains the pass to only consider the relevant features for any given item. F and $\sigma(i)$ are defined as above.

Because our learner considers all feature combinations in the power-set of the relevant features for a given word, it can yield multiple mappings

 $^{{}^{1}\}theta_{N}$ is almost always a small fraction of N unless N is very small: for N = 5, one rule covering 2 items and the other rule covering 3 are *both* productive. We are not aware of any empirical studies attesting to this effect but will assume that in such cases, the more dominant rule is the productive one.

corresponding to the same morphological process. For example, in Spanish, {1, PL} maps to mos, but so does $\{IND, PRS, 1, PL\}, \{PRS, 1, PL\},$ {POS, IMP, 1, PL} and others. To remedy this, once our learner has a non-empty constrained mapping at a given pass, it finds the intersection of all feature-sets that have mapped to each morphological process. If the mapping from this intersection to the morphological process in question passes the tolerance threshold, our learner is finished for this mapping. Otherwise, features are added by frequency to increase specification until the mapping either runs out of features or passes the tolerance threshold. This allows our model to acquire the minimal set of features that productively maps to a given morphological process.

Feature categories are removed from our learner's consideration once they have been used to constrain a mapping in a pass. After a pass, all endings that were learned in that pass are removed indiscriminately from the inflected forms in the input. For example, most Spanish $\{3 \text{ PL}\}$ forms end in n, so once this ending has been acquired, every inflected form in our data ending with n has this ending removed. We do not check the feature-sets corresponding to the inflected form before removal, which is motivated by children's aggressive segmentation "errors" once they identify productive processes (e.g., removing -*s* from the preposition *versus* to create a verb *vers*; Yang 2016).

2.4 Narrow Mappings and Stem Alternations

Narrow mappings refer to productive processes that reside within exceptions to the more-general mappings learned above. For example, the narrow mapping {3, SG, PFV} = \circ is a productive pattern that supersedes the more general mapping {3, SG} = \emptyset in Spanish, and {PTCP, PRS} = ing is a narrow mapping that supersedes {PRS} = \emptyset in English.

To identify potential narrow mappings, at each pass, the learner counts cases where the morphological processes proposed so far do not yield the correct inflected form. Then for each feature-set where there was at least one failure, it checks to see if a TP-majority failed. It then checks if it can find some suffix that correctly inflects enough instances that meet the feature-specification to pass the tolerance threshold. If it can, it takes the corresponding morphological process to be a narrow mapping.

In cases where there is no single process that corresponds to the narrow mapping, our learner attempts instead to categorize the suffixes based on endings of the lemma. We subdivide the instances based on properties of the ending of the lemma and search for productivity recursively until we have a set of morphological processes that productively describe our data or we only have one lemma fitting a condition. For example, the Spanish imperfect suffixes are conditioned on the stem vowel, *-aba* for *-ar* verbs and *-ía* for *-er* and *-ir* verbs. A morphological process that treats ia or aba as the default and learns the rest as exceptions will not pass the tolerance threshold because the opposing stem classes are always too large, but if the data is subset according to the stem endings, it will pass:

$$extbf{a} o extbf{a}$$
i, e o ia

If our learner can acquire a set of suffixes that are applied productively based on the ending of the lemma, then it takes the most frequent one to be the default. This yields a morphological process like the one for the Spanish imperfect (Table 3).

Features in the current feature category with no mapping in the current pass are treated in much the same way: the learner finds all mappings in an unconstrained pass for which the feature-set includes the feature(s) in question. The mapping with highest count is added to the set of general mappings for the pass, and the others are considered to be possible narrow mappings. For example, our model learns $\{1, SG, PRS\} = 0$ in the first pass in Spanish, since this is the highest-frequency mapping containing $\{1, SG\}$. Other mappings from a super-set of $\{1, SG\}$ are stored as possible narrow mappings.

All narrow mappings are stored separately and verified at the end of learning. Additionally, the suffixes corresponding to these narrow mappings are not removed after each pass. This filters out those that turn out to be outcomes of agglutinative processes our model has yet to learn, and allows the model to acquire these processes.

2.5 Memorizing Irregulars

As well as acquiring productive morphological processes, our learner must also acquire verbs for which there is no productive morphological process determining their inflection. These include suppletive verbs in Spanish and English (e.g., *ir* \sim *fui* and *go* \sim *went*), as well as plausibly but not ultimately predictable stem-changing verbs (e.g., *pedir* \sim *pide* and *sleep* \sim *slept*). Such patterns are simply memorized lexically and do not generalize, which is consistent with children's behavior concerning these processes (Clahsen et al., 1992; Xu and Pinker, 1995).

To accomplish this, our learner attempts to inflect every lemma in our training data based on the morphological processes it has acquired after the main iteration of learning is complete. It checks among the failing feature-sets for those where a TP-majority are failing, and if it has memorized a possible narrow mapping for such a feature-set, this mapping is added to the set of verified morphological processes.

Finally, any remaining forms not covered under one of the productive rules (including the recently verified narrow mappings) are committed to memory. This completes a representational system that contrasts regular forms, which can be inflected via productive processes, with memorized irregular ones that cannot.

3 Experiments

For both Spanish and English, our data consisted of the most frequently-occurring inflected verb forms in child-directed speech taken from the CHILDES corpora (MacWhinney, 2000). Because semantic context is not available in these wordlists, we annotated the words with lemmatizations and features provided in UniMorph 2.0, a Wiktionary-derived list of feature-sets for every lemma-inflected pairing (Kirov et al., 2018). These features provide a rough approximation of children's accurate morphosemantic knowledge (children rarely use morphological forms in inappropriate semantic contexts; §1). Following UniMorph's annotation scheme, we included English participles but not Spanish participles. For a fixed training data set, which corresponds to a child's internal vocabulary, the learning model will by design deterministically produce a set of output rules.² New training data, as long as they are sampled from the high frequency range of child-directed input, which would correspond to the variation across individual children's input data, generally do not lead to significant variation in the output rules. We regard this as an attractive feature as it would account for the general uniformity in the terminal grammars across individual child learners.

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$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			Second Pass				
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	3 pl pst	ed	y→ied,e→ed	367	1		
1 SG PST ed $y \rightarrow ied, d \rightarrow t$ 136 built	3 sg pst	ed	y→ied	139	tried		
		ed	y→ied,e→ed	203	walked		
	1 SG PST	ed	y→ied,d→t	136	built		
IPLPST ed $y \rightarrow ied$ 67 cried	1 pl pst	ed	y→ied	67	cried		

Table 2: Learned mappings for English

3.1 English

For English, we combined CHILDES data from the Manchester, Wells, and Belfast corpora, which together contained 3953 unique inflected forms corresponding to 1285 verb lemmas. We considered the lemma to be the plain infinitive, and removed (cliticized) contractions. We also removed gemination (e.g., converting put \sim putting to put \sim puting) because this orthographic pattern has no (synchronic) morpho-phonological relevance and is also subject to spelling variation (e.g., traveling/travelling). The English dataset is larger than Spanish because irregular verbs in English are notoriously frequent: the -ed rule would not emerge as productive if the vocabulary size was too small. The irregulars in Spanish, by contrast, show no obvious frequency effects (Fratini et al., 2014), so a more modest vocabulary is sufficient to identify the major productive processes. This mirrors the considerably earlier acquisition of Spanish inflectional morphology (González, 1978).

Results for English can be seen in Table 2. Each row in the table lists a set of features, the form they are mapped to, any active alternations, the number of times that set is attested in the data, and an example form. The result of each individual pass is indicated and broad and narrow mappings are listed separately. We acquire {PRS} = \emptyset in the first pass because, as discussed in § 2.2, we only

²The algorithm is actually quite fast. Loading the data for both languages, running the models, and outputting the results in PyCharm takes 6.6 seconds on the first author's consumer-grade laptop.



Figure 1: Spanish lemmas by proportion of attested inflectional categories

consider a suffix to count towards e in the TP if it is not a sub- or super- string of another suffix. Since \emptyset is a sub-string of both s and ing, {PRS} = \emptyset passes the TP. However, when we check for narrow mappings (§ 2.3), we find that both {PTCP, PRS} and {3, SG, PRS} fail to be correctly inflected by \emptyset . These feature sets are subsequently learned as productive narrow mappings, shown in Table 2.

For English, the alternations in the narrow mappings are almost entirely orthographic in nature. For example, orthographic e must be removed when forming the ing participle, and y must be converted to i before ed. Ultimately, there are 765 items that follow spelling alternations which are not sufficiently frequent in the data and thus must be memorized as exceptions (e.g., flies and lying), or are simply irregular (e.g. went). These 765 items correspond to 461 lemmas; identical inflected forms realizing different features are counted as separate exceptions since they are presented separately in the training data (e.g. went is counted twice if it appears in both $\{3, SG, PST\}$ and $\{2,$ SG, PST}). Obviously, these orthographic issues do not arise for children acquiring morphology from speech, and we discuss extensions of our model in \S 4. These items are stored; the associated patterns are deemed unproductive and do not generalize.

3.2 Spanish

For Spanish, we took the 1,000 most-used inflected forms from the CHILDES FernAguado Corpus and removed cliticized reflexives, resulting in 989 inflected forms corresponding to 302 verb lemmas.



Figure 2: Spanish inflectional categories by proportion of attested lemmas

We removed orthographic accents from Spanish verbs for ease of processing and considered the Spanish lemma to be the infinitive form with the r removed. This yielded an extremely sparse dataset as expected from Lignos and Yang (2016). The 302 lemmas exhibit a highly skewed distribution with most verbs appearing in only one or two inflected categories and none achieving even 60% saturation (Figure 1). Additionally, no inflectional category is attested with more than half of all known verbs, and only 37 are attested at all (Figure 2). With no direct evidence at all in their favor, the remaining endings must be inferred from the rest of the data.

Results for Spanish, sorted by pass and mapping type, can be seen in Table 3. Notable here is that the alternations capture differences between conjugations, for example the imperfect for both -er/-ir and -ar verbs in the fourth pass, and several subjunctives for -ar and -ir verbs in the narrow mappings. Additionally, the learner successfully acquired mappings that are quite rare, for example the 2nd person plural imperative -d which only appears twice in the data in sed and mirad. As well as these productive results, the learner also acquired 266 memorized exceptions corresponding to 78 lemmas in Spanish, including ser and haber. This means that just over a quarter of the 302 verb lemmas observed in Spanish were inflected irregularly at least once, contrasting with over a third of the 1285 English lemmas and matching well with the findings of Fratini et al. (2014) discussed above.

In the first pass, we see in Table 3 that we learn the common person+number endings in Spanish.

Broad Mappings			Narrow Mappings						
Features	Default	Alterns.	Ct.	Ex.	Features	Default	Alterns.	Ct.	Ex.
First Pass			First Pass						
3 sg	Ø		227	ama	SBJV PRS 3 SG	e	$i \rightarrow a$	13	ame
3 pl	n		103	aman	pos imp 3 sg	е	$i \rightarrow a$	14	ame
1 pl	mos		51	amamos	IND PST 3 SG PFV	0		72	amo
2 pl	is		10	amais	SBJV PRS 3 PL	an		2	coman
PRS 1 SG	0		163	amo	pos imp 3 pl	an		2	coman
PRS 2 SG	s		129	amas	ind pst 3 pl pfv	ron		23	amaron
	Second Pass		pos imp 1 pl	emos		3	amemos		
IND	Ø		651	ama	SBJV PRS 1 PL	emos		3	amemos
IMP	Ø		127	ama	pos imp 2 pl	d		2	amad
NFIN	r		146	amar	SBJV PRS 1 SG	е	$i \rightarrow a$	14	ame
COND	ria		16	amaria	IND PST 1 SG PFV	е	$i \rightarrow i$	18	ame
Third Pass		COND 2 SG	rias		2	amarias			
PRS	Ø		492	ama	SBJV PRS 2 SG	es	$i \rightarrow as$	33	ames
FUT	ra		20	amara	IND FUT 2 SG	ras		3	amaras
Fourth Pass		IND PST 2 SG IPFV	ias		9	comias			
IPFV	ia	a→aba	65	amaba	IND PST 2 SG PFV	ste		10	amaste
					Second Pass				
					IND FUT 1 PL	re		2	amaremos

Table 3: Learned mappings for Spanish

For $\{1 \text{ sG}\}$ and $\{2 \text{ sG}\}$, we have too many exceptions (namely the 1st person in every non-present tense and the second singular imperative) to acquire generalizable rules based on only person and number, so we follow § 2.3 and take the more-specified rule with highest count. After acquiring these endings, we check for possible sub-regularities. Those that were successfully verified at the end of learning are listed in Table 3, but several others are hypothesized, since the person+number endings learned in the first pass do not fully inflect Spanish verbs in non-present tenses, and the agglutinative endings indicating these tenses are not learned until at least the second pass. Such hypothesized and later-rejected narrow mappings include, for example, $\{3, PL, FUT\}$ = ran.

4 Discussion

Our model can be seen as an operationalization of the Tolerance Principle which has been used extensively, by the means of manual calculation using corpus statistics, to model morphological acquisition in many languages. It provides a mechanistic account of the developmental findings (§1) that children make a near-categorical distinction between unproductive and productive processes (Tables 2 and 3). Further, since our learner explicitly memorizes the irregular forms to which unproductive processes apply (§ 2.4), it, like children, will not generalize these processes beyond its input (Clahsen et al., 2002; Mayol, 2007).

The iterative processing of feature-categories in a descending order of frequency enables the model

to acquire the productive morphological processes of both Spanish and English in approximately the same order as a child. For example, Aguirre and Marrero (2009) show that Spanish-learning children acquire person+number endings early in the acquisition process, and our learner similarly acquires these in the first pass of its learning (Table 3). Likewise, Brown (1973) shows that Englishlearning children acquire the present participle before the past tense, which is less regular; our model learns the narrow mapping for {PTCP, PRS} in the first pass of learning and the narrow mappings for PST in the second.

4.1 Limitations

Though the learner successfully acquires conjugation-dependent endings such as the Spanish imperfect and most of the present subjunctives, there are a few generalizations that it misses. For example, the imperfect subjunctives (in -ra- or -se-) are missing simply because they are not attested in the data; these are among the least common forms in Spanish. Indeed, we are not aware of any study of child Spanish that documents such processes. More subtly, there are some instances where -er forms are not accounted for, such as the {SBJV PRS 3 SG}, where $e \rightarrow a$ is not acquired. This likely results from the training data, as many of the -er verbs in our data are irregular (e.g. ser, haber, tener), which prevents our learner from forming generalizations for such morphological processes. However, this is in line with the relatively late acquisition of the subjunctive by Spanish-learning

children (González, 1978), implying a larger dataset may be necessary to properly acquire the subjunctive. A similar case happens for the {FUT 1 SG}, which does not appear in our data and thus is not attested as a narrow mapping. However, we do learn the attested mapping for {FUT 1 PL}; for this case, note that since re was learned as a second pass narrow mapping, we can agglutinate it with first pass mappings to create, in this case, remos.

The limitations of the Spanish data can also be seen in the distribution of the mappings for the 2nd person singular. We only learn $s = \{2 \text{ SG PRS}\}$ rather than $\{2 \text{ SG}\}$. There are just 9 instances of $\{2 \text{ SG IPFV}\}$, 3 $\{2 \text{ SG FUT}\}$, and many imperatives (which do not take -s): 101 of our 254 $\{2 \text{ SG}\}$ forms are {POS IMP 2 SG}. This means that the s ending cannot generalize to $\{2 \text{ SG}\}$ and we must learn separate mappings, in this case, for the present, future, imperfect, and conditional $\{2 \text{ SG}\}$.

The relatively small size of our Spanish data may be part of the reason that we fail to learn all cases for some of our narrow mappings. Ultimately, a frequency-based sample from child-directed corpora is only a crude proxy: accurate modeling of morphological acquisition must approximate the child's learners of (psychological) vocabulary as closely as possible.

4.2 Future directions

The linguistically interpretable rules which our model learns can be used for morphological segmentation, analysis, and generation. It can be extended to languages with other types of morphological processes, leaving the productivity evaluation component unchanged. Representing word forms as sequences of phonological feature bundles can remove the artificiality of orthography, potentially leading to the discovery of phonologically conditioned morphological rules.

The development of grammar, including morphology, often follows a a U-shaped trajectory marked by near-perfect production, a period of overgeneralization, and a return to near-perfect production (see e.g. Marcus et al. 1992). This has been interpreted as the calibration between rules and exceptions: children gradually accumulate vocabulary items that follow productive rules, which eventually overwhelm the exceptions that do not follow the rules. Our model can straightforwardly account for such developmental process by providing the learner with an incrementally larger amount of data (i.e., more lemma/inflection pairs). The tipping point at which rules become productive would correspond to the dip in the U-shaped learning curve.

Derivational morphology poses interesting challenges for our model. It remains to be seen if simple semantic features such as category transformation are sufficient to enable successful acquisition (e.g., the German DErivBase; Zeller et al. 2013) or finergrained semantic distinctions need to be made: the English deverbal suffix *-er* can realize the agent (e.g., *kicker*) or the instrument (e.g., *cutter*).

Our model requires explicit representation of semantic features in order to identify morphological processes. Some of these features are likely universal while others may need to be constructed on a language-specific basis. An intriguing direction is to explore the extent to which distributional methods can induce semantic relations that hold for morphologically related words (e.g., Luong et al. 2013), as opposed to relying on annotated data such as UniMorph. The induction of semantic relations needn't be comprehensive as long as it is highly accurate: as our model demonstrates, a very modest amount of high quality data may be sufficient for linguistically accurate morphological learning.

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A Appendix

gorithm 1 Overview of our Learning Algorithm
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 $input = [(l_1, i_1, F_1), ..., (l_n, i_n, F_n)]$ for feature category C in order of decreasing frequency do $S = \bigcup_{(l,i,F) \in input, \ F \cap C \neq \emptyset} \mathcal{P}(F \cap C) \times \sigma(i)$ mappings = TP(S)if $mappings = \emptyset$ then for feature category C_2 in order of decreasing frequency do mappings = TP constraining to $C \cup C_2$ if $mappings \neq \emptyset$ then break end if end for end if for any feature-set $c \in C$ without a mapping do Do an unconstrained TP pass Add superset of c with highest count to mappings Add all other supersets to possible narrow mappings end for for $m \in mappings$ do for $(l, i, F) \in input, m \in F$ do if $N/\ln N$ or more instances of F fail to be inflected by m then $n_m =$ new TP mapping constraining to F if n_m correctly inflects F then Add n_m to mappings else $sub = \{(l_i, i_i, F_i) \in input, F_i = F\}$ Divide *sub* based on endings of each l_j while n_m does not correctly inflect \vec{F} and $\forall s \in sub, |s| > 1$ do Increase specificity of endings in sub $n_m = \bigcup_{s \in sub} TP(s)$ end while if n_m correctly inflects F then Add n_m to mappings end if end if end if end for end for end for for (l, i, F) that can't be inflected with learned mappings from passes do if \exists narrow mapping n that inflects $N/\ln N$ instances of F then Verify n and memorize any exceptions to it else Memorize(l, i, F) as an exception end if end for