

Exact Hard Monotonic Attention for Character-Level Transduction

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Abstract

Many common character-level, string-to-string transduction tasks, e.g. grapheme-to-phoneme conversion and morphological inflection, consist almost exclusively of monotonic transduction. Neural sequence-to-sequence models with soft attention, which are non-monotonic, often outperform popular monotonic models. In this work, we ask the following question: Is monotonicity really a helpful inductive bias in these tasks? We develop a hard attention sequence-to-sequence model that enforces strict monotonicity and learns a latent alignment jointly while learning to transduce. With the help of dynamic programming, we are able to compute the exact marginalization over all monotonic alignments. Our models achieve state-of-the-art performance on morphological inflection. Furthermore, we find strong performance on two other character-level transduction tasks. Code is available at <https://github.com/shijie-wu/neural-transducer>.

1 Introduction

Many tasks in natural language can be treated as character-level, string-to-string transduction. The current dominant method is the neural sequence-to-sequence model with soft attention (Bahdanau et al., 2015; Luong et al., 2015). This method has achieved state-of-the-art results in a plethora of tasks, for example, grapheme-to-phoneme conversion (Yao and Zweig, 2015), named-entity transliteration (Rosca and Breuel, 2016) and morphological inflection generation (Cotterell et al., 2016). While soft attention is very similar to a traditional alignment between the source characters and target characters in some regards, it does not explicitly a distribution over alignments. On the other hand, neural sequence-to-sequence models with hard alignment (Xu et al., 2015; Wu et al., 2018)

are analogous to the latent alignment in the classic IBM models for machine translation, which do model the alignment distribution explicitly (Brown et al., 1993).

The standard versions of both soft and hard attention are non-monotonic. However, if we look at the data in grapheme-to-phoneme conversion, named-entity transliteration, and morphological inflection—examples are shown in Fig. 1—we see that the tasks require almost exclusively monotonic transduction. Yet, counterintuitively, the state of the art in high resource morphological inflection is held by non-monotonic models (Cotterell et al., 2017)! Indeed, in a recent controlled experiment, Wu et al. (2018) found non-monotonic models (with either soft attention or hard alignment) outperform popular monotonic models (Aharoni and Goldberg, 2017) in the three above mentioned tasks. However, the inductive bias of monotonicity, if correct, should help learn a better model or, *at least*, learn the same model.

In this paper, we hypothesize that the underperformance of monotonic models stems from the lack of joint training of the alignments with the transduction. Generalizing the model of Wu et al. (2018) to enforce monotonic alignments, we show that, for all three tasks considered, monotonicity is a good inductive bias and jointly learning a monotonic alignment improves performance. We provide an exact, cubic-time, dynamic-programming inference algorithm to compute the log-likelihood and an approximate greedy decoding scheme. Empirically, our results indicate that, rather than the pipeline systems of Aharoni and Goldberg (2017) and Makarov et al. (2017), we should jointly train monotonic alignments with the transduction model, and, indeed, we set the single model state of the art on the task of morphological inflection.¹

¹The state of the art for morphological inflection is held by ensemble systems, much like parsing and other structured

Task	Grapheme-to-phoneme	Transliteration	Morphological Inflection
Tag			N AT+ALL SG
Source	a c t i o n	A A C H E N	l i p u k e
Target	AE K SH AH N	아 험	l i p u k k e e l l e

Figure 1: Example of source and target string for each task. Tag guides transduction in morphological inflection.

2 Hard Attention

2.1 Preliminary

We assume the source string $\mathbf{x} \in \Sigma_{\mathbf{x}}^*$ and the target string $\mathbf{y} \in \Sigma_{\mathbf{y}}^*$ have finite vocabularies $\Sigma_{\mathbf{x}} = \{x_1, \dots, x_{|\Sigma_{\mathbf{x}}|}\}$ and $\Sigma_{\mathbf{y}} = \{y_1, \dots, y_{|\Sigma_{\mathbf{y}}|}\}$, respectively. In tasks where the tag is provided, i.e., labeled transduction (Zhou and Neubig, 2017), we denote the tag as an ordered set $\mathbf{t} \in \Sigma_{\mathbf{t}}^*$ with a finite tag vocabulary $\Sigma_{\mathbf{t}} = \{t_1, \dots, t_{|\Sigma_{\mathbf{t}}|}\}$. We define the set $A = \{1, \dots, |\mathbf{x}|\}^{|\mathbf{y}|}$ to be set of all alignments from \mathbf{x} to \mathbf{y} where an alignment aligns each target character y_i to exactly one source character in \mathbf{x} . In other words, it allows zero-to-one² or many-to-one alignments between \mathbf{x} and \mathbf{y} . For an $\mathbf{a} \in A$, $a_i = j$ refers to the event that y_i is aligned to x_j , the i^{th} character of \mathbf{y} and the j^{th} character of \mathbf{x} .

2.2 0th-order Hard Attention

Hard attention was first introduced to the literature by Xu et al. (2015). We, however, follow Wu et al. (2018) and use a tractable variant of hard attention and model the probability of a target string \mathbf{y} given an input string \mathbf{x} as the following:

$$\begin{aligned}
p(\mathbf{y} | \mathbf{x}) &= \sum_{\mathbf{a} \in A(\mathbf{x}, \mathbf{y})} p(\mathbf{y}, \mathbf{a} | \mathbf{x}) \\
&= \underbrace{\sum_{\mathbf{a} \in A} \prod_{i=1}^{|\mathbf{y}|} p(\mathbf{y}_i | a_i, \mathbf{y}_{<i}, \mathbf{x}) p(a_i | \mathbf{y}_{<i}, \mathbf{x})}_{\text{exponential number of terms}} \\
&= \underbrace{\prod_{i=1}^{|\mathbf{y}|} \sum_{a_i=1}^{|\mathbf{x}|} p(\mathbf{y}_i | a_i, \mathbf{y}_{<i}, \mathbf{x}) p(a_i | \mathbf{y}_{<i}, \mathbf{x})}_{\text{polynomial number of terms}} \quad (1)
\end{aligned}$$

where we show how one can rearrange the terms to compute the function in polynomial time.

prediction tasks. We present the new *best individual* system.

²Zero in the sense of non-character like BOS or EOS

The model above is exactly an 0th-order neuralized hidden Markov model (HMM). Specifically, $p(\mathbf{y}_i | a_i, \mathbf{y}_{<i}, \mathbf{x})$ can be regarded as an emission distribution and $p(a_i | \mathbf{y}_{<i}, \mathbf{x})$ can be regarded as a transition distribution, which *does not* condition on the previous alignment. Hence, we will refer to this model as 0th-order hard attention. The likelihood can be computed in $\mathcal{O}(|\mathbf{x}| \cdot |\mathbf{y}| \cdot |\Sigma_{\mathbf{y}}|)$ time.

2.3 1st-order Hard Attention

To enforce monotonicity, hard attention with conditionally independent alignment decisions is not enough: The model needs to know the previous alignment position when determining the current alignment position. Thus, we allow the transition distribution to condition on previous one alignment $p(a_i | a_{i-1}, \mathbf{y}_{<i}, \mathbf{x})$ and it becomes a 1st-order neuralized HMM. We display this model as a graphical model in Fig. 2. We will refer to it as 1st-order hard attention. Generalizing the 0th-order model, we define 1st-order extension as:

$$\begin{aligned}
p(\mathbf{y} | \mathbf{x}) &= \sum_{\mathbf{a} \in A(\mathbf{x}, \mathbf{y})} p(\mathbf{y}, \mathbf{a} | \mathbf{x}) \\
&= \underbrace{\sum_{\mathbf{a} \in A} \prod_{i=1}^{|\mathbf{y}|} p(\mathbf{y}_i | a_i, \mathbf{y}_{<i}, \mathbf{x}) p(a_i | a_{i-1}, \mathbf{y}_{<i}, \mathbf{x})}_{\text{exponential number of terms}} \\
&= \underbrace{\prod_{i=1}^{|\mathbf{y}|} \sum_{a_{i-1}=1}^{|\mathbf{x}|} \sum_{a_i=1}^{|\mathbf{x}|} p(\mathbf{y}_i | a_i) p(a_i | a_{i-1}) \alpha(a_{i-1})}_{\text{polynomial number of terms}} \quad (2)
\end{aligned}$$

where $\alpha(a_{i-1})$ is the forward probability, calculated using the forward algorithm (Rabiner, 1989) with $\alpha(a_0, \mathbf{y}_0) = 1$, and $p(a_1 | a_0) = p(a_1 | \langle \text{BOS} \rangle, \mathbf{x})$ is the initial alignment distribution. For simplicity, we drop $\mathbf{y}_{<i}$ and \mathbf{x} in $p(\mathbf{y}_i | a_i)$ and $p(a_i | a_{i-1})$. For completeness, we include the

recursive definition of the forward probability:

$$\alpha(a_i) = p(\mathbf{y}_i | a_i) \sum_{a_{i-1}=1}^{|\mathbf{x}|} p(a_i | a_{i-1}) \alpha(a_{i-1})$$

$$\alpha(a_1) = p(\mathbf{y}_1 | a_1) p(a_1 | a_0) \alpha(a_0)$$

Thus, computation of the likelihood in our 1st-order hard attention model is $\mathcal{O}(|\mathbf{x}|^2 \cdot |\mathbf{y}| \cdot |\Sigma_{\mathbf{y}}|)$.

Decoding at test time, however, is hard and we resort to a greedy scheme, described in Alg. 1. To see why it is hard, note that the dependence on $\mathbf{y}_{<i}$ means that we have a neural language model scoring the target string as it is being transduced. Because the dependence is unbounded, there will be no dynamic program that allows for efficient computation.

3 A Neural Parameterization with Enforced Monotonicity

The goal of this section is to take the 1st-order model of §2 and show how we can straightforwardly enforce monotonic alignments. We will achieve this by adding structural zeros to the distribution, which will still allow us to perform efficient inference with dynamic programming. We follow the neural parameterization of Wu et al. (2018). The source string \mathbf{x} is represented by a sequence of character embeddings vectors, which are fed into an encoder bidirectional LSTM (Hochreiter and Schmidhuber, 1997) to produce hidden state representations \mathbf{h}_j^e . The emission distribution $p(\mathbf{y}_i | a_i, \mathbf{y}_{<i}, \mathbf{x})$ depends on these encodings \mathbf{h}_j^e and the decoder hidden states \mathbf{h}_i^d , produced by

$$\mathbf{h}_i^d = \text{LSTM}([\mathbf{e}^d(\mathbf{y}_{i-1}); \mathbf{h}^t], \mathbf{h}_{i-1}^d)$$

where \mathbf{e}^d encodes target characters into character embeddings. The tag embedding \mathbf{h}^t is produced by

$$\mathbf{h}^t = \text{ReLU}(\mathbf{Y} [\mathbf{e}^t(\mathbf{t}_1); \dots; \mathbf{e}^t(\mathbf{t}_{|\Sigma_{\mathbf{t}}|})])$$

where \mathbf{e}^t maps the tag \mathbf{t}_k into tag embedding $\mathbf{h}_k^t \in \mathbb{R}^{d_t}$ or zero vector $\mathbf{0} \in \mathbb{R}^{d_t}$, depends on whether the tag \mathbf{t}_k is presented. Note that $\mathbf{Y} \in \mathbb{R}^{d_t \times |\Sigma_{\mathbf{t}}| d_t}$ is a learned parameter. Also $\mathbf{h}_j^e \in \mathbb{R}^{2d_h}$, $\mathbf{h}_i^d \in \mathbb{R}^{d_h}$ and $\mathbf{h}^t \in \mathbb{R}^{d_t}$ are hidden states.

The Emission Distributon. All of our hard-attention models employ the same emission distribution parameterization, which we define below

$$p(\mathbf{y}_i | a_i, \mathbf{y}_{<i}, \mathbf{x}) = \text{softmax}(\mathbf{W} \mathbf{f}(\mathbf{h}_i^d, \mathbf{h}_{a_i}^e))$$

$$\mathbf{f}(\mathbf{h}_i^d, \mathbf{h}_{a_i}^e) = \tanh(\mathbf{V} [\mathbf{h}_i^d; \mathbf{h}_{a_i}^e])$$

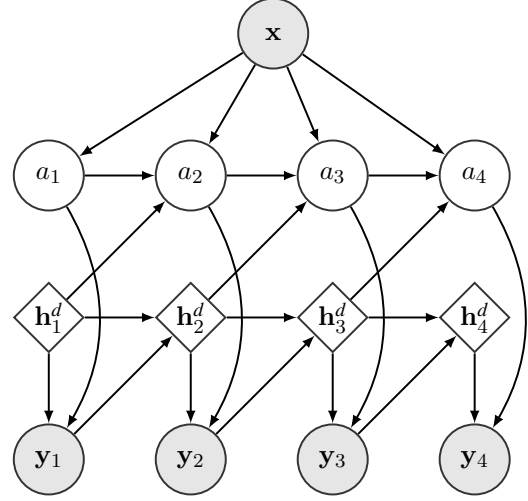


Figure 2: Our monotonic hard-attention model viewed as a graphical model. The circular nodes are random variables and the diamond nodes deterministic variables. We have omitted arcs from \mathbf{x} to y_1, y_2, y_3 and y_4 for clarity (to avoid crossing arcs).

where $\mathbf{V} \in \mathbb{R}^{3d_h \times 3d_h}$ and $\mathbf{W} \in \mathbb{R}^{|\Sigma_{\mathbf{y}}| \times 3d_h}$ are learned parameters.

0th-order Hard Attention. In the case of the 0th-order model, the distribution is computed by a bilinear attention function with eq. (1)

$$p(a_i = j | \mathbf{y}_{<i}, \mathbf{x}) = \frac{\exp(\mathbf{h}_i^{d\top} \mathbf{T} \mathbf{h}_j^e)}{\sum_{j'=1}^{|\mathbf{x}|} \exp(\mathbf{h}_i^{d\top} \mathbf{T} \mathbf{h}_{j'}^e)}$$

where $\mathbf{T} \in \mathbb{R}^{d_h \times 2d_h}$ is a learned parameter.

0th-order Hard Monotonic Attention. We may enforce string monotonicity by zeroing out any non-monotonic alignment *without* adding any additional parameters, which can be done through adding structural zeros to the distribution as follows

$$p(a_i = j | a_{i-1} = j', \mathbf{y}_{<i}, \mathbf{x}) = \frac{\mathbb{1}\{j \geq j'\} \exp(\mathbf{h}_i^{d\top} \mathbf{T} \mathbf{h}_j^e)}{\sum_{j'=1}^{|\mathbf{x}|} \mathbb{1}\{j \geq j'\} \exp(\mathbf{h}_i^{d\top} \mathbf{T} \mathbf{h}_{j'}^e)}$$

These structural zeros prevent the alignments from jumping backwards during transduction and, thus, enforce monotonicity. The parameterization is identical to the 0th-order model up to the enforcement of the hard constraint with eq. (2).

1st-order Hard Monotonic Attention. We may also generalize the 0th-order case by *adding more parameters*. This will equip the model with a more expressive transition function. In this case, we take

Algorithm 1 Greedy decoding. (N is the maximum length of target string.)

```

1: for  $i = 1, \dots, N$  do
2:   if  $i = 1$  then
3:      $y_i^* = \operatorname{argmax}_{y_i} \sum_{a_i=1}^{|\mathbf{x}|} p(y_i | a_i) p(a_i | a_{i-1}) \alpha(a_0)$  ▷ Greedy decoding
4:      $\alpha(a_1) = p(y_1^* | a_1) p(a_1 | a_0) \alpha(a_0)$  ▷ Forward probability
5:   else
6:      $y_i^* = \operatorname{argmax}_{y_i} \sum_{a_i=1}^{|\mathbf{x}|} p(y_i | a_i) \sum_{a_{i-1}=1}^{|\mathbf{x}|} p(a_i | a_{i-1}) \alpha(a_{i-1})$  ▷ Greedy decoding
7:      $\alpha(a_i) = p(y_i^* | a_i) \sum_{a_{i-1}=1}^{|\mathbf{x}|} p(a_i | a_{i-1}) \alpha(a_{i-1})$  ▷ Forward probability
8:   if  $y_i^* = \text{EOS}$  then
9:     return  $\mathbf{y}^*$ 

```

the 1st-order hard attention to be an offset-based transition distribution similar to Wang et al. (2018):

$$p(a_i | a_{i-1}, \mathbf{y}_{<i}, \mathbf{x}) = \begin{cases} \operatorname{softmax}(\mathbf{U}[\mathbf{h}_i^d; \mathbf{T} \mathbf{h}_{a_{i-1}}^e]) & 0 \leq \Delta \leq w \\ 0 & \text{otherwise} \end{cases}$$

where $\Delta = a_i - a_{i-1}$ is relative distance to previous attention position and $\mathbf{U} \in \mathbb{R}^{(w+1) \times 2d_h}$, a learned parameter. Note that, as before, we also enforce monotonicity as a hard constraint in this parameterization.

4 Related Work

There have been previous attempts to look at monotonicity in neural transduction. Graves (2012) first introduced the monotonic neural transducer for speech recognition. Building on this, Yu et al. (2016) proposes using a separated `shift/emit` transition distribution to allow more expressive model. Like us, they also consider morphological inflection and outperform a (weaker) soft attention baseline. Rastogi et al. (2016) offer a neural parameterization of a finite-state transducer, which implicitly encodes monotonic alignments. Instead of learning the alignments directly, Aharoni and Goldberg (2017) take the monotonic alignments from an external model (Sudoh et al., 2013) and train the neural model with these alignments. In follow-up work, Makarov et al. (2017) show this two-stage approach to be effective, winning the CoNLL-SIGMORPHON 2017 shared task on morphological inflection (Cotterell et al., 2017). Raffel et al. (2017) propose a stochastic monotonic transition process to allow sample-based *online* decoding.

5 Experiments

5.1 Experiments Design

Tasks. We consider three character-level transduction tasks: grapheme-to-phoneme conversion (Weide, 1998; Sejnowski and Rosenberg, 1987), named-entity transliteration (Zhang et al., 2015) and morphological inflection in high-resource setting (Cotterell et al., 2017).

Empirical Comparison. We compare (i) soft attention without input-feeding (SOFT) (Luong et al., 2015), (ii) 0th-order hard attention (0-HARD) (Wu et al., 2018), (iii) 0th-order monotonic hard attention (0-MONO) and (iv) 1st-order monotonic hard attention (1-MONO). The SOFT, 0-HARD and 0-MONO models have an *identical* number of parameters, but the 1-MONO has more. All of them have approximately 8.6M parameters. Experimental details and hyperparameters may be found in App. A.

5.2 Experimental Findings

Finding #1: Morphological Inflection. The first empirical finding in our study is that we achieve single-model, state-of-the-art performance on the CoNLL-SIGMORPHON 2017 shared task dataset. The results are shown in Tab. 2. We find that the 1-MONO ties with the 0-MONO system, indicating the additional parameters do not add much. Both of these monotonic systems surpass the non-monotonic system 0-HARD and SOFT. We also report comparison to other top systems at the task in Tab. 1. The previous state-of-the-art model, Bergmanis et al. (2017), is a non-monotonic system that outperformed the monotonic system of Makarov et al. (2017). However, Makarov et al. (2017) is a pipeline system that took alignments from an existing aligner; such a system has no manner, by which it can recover from poor initial

Morphological Inflection	ACC
Silfverberg et al. (2017)	93.0
SOFT	93.4
Makarov et al. (2017)	93.9
0-HARD	94.5
Bergmanis et al. (2017)	94.6
Makarov and Clematide (2018)	94.6
0-MONO	94.8
1-MONO	94.8

Table 1: Average dev performance on morphological inflection of our models against single models from the 2017 shared task. All systems are single model, i.e., *without* ensembling. Why dev? No participants submitted single-model systems for evaluation on test and the best systems were not open-sourced, constraining our comparison. Note we report numbers from their paper.³

alignment. We show that *jointly* learning monotonic alignments lead to improved results.

Finding #2: Effect of Strict Monotonicity. The second finding is that by comparing SOFT, 0-HARD, 0-MONO in Tab. 2, we observe 0-MONO outperforms 0-HARD and 0-HARD in turns outperforms SOFT in all three tasks. This shows that monotonicity should be enforced strictly since strict monotonicity does not hurt the model. We contrast this to the findings of Wu et al. (2018), who found the non-monotonic models outperform the monotonic ones; this suggests strict monotonicity is more helpful when the model is allowed to learn the alignment distribution jointly.

Finding #3: Do Additional Parameters Help? The third finding is that 1-MONO has a more expressive transition distribution and, thus, outperforms 0-MONO and 0-HARD in G2P. However, it performs as well as or worse on the other tasks. This tells us that the additional parameters are not always necessary for improved performance. Rather, it is the hard constraint that matters—not the more expressive distribution. However, we remark that enforcing the monotonic constraint does come at an additional computational cost: an additional factor $\mathcal{O}(|x|)$.

6 Conclusion

We expand the hard-attention neural sequence-to-sequence model of Wu et al. (2018) to enforce monotonicity. We show, empirically, that enforcing monotonicity in the alignments found by

³Some numbers are obtained by contacting authors.

	Trans		G2P		MorInf	
	ACC	MFS	WER	PER	ACC	MLD
SOFT	40.4	0.893	29.3	0.071	92.9	0.157
0-HARD	41.1*	0.894	29.2*	0.070	93.8*	0.126
0-MONO	41.2*	0.895	29.0* [×]	0.072	94.4*[×]	0.113
1-MONO	40.8	0.893	28.2*[×]†	0.069	94.4*[×]	0.116

Table 2: Average test performance of named-entity transliteration (Trans), grapheme-to-phoneme conversion (G2P) and morphological inflection (MorInf). First group has exactly same number of parameter while the second group has slightly more parameter. *, [×] and [†] indicate statistical significant improvement against SOFT, 0-HARD and 0-MONO on language-level paired permutation test ($p < 0.05$).

hard attention models helps significantly, and we achieve state-of-the-art performance on the morphological inflection using data from the CoNLL-SIGMORPHON 2017 shared task. We isolate the effect of monotonicity in a controlled experiment and show monotonicity is a useful hard constraint for three tasks, and speculate previous underperformance is due to a lack of joint training.

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A Experimental Details

A.1 Tasks.

We ask the authors of Wu et al. (2018) for the split data of grapheme-to-phoneme conversion (CMU-Dict (Weide, 1998) and NetTalk (Sejnowski and Rosenberg, 1987)) and NEWS 2015 shared task on named-entity transliteration. In named-entity transliteration, we only run experiments on 11 language pairs.⁴

Grapheme-to-Phoneme Conversion is evaluated by word error rate (WER) and phoneme error rate (PER) (Yao and Zweig, 2015), where PER is the edit distance divided by the length of the phonemes. Named-entity transliteration is evaluated by word accuracy (ACC) and mean F-score (MFS) (Zhang et al., 2015). F-score is computed by

$$\begin{aligned} \text{LCS}(c, r) &= \frac{1}{2}(|c| + |r| - \text{ED}(c, r)) \\ R_i &= \frac{\text{LCS}(c_i, r_i)}{|r_i|} \\ P_i &= \frac{\text{LCS}(c_i, r_i)}{|c_i|} \\ \text{FS}_i &= 2 \frac{R_i \times P_i}{R_i + P_i} \end{aligned}$$

where r_i and c_i is the i -th reference and prediction and $\text{ED}(c, r)$ is the edit distance between c and r . Morphological inflection is evaluated by word accuracy (ACC) and average edit distance (MLD) (Cotterell et al., 2017).

A.2 Parameterization.

For completeness, we also include the parameterization of soft attention.

$$\begin{aligned} p(y_i | y_{<i}, \mathbf{x}) &= \text{softmax}(\mathbf{W} \mathbf{f}(\mathbf{h}_i^d, \mathbf{c}_i)) \\ \mathbf{c}_i &= \sum_{j=1}^{|\mathbf{x}|} \alpha_{ij} \mathbf{h}_j^e \\ \alpha_{ij} &= \frac{\exp(e_{ij})}{\sum_{j=1}^{|\mathbf{x}|} \exp(e_{ij})} \\ e_{ij} &= \mathbf{h}_i^{d\top} \mathbf{T} \mathbf{h}_j^e \end{aligned}$$

The dimension of character and tag embedding are 200 and 40, respectively. The encoder and decoder LSTM both have 400 hidden dimensions (d_h). We also have a 2 layer encoder LSTM. We have 0.4 dropout in embedding and encoder LSTM.

⁴Ar-En, En-Ba, En-Hi, En-Ja, En-Ka, En-Ko, En-Pe, En-Ta, En-Th, Jn-Jk and Th-En.

The w in 1st-order hard monotonic attention model is 4.

A.3 Optimization.

The model is trained with Adam (Kingma and Ba, 2015) and the learning rate is 0.001. We halve the learning rate whenever the development log-likelihood increase and we stop early when the learning rate reaches 0.00001. We apply gradient clipping with maximum gradient norm 5. The models are selected by development evaluation metric and decoded greedily since no improvements are observed when using beam search (Wu et al., 2018).