Complex Numerical Reasoning with Numerical Semantic Pre-training Framework

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bupt-reasoning-lab.github.io/CNR-NST
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Abstract

Multi-hop complex reasoning over incomplete knowledge graphs (KGs) has been extensively studied, but research on numerical knowledge graphs (NKGs) remains relatively limited. Recent approaches focus on separately encoding entities and numerical values, using neural networks to process query encodings for reasoning. However, in complex multi-hop reasoning tasks, numerical values are not merely symbols, and they carry specific semantics and logical relationships that must be accurately represented. In this work, we propose a Complex Numerical Reasoning with Numerical Semantic Pre-training Framework **(CNR-NST)**. The CNR-NST framework can perform binary operations on numerical attributes in NKGs, enabling it to infer new numerical attributes from existing knowledge. Our approach effectively handles up to 102 types of complex numerical reasoning queries. On three public datasets, CNR-NST demonstrates SOTA performance in complex numerical queries, achieving an average improvement of over 40% compared to existing methods. Notably, this work expands the query types for complex multi-hop numerical reasoning and introduces a new evaluation metric for numerical answers, which has been validated through comprehensive experiments.

1 Introduction

Complex query answering (CQA) refers to the process of reasoning and performing computations on knowledge graphs (KGs) by combining multiple entities and relationships to retrieve entities that fulfill specific logical conditions (Kotnis and García-Durán, 2019). This field has seen significant advancements, with research increasingly focused on enhancing the accuracy of models in handling intricate query tasks (Zhu et al., 2022; Arakelyan et al., 2021). Despite this progress, real-world KGs

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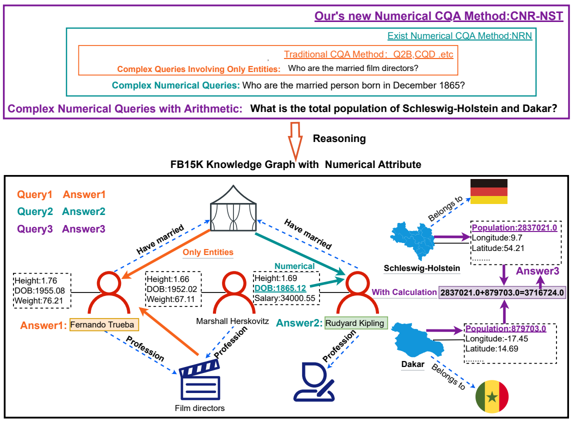

Figure 1: Three types of queries: (1) $Q1$ refers to **Complex Queries Involving Only Entities**, which involve complex reasoning tasks that solely include entities; (2) $Q2$ refers to **Complex Numerical Queries**, which combine both numerical values and entities for reasoning; (3) $Q3$ refers to **Complex Numerical Queries with Arithmetic**, where the query requires performing computations between two numerical values from the NKG.

are not limited to discrete entity-relation knowledge, and they also contain numerous numerical attributes, such as birth dates, event times, and territorial sizes of countries. Numerical knowledge graph (NKG), therefore, offer a more nuanced approach to modeling real-world querys (Xue et al., 2022). Figure 1 presents an example of the FB15K NKG (Kotnis and García-Durán, 2019) illustrating three distinct query types: (1) **Complex Queries Involving Only Entities**, such as *"Q1: Which film directors are married?"*; (2) **Complex Numerical Queries**, such as *"Q2: Who are the married individuals born in August 1955?"*; (3) **Complex Numerical Queries with Arithmetic**, such as *"Q3: What is the combined population of Schleswig-Holstein and Dakar?"*.

Traditional CQA methods are effective for multi-hop queries but encounter challenges in accurately capturing the subtle nuances of numerical semantics (Ren et al., 2024). These methods neglect the

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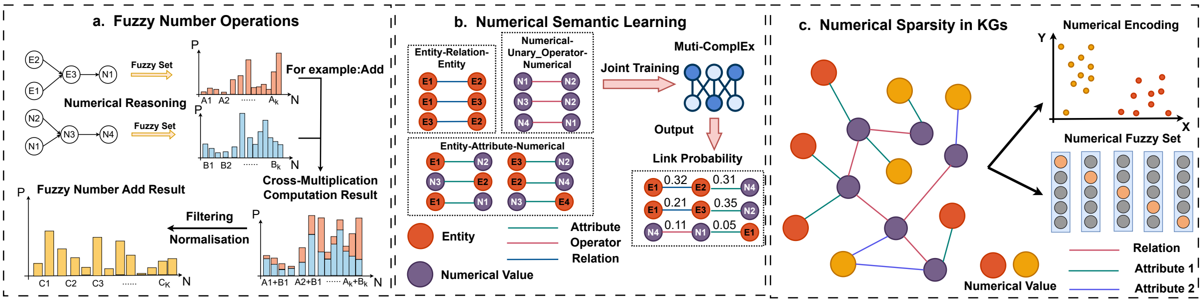

Figure 2: (a): The process of performing operations on two fuzzy numbers derived from reasoning; (b): The pre-training process for learning numerical semantics in CNR-NST; (c): Numerical sparsity in the KG and two numerical representation methods: direct encoding and fuzzy sets.

inherent meanings of numerical attributes. Existing approaches, such as the Numerical Reasoning Network (NRN) (Bai et al., 2023a), present a framework that encodes entities and numerical values separately. However, when using Sinusoidal (Sundararaman et al., 2020) and DICE (Vaswani et al., 2017) encoding methods for numerical values, this approach encounters issues with sparsity in the KG. Many encoding regions lack corresponding numerical mappings in the KG, leading to incomplete learning of numerical semantics.

Handling complex numerical query tasks presents significant challenges. First, current methods are constrained by reasoning answers limited to the NKG, unable to compute or infer new numerical answers from multi-values ($Q3$), while inferring new numerical attributes from existing values is essential for practical applications. Second, numerical values are continuous, with semantics influenced by units, ranges, precision, and context (Ren et al., 2024; Kim et al., 2023). And numerical data sparsity in KGs introduces additional challenges for complex query resolution (Li et al., 2022).

To tackle the challenges in complex numerical reasoning, we propose the Complex Numerical Reasoning with Numerical Semantic Pre-training Framework (CNR-NST), which consists of the following three components.

- **Numerical Binary Operation Operator** As shown in Figure 2(a), CNR-NST introduces new numerical query operators to handle queries with numerical answers absent from the original KG, supporting 102 distinct query types. Using relevant theorems in fuzzy mathematics, operations on two fuzzy numbers within the real number domain are enabled. By representing numerical values as fuzzy sets, these operations can be effectively mapped to mathematical calculations between two numerical attributes in the KG. The mathematical proof of the mapping between real number operations and fuzzy set operations is provided in Appendix A.1.
- **Numerical Semantic Learning** To effectively capture the semantics of numerical values, we utilize a joint predictor learning the relationships between entity attributes and their numerical values, thus facilitating knowledge transfer across tasks. Figure 2(b) provides a comprehensive overview of this architecture.
- **Complex Numerical Reasoning and Computation** CNR-NST represents numerical values and entities using fuzzy sets, avoiding training neural operators in an unrestricted numerical embedding space (Figure 2(c)). Fuzzy sets capture the uncertainty of numerical values during complex reasoning and effectively represent the inherent fuzzy relationships within numerical data. Intermediate variable values in the reasoning process reflect probability scores of corresponding entities or numerical values, significantly enhancing numerical inference accuracy.

In summary, our contributions are:

- We introduce the binary operation operator for the first time in complex numerical queries, defining new operators and query types.
- We introduce the CNR-NST framework, which learns various entity-numerical relationships during the pre-training stage.
- Compared to the current SOTA, CNR-NST achieves substantial performance gains across all 102 types of complex numerical queries.

2 Preliminaries

Numerical Knowledge Graph NKG is the KG with numerical attributes, defined as $\mathcal{G}$ =

Notations	Definitions
$\mathcal{G}$	NKG
$\mathcal{V}$	The set of entity in the NKG
$\mathcal{R}$	The set of relation in the NKG
$\mathcal{A}$	The set of attribute in the NKG
$\mathcal{F}$	The set of numerical relation in the NKG
$\mathcal{N}$	The set of numbers in the NKG
$\mathcal{X}$	$= \mathcal{V} \cup \mathcal{N}$, the set of universal vertex in the NKG
$\mathcal{E}$	$= \mathcal{R} \cup \mathcal{A} \cup \mathcal{F}$, the set of universal edge in the NKG
$\mathcal{T}$	the triples in the NKG
$X \in \mathcal{X}$	Any vertex (including entities and numerical values) in the NKG
$r(V, V')$	The entity V is associated with the entity V' by the relationship r
$a(V, N)$	The entity V is associated with the numberical N by the attribute a
$f(N, N')$	The numberical N is associated with the numberical N' by the numerical relational f

Table 1: Definition of mathematical symbols.

$(\mathcal{V}, \mathcal{N}; \mathcal{R}, \mathcal{A}, \mathcal{F}; \mathcal{T})$, which contains entity $V \in \mathcal{V}$, attribute value $N \in \mathcal{N}$, relation $r \in \mathcal{R}$, attribute $a \in \mathcal{A}$, numerical relation $f \in \mathcal{F}$, and triples $(h, r, t) \in \mathcal{T} \subset (\mathcal{V} \times \mathcal{R} \times \mathcal{V}) \cup (\mathcal{V} \times \mathcal{A} \times \mathcal{N}) \cup (\mathcal{N} \times \mathcal{F} \times \mathcal{N})$. The difference from KG (Pai and Costabello, 2021) is that $\mathcal{N}$, $\mathcal{A}$, $\mathcal{F}$ have been added.

Numerical Complex Query Answering The complex query Q on a NKG $\mathcal{G}$ can be defined as:

$$q\left[X_?\right] = V_1, \ldots, V_i \in \mathcal{V}, N_1, \ldots, N_j \in \mathcal{N} : c_1 \vee c_2 \vee \cdots \vee c_n \quad (1)$$

$$c_i = e_{i,1} \wedge e_{i,2} \wedge \cdots \wedge e_{i,m} \quad (2)$$

Here, X refers to a vertex, which can be any entity or numerical value. And c_i represents a conjunction of several atomic logical expressions $e_{i,j}$, where each $e_{i,j}$ can be one of the following expressions:

$$e_{i,j} = r\left(V, V'\right), V, V' \in \left\{V_1, \ldots, V_i\right\}, V \neq V', r \in \mathcal{R} \quad (3)$$

$$e_{i,j} = a\left(V, N\right), V \in \left\{V_1, \ldots, V_i\right\}, N \in \left\{N_1, \ldots, N_j\right\}, a \in \mathcal{A} \quad (4)$$

$$e_{i,j} = f\left(N, N'\right), N, N' \in \left\{N_1, \ldots, N_j\right\}, N \neq N', \\ f \in \mathcal{F} = \left\{\leq, \geq, =, +, -, \times, \div, \ldots\right\}$$

In the above equations, V represents a subset of entity $\mathcal{V}$, and N represents a subset of the numerical attribute set $\mathbb{N}$. The binary function r_i determines whether a class i relationship exists between two entities. The function a_j determines whether an entity possesses a value for attribute j. The function f checks whether a filtering condition, such as greater than, less than, or addition, subtraction, multiplication, and division, is satisfied between two numerical values.

3 Methodology

In this section, we introduce the CNR-NST framework. We first describe the construction of the entity-numerical knowledge graph embedding (KGE) model, followed by an explanation of how the adjacency matrix is used for reasoning and computation in multi-hop queries. Figure 3 provides an overview of the CNR-NST framework.

3.1 Constructing Numerical and Entity Adjacency Matrices

Pre-training Framework To address complex queries involving both entities and numerical values, we propose the Multi-ComplEx joint training framework (as shown in Figure 3), based on the ComplEx model (Trouillon et al., 2016), and introduce three link predictors: $r\left(V, V'\right) \in \mathbb{R}$, $a\left(V, N\right) \in \mathbb{A}$, and $f\left(N, N'\right) \in \mathbb{F}$, which effectively capture relationships among entities, relations, attributes, and numerical values. The architecture designed to learn numerical size relationships is presented in Appendix A.2. These triples are jointly trained with the various types of (h, r, t) triples from the KG training set, and the scoring function for the triples is as follows:

$$f\left(h, r, t\right) = \sum_{r_j \in \{\mathbb{R}, \mathbb{A}, \mathbb{G}\}} \beta_j Re\left(\langle h, r_j, \bar{t} \rangle\right) \quad (5)$$

where β_j represents the weight parameters under different types of triple relationships. Among them, h is the head entity, r_j is the relationship, t is the tail entity, and $\bar{t}$ represents the conjugate complex number of the complex vector t. Re refers to the real part of the complex vector. For any given triple (h, r, t), the Loss Function for link prediction training is defined as follows:

$$\mathcal{L}_X = -\sum_{(h,r,t) \in (\mathbb{R} \times \mathbb{A} \times \mathbb{F})} \log \sigma\left(f_X\left(h, r, t\right)\right) - \sum_{\left(h', r, t\right) \in \left(\mathbb{R}' \times \mathbb{A}' \times \mathbb{F}'\right)} \log \sigma\left(f_X\left(h', r, t\right)\right) \quad (6)$$

$$\mathcal{L}_{joint} = \sum_{X \in (\mathbb{R} \cup \mathbb{A} \cup \mathbb{F})} \beta_X \cdot \mathcal{L}_X \quad (7)$$

where X represents various relationships between numerical attributes and entities, σ denotes a hyperbolic tangent function, and f_X is the cross-entropy loss function. The training details of the joint training framework are provided in Appendix A.2.

As demonstrated by the experiments in Section 4.4, the Multi-ComplEx joint training framework

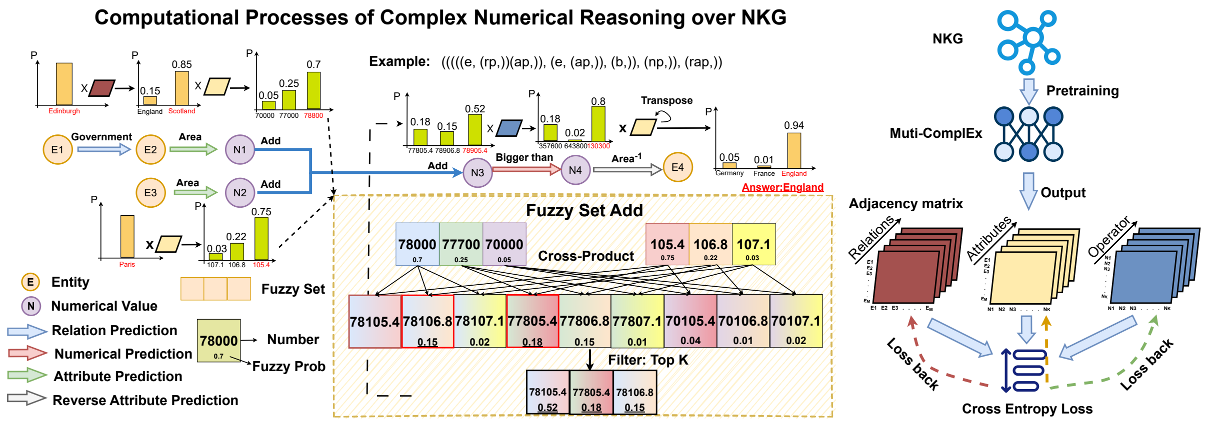

Figure 3: Overview of the CNR-NST framework. For complex multi-hop queries, the intermediate variable at each step is quantified by a score computed via a pre-trained KGE model.

provides substantial improvements in numerical link prediction compared to methods that train entity and numerical relationships separately.

Scoring Function Using a Multi-ComplEx model, we can score the likelihood of three atomic formulas $f_r(v_i, v_j)$, $f_a(v_i, n_j)$, and $f_f(n_i, n_j)$. This is achieved by employing neural link predictors f_r, f_a, f_f to infer missing edges in the KG in the $\mathcal{V}\times\mathcal{R}\times\mathcal{V}, \mathcal{V}\times\mathcal{A}\times\mathcal{N}, \mathcal{N}\times\mathcal{F}\times\mathcal{N}$ space. The probabilities of all known triples within the KG are set to 1, while the probability values of triples outside the known KG are modeled to follow the original distribution (Arakelyan et al., 2021). We define the neural adjacency matrix $M_R \in [0,1]^{|\mathcal{R}|\times|\mathcal{V}|\times|\mathcal{V}|}$, $M_A \in [0,1]^{|\mathcal{A}|\times|\mathcal{V}|\times|\mathcal{N}|}$, $M_F \in [0,1]^{|\mathcal{G}|\times|\mathcal{N}|\times|\mathcal{N}|}$ as:

$$\hat{R}(x_i, x_j) = \frac{\exp\left(\mathcal{E}(x_i, x_j)\cdot N_X\right)}{\sum_{x\in\mathcal{V}\cup\mathcal{N}}\exp\left(\mathcal{E}(x_i, x_j)\right)} \tag{8}$$

$$N_X = |\{(x_i, \mathcal{E}, x_j) \in \mathcal{F}_{train} | x_i, x_j \in \mathcal{V}\cup\mathcal{N})\}|$$

$$\hat{R}(x_i, x_j) = \begin{cases} 1 & \text{if } (x_i, \mathcal{E}, x_j) \in \mathcal{F}_{train} \\ \min\left\{\hat{R}(x_i, x), 1-\epsilon\right\} & \text{otherwise} \end{cases} \tag{9}$$

where x represents either a numerical value or an entity, and $\mathcal{E}$ refers to one of the three relationships: $\mathcal{R}$, $\mathcal{A}$, $\mathcal{F}$, $\mathcal{F}_{train}$ represents the training set in the KG, and x_i represents an entity or a numerical value. The expression $(x_i, \mathcal{E}, x_j)$ refers to any triple in one of the three categories. The value of ϵ is typically set to 0.0001. Also note that the matrix M contains a large number of zeros, and it will be stored as a sparse matrix, significantly reducing storage space.

3.2 Fuzzy Representation and Operator Definition

Fuzzy Set Representation CNR-NST utilizes fuzzy sets to address the inherent uncertainty of numerical values. Specifically, for an anchored entity v or an anchored numerical value n, we represent them using an initialization vector $[0, 0, \ldots, 1, \ldots, 0]$, where the position corresponding to the entity or numerical value is set to 1, with all other positions set to 0. For intermediate entities or numerical values, we use fuzzy vectors $v_{1,\ldots,k} \in [0,1]^{|V|}$ ($n_{1,\ldots,j} \in [0,1]^{|N|}$), respectively, to represent their states. The uncertainty present in complex queries can then be quantified using a membership function.

$$\mathcal{U}(Q) = \{(x, \mu_A(x)) | x \in X\} \tag{10}$$

where the element x represents an entity or a numerical value, $\mathcal{U}(Q)$ denotes the probability that x satisfies the query Q, and the membership function is μ_A.

Numerical Operators There are three types of numerical operators: attribute projection, reverse attribute projection, and numerical projection. Let $V^*(X = x)$ represent the maximum truth value of a subquery rooted at X when x is assigned as an entity or numerical value. We recursively compute the $\mathbf{V}^*(X) = [V^*(X = x)]_{x\in\mathcal{N}} \in [0,1]^{|N|}$ for each node in the query tree to maximize the overall truth value of the query tree, ultimately deriving the truth value $\mathbf{V}^*(x_?)$ at the root node.

When a node is connected to its child node by an edge, the fuzzy set of the node is calculated as follows:

$$\mathbf{V}^*(x_?) = \max_j\left(\left(V^*(x_k)^T \cdots \times |X|\right) \odot M_x\right) \tag{11}$$

where max_j represents the maximum value in the column, and x can represent either an entity or a numerical value. M_x refers to the entity-value probability distribution matrix M_a, value-entity probability distribution matrix M_a^T or the value-value probability distribution matrix M_f.

Numerical complex queries also involve entity-related operators, such as relation projection, intersection, and union. The mathematical formulations of these operators are provided in Appendix B.3.

3.3 Fuzzy Reasoning and Computation

In this section, we introduce how numerical complex queries are answered using the numerical atomic queries defined earlier. Starting from the anchor node X_{anchor}, which can represent either an entity or a numerical value, numerical reasoning and computation are executed based on the types of edges within the query computation tree.

Numerical Reasoning At each step of numerical complex reasoning, CNR-NST assigns a score $\mathbf{V}^*(X_?) \in [0,1]^{|x|}$ to each node X, where this score reflects the likelihood of the query tree with $X_?$ as the endpoint being satisfied. CNR-NST then determines the optimal entity assignments by back-propagating through the query tree:

$$\mathbf{V}_i = \varphi_i(V_{i1}, V_{i2}, \cdots, V_{im}) \tag{12}$$

where $\mathbf{V}_i$ refers to the fuzzy representation of non-anchor nodes, and φ denotes the various prediction functions applied to numerical values or entities. Specifically, when V represents a numerical value, $V_{im} = \mu_m(n_m)$.

Numerical Computation It is important to distinguish between numerical computation and numerical reasoning. Numerical computation involves performing arithmetic operations, such as addition, subtraction, multiplication, or division, on numerical values in the KG. In the CNR-NST, when handling numerical complex queries, the numerical value at a given step is not a precise value but instead a fuzzy set $[0,1]^{|x|}$. We interpret coordinate-corresponding numerical values as membership functions, converting the fuzzy set into a fuzzy number.

Let $\mathcal{R}$ be the real number domain, and let the mapping $\mathcal{R} \cdot \mathcal{R} \to \mathcal{R}$ be a binary operation on the real number domain (Mordeson, 2001). From this mapping, a new mapping $F(R) \cdot F(R) \to F(R)$ can be induced. Based on the extension principle in fuzzy mathematics, we can easily derive the following theorem:

$$\underline{A} \otimes \underline{B} = \int_R \bigvee_{x \otimes y = z} \left(\mu_{\underline{A}}(x) \bigwedge \mu_{\underline{B}}(y)\right) / z \tag{13}$$

Here, $\otimes$ can represent the four basic arithmetic operations on real numbers. For convenience, we often discretize the real number domain for processing, so the above expression is transformed into:

$$\underline{A} \otimes \underline{B} = \sum_z \frac{\bigvee_{x \otimes y = z} \left(\mu_{\underline{A}}(x) \bigwedge \mu_{\underline{B}}(y)\right)}{z} \tag{14}$$

As a result of the above derivation, we obtain a new fuzzy number and a membership set with a size of $|N|^2$. To prevent exponential growth in dimensionality during multiple numerical operations, we retain only the components with the highest membership values. This approach ensures that the dimensionality of numerical fuzzy sets remains stable throughout the reasoning process.

3.4 Performance Complexity Analysis

Space Complexity CNR-NST's memory usage includes the composite neural adjacency matrix M_x, containing $|V|^2|\mathcal{R}| + |N|^2|\mathcal{F}| + |V||N||\mathcal{A}|$ elements. However, as described in Section 4.1, most values in the neural matrix M_x can be filtered by a threshold ϵ. Experiments show M_x can be stored on a single 24GB GPU.

Time Complexity In the numerical reasoning process, each variable is computed with a complexity of $O\left(\left|\mathcal{X}^2\right|\right)$, where $\mathcal{X}$ refers to either numerical values or entities. However, since each variable contains a large number of zero elements, the actual complexity of a single query, as derived from Equation 6, is $O(|\mathcal{X}| \cdot N \cdot |\mathbf{V}^*(X_k) > 0|)$, where N represents the number of projections.

4 Experiments

4.1 Experimental Setup

Datasets We conducted experiments on three datasets: FB15K, DB15K, and YAGO15K (Kotnis and García-Durán, 2019) (see Appendix B for details). We used the 8 major categories and 92 subtypes of multi-hop numerical queries introduced by NRN (Bai et al., 2023a). A full statistical overview of these numerical queries is in Appendix B.4.

Baselines We selected the LitCQD (Demir et al., 2023) model and the three numerical reasoning models from NRN (Bai et al., 2023a) as our baselines. However, because the calculation of average MRR values in NRN relies heavily on the query sampling method, and their original dataset did not include all 92 query types, we expanded the dataset

Dataset	Method	1rp	2p	ppp	2pi	3p3i	2pip	pppi	2pu	2pup
FB15K-237	LitCQD	34.76	11.43	7.38	37.47	43.71	16.66	23.52	4.25	4.55
	CNR-NST	**35.36**	**18.11**	**15.94**	**43.53**	**59.02**	**27.55**	**36.11**	**16.55**	**16.63**
FB15K	LitCQD	85.22	44.29	23.54	74.23	71.11	60.17	51.34	13.94	9.66
	CNR-NST	**85.24**	**64.41**	**52.61**	**74.78**	**71.18**	**69.55**	**70.18**	**74.79**	**60.24**
DB15K	LitCQD	**38.55**	27.71	19.01	59.17	74.04	36.85	53.84	11.34	16.78
	CNR-NST	36.89	**30.01**	**24.18**	**60.47**	**76.46**	**38.45**	**54.25**	**24.29**	**27.23**
YAGO15K	LitCQD	51.93	16.29	8.74	48.61	**66.21**	20.91	34.66	4.24	7.29
	CNR-NST	**54.51**	**17.14**	**9.33**	**54.71**	64.58	**21.29**	**36.32**	**21.88**	**13.04**
Dataset	**Method**	**nr**	**nrp**	**nrpi**	**1ap(MAE)**	**1ap(MSE)**	**pa(MAE)**	**pa(MSE)**	**2pa(MAE)**	**2pa(MSE)**
FB15K-237	LitCQD	0.94	1.93	8.73	0.077	0.024	0.051	0.008	0.039	0.004
	CNR-NST	**3.73**	**7.62**	**13.92**	**0.051**	**0.011**	**0.018**	**0.002**	**0.011**	**0.0007**
FB15K	LitCQD	0.05	0.61	5.72	0.376	0.223	0.386	0.228	0.414	0.265
	CNR-NST	**1.61**	**9.22**	**16.19**	**0.048**	**0.011**	**0.029**	**0.006**	**0.032**	**0.006**
DB15K	LitCQD	0.19	1.63	**31.68**	0.042	**0.015**	0.039	0.008	0.039	0.006
	CNR-NST	**1.48**	**3.64**	14.59	**0.042**	0.017	**0.017**	**0.003**	**0.013**	**0.001**
YAGO15K	LitCQD	0.14	0.99	**16.35**	**0.049**	**0.007**	0.062	0.011	0.079	0.013
	CNR-NST	**2.67**	**1.45**	8.55	0.061	0.013	**0.061**	**0.010**	**0.058**	**0.008**

Table 2: MRR (%) and MAE, MSE for the types of queries that LitCQD can support.

accordingly. We took into account the differences in average calculation methods and the number of queries, and conducted comprehensive comparative experiments to ensure consistency and accuracy.

Evaluation Protocol For each complex numerical query, answers are labeled "easy" or "hard" based on whether they can be directly inferred from existing graph edges. For instance, on the test set, easy answers are derivable from training/validation graphs, while hard answers require reasoning over missing edges. We evaluate complex multi-hop queries using MRR and Hits@K on the test set (Zhang et al., 2019).

Implementation Details We first trained multiple KGE models on the training graph, adopting an extended Multi-ComplEx model, based on ComplEx (Trouillon et al., 2016) and incorporating N3 regularization (Lacroix et al., 2018). Next, we derived several computational neural adjacency matrices M_x from these KGE models. To reduce the memory footprint of M_x, we applied an adjacency matrix filter A to remove values below a specific threshold, converting the matrix into a sparse format that fits storage on a single NVIDIA A40 GPU.

4.2 Main Results

Tables 2 and 3 present CNR-NST's performance in complex numerical query reasoning on four public datasets. Table 2 compares it with LitCQD, showing CNR-NST supports more query types and outperforms LitCQD. Table 3 compares it with three benchmarks, demonstrating it exceeds baselines in nearly all query types (without specialized training), with over 100% improvements in many cases. Table 4 highlights CNR-NST's average 300%+ performance gain in numerical answer queries vs. baselines. We attribute this improvement to the pre-training framework, which predicts link probabilities between entities and numerical values and enhances generalization for complex multi-hop queries (experimental verification in Section 4.4). The multiple entity-numerical value relationships serve as extra constraints in numerical reasoning, narrowing the reasoning space and boosting accuracy. Moreover, fuzzy sets inherently address the numerical fuzziness in complex reasoning, further improving CNR-NST's task performance.

4.3 Answering Complex Numerical Computation Query

The 92 aforementioned query types are limited to single-numerical reasoning. This paper introduces 5 new query types that perform calculations and reasoning on two or more numerical values in the KG, further diversifying multi-hop queries. Unlike previous methods confined to the discrete numerical domain in KG, we for the first time extend KG numerical reasoning to the real number domain, enabling more accurate modeling of real-world KG queries.

Dataset	$Query_N$	Avg_M	Method	Avg_{All}	1p	2p	2i	3i	pi	ip	2u	up	2b	3b	bp	pb	2pb
FB15K	77	Avg_W	GQE+NRN	20.04	31.69	8.15	33.99	41.19	23.10	10.10	7.57	4.51	-	-	-	-	-
			Q2B+NRN	22.36	37.01	7.74	36.88	42.72	24.25	10.09	15.61	4.54	-	-	-	-	-
			Q2P+NRN	24.44	42.75	12.87	33.71	38.75	23.14	13.23	23.16	7.89	-	-	-	-	-
			CNR-NST	**40.78**	**60.26**	**38.35**	**51.49**	**54.01**	**36.89**	**36.16**	**24.91**	**24.19**	-	-	-	-	-
		Avg	GQE+NRN	11.76	11.19	4.24	18.85	34.66	7.52	11.47	3.41	2.73	-	-	-	-	-
			Q2B+NRN	12.83	14.33	3.87	21.08	34.36	8.13	12.74	5.02	3.09	-	-	-	-	-
			Q2P+NRN	11.62	12.20	3.31	19.43	33.57	5.52	10.85	5.52	2.56	-	-	-	-	-
			CNR-NST	**29.64**	**41.11**	**19.17**	**37.37**	**45.44**	**27.14**	**26.27**	**22.91**	**17.69**	-	-	-	-	-
	92		GQE+NRN	15.05	9.79	4.59	27.83	45.44	8.67	17.91	3.32	2.85	-	-	-	-	-
			Q2B+NRN	16.20	12.04	4.16	28.21	**48.07**	18.93	9.99	5.02	3.14	-	-	-	-	-
			Q2P+NRN	10.34	9.65	3.47	17.50	26.41	11.96	5.47	5.65	2.58	-	-	-	-	-
			CNR-NST	**29.00**	**32.32**	**23.02**	**34.93**	36.69	**31.15**	**28.97**	**25.18**	**19.74**	-	-	-	-	-
	102		CNR-NST	**22.78**	**32.32**	**23.02**	**34.93**	36.69	**31.15**	**28.97**	**25.18**	**19.74**	**11.16**	**11.51**	**10.29**	**18.96**	**12.24**
DB15K	77	Avg_W	GQE+NRN	10.96	10.29	2.53	20.14	35.46	12.50	2.52	2.08	2.14	-	-	-	-	-
			Q2B+NRN	11.89	10.96	2.71	22.60	37.44	13.81	3.05	2.41	2.13	-	-	-	-	-
			Q2P+NRN	12.98	14.71	3.81	23.75	36.66	14.47	2.96	4.63	2.81	-	-	-	-	-
			CNR-NST	**23.46**	**22.54**	**16.98**	**36.60**	**46.59**	**30.67**	**14.91**	**8.40**	**10.96**	-	-	-	-	-
		Avg	GQE+NRN	10.91	3.50	2.72	17.60	43.30	11.28	5.54	1.40	1.91	-	-	-	-	-
			Q2B+NRN	12.01	4.15	2.78	19.12	47.88	12.53	6.31	1.36	1.98	-	-	-	-	-
			Q2P+NRN	11.90	4.72	3.01	13.52	**50.03**	13.81	5.21	2.51	2.41	-	-	-	-	-
			CNR-NST	**18.77**	**15.69**	**10.72**	**30.03**	45.61	**20.84**	**14.47**	**6.21**	**6.64**	-	-	-	-	-
	92		GQE+NRN	14.73	3.33	2.99	23.47	**58.45**	19.14	7.12	1.38	1.97	-	-	-	-	-
			Q2B+NRN	15.26	3.92	3.21	25.16	57.24	20.83	8.13	1.41	2.19	-	-	-	-	-
			Q2P+NRN	10.90	4.71	2.99	18.54	37.47	13.98	4.84	2.46	2.20	-	-	-	-	-
			CNR-NST	**22.33**	**16.19**	**10.68**	**32.16**	55.56	**29.94**	**15.77**	**9.39**	**8.96**	-	-	-	-	-
	102		CNR-NST	**22.41**	**16.19**	**10.68**	**32.16**	55.56	**29.94**	**15.77**	**9.39**	**8.96**	**12.27**	**11.39**	**5.72**	**21.49**	**16.94**
YAGO15K	77	Avg_W	GQE+NRN	15.60	14.79	4.23	35.68	39.94	18.29	5.65	4.23	1.96	-	-	-	-	-
			Q2B+NRN	18.78	21.40	4.59	39.72	45.16	19.62	7.90	9.05	2.82	-	-	-	-	-
			Q2P+NRN	14.82	22.97	5.70	25.70	29.04	14.38	5.40	**11.87**	3.51	-	-	-	-	-
			CNR-NST	**27.71**	**31.11**	**16.32**	**44.30**	**54.98**	**33.64**	**21.48**	10.13	**9.71**	-	-	-	-	-
		Avg	GQE+NRN	15.53	2.72	3.20	26.72	62.76	18.01	6.68	1.70	2.47	-	-	-	-	-
			Q2B+NRN	16.00	4.14	3.10	26.30	62.50	20.83	6.81	1.88	2.45	-	-	-	-	-
			Q2P+NRN	17.99	7.80	3.73	36.62	59.57	22.35	7.54	3.35	2.96	-	-	-	-	-
			CNR-NST	**26.00**	**19.55**	**11.05**	**45.58**	**68.13**	**34.74**	**16.45**	**6.31**	**6.20**	-	-	-	-	-
	92		GQE+NRN	16.92	2.59	3.48	31.13	62.28	25.19	6.63	1.61	2.47	-	-	-	-	-
			Q2B+NRN	17.64	3.20	3.03	32.52	**65.53**	25.53	7.40	1.60	2.34	-	-	-	-	-
			Q2P+NRN	16.50	6.21	3.71	28.30	53.39	25.51	8.58	3.28	3.03	-	-	-	-	-
			CNR-NST	**22.97**	**15.50**	**11.24**	**38.74**	55.22	**34.23**	**16.44**	**5.90**	**6.45**	-	-	-	-	-
	102		CNR-NST	**19.51**	**15.50**	**11.24**	**38.74**	**55.22**	**34.23**	**16.44**	**5.90**	**6.45**	**11.71**	**12.49**	**5.92**	**18.94**	**20.79**

Table 3: MRR (%) of all test set query types. $Query_N$: total query types. Avg_M: average calculation method. Avg_W: weighted average (by query type count). Avg: direct subclass average. See Appendix A.6 for details.

As shown in Table 3, we define five new query types: 2b, 3b, bp, pb, and 2pb. The b operator denotes binary operations (addition, subtraction, multiplication, division) on two numerical values. Since the answers to these query types are not present in the original KG, they generate numerical subqueries. We adopt the new evaluation metrics introduced in Section 4.4. Detailed descriptions of these query types are provided in Appendix D.

4.4 Supplementary Experimental Details

New Evaluation Metrics for Numerical-Type Answers In complex multi-hop numerical reasoning, many queries return numerical answers. Previous methods used entity-based evaluation metrics for these queries, which are limited: they ignore differences between numerical values (*e.g.,* their continuity) and fail to evaluate queries where numerical answers are absent from the original KG. To resolve this, we propose a new numerical evaluation metric analogous to Mean Reciprocal Rank (MRR), denoted as $MRR_{0.001}$. Instead of ranking by exact numerical match, we determine rank based on the probability of numerical nodes whose relative error against the correct answer is below a threshold (usually 0.001), with ranking calculations applied only to hard answers.

As shown in Table 4, we re-evaluated the numerical answers across the 102 sub-queries handled by CNR-NST and compared the results with the baseline model, Q2P. CNR-NST significantly outperformed Q2P on the new evaluation metric: while it may not always generate completely accurate answers in numerical reasoning, its performance improves substantially within a permitted error margin, indicating that the inferred answers are often very close to the correct values. Furthermore, experimental results show that our approach offers a significant advantage over baseline models in numerical reasoning tasks, with an average performance improvement of 200%.

Sensitivity Analysis of the New MRR Metric At the beginning of this subsection, we propose a

Dataset	Metric	Method	Avg_{All}	1p	2p	2i	3i	pi	ip	2u	up
FB15K	MRR	Q2P+NRN	4.91	0.42	0.86	9.06	17.75	7.39	3.17	0.14	0.47
		CNR-NST	**20.01**	**17.77**	**15.33**	**27.87**	**37.84**	**28.70**	**18.38**	**4.25**	**9.91**
	$\text{MRR}_{0.001}$	Q2P+NRN	1.11	0.45	0.57	1.44	3.52	1.22	0.46	0.34	0.90
		CNR-NST	**24.94**	**22.54**	**16.41**	**36.26**	**46.98**	**37.83**	**22.61**	**7.66**	**9.26**
DB15K	MRR	Q2P+NRN	2.91	0.22	0.49	6.56	10.25	3.29	2.12	0.06	0.31
		CNR-NST	**12.84**	**6.77**	**5.93**	**17.28**	**26.64**	**21.03**	**10.17**	**7.81**	**7.07**
	$\text{MRR}_{0.001}$	Q2P+NRN	10.67	4.96	6.29	4.58	5.60	9.66	13.69	18.28	22.27
		CNR-NST	**24.79**	**18.92**	**15.45**	**25.62**	**31.61**	**35.88**	**24.90**	**23.24**	**22.70**
YAGO15K	MRR	Q2P+NRN	10.63	0.51	1.17	20.31	36.20	16.97	8.24	0.31	1.31
		CNR-NST	**17.48**	**12.31**	**11.80**	**25.16**	**37.16**	**25.87**	**19.38**	**3.16**	**5.02**
	$\text{MRR}_{0.001}$	Q2P+NRN	19.26	5.66	**21.09**	9.48	10.42	13.06	21.80	**39.05**	**33.52**
		CNR-NST	**25.36**	**18.35**	17.39	**32.92**	**37.59**	**34.10**	**22.37**	23.24	16.91

Table 4: MRR and $\text{MRR}_{0.001}$ (%) for the numerical queries. For detailed results, see Appendix A.7.

new MRR metric. To further verify the usability of the metric, we conduct a sensitivity analysis on the threshold value of this metric across multiple datasets, and the specific experimental results can be found in Figure 4. Through experiments, we find the change of the new MRR metric is basically linearly related to the change of its threshold, which also proves the scientificity and practicability of this metric.

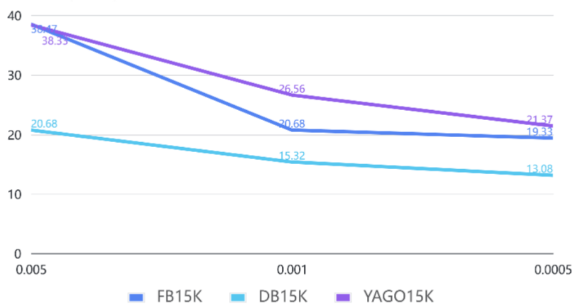

Figure 4: Sensitivity analysis of the new MRR metric and threshold parameter.

Training and Inference Efficiency In Appendix A.8, we present a comparison of the training time of CNR-NST with the baseline model. During the training phase, CNR-NST requires less training time. Although its inference time is longer, the total time for both training and inference remains significantly lower than the time required by NRN.

4.5 Performance of Large Language Models

In this subsection, we utilize several of the currently most powerful large language models (LLMs) to accomplish CQA tasks. The specific approach involves providing the LLM with the query statement structure and the specific entity or relation names, and instructing it to output answers from the candidate options in descending order of probability. The candidate answers consist of the correct answer and several distractor options that share the same attributes as the correct answer. The experimental results demonstrate that GPT-5 achieves the best performance in CQA tasks on complex KGs. However, it still exhibits a significant gap compared with our CNR-NST. The experimental results are shown in Table 5.

Model	1p	2p	3p	2i	2u
Gemini 2.5 Flash	3.20	2.15	3.38	9.78	4.78
Claude Sonnet 4	2.66	2.37	2.28	1.96	1.24
Deepseek-R1	4.32	1.17	0.13	0.54	0.94
GPT-4o	15.76	12.59	7.97	22.31	12.82
GPT-5	34.53	22.54	25.82	47.86	39.15
FNRC	**85.23**	**64.39**	**55.40**	**74.78**	**44.91**

Table 5: Performance of various LLMs on MRR metric.

5 Related Work

Complex Query Answering over KG CQA focuses on reasoning over relationships and entities in a KG to address complex logical queries. Early CQA methods relied on logical reasoning rules to query KGs. Methods like GQE (Hamilton et al., 2018), Q2B (Ren* et al., 2020), Q2P (Bai et al., 2022), ConE (Zhang et al., 2021) and BetaE (Ren and Leskovec, 2020) represent entity sets and queries using geometric shapes or probability distributions, utilizing geometric operations (*e.g.,* intersections, projections) or probabilistic operations for reasoning. Later, CQD (Arakelyan et al., 2021) introduced a framework capable of handling complex logical expressions without explicit training on complex queries. QTO (Bai et al., 2023b) further optimized the query computation tree during complex query processing, improving reasoning accuracy and reducing the search space. FIT (Yin et al., 2024) equips the neural link predictor with fuzzy logic theory and supports complex queries with provable reasoning capabilities.

Numerical Reasoning over KG Numerical reasoning tasks in KGs involve making logical inferences or predictions based on numerical values associated with entities and relationships. Methods such as RAKGE (Kim et al., 2023) , KR-EAR (Lin et al., 2016) , TransEA (Wu and Wang, 2018) and (Lacroix et al., 2018) utilize attribute learning to improve numerical reasoning within KGEs. LiteralE (Kristiadi et al., 2019) enhances KGEs by incorporating textual information through learnable parametric functions. HyNT (Chung et al., 2023) uses the expressive power of Transformers to capture complex relational structures and numerical attributes in KGs. Neural-Num-LP (Wang et al., 2020) learns numerical rules within KGs.

LLMs' Numerical Reasoning Numerical reasoning in LLM involves extracting relevant numerical information from textual descriptions and performing mathematical calculations (Zhang et al., 2023). MathPrompter (Imani et al., 2023) introduces the Chain-of-Thought (CoT) approach, using step-by-step prompting to guide models through solving complex arithmetic problems incrementally. NumeroLogic (Schwartz et al., 2024) defines new numerical formats to handle and execute arithmetic operations. Program-of-Thoughts (PoT) (Chen et al., 2023) employs the Codex model to represent reasoning processes as programs, which are then executed by external systems to perform computations and derive final answers.

Complex Query Answering over NKG Research on CQA with numerical values is still limited. LitCQD (Demir et al., 2023) decomposes complex numerical queries into subqueries, solving them via symbolic and numerical reasoning integration. NRN (Bai et al., 2023a) embeds entities, relations, and numerical attributes into a shared vector space, using neural networks to learn and reason about numerical relationships.

6 Conclusion

In this paper, we present a novel framework for reasoning and computation on NKG, leveraging a pre-trained multi-relational link predictor to infer over 102 types of complex numerical queries. Extensive experiments on three publicly available KGs demonstrate that our proposed model, CNR-NST, significantly surpasses previous SOTA methods. Furthermore, we introduce additional categories of numerical reasoning tasks and new evaluation metrics for numerical answers, contributing to the broader research of multi-hop numerical reasoning.

Limitations

Although we propose the CNR-NST framework, this work has two limitations. First, relevant experiments lack extremely large KG validation. Second, practical engineering applicability remains unverified. Against this backdrop, future work could focus on two key directions: expanding CNR-NST's supported numerical query variety, and enhancing its applicability in diverse reasoning tasks. These efforts would address current limitations and promote the practical value of CNR-NST.

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Contents

A Mathematical Proof

A.1 Fuzzy Set-based Numerical Computations

The principle of extension is an important concept in fuzzy mathematics. It describes how to extend a precise mathematical concept or operation to fuzzy sets or fuzzy values. For example, in the addition operation of fuzzy numbers, according to the principle of extension, the addition operation of ordinary real numbers is extended to the field of fuzzy numbers, enabling us to handle the addition of fuzzy numbers in a way similar to that of ordinary numbers, but we need to take into account the uncertainty of fuzzy numbers.

This section will demonstrate how mappings in the set of real numbers can be induced into mappings between fuzzy sets.

First, we explain the concept of induction. Let there be a mapping on the real number domain:

$$f : U \to V$$

$$u \mapsto v = f\left(u\right)$$

It means that for every element in U, a unique corresponding element can be found in V through the mapping f. From this, a new mapping can be induced, which we still denote as f:

$$f : P\left(U\right) \to P\left(V\right)$$

$$A \mapsto B = f\left(A\right)$$

$$f\left(A\right) \triangleq \{v|\exists u \in A, let\ f\left(u\right) = v, v \in V\}$$

Among them, the power set $P\left(U\right)$ is a set composed of all subsets of U. The key to how a mapping on a set of real numbers can induce a mapping between fuzzy sets lies in determining the membership function of the fuzzy set.

The definition of the extension principle is given below: Let $f : U \to V$. From f, we can induce two mappings:

$$f : F\left(U\right) \to F\left(V\right), f^{-1} : F\left(V\right) \to F\left(U\right)$$

The membership functions of the induced mappings are as follows:

$$\mu_{f(A_{\sim})}\left(v\right) = \begin{cases} \bigvee_{f(u)=v} \mu_{A_{\sim}}\left(u\right) & \text{if } \exists u \in U, let\ f\left(u\right) = v \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_{f^{-1}(B)}\left(u\right) = \mu_{B_{\sim}}\left(v\right), v = f\left(u\right)$$

Among them,$\mu_{A_{\sim}}(u)$ is the membership degree of the element$u$ in the fuzzy set $A_{\sim}$.

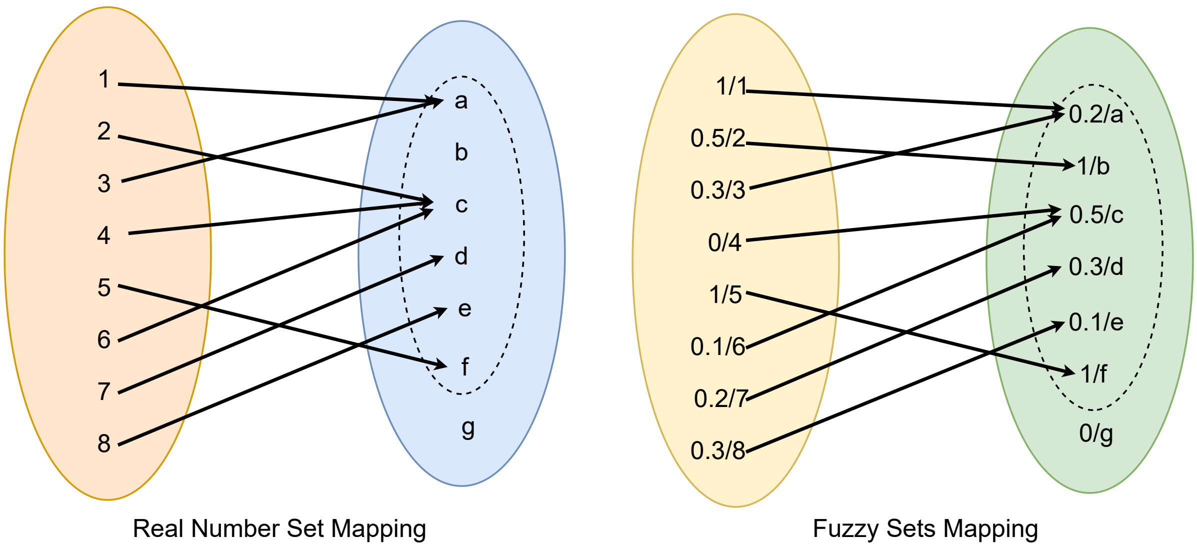


Figure 5: Visualization of the extension principle and the corresponding relationship between the operations of the set of real numbers and fuzzy sets.

A.2 Details of the Training Procedure for the Multi-ComplEx

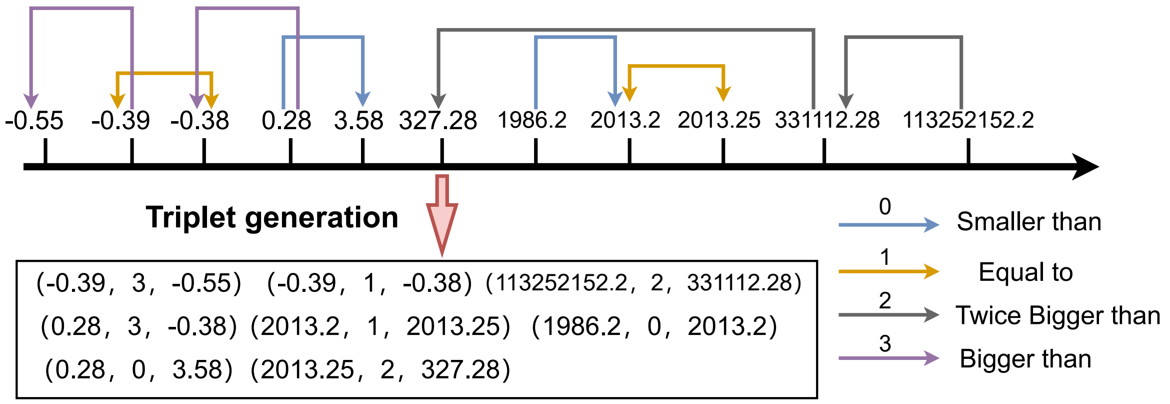


Figure 6: Numerical relationship triplet generation based on the values in the training set for pre-training of the model.

During the joint pre-training stage of CNR-NST, the objective function of the Multi-ComplEx model is defined as follows:

$$f_A(h,r,t) = Re(\langle \mathbf{h}, \mathbf{r_A}, \bar{\mathbf{t}} \rangle)$$

$$f_B(h,r,v) = Re(\langle \mathbf{h}, \mathbf{r_B}, \bar{\mathbf{v}} \rangle)$$

$$f_C(m,r,n) = Re(\langle \mathbf{m}, \mathbf{r_C}, \bar{\mathbf{n}} \rangle)$$

where h, t denotes the entity embeddings, r refers to the embeddings for different types of relations, and v, m, n represents the numerical embeddings.

We employ cross-entropy loss as the optimization objective. For each triplet (h, r, t), the loss function for each predictor is defined as follows:

$$\mathcal{L}_A = -\sum_{(h,r,t)\in\mathbb{R}} \log\sigma(f_A(h,r,t)) + \sum_{(h',r,t)\in\mathbb{R}'} \log\sigma\left(-f_A\left(h',r,t\right)\right)$$

$$\mathcal{L}_B = -\sum_{(h,r,n)\in\mathbb{A}} \log\sigma(f_B(h,r,n)) + \sum_{(h',r,n)\in\mathbb{A}'} \log\sigma\left(-f_B\left(h',r,n\right)\right)$$

$$\mathcal{L}_C = -\sum_{(m,r,n)\in\mathbb{F}} \log\sigma(f_C(m,r,n)) + \sum_{(m',r,n)\in\mathbb{F}'} \log\sigma\left(-f_C\left(m',r,n\right)\right)$$

The loss functions of the three predictors are weighted and combined to form a joint loss function, where the weights are denoted by W_{EAV} and W_{VFV}. These weights can be adjusted based on the task's requirements and importance. The joint loss function is defined as follows:

$$\mathcal{L}_{joint} = \mathcal{L}_A + W_{EAV} \cdot \mathcal{L}_B + W_{VFV} \cdot \mathcal{L}_C$$

To prevent overfitting, regularization terms are often incorporated into the loss function. L1 and L2 regularization can be applied to control the magnitude of the model parameters. For each predictor, the regularization term is defined as follows:

$$\Omega = \lambda\left(\|\mathbf{h}\|^2 + \|\mathbf{r_A}\|^2 + \|\mathbf{t}\|^2 + \|\mathbf{r_B}\|^2 + \|\mathbf{v}\|^2 + \|\mathbf{m}\|^2 + \|\mathbf{r_C}\|^2 + \|\mathbf{n}\|^2\right)$$

The final joint objective function incorporates both the loss function and the regularization terms, and is given by the following expression:

$$\mathcal{L} = \mathcal{L}_{joint} + \Omega$$

A.3 Mathematical Representation of Multi-hop Logical Reasoning

This section will use mathematical expressions to explain and infer various logical operators and mappings involved in complex numerical reasoning.

Let $V^*(X_? = x)$ represent the maximum truth value of the subquery at the root node x when node X is assigned a value x.
Assuming that the root node x is formed by merging sub-node $\{x_?^1, \ldots, x_?^K\}$ by **intersection**, the maximum truth value of the query is given by the following expression:

$$V^*(x_? = e) = \top_{1 \le i \le K} V^*(x_? = e)$$

$$V^*(x_? = n) = \top_{1 \le i \le K} V^*(x_? = n)$$

$$\Rightarrow \mathbf{V}^*(x_?) = \prod_{1 \le i \le K} \mathbf{V}^*\left(x_?^i\right)$$

Similarly, when the root node is merged by **union**, the following expression applies:

$$V^*(x_? = e) = \perp_{1 \le i \le K} V^*(x_? = e)$$

$$V^*(x_? = n) = \perp_{1 \le i \le K} V^*(x_? = n)$$

$$\Rightarrow \mathbf{V}^*(x_?) = 1 - \prod_{1 \le i \le K} \left(1 - \mathbf{V}^*\left(x_?^i\right)\right)$$

When the root node is connected to its child edges through any relational edge, the maximum truth value of the query with this node as the root is expressed as shown in Equation 9 in the main text. Depending on the type of relation, the expression has the following four variations:

$$\mathbf{V}^*(v_?) = \max_j \left(\left(V^*(v_k)^T \cdots \times |v|\right) \odot M_E\right)$$

$$\mathbf{V}^*(n_?) = \max_j \left(\left(V^*(v_k)^T \cdots \times |v|\right) \odot M_A\right)$$

$$\mathbf{V}^*(v_?) = \max_j \left(\left(V^*(n_k)^T \cdots \times |n|\right) \odot M_A^{-1}\right)$$

$$\mathbf{V}^*(n_?) = \max_j \left(\left(V^*(n_k)^T \cdots \times |n|\right) \odot M_F\right)$$

A.4 Supplementary Task Definitions

Numerical FOL Query A complex numerical query is defined in existential positive first-order logic form, which can be recursively defined as:
1. Atomic Formulas: If $t_1, t_2, \ldots, t_n$ are variables or constants, and P is an n-ary predicate, then $P(t_1, t_2, \ldots, t_n)$ is an atomic formula.
2. Compound formulas: It can be constructed using logical connectives $\wedge$ (and), $\vee$ (or), and quantifiers $\forall$ (for all) and $\exists$ (there exists).

Numerical Complex Query Answering As every logic query can be converted into a disjunctive normal form, the complex query Q on a KG with numeric literals ($\mathcal{G}$) can be defined as:

$$q[X_?] = V_1, \ldots, V_i \in \mathcal{V}, N_1, \ldots, N_j \in \mathcal{N} : c_1 \vee c_2 \vee \cdots \vee c_n$$

$$c_i = e_{i,1} \wedge e_{i,2} \wedge \cdots \wedge e_{i,m}$$

Here, c_i represents a conjunction of several atomic logical expressions $e_{i,j}$, where each $e_{i,j}$ can be one of the following expressions:

$$e_{i,j} = r(V, V'), \text{ with } V, V' \in \{V_1, \ldots, V_i\}, V \neq V', r \in \mathcal{R}$$

$$e_{i,j} = a(V, N), \text{ with } V \in \{V_1, \ldots, V_i\}, N \in \{N_1, \ldots, N_j\}, a \in \mathcal{A}$$

$$e_{i,j} = f\left(N, N'\right), \text{ with } N, N' \in \{N_1, \ldots, N_j\}, N \neq N', \\ f \in \{\leq, \geq, =, +, -, \times, \div, \ldots\}$$

In the above equation, the variable E represents a subset of entity $\mathcal{V}$, and the variable C represents a subset of the numerical attribute set $\mathbb{N}$. The binary function r_i determines whether a class i relationship exists between two entities. The function a_j determines whether an entity possesses a value for attribute j. The function f checks whether a filtering condition, such as greater than or less than, is satisfied between two values. The function b_f determines whether a quadratic operation between the first two values yields an answer.

The objective of complex reasoning is to find a valid assignment for the variables such that the query $q\left[X_?\right]$ holds true. The incompleteness of the KG introduces uncertainty, which implies $e_{i,j}$ is no longer a binary variable. Instead, it stands for the likelihood that the correspondence holds, and its generalized truth value ranges between [0,1]. For this reason, as follows (Arakelyan et al., 2021), we formalise Eq.(1) as an optimisation problem:

$$q\left[X_?\right] = X_?.V_1, \ldots, V_i \in \mathcal{V}, N_1, \ldots, N_j \in \mathcal{N} = \\ \arg\max\left(e_{1,1}\top \ldots \top e_{1,m}\right) \perp \ldots \perp \left(e_{n,1}\top \ldots \top e_{n,m}\right)$$

where $e_{n,m}$ is the probability score inferred by Multi-ComplEx based on the corresponding atomic formula. $\perp$ and $\top$ are generalisations of fuzzy logic over [0,1] for conjunctive and disjunctive extraction, and we chose the product t-norm and t- connorm (Hájek, 2001) as natural connectives in fuzzy logic in this paper.

A.5 Reasoning Process of Fuzzy Sets in the Application of Logical Operators

Based on Formula in the main text, the calculation process of logical operators in the query for CNR-NST is as follows.

If root $V^*\left(x_?\right)$ is aggregated from multiple child nodes $\left\{V_?^1, V_?^2 \ldots V_?^K\right\}$, then:

$$\mathbf{V}^*\left(x_?\right) = \prod_{1\leq i\leq K} V_i^*\left(x_?\right)$$

where x represents any node. For other definitions, you can refer to Formula 8 in the main text.

Similarly, the union of the child nodes is shown as follows:

$$\mathbf{V}^*\left(x_?\right) = 1 - \prod_{1\leq i\leq K}\left(1 - V_i^*\left(x_?\right)\right)$$

A.6 Main Experimental Results

Here, we present the MRR results for all 102 sub-queries on our dataset. We applied two sampling methods on the public datasets FB15K, DB15K, and YAGO15K, resulting in 77 query types using the NRN sampling method and 92 query types using our improved method. Additionally, we provide detailed results for the 10 extended numerical computation query types.

The detailed data shows that, while Q2B+NRN and GQE+NRN exhibit decent performance on some sub-tasks involving intersection, they perform poorly on other tasks. Q2P+NRN demonstrates better overall performance, but our CNR-NST model significantly outperforms Q2P+NRN in both average performance and numerical query tasks (see Section 4.2 of the main text for average performance).

The definition of query subclass names: ”p” stands for ”Relation Prediction”, ”a” stands for ”Attribute Prediction”, ”r” stands for ”Reverse Attribute Prediction”, ”n” stands for ”Numerical Prediction”, ”u” stands for ”Union”, ”i” stands for ”Intersection”, and ”b” stands for ”Binary Operator”.

Query_num	Method	1rap	1rp	1ap	1np	pa	ar	rp	ra	an	2n	2p	nr
77	GQE+NRN	1.62	30.64	-	1.31	9.30	0.25	12.40	1.12	1.07	2.29	6.81	0.71
	Q2B+NRN	2.16	39.56	-	1.26	9.66	0.26	8.89	0.70	0.99	2.67	7.09	0.71
	Q2P+NRN	1.79	40.95	-	0.06	13.72	0.64	11.07	1.01	0.10	0.09	7.80	0.70
	CNR-NST	7.95	85.21	-	30.15	31.51	6.13	40.93	4.43	2.04	2.69	64.21	1.43
92	GQE+NRN	1.36	31.18	5.22	1.41	10.91	0.25	13.23	1.32	2.04	1.61	6.69	0.72
	Q2B+NRN	2.21	39.25	5.14	1.57	11.17	0.35	8.76	0.62	2.30	2.42	6.84	0.81
	Q2P+NRN	0.26	37.52	0.78	0.05	2.97	0.07	13.01	0.30	0.12	0.06	10.75	0.44
	CNR-NST	7.98	85.23	4.79	31.31	31.61	6.16	46.89	1.61	17.46	14.49	64.39	1.61

Query_num	Method	2pi	2ai	pri	rpi	2ri	2ni	nai	ani
77	GQE+NRN	27.65	-	29.97	30.11	4.80	1.70		
	Q2B+NRN	31.41	-	31.18	31.41	9.64	1.77		
	Q2P+NRN	37.22	-	35.73	36.05	25.85	0.02		
	CNR-NST	73.21	-	49.12	45.69	16.53	2.30		
92	GQE+NRN	27.83	58.17	30.11	30.70	3.61	1.30	35.64	35.27
	Q2B+NRN	31.29	52.34	31.17	31.75	6.52	1.44	35.91	35.22
	Q2P+NRN	33.30	22.45	30.89	30.50	0.98	0.21	10.76	10.91
	CNR-NST	74.78	35.78	49.23	49.75	16.86	8.93	23.52	20.61

Query_num	Method	2na3i	3n3i	r2p3i	rpr3i	nan3i	n2a3i	2rp3i	3r3i	prp3i	p2r3i	3p3i	2pr3i	2an3i	3a3i	a2n3i	ana3i
77	GQE+NRN	-	1.98	41.98	38.00	-	-	37.26	-	41.96	41.90	32.65	41.55	-	-	-	-
	Q2B+NRN	-	2.31	43.18	39.67	-	-	34.31	-	43.35	34.45	35.37	42.25	-	-	-	-
	Q2P+NRN	-	0.03	47.59	56.41	-	-	47.67	-	47.54	42.33	39.81	47.54	-	-	-	-
	CNR-NST	-	5.89	54.02	52.10	-	-	48.90	-	54.30	61.05	71.02	54.24	-	-	-	-
92	GQE+NRN	49.98	1.74	42.07	29.54	49.99	83.45	40.36	0.30	41.67	41.54	32.90	42.67	49.53	87.23	49.99	84.09
	Q2B+NRN	50.00	1.68	43.23	37.22	49.40	84.72	27.39	0.48	42.58	39.52	35.27	42.27	85.77	95.19	49.76	84.70
	Q2P+NRN	15.83	0.22	43.53	36.49	15.70	24.81	28.90	7.14	42.92	39.83	37.79	43.99	26.90	15.88	15.76	26.93
	CNR-NST	15.21	3.64	60.49	59.37	14.86	25.08	53.95	2.63	42.12	61.14	71.18	59.59	27.14	49.98	14.88	25.79

Query_num	Method	pria	aair	ppia	ppip	prip	rpip	rrip	rpia	aain	nain	nair	anin	nnin	nnir	anir	rria
77	GQE+NRN	26.90	0.18	13.31	11.80	12.33	12.77	11.07	26.38	1.05	0.75	1.22	0.62	1.67	1.03	0.21	0.04
	Q2B+NRN	26.66	0.19	15.34	12.49	12.94	13.28	14.80	27.87	1.34	0.80	0.26	0.66	2.14	0.98	0.26	0.05
	Q2P+NRN	17.44	0.34	17.97	13.58	16.71	16.02	18.97	15.31	0.08	0.07	0.61	0.07	0.09	0.32	0.62	100.00
	CNR-NST	35.80	8.96	37.60	70.38	65.06	71.14	77.86	35.13	2.45	1.34	3.35	1.28	0.98	0.71	3.28	5.04
92	GQE+NRN	26.80	0.16	18.85	12.13	12.41	12.58	10.24	24.98	5.64	6.13	0.20	5.97	1.48	0.98	0.17	0.04
	Q2B+NRN	24.23	0.14	20.20	12.48	12.98	13.55	14.60	27.47	9.29	7.30	0.24	7.44	1.57	0.93	7.30	0.06
	Q2P+NRN	8.36	0.08	6.34	14.60	15.71	15.21	15.64	7.94	0.72	0.97	0.07	0.90	0.09	0.77	0.06	0.03
	CNR-NST	39.33	8.96	41.38	69.55	71.99	72.22	78.19	43.29	12.59	8.03	2.98	8.22	4.58	0.58	1.23	0.43

Query_num	Method	pppi	ppri	arpi	rppi	arri	rpri	nnai	pani	nnni	anni	anai	rani	nrpi	raai	nrri	paai
77	GQE+NRN	17.71	19.05	26.19	21.09	2.58	21.33	-	9.89	1.97	1.00	-	2.31	10.80	-	3.71	-
	Q2B+NRN	22.02	19.31	27.32	23.15	5.30	20.02	-	11.51	2.59	1.23	-	2.85	13.09	-	4.44	-
	Q2P+NRN	24.88	22.35	36.27	47.54	13.85	28.54	-	18.28	0.03	0.02	-	3.02	19.05	-	26.01	-
	CNR-NST	70.71	33.50	46.23	61.24	21.40	42.30	-	8.83	1.44	2.05	-	15.24	16.25	-	6.44	-
92	GQE+NRN	17.70	19.55	25.40	21.37	2.22	21.24	39.87	15.34	1.65	7.43	41.95	2.60	11.08	5.79	2.72	50.67
	Q2B+NRN	21.18	20.63	26.84	23.03	4.38	21.07	40.35	16.93	1.95	7.27	42.54	1.69	12.78	5.92	4.34	51.96
	Q2P+NRN	23.36	20.94	26.58	23.88	1.17	21.56	16.41	5.26	0.28	1.27	14.07	0.28	12.08	3.26	2.67	18.31
	CNR-NST	70.18	37.36	48.38	66.26	10.02	40.18	37.24	7.39	7.23	8.57	39.58	2.05	16.19	48.42	2.63	56.79

Query_num	Method	2pu	rpu	2ru	pru	2au	2nu	anu	nau
77	GQE+NRN	7.97	8.14	0.55	7.97	1.19	0.52	0.48	0.46
	Q2B+NRN	12.65	11.95	0.35	11.85	1.31	0.62	0.74	0.67
	Q2P+NRN	19.43	19.87	0.60	18.98	0.45	0.11	0.08	0.09
	CNR-NST	75.52	48.03	1.12	48.03	4.10	1.01	2.77	2.75
92	GQE+NRN	7.91	7.73	0.57	7.77	0.96	0.55	0.54	0.52
	Q2B+NRN	12.66	11.87	0.42	11.78	1.17	0.67	0.82	0.77
	Q2P+NRN	14.72	14.74	0.09	15.10	0.11	0.15	0.16	0.15
	CNR-NST	74.78	52.43	2.02	52.28	1.19	4.15	7.40	7.22

Query_num	Method	2pup	2pua	prup	prua	anun	anur	2aur	2aun	naur	naun	2nun	2nur	2rup	2rua	rpua	rpup
77	GQE+NRN	5.55	5.26	5.71	3.77	0.83	0.59	0.87	0.33	0.65	0.83	0.72	0.75	7.11	0.94	3.83	5.92
	Q2B+NRN	5.87	6.47	6.12	5.45	1.10	0.54	0.70	0.45	0.58	1.02	0.89	0.65	6.92	1.02	5.56	6.10
	Q2P+NRN	6.03	10.75	7.55	9.31	0.09	0.62	3.39	0.10	0.62	0.08	0.08	0.58	8.15	1.13	8.05	6.91
	CNR-NST	59.62	19.31	52.35	19.72	0.97	1.46	2.43	0.91	1.47	0.95	0.85	1.12	46.03	1.22	20.17	54.58
92	GQE+NRN	5.51	5.97	5.67	4.22	0.99	0.65	1.05	0.71	0.67	0.96	0.64	0.76	6.76	0.92	4.22	5.86
	Q2B+NRN	5.51	6.89	6.05	5.00	1.64	0.54	0.71	1.05	0.62	1.58	0.88	0.68	6.79	1.09	5.11	6.09
	Q2P+NRN	9.17	1.00	8.72	0.88	0.18	0.36	0.75	0.15	0.41	0.22	0.15	0.46	8.78	0.39	0.79	8.87
	CNR-NST	60.23	25.69	56.93	22.68	3.25	0.95	2.37	5.61	0.95	3.53	2.33	0.73	49.98	1.46	22.62	56.55

Query_num	Method	2b	3b	bp		pb		2pb			
		aab	aaab	aabr	aabn	raab	paab	rarab	rapab	parab	papab
102	CNR-NST	18.39	17.56	4.08	5.64	16.62	28.24	7.23	14.68	15.53	26.93

Table 6: Detailed MRR results (%) in the FB15K test set.

Query_num	Method	1rap	1rp	1ap	1np	pa	ar	rp	ra	an	2n	2p	nr
77	GQE+NRN	1.48	5.35	-	1.32	6.81	0.48	7.81	1.88	0.24	0.35	7.65	0.34
	Q2B+NRN	1.63	9.37	-	1.41	6.87	0.61	6.53	1.72	0.28	0.35	8.26	0.23
	Q2P+NRN	0.12	26.72	-	0.01	10.58	0.02	14.06	0.46	0.02	0.01	3.49	0.05
	CNR-NST	2.96	54.60	-	21.10	13.27	1.71	18.40	4.56	19.41	7.66	22.39	1.01
92	GQE+NRN	1.18	5.85	1.83	1.49	5.87	0.37	8.21	1.93	1.97	0.35	8.78	0.34
	Q2B+NRN	1.53	8.76	1.48	1.04	5.16	0.41	6.21	1.30	2.27	0.34	8.28	0.26
	Q2P+NRN	0.56	23.25	0.65	0.37	3.24	0.35	9.18	0.57	0.71	0.16	15.22	0.25
	CNR-NST	2.71	50.98	2.42	22.21	13.40	1.83	20.81	5.02	21.62	7.14	22.58	1.10

Query_num	Method	2pi	2ai	pri	rpi	2ri	2ni	nai	ani
77	29.89	-	34.94	37.07	30.80	0.90	-	-	
	Q2B+NRN	25.69	-	33.35	32.55	39.12	0.77	-	-
	Q2P+NRN	35.52	-	58.84	56.40	30.96	0.01	-	-
	CNR-NST	46.65	-	56.10	51.93	63.00	10.20	-	-
92	29.92	60.58	30.57	32.34	31.01	0.62	32.59	31.44	
	Q2B+NRN	24.91	67.49	28.86	35.57	37.78	0.83	33.54	31.21
	Q2P+NRN	39.24	39.37	34.80	39.88	31.23	0.29	21.16	20.42
	CNR-NST	42.30	45.42	50.36	53.06	63.52	5.91	24.64	24.69

Query_num	Method	2na3i	3n3i	r2p3i	rpr3i	nan3i	n2a3i	2rp3i	3r3i	prp3i	p2r3i	3p3i	2pr3i	2an3i	3a3i	a2n3i	ana3i
77	GQE+NRN	-	0.84	57.55	83.64	-	-	88.75	100.00	53.10	85.00	45.20	50.79				
	Q2B+NRN	-	0.82	53.73	84.85	-	-	89.58	100.00	55.80	86.11	40.52	51.10				
	Q2P+NRN	-	0.01	80.39	77.47	-	-	100.00	50.00	78.75	100.00	40.03	100.00				
	CNR-NST	-	2.48	70.65	83.33	46.18	-	88.89	100.00	76.04	87.18	55.48	71.13				
92	GQE+NRN	46.21	0.63	65.17	89.29	46.18	80.63	87.50	20.00	69.88	100.00	45.61	67.49	80.70	77.32	45.81	82.00
	Q2B+NRN	47.56	0.83	65.45	80.36	45.73	81.23	70.83	100.00	61.02	100.00	40.68	63.45	80.25	85.74	45.88	79.57
	Q2P+NRN	32.37	3.21	63.70	73.81	32.29	46.26	61.11	100.00	65.86	80.00	55.13	64.96	46.69	48.73	31.91	48.12
	CNR-NST	21.41	6.62	76.67	92.86	22.50	48.66	70.83	100.00	80.44	72.22	53.36	70.44	48.43	55.60	21.40	42.10

Query_num	Method	pria	aair	ppia	ppip	prip	rpip	rrip	rpia	aain	nain	nair	anin	nnin	nnir	anir	rria
77	GQE+NRN	19.95	0.53	16.69	14.02	12.88	11.43	7.56	14.03	0.12	0.09	0.33	0.10	0.31	0.30	0.32	8.15
	Q2B+NRN	17.51	0.40	19.46	11.36	14.26	14.98	11.94	10.86	0.11	0.08	0.39	0.09	0.30	0.23	0.38	6.67
	Q2P+NRN	35.66	0.05	18.10	5.07	8.69	8.47	7.38	20.28	0.01	0.01	0.03	0.01	0.01	0.05	0.03	0.02
	CNR-NST	42.49	1.72	26.49	23.29	26.71	26.85	23.82	38.54	7.69	11.20	1.53	10.72	4.76	0.88	1.38	15.14
92	GQE+NRN	14.89	0.17	16.33	13.81	12.00	12.07	8.84	14.66	4.97	6.13	0.28	6.27	0.24	0.29	0.29	3.12
	Q2B+NRN	14.46	0.16	17.71	12.73	11.39	12.32	10.88	18.34	5.44	5.30	0.16	5.79	0.37	0.31	0.31	2.74
	Q2P+NRN	20.92	0.31	15.09	19.22	17.44	17.29	16.31	16.56	4.35	3.78	0.27	3.91	0.13	0.25	0.25	1.19
	CNR-NST	39.19	1.58	24.12	26.16	24.42	26.62	25.56	46.20	9.70	10.82	1.57	11.57	4.54	0.77	1.37	8.90

Query_num	Method	pppi	ppri	arpi	rppi	arri	rpri	nnai	pani	nnni	anni	anai	rani	nrpi	raai	nrri	paai
77	GQE+NRN	23.16	32.30	27.97	24.52	-	28.02	40.38	-	5.86	0.83	-	0.23	3.03	11.56	-	18.28
	Q2B+NRN	19.72	35.65	33.90	26.07	-	29.72	41.82	-	6.12	0.85	-	0.50	2.83	20.62	-	32.11
	Q2P+NRN	15.15	52.74	39.31	24.52	-	33.18	49.27	-	12.74	0.01	-	0.01	1.22	12.95	-	27.96
	CNR-NST	39.56	52.59	51.24	43.73	-	69.50	55.74	-	12.20	5.54	-	30.25	8.06	17.27	-	31.27
92	GQE+NRN	22.79	31.00	35.16	23.93	46.33	34.95	43.85	30.71	10.13	0.66	7.68	38.43	3.48	12.20	42.01	19.85
	Q2B+NRN	19.49	40.11	28.30	23.21	42.91	32.96	43.40	30.56	8.29	0.66	6.02	36.53	2.89	20.32	39.33	33.45
	Q2P+NRN	32.87	38.29	38.75	32.35	36.01	31.80	40.73	19.40	9.35	0.54	5.88	27.28	1.26	27.28	36.01	30.28
	CNR-NST	38.01	53.34	44.27	42.35	53.94	62.77	50.33	30.62	13.82	5.04	7.14	36.67	7.45	17.01	52.27	32.65

Query_num	Method	2pu	rpu	2ru	pru	2au	2nu	anu	nau
77	GQE+NRN	4.79	3.70	0.40	3.70	0.39	0.31	0.16	0.16
	Q2B+NRN	4.27	3.82	0.34	4.33	0.49	0.71	0.53	0.58
	Q2P+NRN	13.76	15.35	0.04	16.03	0.03	0.01	0.05	0.01
	CNR-NST	14.63	9.63	1.14	9.80	1.10	2.91	5.36	5.88
92	GQE+NRN	4.50	3.39	0.39	3.46	0.41	0.39	0.16	0.18
	Q2B+NRN	3.31	3.69	0.24	3.99	0.37	0.57	0.31	0.33
	Q2P+NRN	11.63	7.07	0.23	6.07	0.14	0.25	0.45	0.42
	CNR-NST	14.86	9.40	1.28	9.06	1.17	1.99	4.79	4.67

Query_num	Method	2pup	2pua	prup	prua	anun	anur	2aur	2aun	naur	naun	2nun	2nur	2rup	2rua	rpua	rpup
77	GQE+NRN	5.90	3.75	5.71	3.19	0.25	0.24	4.24	0.11	0.27	0.25	0.46	0.42	3.79	1.68	3.49	5.75
	Q2B+NRN	5.07	3.11	6.03	3.57	0.37	0.21	3.60	0.13	0.22	0.39	0.95	0.34	4.26	1.64	3.54	5.68
	Q2P+NRN	3.00	3.36	5.36	3.13	0.01	0.06	18.16	0.01	0.08	0.01	0.01	0.05	7.57	0.60	3.62	4.88
	CNR-NST	16.57	10.43	14.50	9.21	2.03	0.49	1.35	4.45	0.54	2.60	1.58	0.53	11.15	2.26	8.28	13.25
92	GQE+NRN	7.70	3.68	6.06	4.11	0.71	0.24	4.16	0.45	0.27	0.69	0.40	0.42	3.32	1.90	3.27	5.11
	Q2B+NRN	6.27	2.69	6.23	2.94	1.11	0.15	2.12	0.40	0.18	0.91	0.87	0.21	3.50	1.48	3.14	5.28
	Q2P+NRN	9.82	2.72	8.84	1.92	0.58	0.20	5.08	0.24	0.23	0.56	0.45	0.34	5.84	1.92	2.15	7.66
	CNR-NST	18.20	10.56	15.05	7.89	2.54	0.44	1.38	3.98	0.33	2.39	1.47	0.33	12.44	2.53	8.83	14.86

Query_num	Method	2b	3b	bp		pb		2pb			
		aab	aaab	aabr	aabn	raab	paab	rarab	rapab	parab	papab
102	CNR-NST	11.71	12.49	0.85	11.00	13.49	24.40	13.29	21.67	25.85	22.33

Table 8: Detailed MRR results (%) in the YAGO15K test set.

A.7 Detailed Experimental Results of the New Evaluation Metrics

Dataset	Method	1ap	1np	an	pa	ra	2n	2ai	ani	2ni	nai
FB15K	Q2P+NRN	0.79	0.11	0.12	0.97	1.07	0.11	1.11	2.02	0.31	2.31
	CNR-NST	20.56	24.51	25.70	15.66	8.96	15.30	66.03	26.65	27.15	25.21
DB15K	Q2P+NRN	8.87	1.06	2.83	7.83	12.01	2.48	8.87	2.96	3.58	2.89
	CNR-NST	17.41	20.42	17.19	17.65	17.77	9.19	46.09	25.03	17.95	13.42
YAGO15K	Q2P+NRN	4.53	6.78	20.51	5.93	2.81	55.10	5.32	4.81	21.52	6.26
	CNR-NST	19.06	17.64	21.23	19.01	12.12	17.20	45.76	31.11	20.68	34.12

Dataset	Method	paai	pani	anni	anai	nnai	nnni	rani	raai
FB15K	Q2P+NRN	1.72	2.02	0.45	1.32	1.37	0.40	0.91	1.53
	CNR-NST	63.62	25.76	27.27	46.29	39.46	18.67	19.96	61.60
DB15K	Q2P+NRN	7.74	6.78	3.00	3.28	3.14	5.30	20.35	27.71
	CNR-NST	56.27	21.29	26.04	44.89	51.48	14.22	29.59	43.30
YAGO15K	Q2P+NRN	6.95	8.26	24.14	5.22	4.88	45.47	4.43	5.13
	CNR-NST	55.52	20.46	22.99	38.58	38.17	21.62	17.95	57.47

Dataset	Method	2pia	pria	2ain	anin	rpia	2ria	2nin	nain
FB15K	Q2P+NRN	1.42	0.96	0.11	0.13	0.72	0.03	0.12	0.13
	CNR-NST	20.63	35.04	22.28	28.54	32.89	6.42	12.02	23.07
DB15K	Q2P+NRN	6.25	16.97	7.51	3.25	20.18	44.58	6.49	4.30
	CNR-NST	23.59	34.34	7.39	16.13	40.18	56.93	8.53	12.08
YAGO15K	Q2P+NRN	6.20	2.91	27.87	39.73	1.98	0.81	56.38	38.59
	CNR-NST	23.67	22.41	15.44	26.53	26.21	22.40	17.42	24.89

Dataset	Method	2na3i	3n3i	nan3i	n2a3i	2an3i	3a3i	a2n3i	ana3i
FB15K	Q2P+NRN	2.85	0.50	3.18	2.05	2.86	11.27	3.11	2.37
	CNR-NST	27.19	32.68	36.71	53.68	57.51	78.05	35.18	54.86
DB15K	Q2P+NRN	2.14	7.04	1.99	8.00	6.37	7.92	3.59	7.72
	CNR-NST	19.93	23.69	20.74	35.90	32.97	54.71	25.70	39.22
YAGO15K	Q2P+NRN	7.42	36.03	7.73	7.22	6.53	5.23	7.68	5.50
	CNR-NST	31.78	26.41	28.08	47.05	45.61	55.37	28.83	37.60

Dataset	Method	aau	2nu	nau	2pua	prua	anun	2aun	naun	2nun	2rua	rpua
FB15K	Q2P+NRN	0.47	0.29	0.26	1.93	1.66	0.28	0.28	0.36	0.34	0.74	1.64
	CNR-NST	4.91	6.22	11.86	11.29	14.05	5.25	10.32	6.30	3.15	11.44	12.25
DB15K	Q2P+NRN	3.76	27.26	23.82	10.92	22.80	23.24	27.93	22.11	21.73	26.77	22.64
	CNR-NST	6.51	20.87	24.99	16.80	19.57	20.23	29.34	24.94	17.87	26.63	26.22
YAGO15K	Q2P+NRN	2.96	57.48	56.71	6.34	5.57	60.38	62.93	59.76	63.90	3.08	6.18
	CNR-NST	9.37	26.67	33.67	13.43	13.25	20.12	33.21	20.13	14.54	8.79	11.80

Table 9: Detailed $MRR_{0.001}$ results (%) in all test set.

A.8 Detailed Experimental Results of the Training and Inference Efficiency

As shown in Table 10, during the training phase, CNR-NST only requires training on single-hop queries involving numerical values and entities, resulting in shorter training times compared to NRN, which must train on the entire dataset. CNR-NST's inference process requires calling a pre-trained prediction model at each step of the reasoning chain. In contrast, NRN has fixed neural network dimensions, which allows for faster inference. However, even on a dataset with 400,000 queries, CNR-NST completes the inference in just four hours, which is still significantly less time than what NRN requires for training.

Method	Training_time	1p	2p	2i	3i	pi	ip	2u	up	2b	3b	bp	pb	2pb
Q2P+NRN	32.40	0.13	0.14	0.25	0.33	0.24	0.26	0.28	0.29	-	-	-	-	-
CNR-NST	0.22	3.06	15.83	4.53	10.09	21.19	16.66	4.54	39.51	6.07	7.75	603.78	80.81	528.20

Table 10: Training and inference times on the FB15K dataset (training time in hours, inference time in ms/query).

A.9 Ablation Study of the Pre-training Framework

Dataset	Pre-training Framework	Avg_{Test}	ERE_{Test}	EAV_{Test}	VFV_{Test}
FB15K	ComplEx	37.21	81.96	5.51	24.15
	Multi-ComplEx	**60.61**	**82.23**	**6.78**	**92.83**
DB15K	ComplEx	27.42	47.22	6.97	28.08
	Multi-ComplEx	**51.63**	**47.25**	**11.26**	**96.36**
YAGO15K	ComplEx	23.25	46.46	1.98	21.32
	Multi-ComplEx	**44.42**	**48.34**	**3.39**	**81.53**

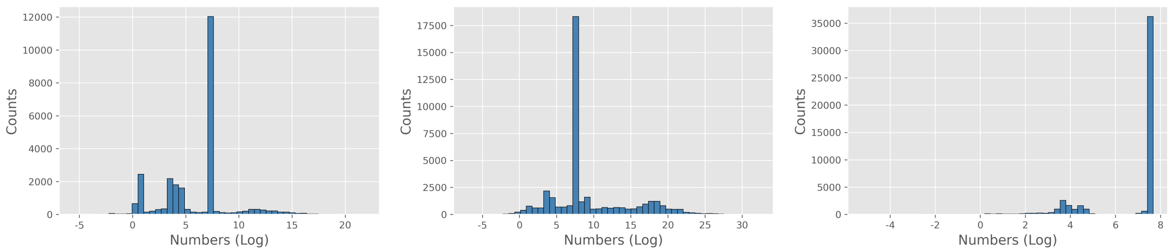
Table 11: MRR results (%) for the predictors (ERE, EAV, VFV) on the validation and test sets.

As shown in Table 11, we performed an ablation study in the pre-training framework. In the single ComplEx model, the three types of link predictors share parameters, whereas in the Multi-ComplEx model, parameters are not shared. Results show the Multi-ComplEx model significantly outperforms the ComplEx model in numerical prediction tasks. Moreover, knowledge embedded in entity relationships greatly enhances numerical prediction accuracy and improves complex numerical query performance.

B Dataset Statistics

B.1 Numerical Sparsity of the Dataset

In this section, we present the numerical sparsity in the three public datasets FB15K, DB15K, and YAGO15K.



FB15K DB15K YAGO15K

Figure 7: Numerical distribution across the three public datasets.

As shown in the three figures above, the horizontal axis represents the logarithmic transformation of actual values, while the vertical axis shows the frequency of occurrences. We observe that most values are concentrated in a specific region, yet the overall range of values remains broad, highlighting the numerical sparsity characteristic of NKGs.

B.2 Knowledge Graph Contruction

Table 12 presents the detailed information for constructing KGs from the original datasets, from which we will randomly sample to form queries.

Dataset	Data Split	Nodes_{N}	Rel_{N}	Attr_{N}	$\text{Rel}_{\text{Edges}}$	$\text{Attr}_{\text{Edges}}$	$\text{Num}_{\text{Edges}}$
FB15K	Training	25,106	1345	15	947,540	20,248	27,020
	Validation	26,108			1,065,982	22,779	27,376
	Testing	27,144			1,184,426	25,311	27,389
DB15K	Training	31,980	279	30	145,262	33,131	25,495
	Validation	34,191			161,978	37,269	25,596
	Testing	36,358			178,394	41,411	25,680
YAGO15K	Training	32,112	32	7	196,616	21,732	26,616
	Validation	33,078			221,194	22,748	26,627
	Testing	36,358			245,772	23,520	26,631

Table 12: The statistics of the three KGs.

B.3 Dataset Information for Pre-training

Table 13 shows the training dataset information used for our pre-training framework Multi-ComplEx. The VFV triples are as defined by us, and the dataset splits are identical to those in Table 14.

Dataset	Data Split	ERE Triad	EAV Triad	VFV Triad
FB15K	Training	473,770	20,248	220,421
	Validation	59,221	2,531	27,553
	Testing	59,222	2,532	27,553
DB15K	Training	79,222	33,145	296,006
	Validation	9,903	4,143	37,001
	Testing	9,903	4,144	37,001
YAGO15K	Training	98,308	18,816	229,804
	Validation	12,289	2,352	28,725
	Testing	12,289	2,352	28,726

Table 13: Details of the datasets used for pre-training.

B.4 Sample Queries from Graph

Figure 8 provides a visual representation of the structure for most of the 102 query types. Tables 14 and 15 present the distribution of 13 major query types and 102 subquery types sampled from the three datasets. Note that these quantities only include the test set. Since CNR-NST does not require training on complex queries, we only sample from the test set and conduct experiments. Supplementary explanation: in the Query_Name column of the table, ”p” stands for relation prediction, ”ap” stands for attribute prediction, abbreviated as ”a”, ”np” stands for numerical prediction, abbreviated as ”n”, and ”rap” stands for reverse attribute prediction, abbreviated as ”r”. For example, PAAi refers to the intersection operation between a two-hop query involving one relation prediction and one attribute prediction and a single-hop query of one attribute prediction, with the final answer being a single numerical value. In contrast, PRuA denotes an operation where the union of one relation prediction and one reverse attribute prediction is followed by an attribute prediction, and the final answer consists of multiple numerical values. Table 16 shows the percentage of inference paths present on the top edge of the test set for each query type sampled on our dataset, reflecting to some extent the difficulty of the queries we sampled.

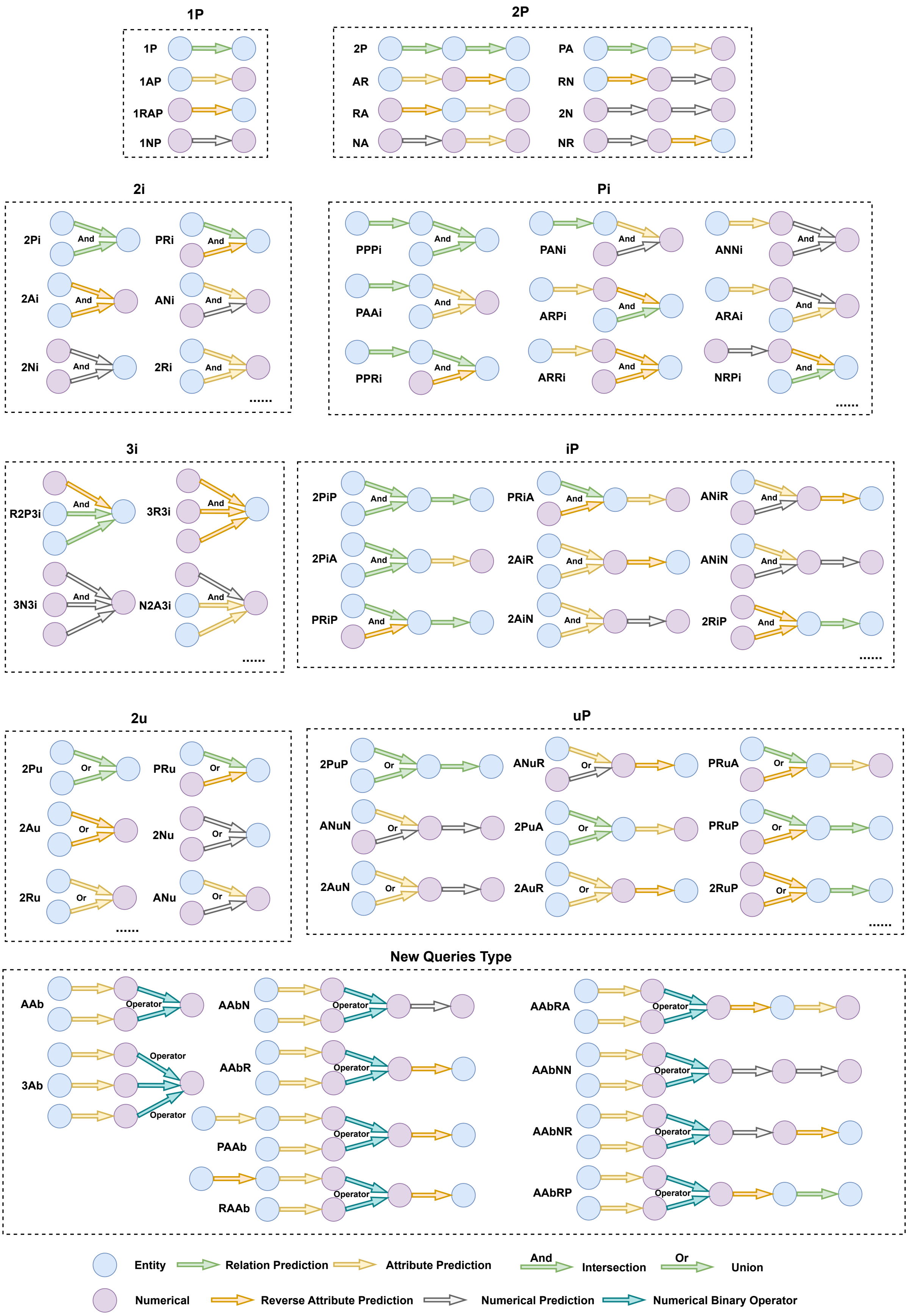


Figure 8: The visualization includes 102 complex numerical reasoning query types, displaying a subset. These structures can combine to form more intricate queries. At the bottom, we present newly defined numerical computation queries.

Type	Query Name	Query Structure Definition	FB15K	DB15K	YAGO15K
1p	1np	('nv', ('np',))	1737	3894	1271
	1ap	('e', ('ap',))	2306	3407	2326
	1rap	('nv', ('rap',))	1059	1085	1133
	1p	('e', ('rp',))	19953	3029	6208
2p	an	(('e', ('ap',)), ('np',))	2735	4070	1641
	2p	(('e', ('rp',)), ('rp',))	18355	2088	1273
	nr	(('nv', ('np',)), ('rap',))	2329	2197	1277
	2n	(('nv', ('np',)), ('np',))	2146	4655	2294
	ra	(('nv', ('rap',)), ('ap',))	587	2481	924
	pa	(('e', ('rp',)), ('ap',))	5257	4833	2988
	ar	(('e', ('ap',)), ('rap',))	4570	5072	3662
	rp	(('nv', ('rap',)), ('rp',))	5549	2615	2470
2i	ani	(('e', ('ap',)), ('nv', ('np',)), ('i',))	7176	14727	10576
	2pi	(('e', ('rp',)), ('e', ('rp',)), ('i',))	25720	1192	2574
	nai	(('nv', ('np',)), ('e', ('ap',)), ('i',))	7473	14829	10607
	2ni	(('nv', ('np',)), ('nv', ('np',)), ('i',))	9196	13349	7765
	2ai	(('e', ('ap',)), ('e', ('ap',)), ('i',))	1976	2307	3343
	rpi	(('nv', ('rap',)), ('e', ('rp',)), ('i',))	5221	229	132
	pri	(('e', ('rp',)), ('nv', ('rap',)), ('i',))	5161	214	135
	2ri	(('nv', ('rap',)), ('nv', ('rap',)), ('i',)	109	896	61
3i	nan3i	(('nv', ('np',)), ('e', ('ap',)), ('nv', ('np',)), ('i',))	12747	21454	15296
	2pr3i	(('e', ('rp',)), ('e', ('rp',)), ('nv', ('rap',)), ('i',))	3515	19	52
	n2a3i	(('nv', ('np',)), ('e', ('ap',)), ('e', ('ap',)), ('i',))	3703	3735	5989
	r2p3i	(('nv', ('rap',)), ('e', ('rp',)), ('e', ('rp',)), ('i',))	3527	16	45
	a2n3i	(('e', ('ap',)), ('nv', ('np',)), ('nv', ('np',)), ('i',))	12641	21321	15287
	3p3i	(('e', ('rp',)), ('e', ('rp',)), ('e', ('rp',)), ('i',))	18859	294	898
	p2r3i	(('e', ('rp',)), ('nv', ('rap',)), ('nv', ('rap',)), ('i',))	93	6	6
	2na3i	(('nv', ('np',)), ('nv', ('np',)), ('e', ('ap',)), ('i',))	12459	21500	15285
	3a3i	(('e', ('ap',)), ('e', ('ap',)), ('e', ('ap',)), ('i',))	1977	1327	2454
	ana3i	(('e', ('ap',)), ('nv', ('np',)), ('e', ('ap',)), ('i',))	3625	3723	5921
	3n3i	(('nv', ('np',)), ('nv', ('np',)), ('nv', ('np',)), ('i',))	11444	11950	12818
	prp3i	(('e', ('rp',)), ('nv', ('rap',)), ('e', ('rp',)), ('i',))	3562	24	46
	2an3i	(('e', ('ap',)), ('e', ('ap',)), ('nv', ('np',)), ('i',))	3724	3716	5874
	2rp3i	(('nv', ('rap',)), ('nv', ('rap',)), ('e', ('rp',)), ('i',))	97	5	4
	rpr3i	(('nv', ('rap',)), ('e', ('rp',)), ('nv', ('rap',)), ('i',))	97	2	7
	3r3i	(('nv', ('rap',)), ('nv', ('rap',)), ('nv', ('rap',)), ('i',))	1	17	4
ip	2pip	((('e', ('rp',)), ('e', ('rp',)), ('i',)), ('rp',))	14745	783	828
	nair	((('nv', ('np',)), ('e', ('ap',)), ('i',)), ('rap',))	4849	4748	4436
	2pia	((('e', ('rp',)), ('e', ('rp',)), ('i',)), ('ap',))	7104	3468	1960
	2nir	((('nv', ('np',)), ('nv', ('np',)), ('i',)), ('rap',))	8581	4588	6937
	nain	((('nv', ('np',)), ('e', ('ap',)), ('i',)), ('np',))	5846	9170	5772
	2nin	((('nv', ('np',)), ('nv', ('np',)), ('i',)), ('np',))	5064	8043	6116
	2air	((('e', ('ap',)), ('e', ('ap',)), ('i',)), ('rap',))	4196	4068	3189
	rpip	((('nv', ('rap',)), ('e', ('rp',)), ('i',)), ('rp',))	5910	754	740
	anin	((('e', ('ap',)), ('nv', ('np',)), ('i',)), ('np',))	5906	9225	5898
	prip	((('e', ('rp',)), ('nv', ('rap',)), ('i',)), ('rp',))	6020	814	723
	2rip	((('nv', ('rap',)), ('nv', ('rap',)), ('i',)), ('rp',))	1683	996	464
	2ain	((('e', ('ap',)), ('e', ('ap',)), ('i',)), ('np',))	2262	2289	1981
	anir	((('e', ('ap',)), ('nv', ('np',)), ('i',)), ('rap',))	4851	4581	4484
	rpia	((('nv', ('rap',)), ('e', ('rp',)), ('i',)), ('ap',))	434	157	22
	pria	((('e', ('rp',)), ('nv', ('rap',)), ('i',)), ('ap',))	379	168	23
	2ria	((('nv', ('rap',)), ('nv', ('rap',)), ('i',)), ('ap',))	11	459	13

Table 14: The number and types of queries sampled from the three datasets. (1)

Type	Query Name	Query Structure Definition	FB15K	DB15K	YAGO15K
pi	anni	((('e', ('ap',)), ('np',)), ('nv', ('np',)), ('i',))	7960	13633	9102
	rppi	((('nv', ('rap',)), ('rp',)), ('e', ('rp',)), ('i',))	6511	618	1381
	nrpi	((('nv', ('np',)), ('rap',)), ('e', ('rp',)), ('i',))	13161	2829	5621
	paai	((('e', ('rp',)), ('ap',)), ('e', ('ap',)), ('i',))	1333	1245	1349
	2nni	((('nv', ('np',)), ('np',)), ('nv', ('np',)), ('i',))	6975	11904	11132
	pani	((('e', ('rp',)), ('ap',)), ('nv', ('np',)), ('i',))	10097	10100	7629
	2nai	((('nv', ('np',)), ('np',)), ('e', ('ap',)), ('i',))	4537	11300	7658
	anai	((('e', ('ap',)), ('np',)), ('e', ('ap',)), ('i',))	4468	9727	7520
	2ppi	((('e', ('rp',)), ('rp',)), ('e', ('rp',)), ('i',))	16875	541	1063
	nrri	((('nv', ('np',)), ('rap',)), ('nv', ('rap',)), ('i',))	1375	3247	1291
	2pri	((('e', ('rp',)), ('rp',)), ('nv', ('rap',)), ('i',))	5806	91	64
	rani	((('nv', ('rap',)), ('ap',)), ('nv', ('np',)), ('i',))	804	3258	1398
	arpi	((('e', ('ap',)), ('rap',)), ('e', ('rp',)), ('i',))	2933	139	131
	rpri	((('nv', ('rap',)), ('rp',)), ('nv', ('rap',)), ('i',))	1467	570	112
	arri	((('e', ('ap',)), ('rap',)), ('nv', ('rap',)), ('i',))	104	732	73
	raai	((('e', ('rap',)), ('ap',)), ('nv', ('ap',)), ('i',))	65	11	42
2u	pru	(('e', ('rp',)), ('nv', ('rap',)), ('u',))	14157	5705	6783
	2pu	(('e', ('rp',)), ('e', ('rp',)), ('u',))	19129	3734	2964
	anu	(('e', ('ap',)), ('nv', ('np',)), ('u',))	12581	23056	20887
	2nu	(('nv', ('np',)), ('nv', ('np',)), ('u',))	11988	21767	11051
	rpu	(('nv', ('rap',)), ('e', ('rp',)), ('u',))	14195	5591	6641
	nau	(('nv', ('np',)), ('e', ('ap',)), ('u',))	12757	23000	21086
	2ru	(('nv', ('rap',)), ('nv', ('rap',)), ('u',))	9359	7740	14718
	2au	(('e', ('ap',)), ('e', ('ap',)), ('u',))	2548	5497	3840
up	2aur	((('e', ('ap',)), ('e', ('ap',)), ('u',)), ('rap',))	3403	2913	3922
	rpup	((('nv', ('rap',)), ('e', ('rp',)), ('u',)), ('rp',))	6871	1718	1259
	2aun	((('e', ('ap',)), ('e', ('ap',)), ('u',)), ('np',))	6705	12892	10000
	naur	((('nv', ('np',)), ('e', ('ap',)), ('u',)), ('rap',))	7195	5387	12506
	2pup	((('e', ('rp',)), ('e', ('rp',)), ('u',)), ('rp',))	10171	1022	574
	rpua	((('nv', ('rap',)), ('e', ('rp',)), ('u',)), ('ap',))	3494	3352	1770
	anun	((('e', ('ap',)), ('nv', ('np',)), ('u',)), ('np',))	7186	14110	9244
	2nur	((('nv', ('np',)), ('nv', ('np',)), ('u',)), ('rap',))	8816	6578	9445
	anur	((('e', ('ap',)), ('nv', ('np',)), ('u',)), ('rap',))	7071	5473	12582
	2rup	((('nv', ('rap',)), ('nv', ('rap',)), ('u',)), ('rp',))	4336	2610	2807
	naun	((('nv', ('np',)), ('e', ('ap',)), ('u',)), ('np',))	7337	14341	9114
	2pua	((('e', ('rp',)), ('e', ('rp',)), ('u',)), ('ap',))	6981	3095	1245
	2nun	((('nv', ('np',)), ('nv', ('np',)), ('u',)), ('np',))	7575	14475	8895
	prup	((('e', ('rp',)), ('nv', ('rap',)), ('u',)), ('rp',))	6779	1673	1204
	prua	((('e', ('rp',)), ('nv', ('rap',)), ('u',)), ('ap',))	3631	3295	1768
	2rua	((('nv', ('rap',)), ('nv', ('rap',)), ('u',)), ('ap',))	864	4971	2133
2b	aab	(('e', ('ap',)), ('e', ('ap',)), ('b',))	1997	1997	1997
3b	3ab	(('e', ('ap',)), ('e', ('ap',)), ('e', ('ap',)), ('b',))	1997	1997	1997
2abp	aabr	((('e', ('ap',)), ('e', ('ap',)), ('b',)), ('rap',))	885	431	1690
	aabn	((('e', ('ap',)), ('e', ('ap',)), ('b',)), ('np',))	1110	1565	305
p2ab	raab	((('nv', ('rap',)), ('ap',)), ('e', ('ap',)), ('b',))	349	862	1225
	paab	((('e', ('rp',)), ('ap',)), ('e', ('ap',)), ('b',))	1647	1134	771
2p2ab	rarab	((('nv', ('rap',)), ('ap',)), (('e', ('rp',)), ('ap',)), ('b',))	510	365	826
	rapab	((('nv', ('rap',)), ('ap',)), (('nv', ('rap',)), ('ap',)), ('b',))	99	504	481
	parab	((('e', ('rp',)), ('ap',)), (('nv', ('rap',)), ('ap',)), ('b',))	519	517	457
	papab	((('e', ('rp',)), ('ap',)), (('e', ('rp',)), ('ap',)), ('b',))	866	608	230

Table 15: The number and types of queries sampled from the three datasets. (2)

Dataset	AVG	2p	2i	3i	pi	ip	2u	up
FB15K	0.7537	0.6505	0.9123	0.9007	0.7506	0.5644	0.7364	0.7612
DB15K	0.6298	0.5488	0.6892	0.5695	0.5483	0.6915	0.6178	0.7433
YAGO15K	0.6048	0.5627	0.7651	0.6628	0.6924	0.5106	0.6272	0.4128

Table 16: Percentage of edges on the inference path that exist in the test set.

C Model Hyperparameter Settings and Training Details

The hyperparameter settings for CNR-NST are shown in Table 17. W_{rel} refers to the score weight of the relation, W_{EAV} refers to the task score weight in joint training, and Thrshd refers to the threshold of the corresponding adjacency matrix. $\text{Thrshd}_{\text{fuzzy}}$ refers to the filtering value for numerical fuzzy sets during binary operations, which is used to speed up the inference process.

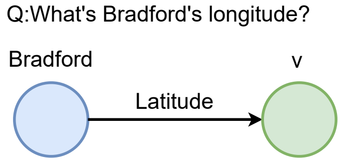
	Training Epoch	Learning Rate	Ranking	Decay1	Decay2	W_{rel}	W_{EAV}	W_{VFV}
Multi-ComplEx	100	0.05	500	0.9	0.999	4	5	0.1
	Fraction	$\text{Thrshd}_{\text{ERE}}$	$\text{Thrshd}_{\text{EAV}}$	$\text{Thrshd}_{\text{VFV}}$	$\text{Thrshd}_{\text{fuzzy}}$	$\text{Neg}_{\text{Scale}}$		
Reasoning Model	10	0.001	0.0001	0.0001	0.001	6		

Table 17: Hyperparameter settings of CNR-NST.

D Examples of Numeric Queries

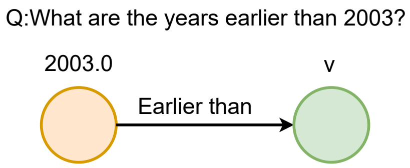
This section presents a subset of query types from the 102 sub-queries that involve numerical reasoning and computation. We selected several queries with real-world significance to demonstrate that most of our query types are capable of reflecting real-world scenarios.

For some query answers (*e.g.,* 1ap), there is only one correct answer, but we still present the top 5 inferred answers. Additionally, for queries with too many answers, we skipped some of the correctly predicted easy answers to focus on demonstrating that our model can still infer the correct hard answers.



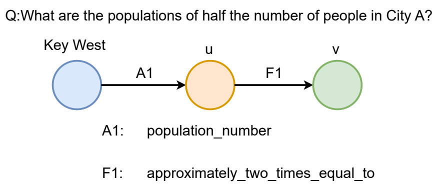
Logical Expression:	$Q[N_?] = N_?, \exists N_?, a_1(e_1, N_?)$		
Rank	Query Answers	Correctness	Answer type
1	53.69	✗	-
2	53.8	✓	Hard
3	51.5	✗	-
4	51.48	✗	-
5	51.51	✗	-

Figure 9: Intermediate variable assignments and ranks for example 1ap query.



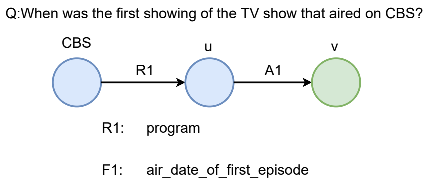
Logical Expression:	$Q[N_?] = N_?, \exists N_?, f_1(n_1, N_?)$		
Rank	Query Answers	Correctness	Answer type
1	2002.17	✓	Easy
2	1988.33	✓	Easy
3	72202	✗	-
4	2003	✗	-
5	1920.92	✗	-
6	1963.92	✓	Easy
7	10934.93	✗	-
8	1911.5	✗	-
9	1950.83	✓	Easy
10	1977.33	✓	Easy
11	1948.67	✓	Easy
12	1974.5	✓	Easy
13	1948.92	✓	Hard

Figure 10: Intermediate variable assignments and ranks for example 1np query.



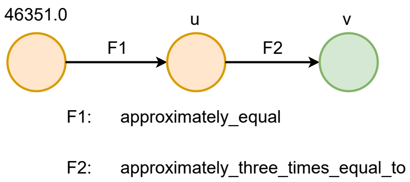
Logical Expression:	$Q[N_?] = N_?, \exists N_a, a_1(e_1, N_a) \wedge f_1(N_a, N_?)$		
Rank	Query Answers	Correctness	Answer type
1	53827.00	✓	Easy
2	52966.00	✓	Easy
3	53326.00	✓	Easy
4	53066.00	✓	Easy
5	53025.00	✓	Hard
6	53374.00	✓	Easy
7	53437.00	✓	Easy
8	53483.00	✗	-
9	53838.00	✓	Easy
10	53311.00	✓	Easy

Figure 11: Intermediate variable assignments and ranks for example an query.



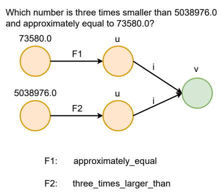
Logical Expression:	$Q[N_?] = N_?, \exists E_a, r_1(e_1, E_a) \wedge a_1(E_a, N_?)$		
Rank	Query Answers	Correctness	Answer type
1	1978.32	✔	Easy
2	1992.75	✔	Easy
3	1985.75	✔	Easy
4	1993.67	✔	Easy
5	1990.75	✔	Easy
.....	.....	.....	.....
26	1960.83	✔	Hard
27	1996.75	✔	Hard
28	2000.42	✗	-
29	1959.83	✔	Easy
30	1972.75	✔	Easy

Figure 12: Intermediate variable assignments and ranks for example pa query.



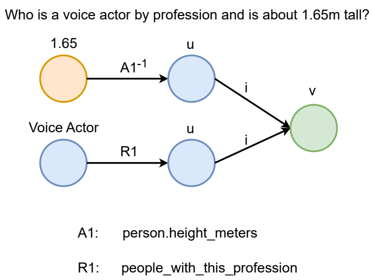
Logical Expression:	$Q[N_?] = N_?, \exists N_a, f_1(n_1, N_a) \wedge f_2(N_a, N_?)$		
Rank	Query Answers	Correctness	Answer type
1	137776	✔	Hard
2	138296	✔	Easy
3	137555	✔	Easy
4	139790	✔	Easy
5	139390	✗	-

Figure 13: Intermediate variable assignments and ranks for example 2n query.



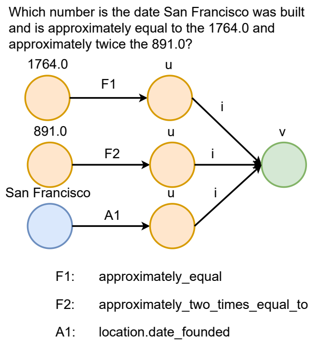
Logical Expression:	$Q[N_?] = N_?, \exists N_?, f_1(n_1, N_?) \wedge f_2(n_2, N_?)$		
Rank	Query Answers	Correctness	Answer type
1	73615.00	✔	Hard
2	73485.00	✔	Easy
3	73208.00	✔	Easy
4	-77.91	✗	-
5	-85.17	✗	-

Figure 14: Intermediate variable assignments and ranks for example 2ni query.



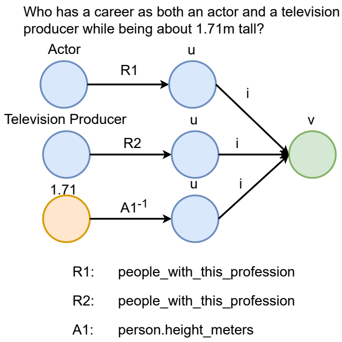
Logical Expression:	$Q[E_?] = E_?, \exists E_?, a_1^{-1}(n_1, E_?) \wedge r_1(e_1, E_?)$		
Rank	Query Answers	Correctness	Answer type
1	Debi Mazar	✔	Easy
2	Andrea Bowen	✔	Easy
3	Ian Holm	✔	Easy
4	Breckin Meyer	✔	Easy
5	Mel Brooks	✔	Easy
6	Christina Applegate	✔	Easy
7	Roy Kinnear	✔	Easy
8	Nathan Lane	✔	Hard
9	Molly Shannon	✗	-
10	Common	✗	-
11	Anna Paquin	✔	Hard

Figure 15: Intermediate variable assignments and ranks for example rpi query.



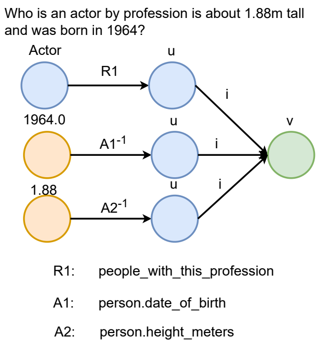
Logical Expression: $Q[N_?] = N_?, \exists N_?, a_1(e_1, N_?) \wedge f_1(n_1, N_?) \wedge f_2(n_2, N_?)$			
Rank	Query Answers	Correctness	Answer type
1	1776.50	✔	Hard
2	1763.83	✗	-
3	1765.00	✗	-
4	1775.67	✗	-
5	1776.67	✗	-

Figure 16: Intermediate variable assignments and ranks for example 2na3i query.



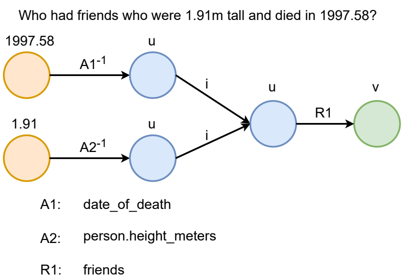
Logical Expression: $Q[E_?] = E_?, \exists E_?, r_1(e_1, E_?) \wedge r_2(e_2, E_?) \wedge a_1^{-1}(n_1, E_?)$			
Rank	Query Answers	Correctness	Answer type
1	Ellen DeGeneres	✔	Easy
2	Candice Bergen	✔	Easy
3	Joan Chen	✔	Easy
4	Martin Lawrence	✔	Easy
5	Martin Short	✔	Easy
6	Kirstie Alley	✔	Hard

Figure 17: Intermediate variable assignments and ranks for example 2pr3i query.



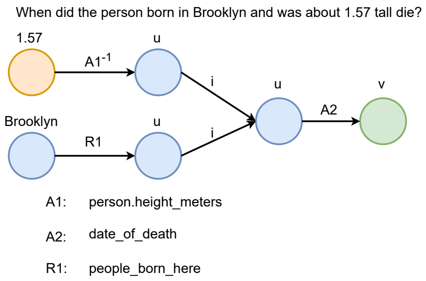
Logical Expression: $Q[E_?] = E_?, \exists E_?, r_1(e_1, E_?) \wedge a_1^{-1}(n_1, E_?) \wedge a_2^{-1}(n_2, E_?)$			
Rank	Query Answers	Correctness	Answer type
1	Brendan Coyle	✔	Easy
2	Benjamin Bratt	✔	Hard
3	Til Schweiger	✗	-
4	Iqbal Theba	✗	-
5	Ray Romano	✗	-

Figure 18: Intermediate variable assignments and ranks for example p2r3i query.



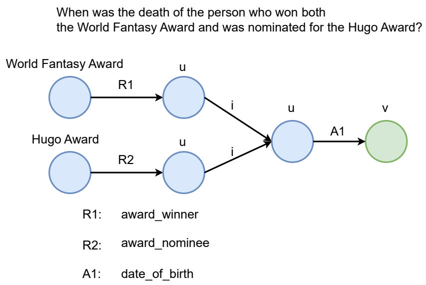
Logical Expression: $Q[E_?] = E_?, \exists E_?, r_1(E_1, E_?) \wedge (\exists E_1, a_1^{-1}(n_1, E_1) \wedge a_2^{-1}(n_2, E_1))$			
Rank	Query Answers	Correctness	Answer type
1	Henry Fonda	✔	Easy
2	Robert Taylor	✔	Easy
3	Myrna Loy	✔	Easy
4	Clark Gable	✔	Easy
5	Robert Young	✔	Easy
6	Joan Crawford	✔	Easy
7	Loretta Young	✔	Easy
8	James Stewart	✗	-
9	Bette Davis	✔	Hard
10	Lucille Ball	✗	-

Figure 19: Intermediate variable assignments and ranks for example 2rip query.



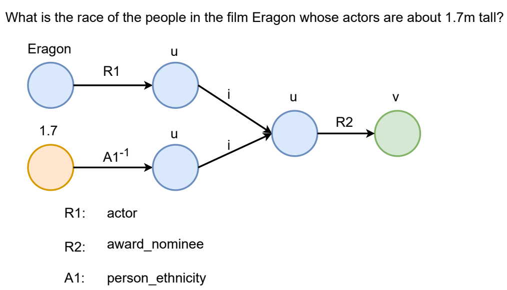
Logical Expression:	$Q[N_?] = N_?, \exists N_?, a_1(E_1, N_?) \wedge \left(\exists E_1, a_2^{-1}(n_1, E_1) \wedge r_1(e_1, E_1)\right)$		
Rank	Query Answers	Correctness	Answer type
1	2014.75	✔	Easy
2	2014.33	✔	Hard
3	2014.17	✗	-
4	2001.67	✗	-
5	2011.42	✗	-

Figure 20: Intermediate variable assignments and ranks for example rpia query.



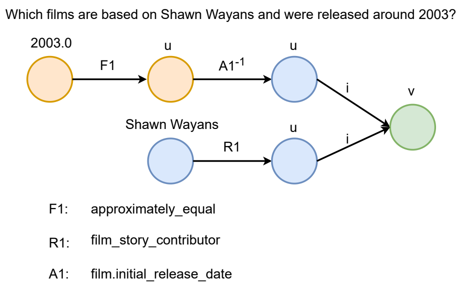
Logical Expression:	$Q[N_?] = N_?, \exists N_?, a_1(E_1, N_?) \wedge (\exists E_1, r_1(e_1, E_1) \wedge r_2(e_2, E_1))$		
Rank	Query Answers	Correctness	Answer type
1	1916.67	✔	Easy
2	1929.83	✔	Easy
3	1934.43	✔	Hard
4	1911	✔	Hard
5	1947.75	✗	-

Figure 21: Intermediate variable assignments and ranks for example 2pia query.



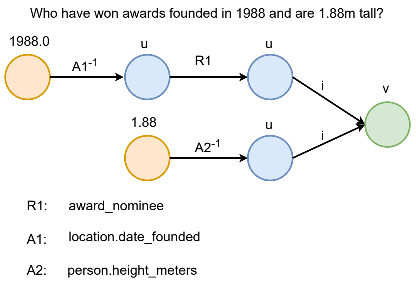
Logical Expression:	$Q[E_?] = E_?, \exists E_?, r_1(E_1, E_?) \wedge \left(\exists E_1, a_1^{-1}(n_1, E_1) \wedge r_2(e_2, E_1)\right)$		
Rank	Query Answers	Correctness	Answer type
1	English people	✔	Easy
2	Jewish people	✔	Easy
3	Ashkenazi Jews	✔	Easy
4	Hungarians	✔	Hard
5	White Americans	✗	-

Figure 22: Intermediate variable assignments and ranks for example rpip query.



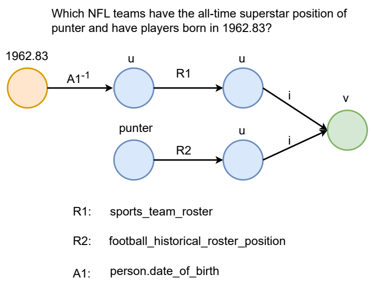
Logical Expression:	$Q[E_?] = E_?, \exists E_?, r_1(E_1, E_?) \wedge \left(\exists E_?, f_1(n_1, N_1) \wedge a_1^{-1}(N_1, E_?)\right)$		
Rank	Query Answers	Correctness	Answer type
1	Scary Movie 3	✔	Easy
2	Pride & Prejudice	✔	Easy
3	Scary Movie 4	✔	Hard
4	Scary Movie	✗	-
5	Kill Bill Volume 1	✗	-

Figure 23: Intermediate variable assignments and ranks for example nrpi query.



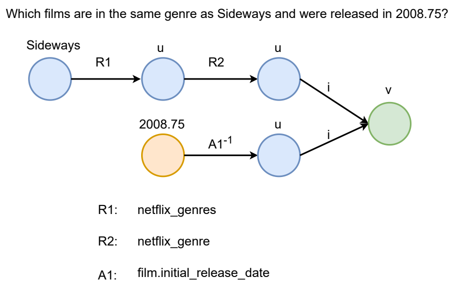
Logical Expression:	$Q[E_?] = E_?, \exists E_?, a_1^{-1}(n_1, E_?) \wedge (\exists E_?, a_2^{-1}(n_2, E_1) \wedge r_1(E_1, E_?))$		
Rank	Query Answers	Correctness	Answer type
1	James Coburn	✔	Easy
2	Ray McKinnon	✔	Easy
3	Djimon Hounsou	✔	Hard
4	Morgan Freeman	✔	Hard
5	Danny Huston	✗	-

Figure 24: Intermediate variable assignments and ranks for example rpri query.



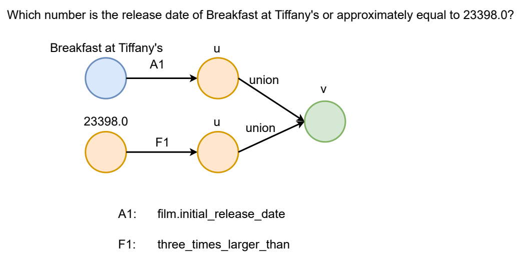
Logical Expression:	$Q[E_?] = E_?, \exists E_?, r_1(e_1, E_?) \wedge (\exists E_?, a_2^{-1}(n_2, E_1) \wedge r_2(E_1, E_?))$		
Rank	Query Answers	Correctness	Answer type
1	New England Patriots	✔	Easy
2	Buffalo Bills	✔	Easy
3	Los Angeles Chargers	✔	Easy
4	Chicago Bears	✔	Hard
5	Toronto Argonauts	✗	-

Figure 25: Intermediate variable assignments and ranks for example rppi query.



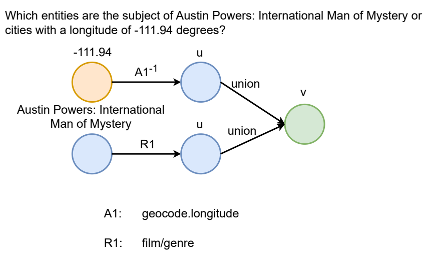
Logical Expression:	$Q[E_?] = E_?, \exists E_?, a_1^{-1}(n_1, E_?) \wedge (\exists E_?, r_1(e_1, E_1) \wedge r_2(E_1, E_?))$		
Rank	Query Answers	Correctness	Answer type
1	The Brothers Bloom	✔	Easy
2	Zack and Miri Make a Porno	✔	Easy
3	Nick and Norah's Infinite Playlist	✔	Easy
4	Management	✔	Hard
5	Good	✗	-

Figure 26: Intermediate variable assignments and ranks for example ppri query.



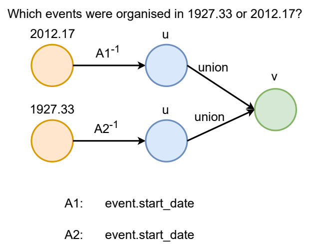
Logical Expression:	$Q[N_?] = N_?, \exists N_?, a_1(e_1, N_?) \vee f_1(n_1, N_?)$		
Rank	Query Answers	Correctness	Answer type
1	1961.83	✔	Easy
2	23260.00	✔	Easy
3	23504.00	✔	Easy
4	23307.00	✔	Easy
5	23398.00	✔	Easy
6	23549.00	✔	Hard

Figure 27: Intermediate variable assignments and ranks for example anu query.



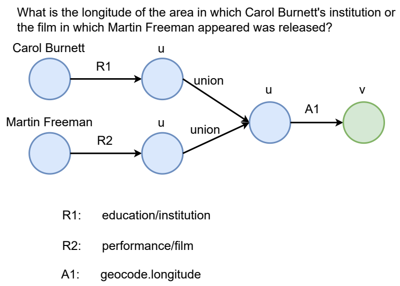
Logical Expression:	$Q[E_?] = E_?, \exists E_?, r_1(e_1, E_?) \vee a_1^{-1}(n_1, E_?)$		
Rank	Query Answers	Correctness	Answer type
1	action film	✔	Easy
2	comedy	✔	Easy
3	spy film	✔	Easy
4	parody	✔	Easy
5	Tempe	✔	Easy
6	Gullaug	✔	Hard

Figure 28: Intermediate variable assignments and ranks for example rpu query.



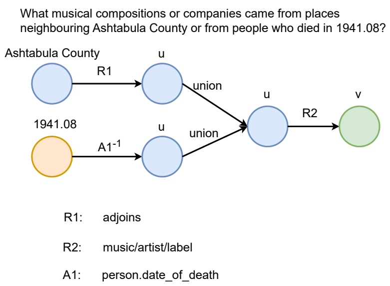
Logical Expression:	$Q[E_?] = E_?, \exists E_?, a_1^{-1}(n_1, E_?) \vee a_2^{-1}(n_2, E_?)$		
Rank	Query Answers	Correctness	Answer type
1	84th Academy Awards	✔	Easy
2	Chinese Civil War	✔	Easy
3	54th Annual Grammy Awards	✔	Easy
4	62nd Berlin International Film Festival	✔	Easy
5	49th Annual Grammy Awards	✗	-
6	38th People's Choice Awards	✗	-
7	80th Academy Awards	✗	-
8	60th Academy Awards	✗	-
9	65th British Academy Film Awards	✔	Hard

Figure 29: Intermediate variable assignments and ranks for example 2ru query.



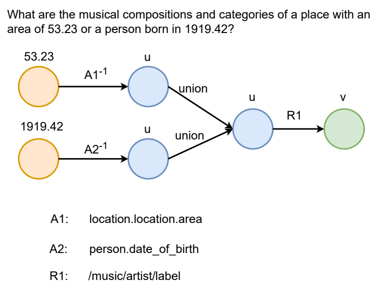
Logical Expression:	$Q[N_?] = N_?, \exists N_?, a_1(E_1, N_?) \wedge (\exists E_1, r_1(e_1, E_1) \vee r_2(e_2, E_1))$		
Rank	Query Answers	Correctness	Answer type
1	-118.34	✔	Easy
2	-118.44	✔	Hard
3	-97.74	✗	-
4	-87.67	✗	-
5	-72.93	✗	-

Figure 30: Intermediate variable assignments and ranks for example 2pua query.



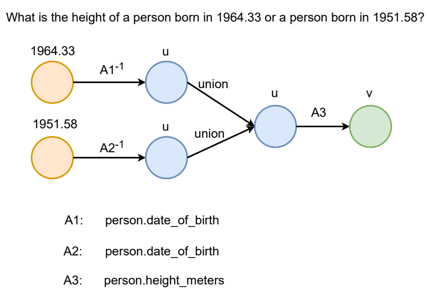
Logical Expression:	$Q[E_?] = E_?, \exists E_?, r_1(E_1, E_?) \wedge (\exists E_1, r_1(e_1, E_1) \vee a_1^{-1}(n_1, E_1))$		
Rank	Query Answers	Correctness	Answer type
1	Virgin Records	✔	Easy
2	Columbia Records	✔	Easy
3	Vanguard Records	✔	Easy
4	Koch Entertainment	✔	Easy
5	Warner Bros. Records	✗	-
6	MCA Records	✔	Hard

Figure 31: Intermediate variable assignments and ranks for example rpup query.



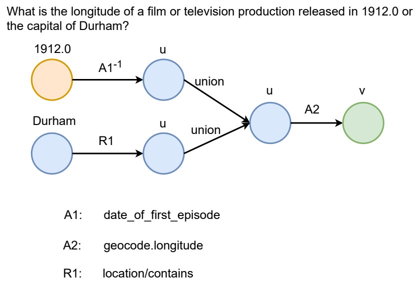
Logical Expression:	$Q[E_?] = E_?, \exists E_?, r_1(E_1, E_?) \wedge (\exists E_1, a_1^{-1}(n_1, E_1) \vee a_2^{-1}(n_2, E_1))$		
Rank	Query Answers	Correctness	Answer type
1	Columbia Records	✔	Easy
2	Vanguard Records	✔	Hard
3	Warner Bros. Records	✗	-
4	EMI	✗	-

Figure 32: Intermediate variable assignments and ranks for example 2rup query.



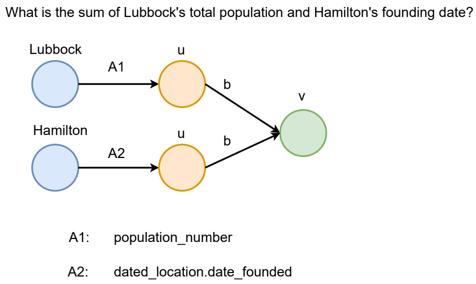
Logical Expression:	$Q[N_?] = N_?, \exists N_?, a_1(E_1, N_?) \wedge (\exists E_1, a_2^{-1}(n_1, E_1) \vee a_3^{-1}(n_2, E_1))$		
Rank	Query Answers	Correctness	Answer type
1	1.75	✔	Easy
2	1.70	✔	Easy
3	1.88	✔	Easy
4	1.68	✔	Easy
5	1.85	✔	Easy
6	1.78	✔	Easy
7	1.82	✔	Easy
8	1.73	✗	-
9	1.65	✔	Hard

Figure 33: Intermediate variable assignments and ranks for example 2rua query.



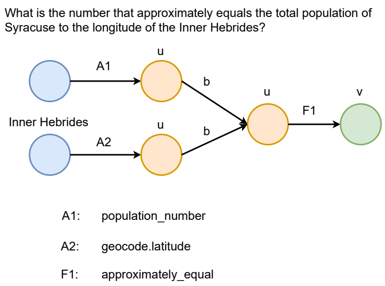
Logical Expression:	$Q[N_?] = N_?, \exists N_?, a_1(E_1, N_?) \wedge \left(\exists E_1, a_2^{-1}(n_1, E_1) \vee r_1(e_1, E_1)\right)$		
Rank	Query Answers	Correctness	Answer type
1	-73.96	✔	Easy
2	-1.57	✔	Hard
3	-122.25	✗	-
4	-74.01	✗	-
5	-117.07	✗	-

Figure 34: Intermediate variable assignments and ranks for example rpua query.



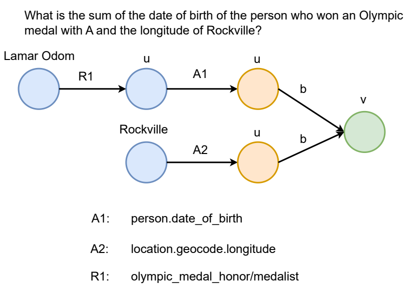
Logical Expression:	$Q[N_?] = N_?, \exists N_? = N_1 + N_2 \mid \exists N_1, a_1(e_1, N_1), \exists N_2, a_2(e_2, N_2)$			
Rank	Query Answers	Accurate Answers	Correctness	Answer type
1	237858.00	237881	✔	Hard
2	237905.00	-	✗	-

Figure 35: Intermediate variable assignments and ranks for example aab query.



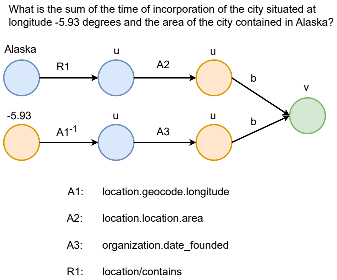
Logical Expression:	$Q[N_?] = N_?, \exists N_?, (f_1(N_3, N_?)) \wedge (\exists N_3 = N_1 + N_2 \mid \exists N_1, a_1(e_1, N_1), \exists N_2, a_2(e_2, N_2))$		
Rank	Query Answers	Correctness	Answer type
1	7981900.00	✔	Easy
2	8041685.00	✔	Easy
3	8053574.00	✔	Easy
4	8015348.00	✔	Easy
5	8081957.00	✔	Easy
6	8084000.00	✔	Hard
7	7977000.00	✔	Hard

Figure 36: Intermediate variable assignments and ranks for example aabn query.



Logical Expression:	$Q[N_?] = N_?, \exists N_?, (f_1(N_3, N_?)) \wedge (\exists N_3 = N_1 + N_2 \mid \exists N_1, r_1(e_1, E_1) \wedge a_1(E_1, N_1), \exists N_2, a_2(e_2, N_2))$			
Rank	Query Answers	Accurate Answers	Correctness	Answer type
1	1907.85	1907.27	✔	Hard
2	1898.35	1898.35	✔	Easy
3	1899.18	1899.18	✔	Easy
4	1904.93	1904.93	✔	Easy
5	1899.27	-	✗	-

Figure 37: Intermediate variable assignments and ranks for example paab query.



Logical Expression:	$Q[N_?] = N_?, \exists N_?, (f_1(N_3, N_?)) \wedge (\exists N_3 = N_1 + N_2 \vert \exists N_1, r_1(e_1, E_1) \wedge a_1(E_1, N_1), \exists N_2, r_2(e_2, E_2) \wedge a_2(E_2, N_2))$			
Rank	Query Answers	Accurate Answers	Correctness	Answer type
1	8885.10	8885.10	✔	Easy
2	67272.10	67272.10	✔	Easy
3	43322.48	43322.48	✔	Easy
4	6928.23	6928.23	✔	Hard

Figure 38: Intermediate variable assignments and ranks for example parab query.