Tiny Budgets, Big Gains: Parameter Placement Strategy in Parameter Super-Efficient Fine-Tuning

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Abstract

In this work, we propose FoRA-UA, a novel method that, using only 1–5% of the standard LoRA's parameters, achieves state-of-the-art performance across a wide range of tasks. Specifically, we explore scenarios with extremely limited parameter budgets and derive two key insights: (1) fix-sized sparse frequency representations approximate small matrices more accurately; and (2) with a fixed number of trainable parameters, introducing a smaller intermediate representation to approximate larger matrices results in lower construction error. These findings form the foundation of our FoRA-UA method. By inserting a small intermediate parameter set, we achieve greater model compression without sacrificing performance. We evaluate FoRA-UA across diverse tasks, including natural language understanding (NLU), natural language generation (NLG), instruction tuning, and image classification, demonstrating strong generalisation and robustness under extreme compression.[1]

1 Introduction

Ever since the success demonstrated by prior Parameter-Efficient Fine-Tuning (PEFT) work (Houlsby et al., 2019; Hu et al., 2022, *inter alia*), it has become standard practice to apply some form of PEFT technique when fine-tuning large language models (LLMs).

These techniques, including Adapters (Houlsby et al., 2019), Low-Rank Adaption (LoRA; Hu et al., 2022) and Prefix-Tuning (Li and Liang, 2021), allow us to update only a fraction of the original model's weights while achieving comparable domain or task adaptation to full-model fine-tuning. This leads to substantial savings in memory and computational resources. PEFT approaches are thus crucial for the rapid development of new LLMs and LLM-based methods, playing a key role in democratising the field of Artificial Intelligence (AI) and Natural Language Processing (NLP).

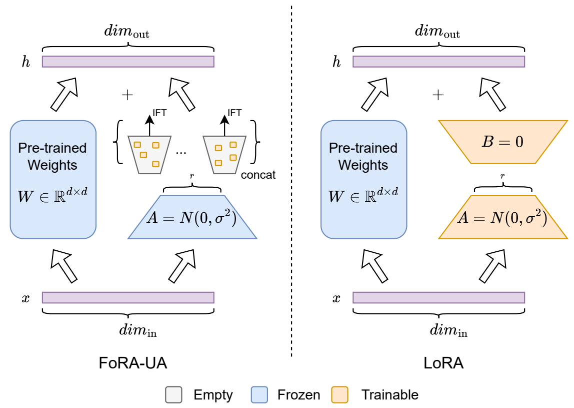

Figure 1: Illustration of the FoRA-UA Architecture. Compared to traditional LoRA approaches, FoRA-UA incorporates a number of smaller matrices for future parameter comparison without sacrificing performance.

Nevertheless, the speed at which LLMs are scaling up is unprecedented and unanticipated. It is now commonplace for a "standard" model to comprise hundreds of billions of parameters. For example, the LLaMA 3.1 (Grattafiori et al., 2024) model contains 405B active parameters, and the flagship Qwen 3 model has 235B (Yang et al., 2025a). Ironically, versions with only a few billion parameters are now referred to as "small language models," even though BERT (Devlin et al., 2019) and GPT-2 (Radford et al., 2019) were originally considered "large language models." Therefore, the current rate of compression in traditional LoRA approaches is not sufficient. Existing models are already reaching record-breaking sizes, and there are very few signs that the trend of scaling is slowing down. There is also a recent push towards mobile computing (Xue et al., 2024; Chen and Li, 2024) which prerequisites memory-efficient LLM inference. Furthermore, effective PEFT methods are key to democratising NLP research, enabling members of the research

[1]Code is available at https://github.com/zhaojinm/FoRA-UA.

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community with limited computational resources to participate in cutting-edge work.

Recent trends in parameter-efficient fine-tuning have pushed the limits of efficiency, with some methods (Kopiczko et al., 2024; Gao et al., 2024; Wu et al., 2024b) using even fewer trainable parameters than the lower bound of LoRA with rank=1. This raises a fundamental question: *how should we allocate a fixed amount of trainable parameters under extremely limited budgets?* To explore this, we begin with FourierFT (Gao et al., 2024) as our baseline, and conduct a series of experiments to investigate how different structural choices affect approximation quality and downstream performance. These experiments lead to three key observations that motivate our method design:

Finding 1: Allocating the fixed amount of trainable parameters in smaller matrices leads to more efficient approximation.

Finding 2: Approximating via a smaller, intermediate representation reduces reconstruction error compared to direct approximation.

Finding 3: Performing IFT on multiple small matrices and combining them leads to equal effectiveness theoretically; and, surprisingly, better downstream task performance in practice.

Therefore, instead of using one large matrix to approximate the entire process, why not use multiple smaller ones? The two aforementioned findings form the foundation of our novel method sparse **Fo**urier l**o**w **Ra**nk with **U**niversal shared **A**daptor (FoRA-UA) method. We employ multiple smaller matrices in parallel to approximate the full matrix. Moreover, many of the weights in the smaller matrices do not need to be initialised and can be left empty.

As a result, FoRA-UA achieve the state-of-the-art space compression without any sacrifice in performance. With only 1–5% of original LoRA's number of parameters, we still reproduce the state-of-the-art performance across a wide range of downstream tasks spanning natural language understanding (NLU), natural language generation (NLG), instruction tuning, and image classification, demonstrating its strong generalization and robustness under extreme compression.

Contribution We make three key contributions.

1. We explore scenarios of LoRA-based PEFT with an extremely limited memory budget (§3). We demonstrate two key findings empirically and theoretically that establish the foundation of FoRA-UA. These novel insights have important implications for the broader PEFT community.
2. We introduce FoRA-UA (§4), a novel PEFT method that uses the fewest parameters while still achieving state-of-the-art performance. To date, FoRA-UA achieves the best trade-off between performance and trainable parameters.
3. We thoroughly evaluate FoRA-UA across a wide range of downstream NLP tasks (§5), and even explore multimodal use cases of LLMs. This comprehensive evaluation demonstrates the effectiveness and robustness of our method, showing that it can bring significant performance gains with only a tiny budget.

2 Related Work

Parameter Efficient Fine-Tuning A foundational PEFT method is Adapter Tuning (Houlsby et al., 2019), which inserts small layers into the model while keeping the pre-trained weights frozen. This enables efficient task adaptation with minimal parameter updates. BitFit (Ben Zaken et al., 2022) takes a more extreme approach by only fine-tuning bias terms, demonstrating that even such minimal modifications can yield competitive results.

LoRA (Hu et al., 2022) further improves efficiency by introducing trainable low-rank matrices to modify model weights. Prompt Tuning (Lester et al., 2021; Liu et al., 2022; Jin et al., 2024) and Prefix-Tuning (Li and Liang, 2021) shift the focus from weight updates to learnable input prompts, allowing models to adapt without modifying their core parameters. Zhong et al. (2025) add a nonlinear layer between the original weight and updated weight. Other efficient methods are commenly used such as LLM embedding/steering (Deng et al., 2025a; Cheng et al., 2025; Gan et al., 2025), distillation (Dong et al., 2024b,c,a), KV cache during inference (Li et al., 2025a), knowledge editing (Deng et al., 2025b), token pruning (Yang et al., 2025c; Liu et al., 2025; Hu et al., 2025), finetune partial layers (Fan et al., 2025a) and chunk-wise gradient pruning(Li et al., 2025b).

LoRA and its Variations After their initial success, several extensions of LoRA (Hu et al., 2022) have been proposed. One notable extension is AdaLoRA (Zhang et al., 2023b) which updates the rank dynamically during fine-tuning based on the importance of the parameters. Another variant, DyLoRA (Valipour et al., 2023), extends LoRA by introducing dynamic matrix updates. Dynamic updating has also been integrated into some work (Liu et al., 2024c; Wang et al., 2024; Ding et al., 2023). LoRA+ (Hayou et al., 2024) proved that to achieve optimal, the learning rates for A and B should be different, with A's learning rate being much smaller than B's. LoRA-FA (Zhang et al., 2023a) freeze A and only fine-tune B. However, unlike our method, LoRA-FA does not share the low-rank adaptation matrices across different linear layers and results in a higher memory cost. Some recent work has integrated Mixture of Experts (MoE) with LoRA (Luo et al., 2024; Dou et al., 2024; Qing et al., 2024), enabling more efficient use of model capacity by activating different subsets of parameters depending on the input. Additionally, some methods (Shen et al., 2024; Ren et al., 2024; Mao et al., 2024; Tian et al., 2024) decompose the adaptation matrix into smaller blocks. There are also other LoRA-based methods (Renduchintala et al., 2024; Liu et al., 2024a; Shi et al., 2024; Zhong and Zhou, 2024; Zhang et al., 2025, *inter alia*) that we do not elaborate on here for brevity, but we refer interested readers to explore them further.

Fourier Transformation in PEFT Fourier transformations (FTs) have recently been explored in PEFT to enhance model generalization and efficiency. Borse et al. (2024) introduces low-rank adaptation in the frequency domain, improving generalization by reducing redundancy and enabling adaptive rank selection. Zeng et al. (2024) leverages Fourier transforms in visual prompt tuning, enhancing robustness across datasets with varying disparities. Unlike LoRA decomposes weights into two low-rank matrices, Gao et al. (2024) directly approximates ΔW using an Inverse Fourier Transformation (IFT).

3 LoRA under Extremely Limited Budget

LoRA (Hu et al., 2022) is a SOTA PEFT method designed to adapt large-scale pre-trained models for downstream tasks while reducing memory and computational overhead. Instead of updating all model parameters, LoRA introduces trainable low-rank matrices to approximate weight updates, effectively reducing the number of learnable parameters.

Formally, given a pre-trained weight matrix $W \in \mathbb{R}^{d\times k}$, LoRA models its update as a low-rank

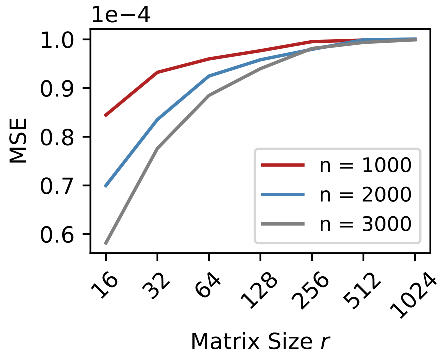


Figure 2: Reconstruction error across different matrix sizes under fixed sparsity.

decomposition,

$$\Delta W = \underline{BA}, \tag{1}$$

where $B \in \mathbb{R}^{d\times r}$ and $A \in \mathbb{R}^{r\times k}$, with $r \ll min(d,k)$ to ensure that the rank of the update remains significantly lower than the original weight matrix. We use underline to denote the *trainable* matrices. The adapted model parameters W' are then represented as:

$$W' = W + \Delta W = W + \underline{BA}. \tag{2}$$

During training, only A and B are updated, while W remains frozen (see the right panel of Figure 1).

Sparse matrices (Ding et al., 2023; Wang et al., 2024) have also been employed in PEFT to further compress the trainable parameter space. Gao et al. (2024) is one of them that uses sparse inverse Fourier transformations to approximate ΔW as:

$$\Delta W = \text{IFT}(\underline{B}) \tag{3}$$

where $B \in \mathbb{R}^{d\times k}$ is a sparse matrix typically with less than 2000 non-zero entries, and IFT stands for the two-dimensional **inverse Fourier transformation**

To better understand how to allocate extremely limited parameter budgets, we designed a series of controlled experiments that reveal several key observations.

3.1 Small Matrices or Large Marices?

Finding 1: *A fixed-size sparse frequency representation (i.e., $B \in \mathbb{R}^{d\times k}$, with n nonzero entries for, e.g., $n = 1000, 2000$) provides more accurate approximations for smaller matrices.*

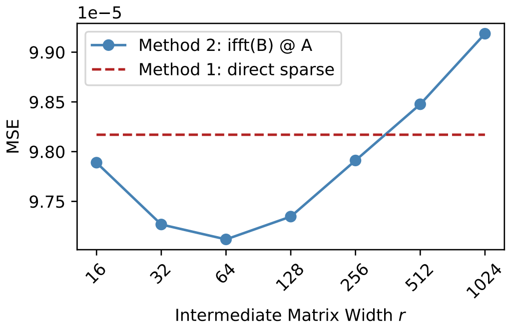


Figure 3: Reconstruction error.

To verify the approximation accuracy of sparse Fourier-domain representations under a fixed budget of non-zero entries, we generated random target matrices $\tilde{B} \in \mathbb{R}^{1024\times r}$ with increasing width $r \in \{16, 32, \dots, 1024\}$, where each entry is sampled from a Gaussian distribution $\mathcal{N}(0; 0.01^2)$.

For each $\tilde{B}$, we initialized a sparse frequency matrix B with exactly $n \in \{1000, 2000, 3000\}$ randomly selected non-zero entries as learnable/trainable parameters. This matrix B is to approximate the Fourier-space representation of the target $\tilde{B}$. Its entries are complex numbers that can be considered as the Fourier coefficients of $\tilde{B}$. The training is performed in the frequency domain directly: the optimal values of the complex coefficient entries are found via gradient descent to minimize the mean squared error (MSE) between $\mathrm{Re}(\mathrm{ifft}(B))$ and the target matrix $\tilde{B}$.

As shown in Figure 2, the reconstruction error increases with the size of the target matrix (i.e., r). This confirms the intuition: larger matrices have higher entry densities, so their Fourier transforms cover wider spectra of frequencies; smaller matrices are the opposite. That is why B better represents smaller $\tilde{B}$ when n is fixed.

3.2 Direct or Indirect Approximation?

Finding 2: *When the number of trainable parameters is fixed, approximating a large matrix via a smaller intermediate representation (e.g., Re($\mathrm{ifft2}(B)) \cdot A$) leads to lower reconstruction error than directly approximating the full matrix.*

We consider the task of reconstructing a structured matrix $W \in \mathbb{R}^{768\times 768}$ under a strict parameter budget: only 1000 trainable entries are allowed. In *Method 1*, we directly approximate W using Equation (3) with 1000 sparse entries in B. In *Method 2*, we instead learn a sparse frequency-domain matrix $B \in \mathbb{C}^{768\times r}$ with the same parameter constraint, and compute $W \approx \mathrm{Re}(\mathrm{ifft2}(B)) \cdot A$, where $A \in \mathbb{R}^{r\times 768}$ is a fixed random projection.

As shown in Figure 3, Method 2 outperforms Method 1 when the intermediate dimension r is small (e.g., $r = 16, 32, 64$). As r increases, the error of Method 2 gradually rises and eventually surpasses the direct method. This supports the hypothesis that, under a fixed parameter budget, it is more effective to allocate capacity toward a compact latent representation than to directly target a large matrix. This trade-off reflects an implicit inductive bias toward low-rank, low-frequency, or compressible structures.

3.3 Split Down Projection

Theorem 3.1. *For any $M \in \mathbb{R}^{a\times b}$, there exist $M_1 \in \mathbb{R}^{a\times b_1}$, $M_2 \in \mathbb{R}^{a\times b_2}$ such that $b_1 + b_2 = b$ and the IFT of M satisfies*

$$\mathrm{IFT}(M) = [\mathrm{IFT}(M_1), \mathrm{IFT}(M_2)]\,, \tag{4}$$

where $[..., ...]$ represents the horizontal concatenation. Similarly, there also exist $M_1' \in \mathbb{R}^{a_1\times b}$, $M_2' \in \mathbb{R}^{a_2\times b}$ such that $a_1 + a_2 = a$ and

$$\mathrm{IFT}(M) = \left[\mathrm{IFT}(M_1'); \mathrm{IFT}(M_2')\right]\,, \tag{5}$$

where $[...; ...]$ represents the vertical concatenation.

Proof. See Appendix G. □

The above theorem shows that further splitting a matrix is theoretically lossless, we also empirically observe in Section 5.7 that such decomposition improve the performance

4 FoRA-UA: Tiny Budget, Big Gains

4.1 Method

Based on the previous findings, our proposed method become:

$$\Delta W = \left[\mathrm{IFT}(\underline{B_1}); ...; \mathrm{IFT}(\underline{B_M})\right] A, \tag{6}$$

Here, $A \in \mathbb{R}^{r\times k}$ is a frozen matrix. $B \in \mathbb{R}^{d\times r}$ is a trainable matrix in the *frequency domain*. This Fourier transformation can improve the training efficiency of our algorithm by allowing it to directly tune the *"spectrum"* encoded in B. We also let the matrix B be **sparse** – during training processes, only the nonzero entries of B need to be updated.

We highlight benefits of our split-down project method:

Computation efficiency: Suppose ΔT is the average timescale to complete an arithmetic operation. Performing the two-dimensional IFT of matrix $B \in \mathbb{R}^{k\times b}$ via a typical fast Fourier transform (FFT) algorithm requires a timescale

$$T_{\text{full}} = [kb\log(kb)]\,\Delta T, \tag{7}$$

while the total time to transform all the B_m's is

$$\begin{aligned} T_{\text{split}} =& [kb_1 \log(kb_1) + \\ & ... + kb_N \log(kb_M)]\Delta T. \end{aligned} \tag{8}$$

One can see that $T_{\text{split}} \leq [kb\log(kb_{\max})]\,\Delta T$, where $b_{\max} = \max(b_1, ..., b_M)$. Hence, T_{split} is shorter than T_{full} because $b_{\max}$ is smaller b.

Flexibility: The method provides the flexibility to optimize each part independently.

4.2 Implementation

The left panel of Figure 1 illustrates the overview of our approach.

We begin by constructing the matrix $A \in \mathbb{R}^{r\times k}$ with its entries randomly drawn from the normal distribution $\mathcal{N}(0, \sigma^2)$. This matrix is frozen during the training steps, meaning the values of its entries remain fixed at all times.

We then initialize a preliminary frequency matrix $B \in \mathbb{R}^{d\times r}$ with entries $B_{i,j}$ set to

$$B_{i,j} = \begin{cases} \mathcal{N}(0,\sigma^2) & \text{if } (i,j) \in S \\ 0 & \text{otherwise} \end{cases}, \tag{9}$$

where S contains n randomly selected integer pairs (i, j) representing the indices of B. The smaller spectral matrices $B_m \in \mathbb{R}^{\frac{d}{M}\times n}$ are obtained by equally splitting B into M pieces.

At each training step, we calculate the IFT of B_m matrices using the (inverse) fast Fourier transform algorithm, and obtain the weight ΔW via Equation 6. We then update the non-zero entries of B_m matrices and optimize ΔW.

4.3 Parameter Counting

Let L denote the number of layers, and d the dimension of internal hidden vectors. For LoRA, the number of trainable parameters for a single target module (e.g. *key, value*) is $L \times r \times d \times 2$, where r is the rank of the low-rank matrices. For our approach, let n represent the number of nonzero entries in the matrices B_i, the number of trainable parameters for a single target module for FoRA-UA is $n \times L$. Empirically, n is typically chosen to be less than 2000. Taking RoBERTa-Large as an example, Table 1 shows the parameters required for fine-tuning. We observe that the number of trainable parameters selected by FoRA-UA is significantly smaller than that of LoRA.

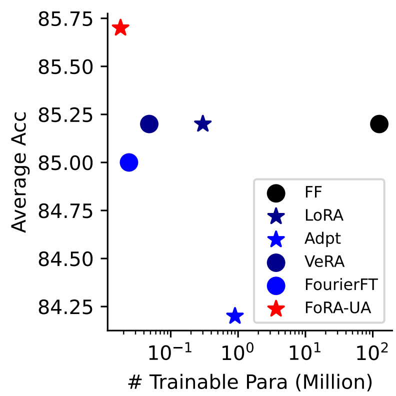


Figure 4: RoBERTa result visualize.

Model	LoRA		FoRA-UA	
	r	#para	n	#para
Rob_large	1	98.3K	200	9.6k
	8	786.4K	500	24k

Table 1: Trainable parameter counting.

5 Results

5.1 Experimental Setup

Benchmark Selection We evaluate our method on a diverse set of tasks spanning multiple domains, including natural language understanding with GLUE (Wang et al., 2018), natural language generation with E2E (Novikova et al., 2017), mathematical reasoning with GSM8k (Cobbe et al., 2021), SingleEq (Koncel-Kedziorski et al., 2015), MultiArith (Roy and Roth, 2015), and SVAMP (Patel et al., 2021), and vision classification with CIFAR100 (Krizhevsky et al., 2009), Food-101 (Bossard et al., 2014), Flowers-102 (Nilsback and Zisserman, 2008) and RESISC45 (Cheng et al., 2017).

Statistics of all benchmarks are listed in Appendix F.

Model Selection We apply our method to a wide variety of model architectures:

1. Encoder-based models such as RoBERTa-base and RoBERTa-large (Liu et al., 2020).

Model	Method	# Trainable Parameters	SST-2 (Acc.)	MRPC (Acc.)	CoLA (MCC)	QNLI (Acc.)	RTE (Acc.)	STS-B (PCC)	Avg.
RoBERTa-base	FF	125M	94.8	90.2	63.6	92.8	78.7	91.2	85.2
	Adpt^{D}	0.3M	$94.2_{\pm0.1}$	$88.5_{\pm1.1}$	$60.8_{\pm0.4}$	$93.1_{\pm0.1}$	$71.5_{\pm2.7}$	$89.7_{\pm0.3}$	83.0
	Adpt^{D}	0.9M	$94.7_{\pm0.3}$	$88.4_{\pm0.7}$	$62.6_{\pm0.6}$	$93.0_{\pm0.2}$	$75.9_{\pm2.2}$	$90.3_{\pm0.4}$	84.2
	LoRA	0.3M	$\mathbf{95.1}_{\pm0.2}$	$89.7_{\pm0.7}$	$63.4_{\pm1.2}$	$\mathbf{93.3}_{\pm0.3}$	$78.4_{\pm0.8}$	$91.5_{\pm0.2}$	85.2
	AdaLoRA	0.3M	$94.5_{\pm0.2}$	$88.7_{\pm0.6}$	$62.0_{\pm0.4}$	$93.1_{\pm0.2}$	$\mathbf{81.0}_{\pm0.6}$	$90.5_{\pm0.2}$	85.0
	VeRA	0.048M	$94.6_{\pm0.1}$	$89.5_{\pm0.5}$	$65.6_{\pm0.8}$	$91.8_{\pm0.2}$	$78.7_{\pm0.7}$	$90.7_{\pm0.2}$	85.2
	FourierFT	0.024M	$94.2_{\pm0.3}$	$90.0_{\pm0.8}$	$63.8_{\pm1.6}$	$92.2_{\pm0.2}$	$79.1_{\pm0.5}$	$90.8_{\pm0.2}$	85.0
	RED	0.02M	$93.9_{\pm0.3}$	$89.2_{\pm1.0}$	$61.0_{\pm3.0}$	$90.7_{\pm0.4}$	$78.0_{\pm2.1}$	$90.4_{\pm0.3}$	83.9
	FoRA-UA	0.018M	$94.7_{\pm0.2}$	$\mathbf{90.8}_{\pm0.8}$	$\mathbf{66.2}_{\pm1.5}$	$91.7_{\pm0.4}$	$79.0_{\pm1.2}$	$\mathbf{91.6}_{\pm0.1}$	**85.7**
RoBERTa-large	FF	356M	96.4	90.9	68.0	94.7	86.6	92.4	**88.2**
	Adpt^{P}	3M	$96.1_{\pm0.3}$	$90.2_{\pm0.7}$	$68.3_{\pm1.0}$	$\mathbf{94.8}_{\pm0.2}$	$83.8_{\pm2.9}$	$92.1_{\pm0.7}$	87.6
	Adpt^{P}	0.8M	$\mathbf{96.6}_{\pm0.2}$	$89.7_{\pm1.2}$	$67.8_{\pm2.5}$	$\mathbf{94.8}_{\pm0.3}$	$80.1_{\pm2.9}$	$91.9_{\pm0.4}$	86.8
	Adpt^{H}	6M	$96.2_{\pm0.3}$	$88.7_{\pm2.9}$	$66.5_{\pm4.4}$	$94.7_{\pm0.2}$	$83.4_{\pm1.1}$	$91.0_{\pm1.7}$	86.8
	Adpt^{H}	0.8M	$96.3_{\pm0.5}$	$87.7_{\pm1.7}$	$66.3_{\pm2.0}$	$94.7_{\pm0.2}$	$72.9_{\pm2.9}$	$91.5_{\pm0.5}$	84.9
	LoRA	0.8M	$96.2_{\pm0.5}$	$90.2_{\pm1.0}$	$68.2_{\pm1.9}$	$\mathbf{94.8}_{\pm0.3}$	$85.2_{\pm1.1}$	$92.3_{\pm0.5}$	87.8
	VeRA	0.061M	$96.1_{\pm0.1}$	$90.9_{\pm0.7}$	$68.0_{\pm0.8}$	$94.4_{\pm0.2}$	$85.9_{\pm0.7}$	$91.7_{\pm0.58}$	87.8
	FourierFT	0.048M	$96.0_{\pm0.2}$	$90.9_{\pm0.9}$	$67.1_{\pm1.4}$	$94.4_{\pm0.4}$	$\mathbf{87.4}_{\pm1.6}$	$91.9_{\pm0.4}$	88.0
	RED	0.05M	$96.0_{\pm0.5}$	$90.3_{\pm1.4}$	$68.1_{\pm1.7}$	$93.5_{\pm0.3}$	$86.2_{\pm1.4}$	$91.3_{\pm0.2}$	87.6
	FoRA-UA	0.024M	$\mathbf{96.6}_{\pm0.3}$	$\mathbf{91.2}_{\pm0.6}$	$\mathbf{69.0}_{\pm1.6}$	$93.9_{\pm0.2}$	$86.9_{\pm1.6}$	$91.9_{\pm0.2}$	**88.3**

Table 2: Performance comparison of different PEFT methods on the GLUE benchmark. Matthew's correlation coefficient is reported for CoLA, Pearson correlation coefficient for STS-B, and accuracy for all other tasks. Results for FF, LoRA, Adapters, AdaLoRA and FourierFT are taken from the Gao et al. (2024), while those for VeRA and Red are sourced from their respective papers (Kopiczko et al., 2024; Wu et al., 2024b).

2. Decoder-based models, including GPT-2 (Radford et al., 2019) and LLaMA 3 (Touvron et al, 2023).

3. Vision Transformers (ViT) (Dosovitskiy et al., 2021).

Baseline Selection We compare our proposed method with the following PEFT baselines: Full fine-tuning (**FF**), **Adapter Tuning** (Houlsby et al., 2019; Pfeiffer et al., 2021; Rücklé et al., 2021), **LoRA** (Hu et al., 2022), **AdaLoRA** (Zhang et al., 2023b), **VeRA** (Kopiczko et al., 2024), **FourierFT** (Gao et al., 2024), and **Red** (Wu et al., 2024b). **DoRA** (Liu et al., 2024a) **LoRA+** (Hayou et al., 2024)[2]

5.2 Natural Language Understanding

The GLUE (Wang et al., 2018) benchmark is a collection of eight diverse datasets designed to evaluate a model's ability to understand and process natural language (Kou et al., 2024b). It includes tasks such as sentiment analysis (SST-2), natural language inference (MNLI, RTE, QNLI), sentence similarity (MRPC, STS-B, QQP), and linguistic acceptability (CoLA).

Implementation We conduct experiments on both RoBERTa-base and RoBERTa-large models. For RoBERTa-base, we set $n = 750$, and for RoBERTa-large, we set $n = 500$. Our experimental setup follows LoRA (Hu et al., 2022), applying weight updates to the query and value matrices, while fully training the classification head. To ensure consistency with prior work like VeRA (Kopiczko et al., 2024) and FourierFT (Gao et al., 2024), we used separate learning rates for the classification head and the adapted layers. We tuned the learning rate, number of epochs, and rank, with the specific hyperparameter choices detailed in Appendix B.

Tasks like MNLI and QQP contain much larger number of data compared to the other six tasks in GLUE. We choose to omit these two, as prior work has done (Kopiczko et al., 2024; Gao et al., 2024). For each dataset, we conduct five independent runs, each using a randomly selected seed (Kou et al., 2025). For each run, we use the best epoch outcome and report the median across the five runs (Kou et al., 2024a).

Results We present the results in Table 2 and Figure 4, where our method achieves the best average performance across all tasks. Notably, our approach requires the fewest trainable parameters among all methods. For RoBERTa-base, our method uses just 6% of the parameters required by LoRA, while for RoBERTa-large, it requires only 3% of LoRA's parameter number.

[2]A more details summarisation about these methods are in Appendix A.

Model	Method	# Trainable Parameters	BLEU	NIST	METEOR	ROUGE-L	CIDEr
GPT2-Medium	FF*	354.92M	65.95	8.52	45.95	69.13	2.35
	Adpt^H*	0.9M	64.31	8.29	44.91	67.72	2.28
	Adpt^P*	0.8M	64.41	8.30	44.74	67.53	2.29
	LoRA	0.4M	66.86	**8.59**	45.94	69.27	**2.41**
	FourierFT	0.048M	64.89	8.38	43.94	67.11	2.20
	RED	0.050M	64.62	8.33	45.14	67.46	2.25
	LoRA	0.098M	64.96	8.40	45.29	68.03	2.32
	VeRA	0.098M	64.50	8.33	45.38	68.77	2.34
	FourierFT	0.098M	64.46	8.33	45.32	67.80	2.28
	FoRA-UA	0.098M	**67.01**	**8.59**	**46.07**	**69.43**	2.40
	FoRA-UA	0.036M	66.15	8.58	45.12	67.70	2.28
GPT2-Large	FT*	774.03M	65.56	8.50	45.40	68.38	2.27
	Adpt^H*	1.8M	65.94	8.46	45.78	68.65	2.23
	Adpt^P*	1.5M	65.53	8.41	45.65	68.46	2.33
	LoRA	0.77M	68.07	8.74	46.20	**69.92**	**2.43**
	VeRA	0.18M	66.72	8.56	46.17	69.30	2.40
	FourierFT	0.07M	66.38	8.55	45.70	68.66	2.33
	RED	0.09M	65.22	8.40	45.59	68.14	2.34
	FoRA-UA	0.05M	67.23	8.66	45.81	68.47	2.40
	FoRA-UA	0.15M	**68.20**	**8.76**	**46.24**	69.65	2.42

Table 3: Performance of different PEFT methods on E2E test set via GPT2-medium and GPT2-large. Results with * are taken from Wu et al. (2024b)

Model	Method	# Trainable Parameters	AddSub	SingleEq	MultiArith	SVAMP
LLaMa2-7B	LoRA	8.4M	80.5	77.2	93.8	42.9
	FoRA-UA	0.1M	75.5	75.6	91.8	42.9
LLaMa3-8B	LoRA	8.4M	89.6	96.7	96.3	75.4
	DoRA	7.1M	90.3	96.1	96	76.8
	LoRA+	6.9M	89.3	96.7	98	68.3
	VeRA	0.4M	88.6	95.7	95.3	71.3
	FoRA-UA	0.1M	85.3	97.0	94.7	72.6

Table 4: Accuracy on math reasoning.

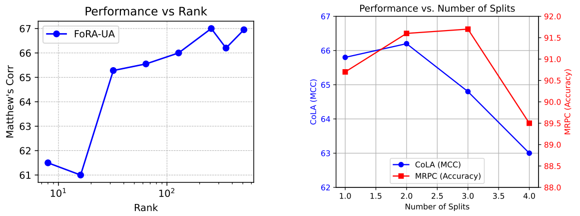
5.3 Natural Language Generation

Implementation We fine-tune the E2E dataset on GPT-2 medium and GPT-2 large models, with detailed hyperparameters provided in Appendix C. Each experiment is conducted three times, and we report the average results. We follow (Li and Liang, 2021) and adopt the script posted by RED (Wu et al., 2024b) and re-ran other PEFT methods for comparison. We tune *key, query, value (i.e. c_attn)* and select the checkpoint with the lowest evaluation loss. For each experiment, we run 3 times and compute the average.

Results Table 3 presents the results on the E2E test set for both GPT2-medium and GPT2-large, demonstrating that our method achieves the smallest parameter size while maintaining competitive performance. For GPT2-medium, our approach outperforms both RED and FourierFT. Additionally, it achieves comparable performance with only 37% of the parameters required by Vera and 9% of those required by Lora. Furthermore, we applied the same number of trainable parameters across different PEFT methods. Specifically, we select $0.098M$ as the number of trainable parameters, which corresponds to the number of parameters Lora needed under the $rank = 1$(smallest) configuration. In this setting, our method surpasses the other three approaches across all five evaluation metrics.

5.4 Reasoning/Instruct Tuning

Implementation We fine-tuned LLaMA2-7B and LLaMA3-8B using the LLM-Adaptor (Hu et al., 2023) framework on the Math-10K dataset. Detailed hyperparameter configurations are provided in Appendix D.



(a) Performance vs. Rank for FoRA-UA on CoLA via RoBERTa-base.

(b) Performance vs. Number of splits.

Figure 5: Comparison of FoRA-UA's effectiveness in terms of rank and number of submatrices(M).

Results Table 4 presents our results across various mathematical benchmarks. Our method achieves performance comparable to LoRA while utilizing only 2% of the parameters that LoRA requires.

5.5 Image Classification

Implementation We use a ViT model pretrained on ImageNet-21K and applied LoRA with a rank of 8 for both ViT-Base and ViT-Large. The only hyperparameter we tuned for our method was the learning rate. Each experiment was run three times, and the results were averaged. For additional hyperparameters, please refer to Appendix E.

Results Table 5 shows that our method achieves performance comparable to full fine-tuning and LoRA in image classification while reducing parameter usage. Specifically, ViT-Base requires only 12.2% of the trainable parameters of LoRA, and ViT-Large utilizes just 6.1% trainable parameters, demonstrating the efficiency of our approach.

5.6 Ablation Study

In this section, we conduct an ablation study to analyze how different factors influence the experimental results. All experiments are repeated five times to ensure statistical reliability. Due to space limitations, we relegate some other studies to Appendix H.

5.6.1 Impact of rank

We conduct these experiments on the CoLA dataset using RoBERTa-base. All experiments were configured with $n = 2000$, and we vary the rank across the following values: $\{16, 32, 64, 128, 256, 350, 512\}$. The results are presented in Figure 5a. From the figure, we observe that performance initially improves as rank increases, with $r = 64$ achieving a Matthew's Correlation of 65.6. However, after $r = 64$, the performance growth slows, indicating diminishing returns. At $r = 256$, performance peaks at 65.55, suggesting an optimal balance between capacity and efficiency. Beyond this point, increasing r further does not lead to improvements. This trend suggests that while higher rank can enhance expressiveness, excessively large ranks may introduce overfitting or optimization instability, leading to degraded generalization.

5.7 Impact of the Number of Splits

To better understand the impact of our split-down projection strategy, we conduct an ablation study on the number of matrix splits used in FoRA-UA on RoBERTa-base and evaluate its effect on downstream performance. As shown in Figure 5b, we observe three key trends. First, introducing a moderate number of splits (e.g., 2 or 3) consistently improves performance over the baseline of a single large matrix. This supports our core motivation that smaller matrices can be more effectively approximated under fixed sparsity constraints. Second, we find that performance degrades sharply when the number of splits becomes too large (e.g., 4), likely because each submatrix receives too few trainable parameters to maintain sufficient representational capacity. Third, the optimal number of splits appears to be task-dependent: while CoLA achieves the highest MCC at 2 splits, MRPC performs best with 3 splits. Please note that the figure does not indicate that our method is unstable. The fluctuations along the y-axis are actually very small; what appears to be a large drop is due to the fact that the difference between the maximum and minimum values is only about 3%.

5.8 Computational Efficiency Analysis

To better understand the computational trade-offs, we compare the FLOPs of different parameter-efficient fine-tuning methods on a single module with hidden size 1024, setting LoRA rank = 16, VeRA rank = 256, and Fora rank = 64. Table 6 summarizes the results. LoRA requires only 16.78M FLOPs, while VeRA incurs a much higher cost of 268M FLOPs due to its larger rank configuration. Our method achieves 67.11M FLOPs, which is higher than LoRA but significantly more efficient than VeRA. This result highlights a favorable balance: although our method introduces additional structural computation compared to LoRA,

Model	Method	# Trainable Parameters	CIFAR100	Food101	Flower102	RESISC45
VIT-B	FF	85.8M	94.53	83.79	98.90	93.07
	LoRA	294.9K	96.21	86.26	100.00	94.81
	FoRA-UA	36K	95.76	83.55	100.00	93.02
VIT-L	FF	303.3M	97.30	88.01	97.06	96.77
	LoRA	786K	96.96	85.69	99.02	96.46
	FoRA-UA	48K	96.64	84.63	100.00	93.49

Table 5: Performance comparison of different PEFT methods on the Image Classification tasks.

Table 6: FLOPs comparison for a single module (hidden size = 1024).

Method	FLOPs
LoRA	16.8 M
VeRA	268.0 M
Ours	67.1 M

the increase remains relatively modest (67.11M vs. 16.78M). At the same time, it avoids the steep overhead of VeRA, which is over **four times** more expensive than ours. Therefore, our approach provides a practical compromise between parameter efficiency and computational cost, offering stronger representational capacity than LoRA without incurring the prohibitive FLOPs of VeRA.

6 Conclusion

We have proposed FoRA-UA, an extremely memory-efficient PEFT method designed to reduce the number of trainable parameters with minimal cost to the performance of other methods. Our experimental results demonstrate that FoRA-UA excels across multiple tasks, including natural language understanding, natural language generation, and image classification while offering superior efficiency compared to existing PEFT methods such as LoRA and VeRA. For instance, with RoBERTa-base, FoRA-UA requires only 6% of the parameters of LoRA, outperforming several current methods.

Limitations

We have demonstrated the effectiveness of our proposed method across a variety of NLP tasks and a representative vision task, we have not yet evaluated its applicability to broader domains such as Code understanding (Wu et al., 2025), VLMs (Yang et al., 2025b; Fan et al., 2025b), RAG (Liu et al., 2024b) or multimodal tasks (Wu et al., 2024a; Li, 2024). Furthermore, although FoRA-UA performs well on standard PEFT benchmarks and model sizes, its scalability to larger-scale model remains an open question. Investigating whether the observed efficiency gains persist in high-capacity, high-throughput settings is an important direction for future work.

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A Baseline Method

We compare our proposed method with the following PEFT baselines:

1. Full fine-tuning involves updating all parameters of a model on a specific dataset to adapt it for a target task, offering maximum flexibility but requiring substantial resources.

2. Adapter Tuning insert adapters between modules such as attention/FNN (Houlsby et al., 2019) and feed-forward (Pfeiffer et al., 2021). Rücklé et al. (2021) removes the adapters that are inactive.

3. LoRA (Hu et al., 2022) is the SOTA of PEFT that update weight by $W = W_0 + BA$.

4. AdaLoRA (Zhang et al., 2023b) improves upon standard LoRA by dynamically adjusting the rank allocation for different layers or weight matrices during fine-tuning, focusing resources on the most impactful areas.

5. VeRA (Kopiczko et al., 2024) is a LoRA extended method that freezes low-rank matrices A and B, and optimizes coefficient vectors $\vec{b}$ and $\vec{d}$.

6. FourierFT (Gao et al., 2024) unlike LoRA that decomposes weight into two matrices, it treats weight changes as spatial domain matrices and learns only sparse spectral coefficients.

7. Red (Wu et al., 2024b) directly edits neural network representations via learnable scaling and biasing vectors.

B Hyperparameters for GLUE

Table 7 contains the hyperparameters we use for GLUE.

C Hyperparameters for E2E

Table 8 contains the hyperparameters we use for GLUE.

D Hyperparameters for Instruct Tuning

Table 9 contains the hyperparameters we use for Instruct Tuning.

E Hyperparameters for Image Classification

Table 10 contains the hyperparameter we use for Image Classification benchmarks.

F Statistics of Benchmark

See table 11 for GLUE, Table 12 for E2E, Table 13 for MTBench, Tabel 14 for image datasets.

G Proof of Theorem

G.1 Observations about the Fourier Transform

The two-dimensional Discrete Fourier Transform (DFT) of an array x_{n_1,n_2} is given by

$$X_{k_1,k_2} = \sum_{n_1=0}^{N_1-1} \left(\omega_{N_1}^{k_1 n_1} \sum_{n_2=0}^{N_2-1} \left(\omega_{N_2}^{k_2 n_2} x_{n_1,n_2} \right) \right), \tag{10}$$

where $\omega_{N_{1,2}} = \exp\left(-i2\pi/N_{1,2}\right)$.

Let A be a matrix representing x_{n_1,n_2},

$$A = \begin{bmatrix} x_{0,0} & x_{0,1} & \dots & x_{0,N_2-1} \\ x_{1,0} & x_{1,1} & \dots & x_{1,N_2-1} \\ \dots & \dots & \dots & \dots \\ x_{N_1-1,0} & x_{N_1-1,1} & \dots & x_{N_1-1,N_2-1} \end{bmatrix}, \tag{11}$$

and let $\tilde{A}$ be the DFT of A,

$$\tilde{A} = \begin{bmatrix} X_{0,0} & X_{0,1} & \dots & X_{0,N_2-1} \\ X_{1,0} & X_{1,1} & \dots & X_{1,N_2-1} \\ \dots & \dots & \dots & \dots \\ X_{N_1-1,0} & X_{N_1-1,1} & \dots & X_{N_1-1,N_2-1} \end{bmatrix}. \tag{12}$$

Meanwhile, let F_{N_1} and F_{N_2} be two linear transformations

$$F_{N_1} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \omega_{N_1} & \omega_{N_1}^2 & \dots & \omega_{N_1}^{N_1-1} \\ 1 & \omega_{N_1}^2 & \omega_{N_1}^4 & \dots & \omega_{N_1}^{2(N_1-1)} \\ \dots & \dots & \dots & \dots & \\ 1 & \omega_{N_1}^{N_1-1} & \omega_{N_1}^{2(N_1-1)} & \dots & \omega_{N_1}^{(N_1-1)^2} \end{bmatrix}, \tag{13}$$

and

$$F_{N_2} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \omega_{N_2} & \omega_{N_2}^2 & \dots & \omega_{N_2}^{N_2-1} \\ 1 & \omega_{N_2}^2 & \omega_{N_2}^4 & \dots & \omega_{N_2}^{2(N_2-1)} \\ \dots & \dots & \dots & \dots & \\ 1 & \omega_{N_2}^{N_2-1} & \omega_{N_2}^{2(N_2-1)} & \dots & \omega_{N_2}^{(N_2-1)^2} \end{bmatrix}. \tag{14}$$

Note that F_{N_1} is N_1-by-N_1 with entries given by $(F_{N_1})_{k_1,n_1} = \omega_{N_1}^{n_1 k_1}$, while F_{N_2} is N_2-by-N_2 with entries $(F_{N_2})_{k_2,n_2} = \omega_{N_2}^{k_2 n_2}$. It is not hard to see that

$$\tilde{A} = F_{N_1} A F_{N_2} \tag{15}$$

and

$$A = (N_1 N_2)^{-1} F_{N_1}^{\dagger} \tilde{A} F_{N_2}^{\dagger}, \tag{16}$$

where $(\cdot)^{\dagger}$ represents the Hermitian transpose. This shows that, for any N_1-by-N_2 matrix A, we have its Fourier transform

$$\mathrm{FT}(A) = F_{N_1} A F_{N_2} \tag{17}$$

and

$$\mathrm{IFT}(A) = (N_1 N_2)^{-1} F_{N_1}^{\dagger} A F_{N_2}^{\dagger}, \tag{18}$$

where the F matrices are defined as above.

G.2 Proof of Theorem 3.1

Without loss of generality, let B be N-by-$2M$, let B_1 and B_2 be N-by-M. Suppose $\mathrm{IFT}(B) =$

Model	Hyperparameter	STS-B	RTE	MRPC	CoLA	SST-2	QNLI
Both	Optimizer	AdamW					
	LR Schedule	Linear					
	Warmup Ratio	0.06					
	Frequency Bias	False					
	Seed	$\{42, 43, 44, 45, 46\}$					
Base	Epochs	80	70	60	80	35	45
	Learning Rate (FoRA-UA)	0.02	0.1	0.01	0.03	0.01	0.01
	Learning Rate (Head)	0.006	0.01	0.006	0.006	0.006	0.006
	Scaling Value	5	1	5	4	5	3
	r	256	64	256	256	256	64
	Max Seq. Len	512					
	Batch Size	32					
	n	2×375					
Large	Epochs	40	60	45	60	15	30
	Learning Rate (FoRA-UA)	0.01	0.08	0.05	0.01	0.01	0.06
	Learning Rate (Head)	0.01	0.005	0.005	0.01	0.01	0.005
	Scaling Value	5	2	4	3	5	2
	r	256	64	256	64	256	64
	Max Seq. Len	512					
	Batch Size	32					
	n	2×250					

Table 7: Hyperparameter setup for the GLUE benchmark.

Model	Hyperparameter	VeRA	FourierFT	RED	FoRA-UA
Medium	Learning Rate	0.02	0.08	0.06	0.1
	Scaling Value	-	300	-	15
	r	1024	-	-	64
	n	-	2000	-	3×500
Large	Learning Rate	0.006	0.08	0.02	0.02
	Scaling Value	-	300	-	15
	r	1024	-	-	32
	n	-	2000	-	3×500
Both	Label Smooth	0.0			
	Weight Decay	0.0001			
	Batch Size	8			
	Optimizer	Adam			
	epoch	5			
	Warmup Step	500			
	Learning Rate Schedule	Linear			
	Seed	{42,43,44}			

Table 8: Hyperparameter setup for the E2E benchmark.

$[\mathrm{IFT}(B_1), \mathrm{IFT}(B_2)]$, the matrices should satisfy the relation

$$\frac{1}{2} B F_{2M}^{\dagger} = \left[B_1 F_M^{\dagger}, B_2 F_M^{\dagger} \right]. \tag{19}$$

Hyperparameter	LoRA	FoRA-UA
LR Schedule	Linear	
Warmup Ratio	0.06	
Batch Size	4	
Optimizer	Adam	
Epoch	1	
Rank	64	128
n	-	3×1000
Scaling Value	16	20
Learning Rate	4e-4	3e-2

Table 9: Hyperparameter setup for the Instruct tuning.

Model	Hyperparameter	CIFAR100	Food101	Flowers102	RESISC45
Base	Learning Rate	0.06	0.06	0.07	0.08
	Head Learning Rate	0.002			
	Scaling Value	15			
	r	64			
	n	1×1500			
Large	Learning Rate	0.03	0.02	0.02	0.03
	Head Learning Rate	0.004			
	Scaling Value	15			
	r	64			
	n	1×1000			
Both	Weight Decay	0.0			
	Batch Size	128			
	Optimizer	Adam			
	epoch	10			
	Seed	{42,43,44}			

Table 10: Hyperparameter setup for the Image Classfication benchmark.

Dataset	Train	Valid	Labels
CoLA	8.5K	1K	2
SST-2	67K	872	2
MRPC	3.7K	408	2
STS-B	7K	1.5K	-
QQP	364K	40K	2
MNLI	393K	20K	3
QNLI	108K	5.4K	2
RTE	2.5K	278	2
WNLI	634	71	2

Table 11: Statistics of the GLUE benchmark. Task types: Acceptability (CoLA), Sentiment (SST-2), Paraphrase (MRPC, QQP), Similarity (STS-B), and Natural Language Inference (MNLI, QNLI, RTE, WNLI).

Dataset	Train	Valid	Test
E2E	42,061	4,672	4,693

Table 12: Statistics of the E2E dataset. The dataset consists of structured meaning representations paired with natural language descriptions.

Note that the LHS term $\frac{1}{2}BF_{2M}^{\dagger}$ can be split in two N-by-M matrices. Therefore, we have B_1 and B_2 as

$$B_1 = \left[\frac{1}{2}BF_{2M}^{\dagger}\right]_{\text{columns } 0 \text{ to } M} F_M, \tag{20}$$

$$B_2 = \left[\frac{1}{2}BF_{2M}^{\dagger}\right]_{\text{columns } M+1 \text{ to } 2M} F_M. \tag{21}$$

See Figure 6 as an example.

Category	# Questions
Writing	10
Roleplay	10
STEM	10
Humanities	10
Coding	10
Reasoning	10
Mathematics	10
Extraction	10
Total	80

Table 13: Statistics of the MT-Bench dataset. The dataset consists of 80 multi-turn questions across eight categories.

Dataset	# Classes	Train	Test
CIFAR-100	100	50K	10K
Food-101	101	75.5K	25.3K
Flowers-102	102	2K	6K
RESISC45	45	31.5K	10.5K

Table 14: Statistics of image classification datasets used in our experiments. CIFAR-100 consists of 100 object categories, Food-101 includes various food items, Flowers-102 contains 102 flower species, and RESISC45 is a remote sensing dataset with 45 scene classes.

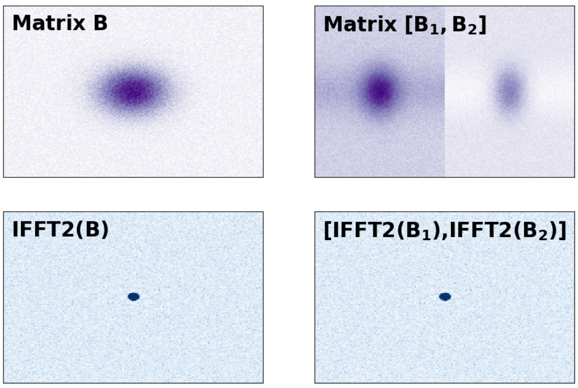

Figure 6: Example of decomposing $\text{IFFT}(B)$ into $[\text{IFFT}(B_1), \text{IFFT}(B_2)]$. The top row shows B and matrix $[B_1, B_2]$. The bottom row, where the colormap shows the absolute values of the IFFT results, confirms that $\text{IFFT}(B) = [\text{IFFT}(B_1), \text{IFFT}(B_2)]$.

Similar proofs also exist for B_1 and B_2 with different sizes and in the case of vertical concatenations.

H More Ablation Studies

H.0.1 Performance *vs.* Trainable Parameters

Figure 8 illustrates the relationship between the number of trainable parameters and model performance on the CoLA dataset. We evaluate FoRA-UA and LoRA, both applied to a RoBERTa-base model. For FoRA-UA, we fix $r = 256$ and vary n across $\{50, 100, 200, 750, 2000, 5000, 10000\}$, while for LoRA, we select rank values $\{1, 2, 4, 6, 8, 15\}$. The x-axis represents the number of trainable parameters on a log scale, and the y-axis reports the corresponding performance. The results indicate that for both FoRA-UA and LoRA, performance improves as the number of trainable parameters increases. However, FoRA-UA consistently outperforms LoRA across all parameter scales, demonstrating its superior efficiency. Notably, FoRA-UA achieves high performance even at relatively small parameter sizes, whereas LoRA requires a larger number of parameters to reach competitive results. This highlights the superior parameter efficiency of FoRA-UA, allowing it to achieve strong results without excessive parameter growth.

H.1 The Importance of A

FourierFT (Gao et al., 2024) simply uses $\text{IFFT}(\Sigma)$ to approximate ΔW. Instead of ours use $\text{IFFT}(\Sigma)$ to approximate the down projection B and then follow the LoRA tradition to learn ΔW. The intuitive behind this is that B is a much smaller matrix than ΔW and a smaller matrix is easier to approximate under the limited trainable parameter condition. To support our statement empirically, we use E2E on GPT2-base as an example. We use the formula $\Delta W = IFT(B)A$ instead of Equation 6 that we use in the experimental part. So the only difference is whether to approximate ΔW directly or approximate B first.

The result is demonstrated in Figure 7. All hypermeter follows Table 8. Note that we pick $n = 2000$ (which is also the #parameter Gao et al. (2024) picked in their experimental part) for FourierFT and $n = 1500$ for FoRA-UA. We can see FoRA-UA consistently achieve better performance across 5 metrics by using 75% #parameters, showing the importance of up projection A.

H.2 Other Transformations

Inverse discrete Fourier transformation of a matrix Σ can also be regarded as matrix multiplications,

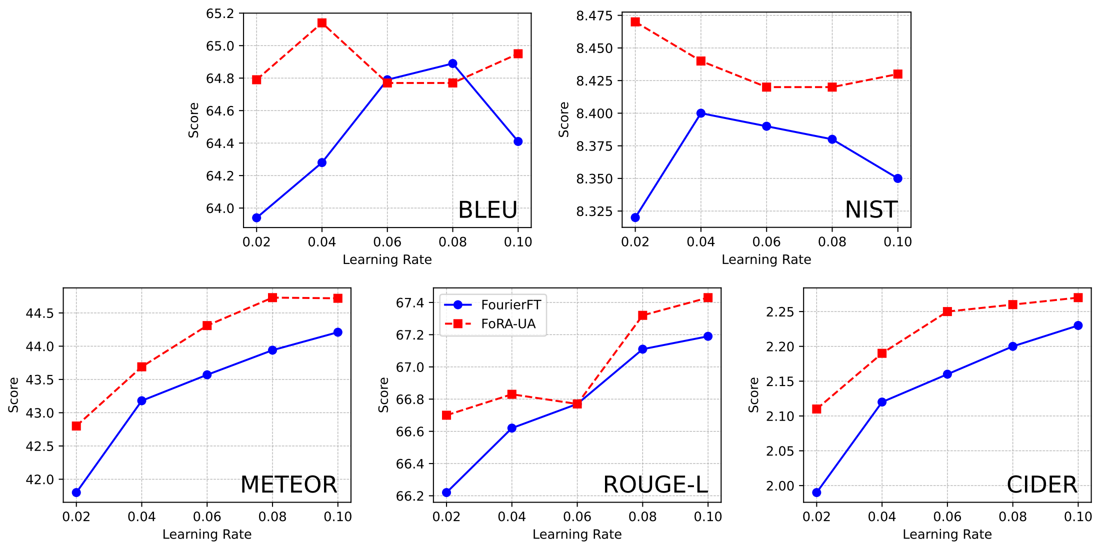

Figure 7: Performance FourierFT and FoRA-UA on E2E, evaluated using 5 different metrics. The solid blue lines with rounded markers represent the scores of FourierFT, while the dashed red lines with squared markers represent the scores of FoRA-UA. FoRA-UA's takes 75% of trainable parameters as FourierFT. This comparison shows that FoRA-UA performs better across all metrics considered, demonstrating the importance of using up projection A.

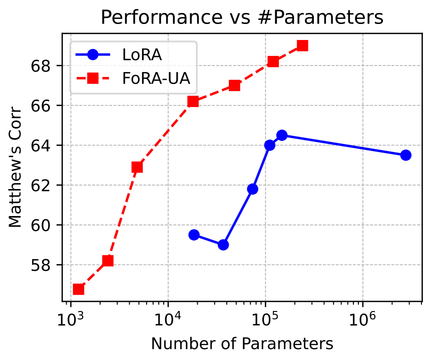

Figure 8: Performance vs. Number of Trainable Parameters for LoRA and FoRA-UA. The x-axis is in the log scale.

$\text{IFFT}(\Sigma) = F_1^\dagger \Sigma F_2^\dagger$. Hence, ΔW can also be written as a product of matrices, $B\Sigma A$, where Σ is trainable, and A and B can take their corresponding values. Results are listed in table 15. We randomly init A and B and try both sparse and dense Σ with a similar number of trainable parameters, ours method archives the best performance and indicates the necessity of the use of inverse Fourier transformations.

Method	MRPC	CoLA
Ours	**91.5**	**67.0**
$B_1\Sigma_1 A_1$	85.8	62.0
$B_2\Sigma_2 A_2$	84.7	60.7

Table 15: Performance comparison of methods on different transformations. The size of B_1 is 768×50 and A_1 is 50×50. Σ_1 is trainable matrix with size 50×50. The size of B_2 is 768×768 and A_2 is 256×768. Σ_2 is trainable *sparse* matrix with size 768×256 and only 2500 nonzero entries.

H.3 Freeze B

We run experiments on MRPC and CoLA with $r = 256$ and $n = 2000$, exploring the impact of freezing different components. As shown in Table 16, freezing A outperforms freezing B, achieving higher scores on both MRPC (91.5) and CoLA (67.0).

Freeze	MRPC	CoLA
A	**91.5**	**67.0**
B	88.5	63.4

Table 16: Performance comparison of methods on freezing A and B on MRPC and CoLA datasets.

H.4 Training Curve

See Figure 9 for Matthew's correlation on CoLA via RoBERTa base. $n = 10000$ for both methods.

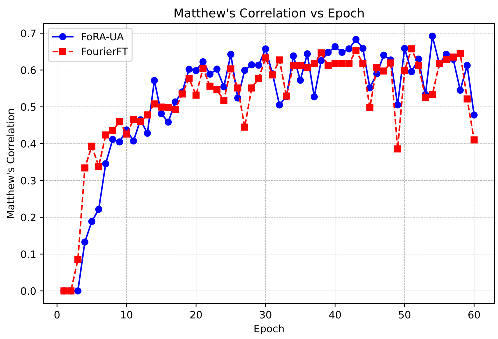

Figure 9: Matthew's correlation comparison between FoRA-UA and FFT.