

Long Chain-of-Thought Fine-tuning via Understanding-to-Reasoning Transition

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Abstract

Reasoning models have demonstrated remarkable performance on complex tasks by generating long reasoning traces prior to producing final answers. However, previous research on long-context scaling in language models has generally focused on managing lengthy input prompts instead of producing long outputs. To leverage the strong long context understanding abilities of current models, we introduce Understanding-to-Reasoning Transition (URT) fine-tuning, a sequence-level curriculum learning framework that gradually shifts a model’s focus from interpreting long chain-of-thoughts to generating them. By incorporating partial reasoning steps in the input context, URT naturally exposes the model to diverse prompt lengths during training, preserving its performance on long-context comprehension while developing advanced reasoning capabilities. Experiments on rigorous reasoning benchmarks, including AIME24 and GPQA Diamond, reveal that our approach surpasses standard fine-tuning by over 10%, while maintaining robust performance on the understanding tasks in RULER.

1 Introduction

Enhancing the long-context capabilities of large language models (LLMs) (Anthropic, 2023; Touvron et al., 2023; Reid et al., 2024) has become both immensely popular and critically important. In recent years, researchers have primarily focused on improving the long-context comprehension abilities of LLMs (Xiong et al., 2023; Zhu et al., 2023; Peng et al., 2023; Gao et al., 2024b), achieving substantial progress on tasks such as summarization and question answering that emphasize the effective utilization and understanding of long inputs. However, with the emergence of reasoning models (OpenAI, 2024; DeepSeek-AI et al., 2025), the focus has noticeably shifted; optimization in

the long-context setting is now aimed at generating long outputs, typically in the form of extensive chain-of-thoughts (CoTs) (Wei et al., 2023).

However, it remains unclear (1) whether current open-source reasoning models that are trained with short prompts can preserve their long-context comprehension capabilities; and (2) whether their abilities to handle long prompts can be effectively leveraged to enhance the learning of long CoTs. To address this gap, our study begins with a preliminary evaluation (§3.1) examining whether popular base LLMs and their reasoning variants can effectively utilize information from lengthy inputs. First, to assess the models’ general long-context capabilities, we adopt the existing benchmark RULER (Hsieh et al., 2024), which measures performance on standardized long-context tasks. In addition to the general domain, we introduce **CoT-U** (Chain-of-Thought Understanding), a new understanding task that evaluates models’ ability to process and reason over lengthy inputs which are human-verified reasoning paths. As illustrated in Figure 1, each input combines a problem statement with its corresponding CoT rationale, testing how effectively models leverage structured reasoning paths. Our evaluation leads to two key observations: (1) Reasoning models (trained predominantly on math problem with limited prompt-length variety) lag behind base models in both CoT-U and general long context tasks, and (2) incorporating partial reasoning steps (tokens) into the input significantly mitigates the difficulty of learning long reasoning chains.

Building on these findings, we propose a new framework for fine-tuning LLMs on challenging chain-of-thought data, termed *Understanding-to-Reasoning Transition* (URT) fine-tuning. In this context, *understanding* refers to the model’s ability to process long-form CoT inputs and assimilate them into accurate final answers. While *reasoning* in our approach involves generating the reasoning steps rather than merely interpreting them. Re-

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cent studies, e.g., Sky-T1 and S1 (DeepSeek-AI et al., 2025; NovaSky Team, 2025; Ye et al., 2025; Muennighoff et al., 2025), have demonstrated that fine-tuning on long CoT data offers a straightforward and cost-effective approach to enabling models to learn extended CoTs. In contrast to these works that finetuned models on complete reasoning chains, URT models the learning process as a gradual transition from understanding to generation. Specifically, URT splits a long reasoning trace into two segments—incorporating part of the trace into the input while requiring the generation of the remaining steps. This means that during training, the input prompt is gradually shortened while the expected output becomes longer. Our approach offers two key benefits. First, it **enhances reasoning** by employing a sequence-level curriculum learning process that bridges the gap between long chain-of-thought understanding and generation by starting with an easier understanding task. Second, it **improves comprehension** by exposing the model to diverse prompt lengths during fine-tuning, thereby avoiding overfitting on short-prompt inputs.

We evaluate our URT fine-tuning method on several challenging reasoning benchmarks, including AIME24 (MAA, 2024), GPQA (Rein et al., 2023), and MATH500 (Hendrycks et al., 2021a), as well as a long-context benchmark, RULER (Hsieh et al., 2024). Our experiments cover various model sizes (8B, 14B, and 32B), and in every case, URT fine-tuning consistently delivers improvements. Notably, our 32B model achieves an improvement of over 10% compared to standard SFT methods. The overall performance of our model is even comparable to that of the teacher models R1-Preview and QwQ-Preview, demonstrating its impressive capacity to acquire and generalize knowledge. In addition, for long-context understanding tasks, our model achieves robust performance on RULER, outperforming QwQ-32B-preview by over 12 points.

2 Related Work

Long-Context Understanding Language models have recently increased their context lengths from 2K to over 128K tokens (Chen et al., 2023; Han et al., 2024; Gao et al., 2024b; Lin et al., 2025). Despite this growth, evaluation and training datasets have mainly concentrated on comprehending long prompts (Li et al., 2024; Jiang et al., 2024). Benchmarks like Scrolls (Shaham et al.,

2022) and LEval (An et al., 2023) assess long-context capabilities through tasks such as question answering (Kočíský et al., 2018; Dasigi et al., 2021), summarization (Zhong et al., 2021; Chen et al., 2022), and synthetic challenges like Needle-in-a-Haystack (gkamradt, 2023; Laban et al., 2024). For training, efforts focus on curating high-quality long-context data from real-world QA datasets, typically following a long-prompt, short-response format (Zhao et al., 2024a; An et al., 2024; Koluguri et al., 2024; Liu et al., 2024; Fu et al., 2024; Wu et al., 2024; Gao et al., 2024a).

Long Chain-of-Thought Reasoning As LLMs’ context lengths expand, research also focuses on generating extended content, such as story writing (Mikhaylovskiy, 2023; Bai et al., 2024). With the release of o1 (OpenAI, 2024), generating long, coherent chains of thought (CoTs) in model responses has become a critical research area (Zhang et al., 2024a; Latif et al., 2024). Enhancements in long CoT generation include Monte Carlo Tree Search (Yao et al., 2023; Zhou et al., 2024; Zhang et al., 2024c), large-scale reinforcement learning (DeepSeek-AI et al., 2025; Pan et al., 2025), and supervised fine-tuning on CoT datasets (Team, 2025; LI et al., 2024). We use supervised fine-tuning for its scalability and strong reasoning performance (Huang et al., 2025; Zhang and Chen, 2024; Muennighoff et al., 2025). Unlike data-centric approaches focused on data diversity and quality (Ye et al., 2025; Min et al., 2024; NovaSky Team, 2025), our work optimizes the training process with a new sequence-level finetuning framework without complex data engineering.

3 Method

This section comprises two primary components: (1) a preliminary evaluation of the long-context capabilities of current models, and (2) our URT-based fine-tuning framework.

3.1 Long Context Understanding Capabilities of Current LLMs

In this section, we describe the details of the long-context understanding evaluation process of current base models and reasoning models.

Evaluation Setup Base models have demonstrated remarkable progress in long-context understanding, as evidenced by their performance on various benchmarks (Zhang et al., 2024b; Song

et al., 2025; Bai et al., 2025). However, their ability to comprehend complex chain-of-thought (CoT) reasoning and the effectiveness of reasoning models for processing long inputs remain largely unexplored. In general long-context ability tests, we utilize RULER (Hsieh et al., 2024) to evaluate these models. We note that there is currently no understanding task featuring long CoT inputs. In this work, we introduce **CoT-U**, a new test dataset designed to assess a model’s ability to retrieve and aggregate information from long chain-of-thought reasoning traces. Given a problem statement accompanied by its multi-step reasoning process as input context, the model must aggregate the useful information to arrive at the correct final answer (see Figure 1).

The construction of CoT-U requires not only the problems but also their corresponding correct reasoning steps. Our experiments are conducted on a dataset comprising 180 math problems and their corresponding solutions, sourced from the Art of Problem Solving (AoPS) platform¹ and spanning the AIME competitions from 2019 to 2024. The AoPS platform provides Olympiad-level math problems accompanied by detailed, human-authored CoTs. In addition to these human-written solutions, we also develop a set of model-generated reasoning paths. Unlike the human solutions, the synthetic paths are enriched with more elaborate explanations and reflective steps, and thus longer and more challenging. Given that o1 (OpenAI, 2024) lacks explicit reasoning tokens, we employ the state-of-the-art open-source model QwQ-32B-Preview from Qwen Team (Bai et al., 2023) to generate these solutions.

From the initial set of 180 generated solutions, we apply rejection sampling to eliminate 56 instances—those with incorrect answers (identified via rule-based matching) and those with correct answers but insufficient or flawed reasoning (determined through human review). In CoT-U, we do not directly include the correct answer in the input. Instead, we remove any sentences that explicitly reveal the ground truth. The model is prompted with the incomplete reasoning process which have already demonstrate the key reasoning steps for deducing the answer. After comprehending this solution, the model only needs to perform simple reasoning to arrive at the final answer. All the data are then subject to manual review to ensure that the

```
<prompt>
A bug walks all day and sleeps all night. On the first day, it starts at point O, facing east, and walks 5 units due east. Each subsequent day, it walks half as far as the previous day in a direction that is 60° counterclockwise ... Find m+n. So I have this problem about a bug that walks different distances each day and rotates 60 degrees counterclockwise every night. It starts at point O, facing east, and walks 5 units east on the first day. Then each subsequent day, it walks half as far as the previous day in a new direction that's 60 degrees counterclockwise from where it was facing the previous night ... Then,  $OP^2 = x^2 + y^2 = 25 + (25 \cdot 3)/9 = 25 + 75/9 = 25 + 25/3 = (75 + 25)/3 = 100/3$  Therefore,  $m = 100$  and  $n = 3$ , so  $m + n = 103$ . wait, but earlier I had a different ... (removed)

User: The provided solution has sufficient information to derive the answer to the question. Please understand the solution and obtain the final answer.
</prompt>
```

Figure 1: An illustrative example from our CoT-U test set, where we remove sentences that directly reveal the final answer and instead include an instruction prompting the model to comprehend the given CoTs.

reasoning steps are logically sound and free from overt mistakes, resulting in a refined test set of 112 cases. Human-generated chains-of-thought average 872 tokens, while AI-generated chains are significantly longer at 16,133 tokens (approximately 18 times longer).

Findings We evaluate the long thought understanding ability of GPT-4o (OpenAI, 2023), Llama3.1-8B (Llama Team, 2024), Qwen2.5-7B/32B (Qwen et al., 2024), and their corresponding long reasoning models: o1-preview (OpenAI, 2024), Qwen2.5-marco-o1 (Zhao et al., 2024b), Qwen2.5-skywork-o1 (o1 Team, 2024), Llama3.1-open-o1², and QwQ-32B-preview on CoT-U. Results are presented in Figure 2 and Figure 3. Our results reveal the following insights:

- (1) All open-source reasoning models exhibit a significant performance degradation in long-context understanding tasks compared to their non-reasoning counterparts.
- (2) Without any hints or reasoning steps provided, open-source reasoning models achieve superior performance in solving mathematical problems. However, when supplied with CoTs generated by other AI systems or humans, their problem-solving capability paradoxically underperforms compared to non-reasoning models.
- (3) Model performance is closely associated with the length of the provided context. Figure 3 illustrates the continuous improvement in performance of Qwen2.5-32B-Instruct and Llama3.1-8B-Instruct as they are supplied with more reasoning steps. Specifically, Qwen2.5-32B-

¹<https://artofproblemsolving.com/>

²<https://github.com/Open-Source-O1/Open-O1>

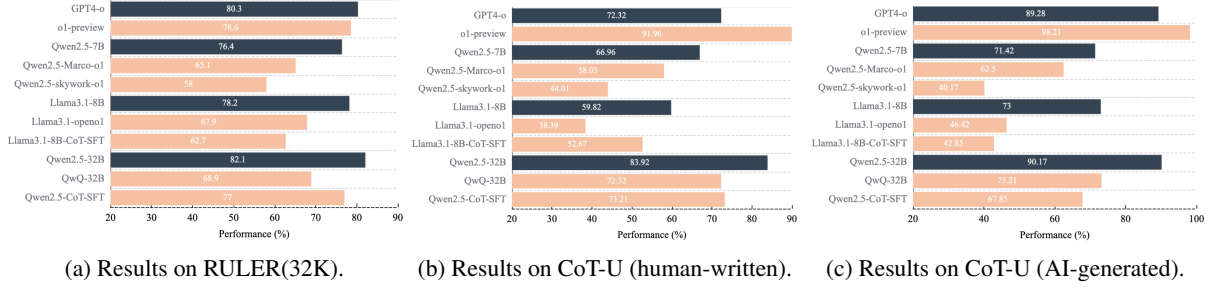


Figure 2: Long context understanding ability of current LLMs. Black bars: base models; orange bars: long reasoning models. Qwen2.5-CoT-SFT and Llama3.1-CoT-SFT is trained by us on the STILL2 dataset (Min et al., 2024).

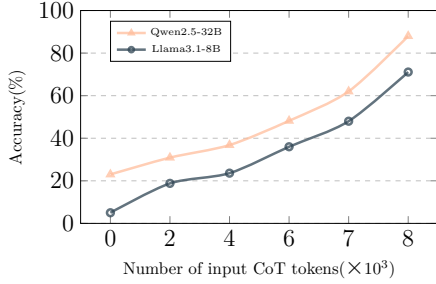


Figure 3: Model performance with varying numbers of provided CoT tokens.

Instruct demonstrates a significant improvement of over 60% compared to directly prompting the model with the problem statement which means the accuracy of solving math problems can be controlled by the number of reasoning tokens.

3.2 Fine-Tuning on Long Thought data

The evaluation results highlights the need for advanced fine-tuning techniques to enhance the long CoT capabilities of reasoning models without compromising their long-context understanding capabilities. This section details our URT fine-tuning method.

Formulation Consider the training dataset defined as

$$D = \{(x^{(i)}, y^{(i)})\}_{i=1}^N,$$

where $x^{(i)}$ is an input instance (e.g., a question or prompt) and $y^{(i)}$ is the corresponding long chain-of-thought response, which includes intermediate reasoning steps and the final answer.

The LLM, parameterized by θ , defines a conditional distribution over outputs as $p(y | x; \theta)$. For a single training instance (x, y) , the loss is defined by:

$$\mathcal{L}(\theta; x, y) = -\log p(y | x; \theta).$$

The overall objective for fine-tuning is to minimize the total loss over the entire dataset:

$$\mathcal{L}(\theta) = \frac{1}{|D|} \sum_{(x^{(i)}, y^{(i)}) \in D} \mathcal{L}(\theta; x^{(i)}, y^{(i)}).$$

This formulation represents the traditional Supervised Fine-Tuning (SFT) method. However, in practice, when dealing with long reasoning chains, the response y is often highly complex and lengthy (Xu et al., 2025; Wang et al., 2024), posing significant challenges for effective model learning.

To overcome the challenges associated with long reasoning chains, we propose Understanding-to-Reasoning Transition Supervised Fine-Tuning (URT-SFT), which involves partitioning the response y into two distinct parts and embedding a segment of y within the input prompt. Specifically, the response y is divided as follows:

$$y = y_{1:t} \circ y_{t+1:T},$$

where: $y_{1:t} = \{y_1, y_2, \dots, y_t\}$, $y_{t+1:T} = \{y_{t+1}, y_{t+2}, \dots, y_T\}$, and $\circ$ denotes the concatenation of sequences. Here, t represents the length of the segment from y that is integrated into the input prompt, which is a hyperparameter. The augmented input x' is then defined as:

$$x' = x \circ y_{1:t},$$

where x is the original input prompt augmented with the first t tokens of the response y .

Consequently, the loss function for a single training instance under URT-SFT is formulated as:

$$\mathcal{L}(\theta; x, y) = -\log p(y_{t+1:T} | x'; \theta).$$

This splitting strategy enables the model to handle long reasoning chains more effectively by reducing the complexity of the output. Furthermore,

Table 1: Examples of different fine-tuning strategies on long reasoning data. During training, the prompt and response are separated by the special token ‘</prompt>’. The lengths of the prompt and response can be adjusted by modifying the placement of </prompt>. Instead of fine-tuning on the entire response, our method begins with an input format that is closer to long-context understanding and gradually transitions to training on the whole thought.

Fine-tuning on whole thought	<prompt> A bug walks all day and sleeps all night... Find $m+n$ </prompt> (no loss on prompt) So I have this problem about a bug that walks different distances each day and...(the whole thought has about 5,120 tokens) the final answer is 103
From long thought understanding to long thought generation	<prompt> A bug walks all day and sleeps all night... Find $m+n$ So I have this problem about a bug that walks different distances...(the input context contains 4,096 tokens) </prompt> Now, $P = 5/0.75 - i(\sqrt{3}/4)$. To divide by a complex number, multiply numerator and denominator by the conjugate of the denominator...(the output contains about 1024 tokens) the final answer is 103
	<prompt> A bug walks all day and sleeps all night... Find $m+n$ So I have this problem about a bug that walks different distances...(the input context contains 2,048 tokens) </prompt> First day: walk 5 units east, so position $Z_1 = 5 + 0i$. Then, rotate 60 degrees...(the output contains about 3072 tokens) the final answer is 103
	Fine-tuning on whole thought (the output contains about 5120 tokens)

in traditional SFT, most inputs x are math problems (Min et al., 2024; NovaSky Team, 2025; LI et al., 2024), which are naturally short. However, after concatenation, x' becomes a long-context input. Exposing the model to a range of input lengths during fine-tuning demonstrably enhances the performance of long reasoning models on long-context tasks (§4.4).

An Example of Decomposing In this section, we demonstrate how we split a long thought, with t set to powers of 2, specifically $\{2,048, 4,096, 8,192\}$. The thought y consists of approximately 5,120 tokens, as illustrated in Table 1. We utilize a simple chat template where the user input is enclosed within <prompt>...</prompt>. Following previous work (Wang et al., 2023; Zheng et al., 2024), we do not calculate loss on the prompt.

In the top block of Table 1, the model is fine-tuned on the entire chain-of-thought y , where the prompt x contains only a brief problem description (e.g., “A bug walks all day and sleeps all night... Find $m + n$ ”), and the full reasoning (about 5,120 tokens) along with the final answer is produced in the response. The subsequent blocks illustrate our method of decomposing the long thought—transitioning from long thought understanding to long thought generation. Given that the input length of 5,120 tokens is below the 8,192 tokens threshold, we experiment with splitting points $t = 2,048$ and 4,096. In our implementation, we first split the text by $\backslash n\backslash n$ and then by length. This ensures that splitting points are at $\backslash n\backslash n$ positions, preventing tokenization errors. In the first row of the second block, the prompt includes a portion of the chain-of-thought (with the context containing

$t = 4,096$ tokens), and the response provides the remaining reasoning steps (approximately 1,024 tokens) together with the final answer. In the second row, we increase the task difficulty by reducing the input context length to around $t = 2,048$ tokens, resulting in a longer response (about 3,072 tokens). Finally, the model is trained on the whole thought without any reasoning steps provided. The training details are in Section §4.1.

Difficulty Control As a sequence-level curriculum learning framework, URT is designed to mimic an easy-to-hard training progression. At the sequence level, we emphasize training examples with longer prompts—providing additional context or intermediate reasoning—paired with shorter responses. This design encourages the model to leverage extended prompts containing rich reasoning information while reducing the burden of generating lengthy responses. At the beginning of every epoch, the training data is shuffled. For identical math problems, training samples that incorporate more reasoning steps are prioritized. It means the model is initially exposed to instances where more intermediate steps help in reaching the final answer. This ordering facilitates a gradual increase in difficulty as training progresses. After each epoch, the model is evaluated on the training set. If it correctly derives a problem’s final answer, that problem is removed, allowing the model to concentrate on unsolved challenges.

4 Experiments

4.1 Training Details

In this section, we describe the data preparation, training process, and hyperparameter settings used in our experiments. Our focus is on the training methodology rather than on curating or annotating data; therefore, we directly use publicly available, open-source dataset provided by STILL2 (Min et al., 2024) where the teacher models are QwQ-32B-preview (Qwen Team, 2024) and R1-lite-preview (DeepSeek Team, 2024).

Data Preparation We utilize the dataset provided in STILL2³ which comprises approximately 4.9K question and chain-of-thought pairs. The majority of the prompts are from the mathematics domain, with a smaller portion from code and science. In the original dataset, there are 4,900 prompts in total, with 2,429 having lengths exceeding 2,048 tokens, 1,071 exceeding 4,096 tokens, and 81 exceeding 8,192 tokens. We then create a decomposed dataset using the method illustrated in Table 1, consisting of 4,814 prompts with diverse input lengths (2,048~8,192).

Training The SFT baselines are trained on 4.9K prompts for 10 epochs, while the URT models are trained on 9.8K prompts for 5 epochs. Additionally, we exclude data that the model can already handle correctly. To validate the effectiveness of our approach, we perform experiments on three base models with sizes ranging from 8B to 32B: Llama3.1-8B-Instruct, Qwen2.5-14B-Instruct, and Qwen2.5-32B-Instruct. We do not modify the models’ tokenizers or chat templates, and we adopt a full-parameter fine-tuning setting. We use Llama Factory (Zheng et al., 2024) for fine-tuning and follow their default SFT hyperparameters. We use a batch size of 64, training in bfloat16 precision. The learning rate is set to 1×10^{-5} , with no warm-up, and it decays to 0 following a cosine schedule. We employ the AdamW optimizer (Loshchilov and Hutter, 2019) with $\beta_1 = 0.9$, $\beta_2 = 0.95$, and a weight decay of 1×10^{-4} . Note that during training we compute the loss only on the reasoning traces from the response, and not on the input (which includes the question and any partial solution). All experiments are conducted on a single machine equipped with 8 NVIDIA A100 GPUs, and we set the maximum training sequence length to 16384.

³https://github.com/RUCAIBox/Slow_Thinking_with_LLMs

Our training pipeline is based on DeepSpeed ZeRO-3 (Aminabadi et al., 2022) and accelerated by FlashAttention-2 (Dao, 2023). For the 32B model, CPU offload is utilized to mitigate GPU out-of-memory. In our experiments, the 8B and 14B models are fine-tuned in under 4 hours, while the 32B model requires approximately 14 hours.

4.2 Evaluation Setup

In this section, we outline the benchmarks and baselines used to evaluate the reasoning capabilities of our models. Specifically, we compare URT with the traditional SFT on widely recognized reasoning tasks covering multiple domains as well as long-context benchmarks. For a detailed description of the benchmarks and baselines employed in this work, please refer to Section A.1.

4.3 Main Results

Long-Reasoning Results Table 2 presents a comprehensive comparison of various models evaluated on three benchmark datasets: MATH500, AIME24, and GPQA-Diamond. The results demonstrate that URT-SFT consistently outperforms the standard SFT approach across all evaluated models. For instance, applying URT-SFT to the Llama3.1-8B-Instruct model results in accuracy improvements of 17.6% on MATH500, 506.1% on AIME24, and 30.6% on GPQA-Diamond, greatly surpassing the SFT baseline. Similarly, URT-SFT outperforms SFT on larger base models like Qwen2.5-14B-Instruct and Qwen2.5-32B-Instruct.

Our proposed URT-SFT method leverages only 5K prompts to fine-tune models, yet achieves performance that rivals or exceeds that of our teacher models R1-preview and QwQ-32B-preview. Specifically, on the AIME24 dataset, our method achieves an accuracy of 56.7%, outperforming r1-preview’s 52.5% and QwQ-32B-preview’s 50.0%. Furthermore, in the GPQA-Diamond benchmark, our 32B model reaches an accuracy of 61.6, surpassing r1-preview by 3 points, demonstrating competitive performance despite the use of fewer prompts. When assessed against contemporary state-of-the-art open-source models, our URT-SFT-trained models also exhibit competitive performance across all benchmarks. On the AIME24 dataset, our approach aligns closely with S1-32B + BF, which is trained on data distilled from the stronger teacher model Gemini 2.0 Flash Thinking (Google Team, 2024). Our 32B model is able to outperform strong open-source baselines STILL2-

Table 2: Performance comparison of various models across three popular benchmarks to assess the long trace reasoning ability of LLMs. **Distilled From** indicates which model provides the training data. The **bold** fonts denote the best performance among our training variants, and we report the gain over the base model. The examples of the input and output for the three benchmarks are shown in Appendix Table 5.

Models	Distilled From	MATH500		AIME24		GPQA-Diamond	
		Acc (%)	Gain (%)	Acc (%)	Gain (%)	Acc (%)	Gain (%)
GPT-4o	N.A.	76.6	-	9.3	-	53.6	-
Claude 3.5 Sonnet	N.A.	78.3	-	16.0	-	65.0	-
o1	N.A.	94.8	-	74.4	-	77.3	-
o1-preview	N.A.	85.5	-	44.6	-	72.3	-
r1	N.A.	97.3	-	79.8	-	71.5	-
r1-preview	N.A.	91.6	-	52.5	-	58.5	-
QwQ-32B-preview	N.A.	90.6	-	50.0	-	65.2	-
<i>Trained on open-source dataset</i>							
Open-o1-Llama3.1-8B	Synthetic data	54.8	-	10.0	-	30.8	-
S1-32B + BF	Gemini Flash Think.	93.0	-	56.7	-	59.6	-
STILL2-32B	r1-preview & QwQ-preview	89.2	-	43.3	-	55.0	-
Sky-T1-32B	QwQ-preview	86.4	-	43.3	-	56.8	-
Llama3.1-8B-Instruct + SFT	r1-preview & QwQ-preview	48.4	-	3.3	-	23.2	-
+ URT-SFT		52.2	+7.9%	13.3	+303.0%	27.2	+17.2%
Qwen2.5-14B-Instruct + SFT	r1-preview & QwQ-preview	57.0	+17.6%	20.0	+506.1%	30.3	+30.6%
+ URT-SFT		78.6	-	13.3	-	43.4	-
Qwen2.5-32B-Instruct + SFT	r1-preview & QwQ-preview	83.2	+6.4%	33.3	+150.4%	53.5	+23.3%
+ URT-SFT		86.0	+9.4%	40.0	+200.7%	60.6	+39.6%
Qwen2.5-32B-Instruct + SFT	r1-preview & QwQ-preview	80.0	-	13.3	-	43.4	-
+ URT-SFT		88.2	+11.5%	43.3	+225.5%	55.1	+26.9%
		90.4	+12.8%	56.7	+326.3%	61.6	+41.9%

32B and Sky-T1-32B, which employ similarly-performing teacher models. The results on the GPQA-Diamond show that our method clearly outperform all previous models when tested with a broader range of domains. We also demonstrate that URT-SFT maintains an advantage over SFT in more tasks (Table 3).

Table 3: Performance across a broader range of tasks, including medicine, commonsense reasoning, code, and math. LCB stands for LiveCodeBench, and PA stands for Putnam-AXIOM.

Model	MedQA	BBH	LCB	PA
Qwen2.5-32B	72.0	50.8	40.8	17.7
+SFT	73.2	58.6	41.3	38.1
+URT-SFT	75.7	59.3	44.5	43.2

Long-Context Results As described in Section 3.1, the model’s ability to understand long reasoning traces deteriorates after extended CoT training, despite the task being a straightforward mathematical one. We hypothesize that this decline is due to the model not encountering long input prompts during training, which hampers its long-context understanding capabilities. To validate this hypothesis, we conducted evaluations using the RULER long-context evaluation suite. The results are presented in Table 4.

Our experiments reveal that, compared to the base model Qwen3.5-32B-Instruct, models trained on a predominantly math problem dataset—both the official Qwen checkpoint QwQ-32B-Preview and models fine-tuned on open-source datasets—exhibit decreased performance. This decline becomes more pronounced as the input length increases from 32K to 64K tokens. However, models fine-tuned with URT are exposed to numerous long prompt inputs during training. Consequently, compared to direct SFT, URT-SFT does not suffer significant performance degradation on long-context tasks and improves performance on long-context tasks by over 10 percentage points compared to QwQ-32B-Preview.

In conclusion, By decomposing long-chain reasoning into manageable segments embedded within prompts, URT-SFT not only simplifies the learning process but also capitalizes on the models’ ability to handle extended contexts, leading to improved accuracy and performance.

4.4 Anaysis

Performance Gains Across Different Difficulty Levels To investigate the scenarios in which URT-SFT enhances standard SFT, we constructed a diverse test set by randomly sampling 200 instances from the MATH dataset across difficulty levels 1 to

Table 4: Long context results of long reasoning models. We test the models on 3 categories: Needle-In-A-Haystack (NIAH), Variable Tracing (VT) and Question Answering (QA) from RULER (Hsieh et al., 2024). We conduct tests using input lengths of 32K and 64K tokens. The demonstrations of the three tasks defined in RULER are shown in Appendix Table 6 and 7.

Models	RULER			Avg.
	NIAH	VT	QA	
Context length = 32K				
Qwen2.5-32B-Instruct	98.0	86.4	62.0	82.1
QwQ-32B-Preview	64.0	82.8	60.0	68.9
Qwen2.5 + SFT	92.0	89.0	50.0	77.0
Qwen2.5 + URT-SFT	95.0	90.4	59.0	81.5
Context length = 64K				
Qwen2.5-32B-Instruct	71.0	67.8	56.0	64.9
QwQ-32B-Preview	52.0	57.4	37.0	48.8
Qwen2.5 + SFT	55.0	70.9	31.0	52.3
Qwen2.5 + URT-SFT	62.0	74.8	45.0	60.6

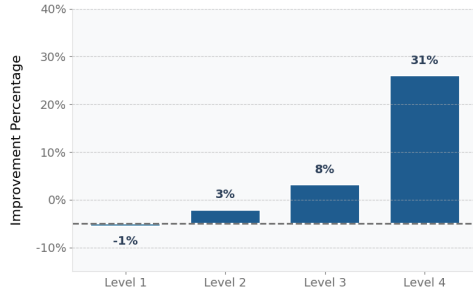


Figure 4: URT-SFT shows a greater performance improvement over SFT on more difficult math problems.

5 and 30 instances from AIME 2024, resulting in a test set with varying levels of difficulty. Specifically, **Level 1** includes problems from MATH difficulty levels 1, 2, and 3; **Level 2** comprises MATH difficulty level 4 problems; **Level 3** consists of MATH difficulty level 5 problems; and **Level 4** contains the 30 AIME 2024 problems. The performance improvements of URT-over SFT across these different difficulty levels are illustrated in Figure 4. Our method demonstrates superior performance on more challenging tasks, highlighting its effectiveness in handling problems of increased complexity.

Reinforcement Learning To evaluate the orthogonality between URT and Reinforcement Learning (RL) fine-tuning, we conduct complementary experiments using Group Relative Policy Optimization (GRPO) (DeepSeek-AI et al., 2025). Specifi-

cally, we fine-tune our models for 1 epoch on the STILL2 training set with a rollout size of 8 and a maximum sequence length of 16,384 tokens per trajectory, following the default configurations of Verl (Sheng et al., 2024) with 8 NVIDIA A100 GPUs. Our results (Figure 5) indicate that RL scaling and URT are orthogonal, with RL further enhancing URT’s performance. Notably, URT demonstrates significantly higher training efficiency compared to RL scaling. For instance, training a 32B model with GRPO consumes over **12 days** on 8 A100 GPUs, suggesting that knowledge distillation may be a more practical alternative for smaller models with academic-level resources.

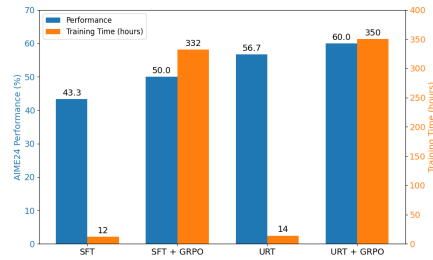


Figure 5: Training time and performance on AIME24 for SFT and URT models further trained with the GRPO.

Inference Time Scaling In our SFT-trained models, we observe inference time scaling behavior similar to that of the o1 model. Specifically, as the number of tokens processed during inference increases, the performance continues to improve. In all experiments, the models were run with a maximum new tokens of 16K.

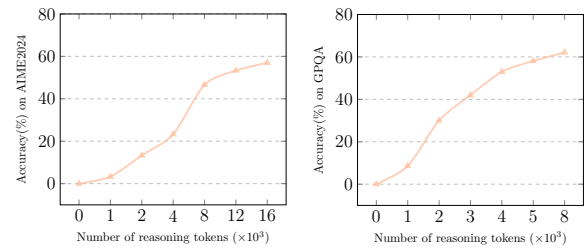


Figure 6: Test-time scaling with URT models on GPQA and AIME24.

5 Conclusion

This paper introduces a new fine-tuning framework called Understanding-to-Reasoning Transition (URT) fine-tuning, which aims to improve the ability of large language models to generate long and coherent CoT reasoning. The core idea is to gradually transition the model’s focus from understanding CoTs to generating them. This is achieved

by incorporating partial reasoning steps into the input context during fine-tuning. The experimental results demonstrate that URT fine-tuning outperforms standard fine-tuning methods, especially on challenging reasoning benchmarks.

Limitations

One limitation of this work is that we have only explored the effectiveness of a training method that gradually transfers from understanding to reasoning within the fine-tuning phase. Due to computational resource constraints, we have not validated the efficacy of our approach within the reinforcement learning framework. Additionally, in terms of long-context understanding, when the input length exceeds 64K tokens, the performance of models trained using URT declines compared to the base model. To further enhance the long reasoning model’s ability to handle lengthy prompts, it may be necessary to incorporate more high-quality long prompt data, rather than relying solely on a training set predominantly consisting of mathematical problems.

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same training dataset as our model with additional refinements and reinforcement learning; and **Open-o1-Llama3.1-8B**, an o1-like model based on Llama3.1-8B-Instruct.

A Appendix

A.1 Benchmarks and Baseline Models

Benchmarks We utilize three widely-recognized reasoning benchmarks to assess the problem-solving skills of our models: (1) AIME24 comprises 30 mathematical problems from the 2024 American Invitational Mathematics Examination. (2) MATH500 ([Hendrycks et al., 2021b](#); [Lightman et al., 2023](#)) is a curated subset of 500 competition-level problems. (3) GPQA Diamond ([Rein et al., 2023](#)) comprises 198 PhD-level STEM questions across Biology, Chemistry, and Physics. We use this benchmark to assess our model’s reasoning ability across STEM domains. We also include MedQA ([Jin et al., 2020](#)), LiveCodeBench-Medium ([Jain et al., 2024](#)) (24.08-24.1), BBH ([Suzgun et al., 2022](#)), and Putnam-AXIOM ([Gulati et al., 2024](#)) to evaluate our method across broader domains, covering medical exams, coding, and commonsense. Since most datasets for long CoT training are dominated by short inputs, we incorporate the RULER ([Hsieh et al., 2024](#)) evaluation suite to assess whether models’ long-context understanding abilities are affected. This suite features long-document comprehension tasks with customizable input lengths.

Baselines We compare our models against several strong baselines to contextualize performance: **o1** and **o1-preview** ([OpenAI, 2024](#)), commercial models by OpenAI known for generating long reasoning traces; **r1** and **r1-preview** ([DeepSeek Team, 2024](#)), developed by DeepSeek with performance comparable to o1; **QwQ-32B-preview** ([Qwen Team, 2024](#)), an open-weight reasoning model from the Qwen Team; **Sky-T1-32B-Preview** ([NovaSky Team, 2025](#)), trained on reasoning data distilled from QwQ-32B-preview; **s1-32B-BF**, trained on data distilled from Google Gemini 2.0 Flash Thinking ([Google Team, 2024](#)) with budget forcing to encourage longer reasoning paths; **STILL2-32B** ([Min et al., 2024](#)), utilizing the

Table 5: The example input prompt and model output for the reasoning benchmarks used in this paper.

MATH500	
<i>Input prompt</i>	Let a be a positive real number such that all the roots of $x^3 + ax^2 + ax + 1 = 0$ are real. Find the smallest possible value of a
<i>Model reponse</i>	Let's tackle this problem step by step. We have the cubic equation: $x^3 + ax^2 + ax + 1 = 0$ And we're told that all its roots are real, with a being a positive real number. We need to find the smallest possible value of a. First, I know that for a cubic equation of the form $x^3 + bx^2 + cx + d = 0$... Thus, the smallest possible value of a is: <code>boxed{3}</code>
AIME 2024	
<i>Input prompt</i>	Find the largest possible real part of $(75 + 117i)z + \frac{96 + 144i}{z}$ where z is a complex number with $z = 4$
<i>Model reponse</i>	Alright, so I've got this problem here. It says: Find the largest possible real part of $(75 + 117i)z + \frac{96 + 144i}{z},$ where z is a complex number with $z = 4$. Okay, let's break this down. First off, z is a complex number with magnitude 4. So, z can be written in polar form as $z = 4e^{i\theta}$, where θ is the argument of z. But maybe it's easier to work with z in terms of its real and imaginary parts. Let's say $z = a + bi$, where a and b are real numbers, and $a^2 + b^2 = 16$, since $z = 4$... Therefore, the maximum value of the real part is: <code>boxed{540}</code>.
GPQA Diamond	
<i>Input prompt</i>	What is the energy of the Relativistic Heavy Ion Collider (RHIC) so that the speed of the nucleus X is equal to $0.96c$? Knowing that X is defined as Li with $A=6$. A. 20 GeV B. 10 GeV C. 36 GeV D. 49 GeV
<i>Model reponse</i>	Iright, I've got this problem about the Relativistic Heavy Ion Collider, or RHIC, and I need to find out what energy it should have so that the speed of a lithium nucleus, specifically Li with $A=6$, is equal to 0.96 times the speed of light. The options are 20 GeV, 10 GeV, 36 GeV, and 49 GeV. First off, I know that in relativity, when objects move at speeds close to the speed of light, their energy isn't just mc^2; there's additional kinetic energy involved. The total energy of a relativistic particle is given by $E = \gamma mc^2$, where γ is the Lorentz factor. The Lorentz factor, γ, is $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$. So, if the speed v is $0.96c$, then v/c is 0.96. Let me calculate γ. $\gamma = \frac{1}{\sqrt{1 - (0.96)^2}}$... This value is closest to 20 GeV, so the correct answer is: <code>boxed{A}</code>

Table 6: The example input prompt and model output for the Needle-in-a-Haystack and Variable Tracing in RULER.

Needle-in-a-Haystack (multi-needle)	
<i>Input prompt</i>	July 2006I've discovered a handy test for figuring out what you're addicted to. Imagine you were going to spend the weekend at a friend's house on a little island off the coast of Maine. There are no shops on the island and you won't be able to leave while you're there. Also, you've never been to this house before, so you can't assume it will have more than any house might. What, besides clothes and toiletries, do you make a point of packing? That's what you're addicted to... The UUID for bd9645ff-0667-1ad1-1e80-b17fa7f1599d is 9dde507b-0676-9ad1-1e80-317fa3b1799c you can't expect an idea to be novel to everyone. Any insight that you have will probably have already been had by at least one of the world's 7 billion people. But it's sufficient if an idea is novel to a lot of readers. Ditto for correctness, importance, and strength. In effect the four components But with Lisp our development cycle was so fast that we could sometimes duplicate a new feature within a day or two of a competitor announcing it in a press release. By the time journalists covering the press release got round to The UUID for b5d648fb-0666-1ad3-5c80-39ff23b379ad is: bcd640fb-0667-1ad5-1c88-337fa3b0699d nThere is a founder community just as there's a VC community. They all know one another, and techniques spread rapidly between them. If one tries a new programming language or a new hosting provider and gets good results, 6 months later half of... Please find the UUID for bd9645ff-0667-1ad1-1e80-b17fa7f1599d from the text above
<i>Model reponse</i>	Alright, I need to find the UUID for bd9645ff-0667-1ad1-1e80-b17fa7f1599d from the provided document. Let me look through the text carefully. I see that there are several UUIDs mentioned, each associated with a specific UUID format. I need to find the one that matches bd9645ff-0667-1ad1-1e80-b17fa7f1599d and note its corresponding UUID. Looking at the first one: The UUID for bd9645ff-0667-1ad1-1e80-b17fa7f1599d is 9dde507b-0676-9ad1-1e80-317fa3b1799c. So, it seems like the UUID for bd9645ff-0667-1ad1-1e80-b17fa7f1599d is 9dde507b-0676-9ad1-1e80-317fa3b1799c. I should double-check to make sure I don't miss any other occurrences. Looking further down, I see another UUID mentioned: The UUID is: bcd640fb-0667-1ad5-1c88-337fa3b0699d. And another one: The UUID for fdc644fb-1667-1ad1-0d80-337fe3b7799d is: bdf6c05b-0427-dad1-0c00-317fa321699d... Thus, the final answer is: boxed{9dde507b-0676-9ad1-1e80-317fa3b1799c}
Variable Tracing	
<i>Input prompt</i>	July 2006I've discovered a handy test for figuring out what you're addicted to. Imagine you were going to spend the weekend at a friend's house on a little island off the coast of Maine. There are no shops on the island and you won't be able to leave while you're there. Also, you've never been to this house before, so you can't assume it will have more than any house might... var GNEXW = 62265 they're going to issue a public report tracking how this specific tranche of money is spent, NFTs are a new territory... var HQPDT = GNEXW you can't expect an idea to be novel to everyone. Any insight that you have will probably have already been had by at least one of the world's 7 billion people. But it's sufficient if an idea is novel to a lot of readers. Ditto for correctness, importance, and strength. In effect the four components var OUNAI = HQPDT But with Lisp our development cycle was so fast that we could sometimes duplicate a new feature within a day or two of a competitor announcing it in a press release. By the time journalists covering the press release got round to var AYWVH = OUNAI nThere is a founder community just as there's a VC community. They all know one another, and techniques spread rapidly between them. var BWYCM = AYWVH If one tries a new programming language or a new hosting provider and gets good results, 6 months later half of... Please find all the 5 variables that are assigned the value 62265 in the text above.
<i>Model reponse</i>	Alright, I need to find all the variables that are assigned the value 62265 in this text. I'll look for lines that have var followed by a variable name, an equals sign, and then 62265. First, I see var GNEXW = 62265. So GNEXW is one variable assigned to 62265. Next, I look further down and find var HQPDT = GNEXW. Wait, HQPDT is being assigned the value of GNEXW, which is 62265. So HQPDT is another variable with the value 62265. Then, I see var BTTDM = 99634. That's not 62265, so I skip that. Later, there's var OUNAI = HQPDT. Since HQPDT is 62265, OUNAI is also 62265... I need to make sure I didn't miss any or count any that shouldn't be included. Let me double-check. - GNEXW = 62265 (direct assignment) - HQPDT = GNEXW (GNEXW is 62265) - OUNAI = HQPDT (HQPDT is 62265) - AYWVH = OUNAI (OUNAI is 62265) - BWYCM = AYWVH (AYWVH is 62265) Looks correct. These are the five variables assigned to 62265. Thus, the final answer is: boxed{GNEXW, HQPDT, OUNAI, AYWVH, BWYCM }

Table 7: An example of the input prompt and corresponding model output for the long-context QA task in RULER.

Long context QA	
<i>Input prompt</i>	The Normans had a profound effect on Irish culture and history after their invasion at Bannow Bay in 1169. Initially the Normans maintained a distinct culture and ethnicity. Yet, with time, they came to be subsumed into Irish culture to the point that it has been said that they became "more Irish than the Irish themselves." The Normans settled mostly in an area in the east of Ireland, later known as the Pale, and also built many fine castles and settlements, including Trim Castle and Dublin Castle. Both cultures intermixed, borrowing from each other's language, culture and outlook. Norman descendants today can be recognised by their surnames. Names such as French, (De) Roche, Devereux, D'Arcy, Treacy and Lacy are particularly common in the southeast of Ireland, especially in the southern part of County Wexford where the first Norman settlements were established. Other Norman names such as Furlong predominate there. Another common Norman-Irish name was Morell (Murrell) derived from the French Norman name Morel. Other names beginning with Fitz (from the Norman for son) indicate Norman ancestry. These included Fitzgerald, FitzGibbons (Gibbons) dynasty, Fitzmaurice. Other families bearing such surnames as Barry (de Barra) and De Búrca (Burke) are also of Norman extraction. Prior to European settlement, the area now constituting Victoria was inhabited by a large number of ..(lengthy documents containing the background for the question)... Victoria was officially created a separate colony in 1851, and achieved self-government in 1855. The Victorian gold rush in the 1850s and 1860s significantly increased both the population and wealth of the colony, and by the Federation of Australia in 1901, Melbourne had become the largest city and leading financial centre in Australasia. Melbourne also served as capital of Australia until the construction of Canberra in 1927, with the Federal Parliament meeting in Melbourne's Parliament House and all principal offices of the federal government being based in Melbourne. Before the St. Elizabeth's flood (1421), the Meuse flowed just south of today's line Merwede-Oude Maas to the North Sea and formed an archipelago-like estuary with Waal and Lek. This system of numerous bays, estuary-like extended rivers, many islands and constant changes of the coastline, is hard to imagine today. From 1421 to 1904, the Meuse and Waal merged further upstream at Gorinchem to form Merwede. For flood protection reasons, the Meuse was separated from the Waal through a lock and diverted into a new outlet called "Bergse Maas", then Amer and then flows into the former bay Hollands Diep. What river originally bounded the Duchy?
<i>Model response</i>	Alright, I'm trying to figure out which river originally bounded the Duchy. I remember that the Duchy of Normandy was established in the 10th century, and it was located in what is now northern France. I think the river in question is the Seine because I recall that Normandy extended down to the Seine River. But I'm not entirely sure, maybe it's another river? Let me think. I know that the Duchy of Normandy was founded by Rollo, a Viking leader, and it was granted to him by the French king in the Treaty of Saint-Clair-sur-Epte in 911. The treaty specified that Rollo and his men could settle in the area between the river Epte and the Atlantic coast. So, the Epte River might have been a boundary, but I don't think that's the main river that bounded the Duchy... Therefore, the river that originally bounded the Duchy of Normandy was the boxed{Epte River}

Table 8: Error bars for the main results. To assess the statistical significance of our results in Table 2, we conduct multiple runs of the model using top-p decoding 16 times with a temperature of 0.6 and top-p=0.95 using our 32B model.

Dataset	SFT		URT	
	Mean	Std Dev	Mean	Std Dev
AIME	45.6	4.85	57.1	4.93
GPQA	55.6	1.72	62.0	1.76
MATH500	87.4	1.04	89.8	0.98