Focus and Higher–Order Unification

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Abstract

Pulman has shown that Higher–Order Unification (HOU) can be used to model the interpretation of focus. In this paper, we extend the unification–based approach to cases which are often seen as a test–bed for focus theory: utterances with multiple focus operators and second occurrence expressions. We then show that the resulting analysis favourably compares with two prominent theories of focus (namely, Rooth's Alternative Semantics and Krifka's Structured Meanings theory) in that it correctly generates interpretations which these alternative theories cannot yield. Finally, we discuss the formal properties of the approach and argue that even though HOU need not terminate, for the class of unification–problems dealt with in this paper, HOU avoids this shortcoming and is in fact computationally tractable.

1 Introduction

In this paper, we argue that Higher–Order Unification (HOU) provides a linguistically adequate tool for modeling the semantics of focus. Building up on (Pulman, 1995), we develop a unification–based analysis of focus which we show favourably compares with two prominent theories of focus, Rooth's Alternative Semantics and Krifka's Structured Meanings theory. For data which is generally viewed as a test–bed for focus theory (utterances with multiple focus operators and second occurrence expressions), we show that contrary to Rooth's and Krifka's theories, the HOU treatment yields a transparent analysis while avoiding under– and over–generation.

2 Focus theory

Focus is a much debated notion. In this paper, we assume a simplified version of Jackendoff's definition: a **focus** is the semantic value of a prosodically prominent element. We take the identification of prosodically prominent elements as given.

To set the stage for this paper, we will briefly review the folklore, i.e. the main issues of focus theory. It is commonly agreed that focus triggers the formation of an additional semantic value which we will call the **Focus Semantic Value** (FSV). The name and definition of the FSV varies from author to author: Jackendoff (Jackendoff, 1972) calls it the *presuppositional set*, Rooth (Rooth, 1992) the *Alternative Set* and Krifka (Krifka, 1992) the *Ground*. In this paper, we assume a definition of the FSV which is in essence Rooth's Alternative set, that is, the set of semantic objects obtained by making an appropriate substitution in the focus position. For instance, the FSV of (1a) is defined as (1b), the set of properties of the form *like–ing y* where y is an individual (in what follows, focus is indicated using upper–case; we also follow Montague's convention that for any type τ, D_τ is the set of objects of type τ and wff_τ is the set of wffs of type τ).

(1) a. *Jon only likes MARY*
      b. $\{\lambda x.l(x,y) \mid y \in D_e\}$

It is also usually agreed that certain linguistic elements **associate with focus** in that the meaning of the utterance containing these elements varies depending on the choice of focus. For instance in (2a–b), the focus operator *only* associates with focus so that the difference in focus between (2a) and (2b) induces a difference in meaning between the two utterances: in a world where Jon introduced Paul to Mary and Sarah, and no other introduction takes place, (2a) is necessarily

false whilst (2b) is true.

(2) a. *Jon only introduced Paul to MARY*
     b. *Jon only introduced PAUL to Mary*

To model this "association–with–focus" phenomenon, the semantics of associating–elements (e.g. focus operators, quantificational adverbs) is made contingent on the FSV which itself, varies with the choice of focus. The following example illustrates this. Suppose that the meaning of *only* is determined by the following rule:

[NP only VP]
$$\hookrightarrow \forall P[P \in FSV \wedge P(NP') \rightarrow P = VP']$$

where NP', VP' represent the meaning of NP and VP respectively, and FSV stands for the focus semantic value of the VP. As we have seen above, the FSV of (1a) is (1b), hence by the above semantic for *only*, the semantics of (1a) is:

$$\forall P[P \in \{\lambda x.l(x,y) \mid y \in D_e\} \wedge P(j)$$
$$\rightarrow P = \lambda x.l(x,m)]$$

Intuitively, the only property of the form *like-ing y* that holds of *Jon* is the property of *like-ing Mary*.

3 The basic analysis

For computing the Focus Semantic Value, we propose to use Higher–Order Unification. More specifically, given (part of) an utterance U with semantic representation Sem and foci $F^1 \ldots F^n$, we require that the following equation, the **ground equation**, be solved:

$$Sem = Gd(F^1) \ldots (F^n)$$

Assuming the typed λ-calculus as our semantic representation language, this equation can be solved by Huet's algorithm (cf. (Huet, 1975)), thus assigning a value to Gd. On the basis of this value, we can then define the FSV, written $\overline{Gd}$, as follows:

Definition 3.1 *(Focus Semantic Value)*
Let Gd be of type $\alpha = \vec{\beta_k} \rightarrow t$ and n be the number of foci $(n \leq k)$, then the Focus Semantic Value derivable from Gd, written $\overline{Gd}$, is $\{Gd(t^1 \ldots t^n) \mid t^i \in wff_{\beta_i}\}$.

As mentioned before, this yields a focus semantic value which is in essence Rooth's Alternative Set[1].

Finally, we assume as in (Pulman, 1995), that foci are stored and discharged non-deterministically as the need arises, thus contributing to the definition of the ground equation. Furthermore, equations are set up at the level at which there are needed e.g. at the VP level in the case of a pre–verbal focus operator.

To illustrate the workings of our approach, we now run through a simple example. Consider (1a). To determine the meaning of *only likes MARY*, the FSV of the VP must be known. Hence the following equation must be solved:

$$\lambda x.l(x,m) = Gd(m)$$

By HOU, the value of Gd is then[2]:

$$Gd = \lambda y \lambda x.l(x,y)$$

And by definition (3.1), the FSV is:

$$\overline{Gd} = \{\lambda x.l(x,y) \mid y \in wff_e\}$$

Assuming the semantic of *only* given above, the semantic representation of (1a) is then:

$$\forall P[P \in \{\lambda x.l(x,y) \mid y \in wff_e\} \wedge P(j)$$
$$\rightarrow P = \lambda x.l(x,m)]$$

In short, we obtain a reading similar to that of Rooth, the difference being in the way the FSV is determined: by HOU in our approach, by means of a semantic definition in Rooth's.

4 Linguistic applications

In this section, we show that the HOU approach favourably compares with Rooth's and Krifka's analysis in that it correctly generates interpretations which these two theories fail to yield. As we shall see, the main reason for this is that the HOU approach makes minimal assumptions about the role syntax plays in determining the FSV. In particular, it relies neither on the use of Quantifier Raising, nor on the assumption of a rule–to–rule definition of the FSV. In this way, it avoids some of the pitfalls these theories encounter.

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[1]Though in fact, our definition is more syntactic than Rooth. In Rooth's approach, the FSV definition is purely semantic whereas in our approach the FSV is indirectly defined by solving equations and the value thus obtained (i.e. the value of Gd) is a *term*, that is, a syntactic object. Hence, our FSV can be more accurately compared to Kratzer's *presupposition skeleton*,. This means that our approach inherits the advantages of Kratzer's approach (cf. (Kratzer, 1991)). In par-

ticular, it adequately captures the interaction of focus with VP ellipsis as illustrated by Kratzer's notorious example: *I only went to TANGLEWOOD because you did.*

[2] Unification yields another possible value of Gd, namely $\lambda y \lambda x.l(x,m)$. In what follows, we assume a restriction similar to the DSP's **Primary Occurrence Restriction** (Dalrymple et al., 1991)'s: the occurrence directly associated with the focus is a primary occurrence and any solution containing a primary occurrence is discarded as linguistically invalid. For instance, m is a primary occurrence in the equation $\lambda x.l(x,m) = Gd(m)$ so that the solution $Gd = \lambda y \lambda x.l(x,m)$ is invalid. For a formal treatment of DSP's Primary Occurrence Restriction and a discussion of how it can be extended to focus, see (Gardent and Kohlhase, 1996).

We begin by a brief summary of Rooth's and Krifka's theories and stress the properties relevant for the present discussion. We then confront the three theories with the data.

4.1 Two alternative theories of focus

Rooth's Alternative Semantics

In Rooth's approach, the FSV is defined by recursion on the truth–conditional structure which is itself derived from LF (i.e. Logical Form, the Government and Binding level of semantic representation). Focus is then seen as introducing a free variable whose value is determined by the current context and is furthermore constrained to be an element or a subset of the FSV. For our purpose, the following characteristics are particularly important:

- Given Rooth's definition of the Alternative Set, a focus operator associates with any focus occurring in its scope.

- Any NP may be subject to Quantifier Raising. Importantly, this includes focused NPs.

- Quantifier Raising may not apply to quantifiers occurring in a scope–island.

Note that Rooth's approach critically relies on *quantifier raising* as a means of moving a focused NP out of the scope of a focus operator. However this only applies if the focus NP is not embedded in a *scope island*.

Krifka's Structured Meanings

Krifka's approach defines a rule–to–rule semantics which assigns to any syntactic constituent, a meaning which can be either a λ–term or a structured meaning, i.e. a tuple of the form $\langle Gd, F \rangle$ where Gd is Krifka's Focus Semantic Value and F is a (possibly complex) focus.

For our purpose, an important characteristic of Krifka's approach is the tight syntax/semantic interaction it presupposes. In particular, the theory requires that a focus operator combines with a syntactic constituent C whose structured semantics $C' = \langle Gd, F \rangle$ provides the focus (F) this operator associates with. In other words, the right–adjacent sibling of a focus operator must contain all and only the foci this operator associates with. As we shall later see, some of the data does not seem to square with this assumption.

4.2 Multiple Focus Operators

Utterances with multiple focus operators[3] are known pathological cases of focus theory:

(3)   a.   *(Jon only$_1$ read the letters that Sarah sent to PAUL$_1$)*

       b.   *Jon also$_2$ only$_1$ read the letters that SUE$_2$ sent to PAUL$_1$.*

In the given context, the preferred reading of (3b) can be glossed as follows: *it is also the case for SUE$_2$, that Jon only$_1$ read the letters she sent to PAUL$_1$ — i.e. Jon didn't read the letters she$_2$ sent to e.g. Peter.* In other words, the preferred reading is that *also$_2$* associates with *SUE$_2$* and *only$_1$* with *PAUL$_1$*.

The HOU analysis

Under the HOU approach, (3b) is analysed as follows. First, the meaning of *only$_1$ read the letters that SUE$_2$ sent to PAUL$_1$* is derived. To determine the FSV of the VP, the ground equation (4b) must be solved for which (4c) is a solution. Applying the semantics of *only* given in section 2, the semantics of (4a) is then as given in (4d)[4].

(4)   a.   *only$_1$ read the letters that SUE$_2$ sent to PAUL$_1$*

       b. $G^1(p) = \lambda x.read(x, l(s, p))$

       c. $G^1 = \lambda y.\lambda x.read(x, l(s, y))$

       d. $\lambda z.\forall P[P \in \overline{\lambda y \lambda x.read(x, l(s, y))} \wedge P(z)$
           $\rightarrow P = \lambda x.read(x, l(s, p))]$

Analysis then proceeds further and the ground equation

$$G^2(s) = \lambda z.\forall P[\ P \in \overline{\lambda y \lambda x.read(x, l(s, y))}$$
$$\wedge P(z) \rightarrow P = \lambda x.read(x, l(s, p))]$$

must be solved to determine the meaning of *also$_2$ only$_1$ read the letters that SUE$_2$ sent to PAUL$_1$*. A possible solution for G^2 is

$$\lambda u.\lambda x.\lambda z.\forall P[\ P \in \overline{\lambda y \lambda x.read(x, l(u, y))}$$
$$\wedge P(z) \rightarrow P = \lambda x.read(x, l(u, p))]$$

Assuming the following semantics for NP *also* VP

$$\exists P[P \in FSV \wedge P(NP') \wedge P \neq VP']$$

we obtain the desired reading

$$\exists P[\ P \in \lambda u \lambda x.\ \overline{only_1\ read\ the\ letters\ that}$$
$$\overline{u\ sent\ to\ Paul_1}$$
$$\wedge P(j) \wedge P \neq \lambda z.z\ \ only_1\ read\ the\ letters$$
$$that\ Sue_2\ sent\ to\ Paul_1]$$

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[3] The subscripts indicates which operators associate with which focus. There are there for clarity only, and have no theoretical import.

[4] For clarity, we have simplified the semantic representation of (3b); nothing hinges on this.

Comparison with Rooth and Krifka

As mentioned in section 4.1, under the Alternative Semantics approach, a focus operator necessarily associates with any focus occurring in its scope. Furthermore in (3b), the scope of $only_1$ is the whole VP *read the letters that SUE_2 sent to $PAUL_1$*. Hence, if no quantifier raising occurs, $only_1$ associates with both SUE_2 and $PAUL_1$. Thus in order to generate the desired reading, SUE_2 must be moved out of the scope of $only_1$. However, since the NP *the letters that SUE_2 sent to $PAUL_1$* is a scope island, quantifier raising is impossible. Hence, the desired reading cannot be generated[5].

Recall that in the Structured Meanings approach, the right-sibling of a focus operator must contain all and only the focus this operator associates with (cf. section 4.1). Hence, to generate the desired reading in (3b), there must exist a syntactic constituent which is right-adjacent to $only_1$ and which contains $PAUL_1$ but not SUE_2[6]; similarly, there must exist a syntactic constituent which is right-adjacent to *also* and which contains SUE_2 but not $PAUL_1$. Given standard assumptions about syntax, such constituents do not exist so that the desired interpretation cannot be generated.

4.3 Second Occurrence Expressions

We call second occurrence expressions (SOE) utterances which partially or completely repeat a previous utterance. Typical cases of SOEs are: corrections (5a), echo-sentences (5b) and variants (5c).

(5) a. A: *Jon only likes MARY.*
         B: *No, PETER only likes Mary.*
     b. A: *Jon only likes MARY.*
         B: *Huhu, Peter only likes Mary.*
     c. A: *Jon only likes MARY.*
         B: *So what? Even PETER only likes Mary.*

An important property of SOEs is that the repeated material is **deaccented**, that is, it is characterised by an important reduction in pitch, amplitude and duration (cf. (Bartels, 1995)). On the other hand, all three theories of focus considered here are based on the assumption that focus is prosodically marked and thus, identifiable. Hence,

the question arises of whether these theories can account for SOEs.

The HOU analysis

Our proposal is to analyse SOEs as involving a deaccented anaphor which consists of the repeated material, and is subject to the condition that its semantic representation must unify with the semantic representation of its antecedent.

This is modeled as follows. Let $SSem$ and $TSem$ be the semantic representation of the source (i.e. antecedent) and target (i.e. anaphoric) clause respectively, and $TP^1 \ldots TP^n$, $SP^1 \ldots SP^n$ be the target and source parallel elements[7], then the interpretation of an SOE must respect the following equations:

$$An(SP^1, \ldots, SP^n) = SSem$$
$$An(TP^1, \ldots, TP^n) = TSem$$

Intuitively, these two equations require that target and source clause share a common semantics An, the semantics of the deaccented anaphor.

Given this proposal, the analysis of (5a) involves three equations:

$$An(j) = \forall P[P \in \overline{\lambda y \lambda x.l(x,y)}$$
$$\wedge P(j) \to P = \lambda x.l(x,m)]$$
$$An(p) = \forall P[P \in \overline{Gd} \wedge P(p) \to P = \lambda x.l(x,m)]$$
$$Gd(F) = \lambda x.l(x,m)$$

Since neither Gd nor $Focus$ are initially given, the third equation above is untyped and cannot be solved by Huet's algorithm[8]. In that situation, we can either assume some delaying mechanism or some extension of Huet's algorithm that can cope with type variables (cf. (Dougherty, 1993; Hustadt, 1991)). Resolution of the first equation yields the following solution:

$$An = \lambda y \forall P[P \in \{\lambda x.l(x,y) \mid y \in wff_e\}$$
$$\wedge P(z) \to P = \lambda x.l(x,m)]$$

By applying An to p, the left-hand side of the second equation is then determined so that the second equation becomes

$$\forall P[P \in \overline{\lambda y \lambda x.l(x,y)} \wedge P(p) \to P = \lambda x.l(x,m)]$$
$$= \forall P[P \in \overline{Gd} \wedge P(p) \to P = \lambda x.like(x,m)]$$

and the value of Gd is identified as being

$$Gd = \lambda y \lambda x.l(x,y)$$

(Note further, that the third equation can now be solved thus yielding the value m for the focus F.) That is, the HOU approach to SOEs allows us to correctly capture that fact that an SOE can

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[5]This point is independently noted in (Rooth, 1992).

[6]This is a simplification: the constituent may in fact contain SUE_2 but this focused NP should already have been bound by some focus operator so that the focus of the whole constituent only includes $PAUL_1$. Since no focus operators occur in this constituent, it follows that such constituent does not exist.

[7]As in (Dalrymple et al., 1991), we take the identification of parallel elements as given — for the moment.

[8]Even though this is not explicitly stated, Pulman's analysis (Pulman, 1995, page 6) faces a similar problem.

inherit its FSV from its source clause (by unification). In (Gardent et al., 1996), we show in more detail how the analysis accounts for the interaction of focus with anaphora and definiteness in the case of a particular instantiation of SOEs, namely corrections.

Comparison with Rooth and Krifka

Under the Alternative Semantics approach, SOEs are captured as follows. It is assumed that the quantification domain of focus operators is a variable whose value is contextually determined. In the standard case (i.e. the case where the focus is prosodically marked), this quantification domain of focus operators is usually identified with the FSV of the VP. However, in the SOE cases, the assumption is that the quantification domain of focus operators is identified with the FSV of the source clause. Thus in (5a), the quantification of *only* in the second clause is identified with the FSV of the preceding utterance i.e. the set of properties of the form *like-ing somebody*.

But now, consider the following example:

(6) a. *Jon only likes MARY.*
    b. * *No, PETER only likes Sarah.*

Clearly, this dialog is ill-formed in that (6b) is no appropriate correction for (6a). However, under the Alternative Semantics approach, it will not be ruled out since the FSV of (6a) provides an appropriate quantification domain for the focus operator in (6b): as required by the semantic of pre-verbal *only*, it is a set of properties whose elements can be identified with the VP semantic value $\lambda x.l(x, m)$. Hence although Rooth's approach captures some cases of SOEs, it does not seem to provide an adequate characterisation of the phenomena at hand.

The Structured Meanings proposal distinguishes between *proper-* and *quasi-*SOEs. Proper-SOEs involve an exact repetition of some previous linguistic material, and are analysed as involving an anaphor which is constrained by the restriction that it be a segmental copy of its antecedent. For instance, the semantics of *only likes Mary* in (5b) is not determined by the semantics of its parts but is instead identified with the semantic value of its antecedent *only likes MARY* in (5a). In contrast, quasi-SOEs only involve *semantic* equivalence between repeating and repeated material (for instance, in a quasi-SOE a repeated element may be pronominalised). Krifka claims that quasi-SOEs have prosodically marked foci and thus do not raise any specific difficulty.

However this theory faces a number of methodological and empirical difficulties. First, it is non-

compositional because the meaning of the deaccented material in proper-SOEs is solely defined by the meaning of its antecedent (rather than the meaning of its parts). Second, the prosodic data is rather unclear: the assumption that quasi-SOE contains a prosodically marked focus is a moot point (cf. (Bartels, 1995)) and if it proves to be false, the analysis fails to account for quasi-SOEs. Third, it is counterintuitive in that it handles separately two classes of data (i.e. quasi- and proper-SOEs) which naturally belong together. Indeed, the HOU approach can be shown to provide a uniform treatment of quasi- and proper-SOEs (cf. (Gardent et al., 1996)).

5 Formal properties of the HOU approach

The unification problem can be stated as follows: Given two terms of a logic $\mathbf{M}$ and $\mathbf{N}$, is there a substitution, σ, of terms for variables that will make the two terms identical (i.e. $\sigma(\mathbf{M}) = \sigma(\mathbf{N})$)?

It is well-known that for Higher-Order Logic (e.g. the typed λ-calculus) the space of solutions can be infinite and furthermore, the HOU problem is only semi-decidable so that the unification algorithm need not terminate for unsolvable problems.

Fortunately, in our case we are not interested in general unification, but we can use the fact that our formulae belong to very restricted syntactic subclasses, for which much better results are known. In particular, the fact that free variables only occur on the left hand side of our equations reduces the problem of finding solutions to higher-order matching, of which decidability has been proven for the subclass of third-order formulae (Dowek, 1992) and is conjectured for the general case. This class, (intuitively allowing only nesting functions as arguments up to depth two) covers all of our examples in this paper. For a discussion of other subclasses of formulae, where higher-order unification is computationally feasible see (Prehofer, 1994).

6 Conclusion

In this paper, we have argued that Higher-Order Unification provides an adequate tool for computing Focus Semantic Values. To this end, we have considered data which is viewed as a testbed for focus theory and shown that, whilst existing theories either under-generate, over-generate or are methodologically unsatisfactory, the HOU approach yields a simple and transparent analysis. There appear to be two main reasons for this.

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First, the HOU analysis makes minimal assumptions about the role syntax is called to play in determining the FSV. It is defined on a purely semantic level in the sense that unification operates on semantic representations, and relies neither on quantifier raising, nor on a rule-to-rule definition of the FSV. As we have seen, this type of approach is a plausible way to avoid under-generation.

Second, the HOU approach permits an equational analysis which can naturally be further constrained by additional equations. The interest of such an approach was illustrated in our treatment of SOEs which we characterise as involving two phenomena: the computation of an FSV, and the resolution of a deaccented anaphor. Not only did we show that this analysis is methodologically and empirically sound, we also showed that it finds a natural realisation in the equational framework of HOU: each linguistic phenomena is characterised by some equation(s) and the equations may mutually constrain each other. For instance, in the case of SOEs, we saw that the equations characterising the deaccented anaphor help determine the unidentified FSV of the utterance containing the unmarked focus.

Clearly, our approach extends to cases of adverbial quantification. For lack of space we could not develop the theory here; let us just point out that von Fintel's criticism (von Fintel, 1995) of semantic approaches to focus, also applies to Krifka's Structured Meanings analysis, but not to the HOU approach presented here. Von Fintel points out that in certain cases of adverbial quantification, a focus operator associates with an *unmarked* focus and does *not* associate with a marked focus occurring in its scope — as should be clear from this article, this is unproblematic for our analysis.

Of course, there are still many open issues. First, how does the proposed analysis interact with quantification? Second, how does it extend to a dynamic semantics (e.g. Discourse Representation Theory)?

7   Acknowledgments

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