A FORMAL REPRESENTATION OF THE THEMATIC-RHEMATIC STRUCTURE OF SENTENCES BASED ON A TYPED λ-CALCULUS

Yoichi UETAKE

Tokyo University of Mercantile Marine *

ABSTRACT: In this paper, we give a formal representation of the thematic-rhematic (T-R) structure of a natural language discourse, based on a typed λ-calculus.

I. INTRODUCTION

In this paper, we give a formal representation of the thematic-rhematic (T-R) structure of a natural language discourse. Some pairs, triples, or in general n-tuples of sentences in a discourse may differ in the place of their information focus. The distribution of this information focus is called the thematic-rhematic (T-R) structure or dichotomy. In English, the use of particles *the* and *a* (*an*) is deeply related to the T-R structure. In general, a noun with the particle *a* constitutes a rheme part of the sentence that appears at the begining of the discourse or text, while that noun with the particle *the* appears in the second, third, *etc.* sentences as themes. In Japanese, the T-R dichotomy is well represented by postpositions *wa* and *ga*. The Korean language has a similar system. Meanwhile, in Slavic languages as Polish, Czech, and Russian, the word order is free and this degree of freedom is used for the representation of the T-R dichotomy. In Chinese, the word order is also used for the T-R dichotomy. Besides *theme* and *rheme*, similar terms as *old-information* and *new-information*, *topic* and *comment*, *topic* and *focus etc.* are used in the literature concerning functional linguistics (see, e.g., Vallduvi). In our analysis, since we do not define these terms explicily, it is not essential which terms are used. We give implicit definition of these concepts *axiomatically*. We consider the problem mainly for Japanese. We propose to use typed λ-calculus to analyse the problem. A logical notation is seen as a typed λ-term. Basic types are T and R. Roughly speaking, T and R stand for a theme part and a rheme part of a sentence, respectively. The difference of T-R dichotomy is given by different types. Thus the same sentence may have different types depending on the situation. For utterances, type inference will be performed. The correctness of a given discourse can be proved by checking the correctness of the types of each utterance. In this paper, we elaborate on this idea.

II. REPRESENTATION BASED ON A TYPED λ-CALCULUS

The purpose of this paper is to propose a formal model for utterance interpretation of the thematic-rhematic structure of a Japanese sentence using a typed λ-calculus. In our analysis, a logical notation is seen as a typed λ-term. Basic types are T and R. Roughly speaking, T and R stand for a theme part and a rheme part of a sentence, respectively. Although we analyse mainly Japanese sentences, the results can be applied to other languages. The T-R dichotomy of a Japanese sentence is represented by the postpositions *wa* and *ga*. For example, the following two sentences are different in T-R dichotomy, and used in diffrent situations: (*a*) *Taroo wa Gakusei desu.* (*Speaking of Taroo, he is a student.*) (*b*) *Taroo ga Gakusei desu.* ((*Of all the people we are talking about*) *Taroo* (*and only Taroo*) *is a student.*) The meaning of both (a) and (b) is *Taroo is a student*, and thus may be writen as *student*(*Taroo*). However this representation is obviously not sufficient for an account of the utterance interpretation of (a) and (b). The NP (noun phrase) of (a) marked with *wa* functions as a theme, i.e., it should have already appeared in the preceding discourse and thus can be considered as an old information. Therefore, in the discourse, sentence (a) should be preceded by a sentence that contains *Taroo* as a rheme (new information). For example, *Taroo* in the following sentence can be considered as a new information: (c) *Taroo ga imasu.* (*Here is Taroo.*) The pair (c), (a) in this order is a correct discourse utterance. On the other hand, the pair (c), (b) cannot be considered correct since *student* functions as a theme in (b) while it has not appeared in the preceding context. As is seen

*Etchujima Koto-ku Tokyo Japan

from (b) and (c), an NP marked with postposition *ga* functions as a rheme (i.e., information focus). To explain the difference between (a) and (b) in the utterance level, we annotate $\lambda x.student(x)$ of (a) and (b) by different typed λ-terms. Roughly speaking we assign $T \rightarrow R$ and R to each $\lambda x.student(x)$ of (a) and (b), respectively. Based on this, if we can show $student(Taroo) : R$ then we say sentence (a) (or (b)) of the discourse is correct. For example, if *Taroo* of (a) has a type T then by the β-reduction of typed λ-calculus, we have $student(Taroo) : R$. For *Taroo* to have a type T, we impose a constraint that *Taroo* must have appeared in a preceeding sentence. Other cases can be treated similarly. See the following descriptions for details. Thus the correctness of the discourse can be proved by checking the correctness of the types of each formula. In general, given a discourse $s_0, s_1, \cdots, s_n$ in logical forms, what we have to show is that $(\vdash s_0 : R)$, $(s_0 : R \vdash s_1 : R)$, $\cdots$, $(s_0 : R, \cdots, s_{n-1} : R \vdash s_n : R)$, succesively.

First consider the following discourse consisting of a single sentence.

$$Taroo\ ga\ imasu.\ (Here\ is\ Taroo.) \tag{1}$$

The meaning of this sentence is:

$$s_0 = here\_is(Taroo) \tag{2}$$

We define this discourse to be *correct* if $s_0 : R$. This is done in the following way: Translate *Taroo ga* into $\lambda f.f(Taroo)$. We let this formula have either type of $T \rightarrow R$ or $R \rightarrow R$ when the proper noun *Taroo* is marked with the postposition *ga*. Thus we have the following translation rules:

$$Taroo\ ga \Longrightarrow \lambda f.f(Taroo) \sqsubseteq s_0 : T \rightarrow R \tag{3.1}$$

$$Taroo\ ga \Longrightarrow \lambda f.f(Taroo) \sqsubseteq s_0 : R \rightarrow R \tag{3.2}$$

This can be writen for short as

$$Taroo\ ga \Longrightarrow \lambda f.f(Taroo) \sqsubseteq s_0 : (T\ or\ R) \rightarrow R \tag{4}$$

In the above, $t \sqsubseteq s_0$ means that t is a typed λ-term component of the logical formula s_0. That is

$$t \sqsubseteq s_0 \ \ iff \ \ (\exists t_1, t_2)\ t_1 t t_2 = s_0 \tag{5}$$

Here t_1 and/or t_2 may be empty. Thus $s_0 \sqsubseteq s_0$. From (3), we have

$$\vdash \lambda f.f(Taroo) \sqsubseteq s_0 : (T\ or\ R) \rightarrow R \tag{6}$$

The verb *imasu* allows a neutral description. A neutral description has the following T-R dichotomy:

$$\frac{Taroo\ ga}{Rheme} \ \frac{imasu.}{Rheme} \tag{7}$$

A sentence of neutral description in the Japanese language was first found and named by Kuroda (1965). This kind of sentence has no theme part. For this kind of verb, we assign a type R and write as follows:

$$\vdash \lambda x.here\_is(x) \sqsubseteq s_0 : R \tag{8}$$

Now by (6) and (8) we can deduce the following *judgement.*

$$\begin{aligned} &e_0 : A_0, e_1 : A_1 \vdash \\ &(\lambda f.f(Taroo))(\lambda x.here\_is(x)) \\ &= (\lambda x.here\_is(x))(Taroo) \\ &= here\_is(Taroo) = s_0 : R \end{aligned} \tag{9}$$

where $e_0 : A_0$ and $e_1 : A_1$ stand for (6) and (8), respectively. Thus $s_0 : R$ has been proved and the correctness of the discourse (1) has been established. To deduce (9), we have of course used the inference rule of the typed λ-calculus given by

$$e_0 : \alpha \rightarrow \beta, e_1 : \alpha \vdash e_0 e_1 : \beta \tag{10}$$

Note that the type used for $(\lambda f.f(Taroo))$ in deduction (9) is $R \rightarrow R$. In general, for a neutral description, β-reduction for $R \rightarrow R$ and R occur. Next we consider the discourse consisting of the following two sentences.

$$Taroo\ ga\ imasu.\ (Here\ is\ Taroo.) \tag{11.1}$$

$$Taroo\ wa\ gakusei\ desu.(Taroo\ is\ a\ student.) \tag{11.2}$$

The T-R dichotomies of the above sentences are as follows:

$$\frac{Taroo\ ga}{Rheme} \ \frac{imasu.}{Rheme} \tag{12.1}$$

$$\frac{Taroo\ wa}{Theme} \ \frac{gakusei\ desu.}{Rheme} \tag{12.2}$$

The NP (noun phrase) of (12.2) marked with *wa* functions as a theme. It should have already appeared in the preceding discourse as a rheme. The discourse (12) satisfies this constraint since *Taroo* appears as a rheme in (12.1) since it is marked with the postposition *ga*. The discourse (12) is actually correct. We now formally state the correctness of (12). The logical forms of (12.1) and (12.2) are given as

$$s_0 = here\_is(Taroo) \tag{13.1}$$

$$s_1 = student(Taroo) \tag{13.2}$$

First we must show $s_0 : R$, however we have already seen this. Thus we show $s_1 : R$. Note that $s_0 = (\lambda x.student(x))(Taroo)$. It is natural to assign $\lambda x.student(x)$ a type $T \rightarrow R$ since (12.2) contains

the postposition *wa*. This postposition is called the thematic *wa*. We write this as follows.

$$wa\ gakusei\ desu \Longrightarrow$$

$$\lambda x.student(x) \sqsubseteq s_1 : T \to R \tag{14}$$

Thus we have

$$\vdash \lambda x.student(x) \sqsubseteq s_1 : T \to R \tag{15}$$

Therefore if *Taroo* has a type T, we have $s_1 : R$ by β-reduction. The NP can be a theme if it has already appeared in the preceding discourse as a rheme. This rule can be written as follows:

$$\lambda f.f(Taroo) \sqsubseteq s_0 : (T\ or\ R) \to R \vdash Taroo \sqsubseteq s_1 : T \tag{16}$$

Now $s_1 : R$ can be shown as follows. By (6) and (16),

$$\vdash Taroo \sqsubseteq s_1 : T \tag{17}$$

Applying the β-reduction rule to (15) and (17), we have $s_1 : R$. Thus the discourse (11) is correct.

In Japanese, the following sentence at the beginning of the discourse is not natural.

Taroo wa gakusei desu.(*Taroo is a student.*) (18)

This is because *Taroo* appears as a theme but it is not preceeded by a sentence in which *Taroo* appears as a rheme. In our formal description, the incorrectness of the discourse (18) is described as a failure of type checking. We define the discourse to be incorrect if either $s_0 : R$ or $s_1 : R$ is not proved. Indeed, $s_0 : R$ where $s_0 = student(Taroo)$ is not proved since we do not have $Taroo \sqsubseteq s_0 : T$.

We now consider the following discourse consisting of two sentences.

Gakusei ga imasu. (19.1)

Taroo ga gakusei desu. (19.2)

The logical forms for (19.1) and (19.2) are given as follows.

$$s_0 = (\exists x)student(x) \wedge here\_is(x) \tag{20.1}$$

$$s_1 = student(Taroo) \tag{20.2}$$

Since *gakusei* (*student*) is marked with the postposition *ga*, and the verb *imasu* allows a neutral description, we have

$$(\exists x)student(x) \wedge here\_is(x) \sqsubseteq s_0 : R \tag{21}$$

From this we have

$$(\exists x)student(x) \sqsubseteq s_0 : R \tag{22}$$

In general we impose the following postulate.

$$A \wedge B \sqsubseteq s_i : R \vdash A \sqsubseteq s_i : R \tag{23}$$

Furthermore we add the following postulate.

$$Qxf(x) \sqsubseteq s_0 : R \vdash \lambda x.f(x) \sqsubseteq s_1 : T \tag{24}$$

where Q stands for a quantifier $\forall$ or $\exists$. This postulate means that a predicate that appeared as a rheme can be treated as a theme in the succeeding sentences. From this and (22) we can deduce

$$\lambda x.student(x) \sqsubseteq s_1 : T \tag{25}$$

We now show $s_1 : R$. First by (4) we have (6). Applying the β- reduction rule (10) to (6) and (25) we have $s_1 = student(Taroo) : R$. Therefore, the discourse (19) is correct. Note that the type used for $\lambda f.f(Taroo)$ is $T \to R$. Compare this with (9).

We now consider the following discourse consisting of a single sentence:

Taroo ga gakusei desu. (26)

In the above sentence type checking fails as follows. Since the postposition *ga* is attached to *Taroo*, we have (6). Therefore, $\lambda x.student(x) \sqsubseteq s_0$ must have a type of either T or R. However this is impossible. Since *gakusei desu* can not be used in a sentence of neutral description, $\lambda x.student(x) \sqsubseteq s_i$ never has a type R. The sentence *x ga gakusei desu* always means that *it is x who is a student* and is used only in the situation where *gakusei* is a theme. According to Kuno (1973), this use of predicate is called the exhaustive-listing. On the other hand, $\lambda x.student(x)$ can have a type T only when *student* has appeared as in (21) in the preceeding context and the postulate (24) can be used. Since (26) does not have a preceeding text, it never happens. Thus it fails to prove $s_0 : R$ and it has been established that (26) is not a correct discourse.

So far we have considered discourses consisting of two sentences. However the above method can be easily extended to a discourse that is consisting of more than three sentences. In this case, the inference rules used over several sentences are modified. For example, (16) can be modified as follows:

$$\lambda f.f(Taroo) \sqsubseteq s_i, i < j : (T\ or\ R) \to R \vdash Taroo \sqsubseteq s_j : T \tag{16'}$$

where s_i denotes the logical form corresponding to the i-th sentence of a discourse. Furthermore, *Taroo* can of course be arbitrary term, and thus we can establish the following more general rule:

$$\lambda f.f(t) \sqsubseteq s_i, i < j : (T\ or\ R) \to R \vdash t \sqsubseteq s_j : T \tag{16''}$$

III. CONCLUSIONS

In this paper, we have given a formal representation of the T-R structure of a natural language discourse. We have proposed using a notion of typed λ-calculus. A logical notation has been seen as a typed λ-term. The correctness of a given discourse can be proved by checking the correctness of the types of each utterance. Although we have analysed mainly Japanese sentences, the results can be applied to other languages by considering adaquate translation rules to encode a given sentence to formal representations.

In Uetake (1993, 1994), the author has proposed another tool for the analysis of the T-R structure. The tool used there is a logical notation called *ontological promiscuity* of Hobbs (1985), which is first-order and nonintensional. Using this description, a proof process of utterance interpretation of a discourse is obtained. It is interesting that two concepts similar to these (i.e., typed λ-calculus and ontological promiscuity) used in the analysis of the T-R structure of a discourse are used in the theory of constructive mathematics (**r**-realizability and constructive type theory). The concept of ontological promiscuity in Uetake(1993, 1994) corresponds to the **r**-realizability and the typed λ-calculus of this paper to the constructive type theory. See Uetake (1994) for more detailed discussion.

One of the reviewers noted that Barbara Partee is recently working on logically reconstructing the Prague school's notion of topic-focus articulation. The author would like to thank him/her for this information.

ACKNOWLEDGEMENTS

The author would like to thank Professor Akira Ishikawa for valuable discussions and comments.

REFERENCES

Beeson, M. J. *Foundations of Constructive Mathematics*, Springer-Verlag.

Bogusławski, A. (1977). *Problems of the Thematic-Rhematic Structure of Sentences*, Państwowe Wydawnictwo Naukowe, Warszawa.

Dirven, R. and V. Fried *Ed.* (1987). *Functionalism in Linguistics, LLSEE*, vol. 20, J. Benjamins Pub. Com.

Hindley, J. R. and J. P. Seldin (1986). *Introduction to Combinators and λ-Calculus*, Cambridge University Press.

Hobbs, J. R. (1985). Ontological Promiscuity, *Proc. of the 23rd Annual Meeting, Association for Computational Linguistics*, pp. 61-69.

Kuno, S. (1973). *The Structure of the Japanese Language*, Cambridge, Mass., MIT Press.

Kuroda, S.-Y. (1965). *Generative Grammatical Studies in the Japanese Language*, Ph.D Dissertation, MIT.

Uetake, Y. (1991-1992). Analysis of the theme and rheme structure of a Japanese sentence, *Lingua Posnaniensis*, vol. XXXIV, pp.125-134.

Uetake, Y. (1993). Two formal representations of the thematic-rhematic structure of sentences, *Proc. of Pacific and Asian Conference on formal and computational Linguistics*, pp. 256-264.

Uetake, Y. (1994). The thematic-rhematic structure of natural languages meets constructive mathematics, preprint presented at the 6th International Workshop *Open Systems and Information Dynamics*, Toruń, Poland, April 6-8.

Vallduvi, E. *Information packaging:A survey*, University of Edinburgh, Human Communication Research Center, Research Paper HCRC/RP-44.