

Making Large Language Models Better Reasoners with Orchestrated Streaming Experiences

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Abstract

Large language models (LLMs) can perform complex reasoning by generating intermediate thoughts under zero-shot or few-shot settings. However, zero-shot prompting always encounters low performance, and the superior performance of few-shot prompting hinges on the manual-crafted demonstrations. In this paper, we present **RoSE (Reasoning with Orchestrated Streaming Experiences)**, a general framework for solving reasoning tasks that can self-improve without complex external efforts. To enable RoSE, we describe an architecture that extends an LLM to store all answered questions and their thoughts in a streaming experience pool then orchestrates helpful questions from the pool to assist in answering new questions. To set up a question-aware orchestration mechanism, RoSE first calculates the similarity of each question in the pool with a new test question. Since the solution to each answered question is not always correct, RoSE will sort the questions according to their similarity with the new question, and then uniformly divide them into multiple buckets. It finally extracts one question from each bucket to make these extracted questions more diverse. To make these extracted questions help RoSE answer new questions as much as possible, we introduce two other attributes of uncertainty and complexity for each question. RoSE will preferentially select the questions with low uncertainty and high complexity from each bucket. We evaluate the versatility of RoSE in various reasoning tasks, LLMs, and CoT methods.

1 Introduction

Large language models (LLMs) (Brown et al., 2020; Thoppilan et al., 2022; Chowdhery et al., 2022; Hoffmann et al., 2022; Ouyang et al., 2022; Zeng et al., 2023; Touvron et al., 2023a; OpenAI, 2023; Sun et al., 2024) have an emerged ability on performing various complex reasoning tasks.

Recently, the chain-of-thought (CoT) prompting technique (Wei et al., 2022) was proposed to have LLMs generate intermediate reasoning paths before generating the final answers. The prompting makes LLMs think deeply before giving an answer and further enhances the reasoning power of LLMs. Besides, the zero-shot CoT prompt (Kojima et al., 2022) "Let's think step by step" also enhances the reasoning power of LLMs without any manual-crafting demonstrations. After the CoT prompting was proposed, more studies tried to manually design better prompts (Zhou et al., 2023; Wang et al., 2023a; Yao et al., 2023a) to further improve the performance of LLMs in reasoning. However, no matter how the prompts change, the goal is to have LLMs generate intermediate reasoning steps.

Recent works such as ReAct (Yao et al., 2023b), Reflexion (Shinn et al., 2023), REMEMBERER (Zhang et al., 2023a), and ExpeL (Zhao et al., 2023) were presented and have demonstrated the feasibility of autonomous agents that are built on top of an LLM core. These methods use LLMs to generate reasoning paths and "actions". These "actions" can be used in API calls and executed in an environment. Besides, some golden feedback will be presented to LLMs during the reasoning process (Shinn et al., 2023; Zhang et al., 2023a) or labeled samples are needed to collect correct or false experiences (Zhao et al., 2023). Overall, these methods still require humans to carefully design some demonstrations and need golden feedback, labeled samples, or external tools to improve the reasoning performance of LLMs.

We investigate how to improve the reasoning performance of LLMs in a more challenging streaming setting without any labeled data, pre-set unlabeled data, feedback signals, and other external help. Inspired by the observation that humans constantly do various exercises to construct a large experience pool in their minds and use the pool to help them quickly and better answer questions in ex-

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ams, we present RoSE, a general framework for solving reasoning tasks with only streaming experiences. The greatest characteristic of RoSE is that it can self-improve by constantly collecting and orchestrating streaming experiences like humans. We build an experience pool for RoSE to store the answered questions and corresponding reasoning paths. We expect these questions can assist LLMs in answering new questions, and construct a novel experience orchestration mechanism to extract helpful questions from the pool for each new reasoning question. To achieve this, we consider three attributes for each question in the pool when orchestrating. First, the solution to each question may be incorrect. If we randomly select some answered questions as demonstrations, LLMs may directly copy the incorrect labels of these questions when they are similar to the questions to be answered. This phenomenon is also known as the *copy effect* (Lyu et al., 2023; Zhang et al., 2023b). To avoid this, we introduce **diversity** so that the extracted questions are distributed from the highest to lowest similarity to the question to be answered. Second, before a question is appended to the pool, we calculate **uncertainty** for it according to the outputs of LLMs. The lower the uncertainty, the more confident RoSE is about its prediction. We first filter questions with higher uncertainty in the pool. However, since the pool is a dynamic system, we also set the dynamic uncertainty threshold to only filter the questions with relatively higher uncertainty in a pool snapshot. Third, one intuition is that the more complex the question, the more it can help RoSE learn how to answer other questions (Fu et al., 2023). Therefore, we introduce the **complexity** as the final attribute. After filtering the questions with high uncertainty, we select the most complex questions as the final demonstrations.

We evaluate the versatility of RoSE on 9 reasoning tasks, 2 LLMs, and different CoT methods. Experimental results show that RoSE significantly improves the reasoning performance of LLMs. The analysis experiments verify the importance of each experience orchestration process and the stability of RoSE across various experimental settings. We summarize our contribution as follows:

- We present RoSE, a general framework for better solving reasoning tasks. We build a novel experience orchestration mechanism by introducing diversity, uncertainty, and complexity to extract more helpful questions to as-

sist LLMs in answering new questions. RoSE can self-improve by constantly answering new questions without complex external effort.

- We verify the versatility of RoSE on 9 reasoning tasks, 2 LLMs, and different CoT methods. Experimental results show that RoSE can significantly improve the reasoning performance of LLMs.
- We conduct extensive further analyses and show that each component of RoSE contributes critically to the improvements and also verify the stability of RoSE across various experimental settings. Code is publicly available at <https://github.com/xyltt/RoSE>.

2 Related Work

2.1 Chain-of-Thought Prompting

Wei et al. (2022) formally presented the CoT prompting in large language models. This technique elicits LLMs to generate a series of intermediate reasoning steps that lead to the final answer to a question using some manual-crafting demonstrations with reasoning steps, so we name it **Few-Shot-CoT**. Kojima et al. (2022) presented that LLMs can also perform CoT reasoning when prompted by a "magic spell" of "Let's think step by step" without any other manual-crafting demonstrations, so we name it **Zero-Shot-CoT**. We categorize prompting methods as zero- and few-shot settings.

Zero-shot Setting Some studies tried to first use zero-shot CoT prompting to obtain the reasoning chain for each unlabeled question and build a retrieval mechanism to retrieve some helpful questions to construct a few-shot prompt. For example, Auto-CoT (Zhang et al., 2023b) uses the k-means clustering method to cluster all the test questions except the current question to be answered, then takes all the questions near each cluster center to construct a few-shot prompt using zero-shot CoT prompting. Plan-and-Solve prompting (Wang et al., 2023a) uses a different zero-shot CoT prompt to elicit LLMs to first decompose a question into sub-questions and then solve each sub-question.

Few-shot Setting Few-shot CoT prompting achieves better performance by eliciting the CoT reasoning ability with effective manual demonstrations. However, designing suitable prompts for all test questions is difficult. Some recent studies mainly focus on manual-crafting more well-

designed prompts instead of addressing this limitation. Zhou et al. (2023) and Khot et al. (2023) presented similar CoT prompts to first decompose a complex question into multiple sub-questions and then solve them one by one. PoT (Chen et al., 2022) uses a CoT prompt to elicit LLMs to generate text and programming language statements where the generated program can be executed by a program interpreter to get the final answer. Fu et al. (2023) presented a complexity-based few-shot CoT prompting method that uses more complex demonstrations (i.e., with more reasoning steps) to obtain better performance than a random few-shot CoT prompt. Yao et al. (2023a) presented a Tree-of-Thought (ToT) prompting method by considering multiple different reasoning paths and self-evaluating choices to decide the next course of action. MoT (Li and Qiu, 2023) obtains the reasoning paths for each unlabeled question using few-shot CoT prompting and filters the questions with low confidence. MemPrompt (Madaan et al., 2022) also uses few-shot prompting to query LLMs and gathers the interaction histories with user feedback to concatenate with the original prompt. Besides, there are many retrieval-based in-context learning methods (Luo et al., 2024) that leverage existing databases and retrieval systems. Unlike these methods, RoSE puts more emphasis on the self-improvement of LLMs without any external data or feedback.

2.2 Reasoning with Language Agents

Some studies built agents to solve reasoning and decision-making tasks. ReAct (Yao et al., 2023b) explores the use of LLMs to generate both reasoning traces and task-specific actions in an interleaved manner. Reflexion (Shinn et al., 2023) is an agent with memory and self-reflection and can be used to solve reasoning and decision-making tasks. ExpeL (Zhao et al., 2023) is an agent that can learn from experiences and insights. However, it needs labeled data to construct experiences and insights. Compared with these agents, RoSE does not require external environments or feedback.

3 Methodology

In this paper, we present RoSE, a framework for collecting and orchestrating streaming experiences to make LLMs self-improve in various reasoning tasks. Our setting is zero-shot (i.e., without any manual-crafting demonstrations) and stream-

ing (i.e., test questions arrive one by one and there are no pre-set unlabeled questions). Figure 1 shows the overview of the proposed framework. RoSE incorporates a streaming experience pool to store the answered questions and their reasoning paths. RoSE will orchestrate the experiences using multiple attributes to extract helpful questions to assist itself in better answering new questions. We construct a novel experience orchestration mechanism for RoSE that considers the diversity, uncertainty, and complexity of questions. In this section, we introduce how RoSE collects streaming experiences and how it orchestrates the collected experiences.

3.1 Streaming Experience Pool

The streaming experience pool is a dynamic system to store the answered questions and their reasoning paths. After answering a new question, RoSE will store it and its reasoning path in the streaming experience pool. Each answered question has two attached attributes of uncertainty and complexity according to the predictions of RoSE. The two attributes will be regarded as important measures to filter collected experiences.

Uncertainty The uncertainty attribute indicates how confident RoSE is in answering a question. As shown in Figure 2, the lower the uncertainty, the more confident RoSE answers the question. RoSE will filter the questions in the experience pool with higher uncertainty to guarantee the correctness of extracted questions. To calculate uncertainty, we make LLMs generate multiple reasoning paths for each question. Each reasoning path has a corresponding predicted answer. Following Li and Qiu (2023), We calculate an entropy to estimate uncertainty according to all predicted answers \mathcal{A} :

$$\mathcal{A}^* = \text{Unique}(\mathcal{A}), \quad (1)$$

$$p(a_i^*) = \sum_{j=1}^m \mathbb{I}(a_i^* = a_j) / m, \quad (2)$$

$$u_{q_t} = - \sum_{i=1}^{|\mathcal{A}^*|} p(a_i^*) \log p(a_i^*), \quad (3)$$

where m is the number of reasoning paths and $\mathcal{A} = [a_1, a_2, \dots, a_m]$ is the corresponding answers of each reasoning path for the test question q_t . $\mathcal{A}^* = [a_1^*, a_2^*, \dots]$ is the set of answers \mathcal{A} . u_{q_t} represents the uncertainty of test question q_t and the higher u_{q_t} is, the more uncertain the LLM is about the question.

Complexity An intuition is that the more complex a question, the more it includes the details

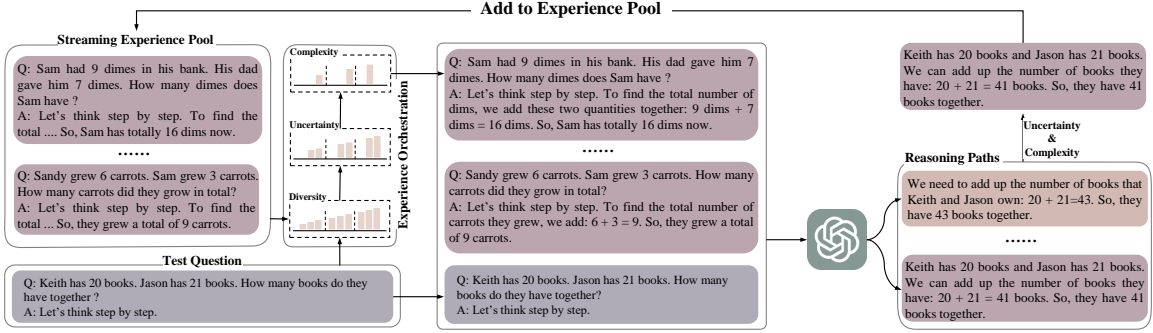


Figure 1: The overview of RoSE

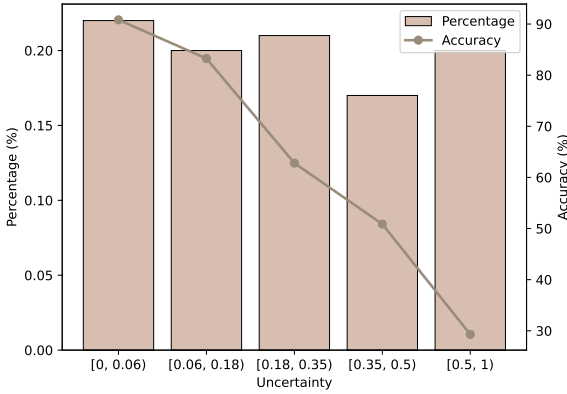


Figure 2: The relation between accuracy and the magnitude of uncertainty value on SVAMP dataset. We normalize the range of uncertainty to $[0, 1]$.

of the reasoning that can better teach LLMs how to reason. Therefore, we introduce the complexity attribute for each question as another important measure when filtering experiences. A natural idea is to use the average complexity of the reasoning paths to represent the complexity of a question. The higher the average path complexity, the more complex the question. For example, when a math word problem is more complex, it may require more columns of equations, resulting in more complex reasoning paths. Therefore, we measure the complexity of a question q as follows:

$$c_q = \sum_{i=1}^{|\mathcal{R}^*|} \text{CountSteps}(r_i) / |\mathcal{R}^*|, \quad (4)$$

where \mathcal{R}^* is the set of reasoning paths corresponding to the most frequent predicted answer and $\text{CountSteps}(\cdot)$ is a function to obtain the number of steps in a reasoning path r . Following Fu et al. (2023), we see a line as one reasoning step.

Experience Collection As just discussed, RoSE generates m reasoning paths for each test question. However, we only select one reasoning path and

add it to the streaming experience pool. To guarantee more reasoning details, we select the path with the most reasoning steps:

$$r^* = \max(\mathcal{R}^*, \text{key} = \text{CountSteps}). \quad (5)$$

Table 1 depicts a demonstration of the collected experiences. RoSE will orchestrate these experiences to better assist itself in answering new questions.

Question	Rationale	Answer	Uncertainty	Complexity
q^1	r^1	a^1	u^1	c^1
q^2	r^2	a^2	u^2	c^2
q^3	r^3	a^3	u^3	c^3
\vdots	\vdots	\vdots	\vdots	\vdots

Table 1: An example of the experiences stored in the experience pool.

3.2 Experience Orchestration

RoSE will orchestrate the collected experiences to assist itself in answering new questions. It first considers the diversity of experiences, and then filters useless questions using the attached attributes of uncertainty and complexity sequentially. Finally, it constructs a CoT prompt using the orchestrated experiences.

Diversity Recent studies found that LLMs will directly copy the wrong labels from the ICL demonstrations (Lyu et al., 2023) or be misled by the wrong predictions in demonstrations (Zhang et al., 2023b) if the demonstrations in prompts are very similar to test questions. Therefore, some recently proposed methods (Zhang et al., 2023b; Li and Qiu, 2023) consider diversity when constructing demonstrations using unlabeled questions. Different from these methods that use k-means clustering, we propose a question-aware approach to maintain

diversity. Specifically, given a test question q_t and the answered questions (q^1, q^2, \dots, q^j) in the experience pool, we first obtain their embedding representations using an off-shelf semantic embedder. Then we calculate the semantic similarity between the answered questions and the test question using their embedding representations. The answered questions are sorted from low to high semantic similarity and uniformly partitioned into k buckets at the dimension of similarity, where k is the number of demonstrations. The process of partitioning is summarized in Algorithm 1. RoSE will select one question in each bucket. This makes the selected questions distribute from low similarity to high similarity to the test question and guarantees the diversity of selected questions. We show that this can perform better than Auto-CoT which uses the k-means clustering method in the latter section.

Uncertainty-based Filtering After partitioning the answered questions into k buckets, RoSE will filter the answered questions with high uncertainty in each bucket. The streaming experience pool is a dynamic system and the uncertainty distribution among all buckets is different in different snapshots. Moreover, the uncertainty distribution is also different for different tasks. Therefore, a fixed filtering threshold does not necessarily work well for every bucket and we can not find an applicable threshold for each task. To ease the awkward situation, we propose to set a dynamic uncertainty threshold for each bucket to guarantee that RoSE only filters out the questions with relatively high uncertainty in each bucket and there are no empty buckets after filtering. Specifically, for each bucket, we adopt the λ times of minimal uncertainty value in the bucket as the threshold and filter out the questions whose uncertainty is higher than the threshold:

$$f(b_i) = \{q \in b_i \mid u_q \leq \lambda \cdot u_i^{min}\}, \quad (6)$$

$$u_i^{min} = \min\{q \in b_i \mid u_q\}, \quad (7)$$

where b_i indicates bucket i and u_i^{min} indicates the minimum uncertainty value of the bucket i .

Complexity-based Filtering The final filtering is complexity-based. As mentioned before, the more complex a question, the more it includes the details of the reasoning that can better teach LLMs how to reason. Therefore, we select the question with the highest complexity from each bucket:

$$q^i = \max(b_i, \text{key} = c_q). \quad (8)$$

Algorithm 1 Partition

Require: $q_t, Q_a = [q^1, q^2, \dots, q^j]$ and k

- 1: Calculate the similarity of each question pair $(q_t, q^1), \dots, (q_t, q^j)$
 - 2: Sort q^1, q^2, \dots, q^j through the magnitude of similarity
 - 3: Uniformly partition Q_a into k buckets at the dimension of similarity, represented by $\mathcal{B} = [b_1, b_2, \dots, b_k]$
 - 4: Remove empty buckets in \mathcal{B}
 - 5: **while** $len(\mathcal{B}) < k$ **do**
 - 6: Select the bucket with the highest number of questions and uniformly partition it into 2 buckets.
 - 7: **end while**
 - 8: **return** \mathcal{B}
-

3.3 Inference

Given a test question q_t , RoSE orchestrates the experiences to extract k experiences from the streaming experience pool and the unit of each experience is a triplet (question, rationale, answer). Finally, it answers the test question in the following manner:

$$o_t = LLM(q^1, r^1, a^1, \dots, q^k, r^k, a^k, q_t) \quad (9)$$

$$r_t, a_t = \text{ParseAnswer}(o_t) \quad (10)$$

4 Experiments

We conduct a series of experiments to compare the proposed RoSE with existing approaches on various reasoning tasks. We find that RoSE robustly improves reasoning capability in different experimental settings and each process of orchestrating experiences is important.

4.1 Experimental Settings

Models We conduct all the main experiments on two large language models including gpt-3.5-turbo-16k-0613 and LLaMA2-13B-Chat (Touvron et al., 2023b). For the semantic embedder, we use all-mpnet-base-v2 (Reimers and Gurevych, 2019). To save the cost, we conduct the most analysis experiments on LLaMA2-13B-Chat unless otherwise specified.

Tasks and Datasets We evaluate RoSE on 9 reasoning tasks. By default, we use the test split for all datasets if the labels are available for evaluation. For StrategyQA, we randomly select 800 samples

Method	Arithmetic						Common Sense			AVG
	AddSub	AQuA	GSM8K	SingleEq	SingleOp	SVAMP	CSQA	Strategy	Date	
<i>GPT-3.5-Turbo-16k-0613</i>										
Zero-Shot-CoT	83.5	55.5	75.8	90.9	90.9	77.5	67.6	65.5	67.5	75.0
Few-Shot-CoT	88.6	55.1	75.4	93.7	90.9	80.6	66.7	68.0	78.3	77.5
Auto-CoT	91.4	52.8	74.4	91.5	93.6	84.9	74.8	62.0	56.6	75.8
Zero-Shot-CoT-SC	85.1	61.8	77.6	93.3	92.5	84.3	72.1	66.3	75.1	78.7
Few-Shot-CoT-SC	89.1	58.7	82.0	94.5	94.8	86.4	68.8	69.9	79.9	80.5
Auto-CoT-SC	89.4	61.8	80.0	92.5	91.6	88.5	77.0	63.9	78.0	80.3
RoSE (Ours)	90.9	70.9	83.9	92.2	95.6	89.2	67.8	71.3	88.6	83.4
<i>LLaMA2-13B-Chat</i>										
Zero-Shot-CoT	14.7	14.2	9.0	18.5	16.2	17.3	33.1	57.4	37.7	24.2
Few-Shot-CoT	37.5	26.0	16.6	43.1	53.2	38.2	24.0	68.1	58.3	40.6
Auto-CoT	58.5	22.4	35.9	69.5	81.0	38.2	61.7	63.0	56.6	54.1
Zero-Shot-CoT-SC	52.4	19.3	31.1	58.9	45.6	50.0	39.1	63.6	36.0	44.0
Few-Shot-CoT-SC	57.5	26.8	31.4	62.6	70.5	57.7	26.1	68.0	54.2	50.5
Auto-CoT-SC	69.9	24.4	48.1	79.9	86.3	63.5	54.7	60.3	55.0	60.2
RoSE (Ours)	79.5	31.5	50.2	81.3	89.5	64.3	62.2	69.4	63.7	65.7

Table 2: Main results for RoSE. "SC" represents self-consistency (Wang et al., 2023b).

from test sets to be evaluated. The detailed statistics of each dataset can be found in Appendix A.

Method Comparison Since we mainly focus on the streaming setting without any labeled data and pre-set unlabeled data, we compare RoSE with Zero-Shot-CoT, Few-Shot-CoT, and Auto-CoT. To make a more fair comparison, we also compare the self-consistency (Wang et al., 2023b) version of these baseline methods. For Auto-CoT, we also adopt the same streaming setting as RoSE.

Implementation Settings We use the temperature $T = 1.0$ when generating diverse reasoning paths and 20 reasoning paths will be generated for each question. We adopt $\lambda = 1.2$ times of minimal uncertainty value in each bucket as the threshold unless otherwise specified. For the methods that do not need to generate multiple diverse reasoning paths, we use the temperature $T = 0$. We conducted all experiments on 8 Nvidia A100 GPUs.

4.2 Main Results

According to the comparison results in Table 2, RoSE performs better than all baselines overall. For the results on GPT-3.5-Turbo, RoSE exceeds Zero-Shot-CoT and Few-Shot-CoT by 8.4 and 5.9 points respectively and exceeds Zero-Shot-CoT-SC and Few-Shot-CoT-SC by 4.7 and 2.9 points respectively. This directly demonstrates that RoSE can self-improve by only the collected streaming experiences. While Few-Shot-CoT prompting

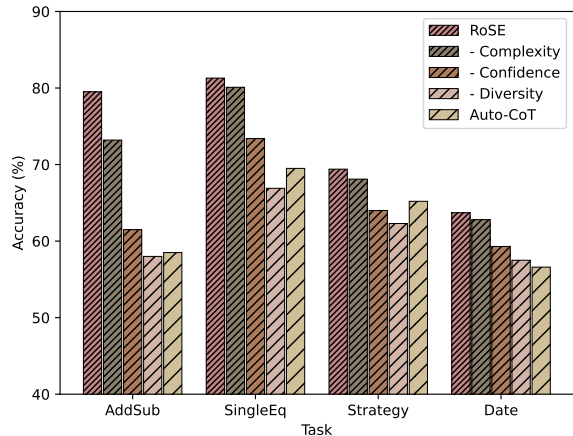


Figure 3: The impact of each orchestration process.

uses demonstrations with human annotations, these demonstrations do not necessarily work for all test questions. However, RoSE has a big advantage over Few-Shot-CoT prompting by orchestrating helpful demonstrations from the experience pool for each test question. RoSE also shows significant improvements to Auto-CoT that only considers the diversity of demonstrations, and this indicates the importance of our proposed well-designed experience orchestration mechanism.

Compared to GPT-3.5-Turbo, LLaMA2-13B-Chat has a big capacity gap on all reasoning tasks. However, RoSE also performs better than all baseline methods overall on LLaMA2-13B-Chat model and the improvement becomes larger than it on GPT-3.5-Turbo. After equipping with RoSE, the

performance of LLaMA2-13B-Chat on multiple tasks approaches GPT-3.5-Turbo, such as SingleEq and StrategyQA.

	Dynamic Threshold			Fixed Threshold		
	1.2	1.4	1.6	0.6	1.2	1.8
AddSub	79.5	78.2	77.7	69.4	73.6	73.4
SingleEq	81.3	80.9	79.7	79.9	81.1	79.8
Strategy	69.4	69.3	68.1	67.1	68.9	68.2
Date	63.7	61.5	62.1	57.7	60.9	60.1

Table 3: The impact of uncertainty threshold.

4.3 Analyses

The Effect of Each Orchestration Process To better understand the contribution of each experience orchestration process, we conduct comprehensive ablation studies on four tasks. The ablation results are shown in Figure 3. We can observe that through the gradual orchestration process from diversity to uncertainty to complexity, the overall performance of RoSE on four datasets is gradually improved. This means that each process we propose increases the helpfulness of the extracted experiences in answering new questions. RoSE that takes uncertainty into account shows a jump in performance compared to the one that does not because the former generates multiple reasoning paths for each question and makes a majority vote among all predicted answers. Besides, RoSE which only considers diversity performs better than Auto-CoT overall. This represents the proposed question-aware diversity maintaining method is superior to the methods that the k-means clustering method used by Auto-CoT.

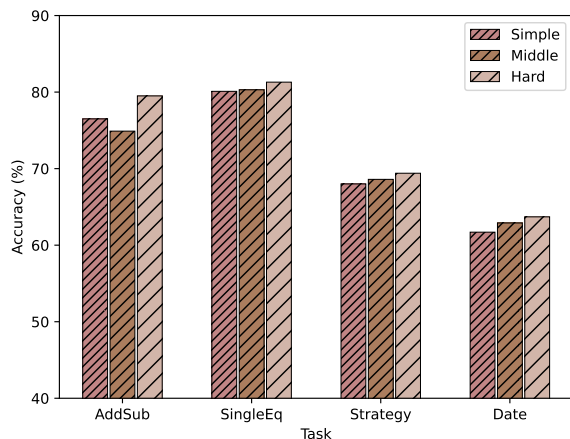


Figure 4: The impact of complexity.

Method	AddSub	SingleEq	Strategy	Date	AVG
<i>Temperature = 0.8</i>					
Zero-Shot-CoT-SC	50.1	57.9	61.6	36.0	51.4
Few-Shot-CoT-SC	54.4	59.8	67.3	53.1	58.7
Auto-CoT-SC	64.1	76.9	63.3	51.3	63.9
RoSE (Ours)	75.4	80.3	68.4	63.4	71.9
<i>Temperature = 1.2</i>					
Zero-Shot-CoT-SC	54.4	59.6	64.3	34.4	53.2
Few-Shot-CoT-SC	62.0	65.2	68.2	55.3	62.7
Auto-CoT-SC	73.1	77.2	60.9	57.8	67.3
RoSE (Ours)	80.3	81.9	69.8	65.9	74.5

Table 4: The results on different temperatures.

Method	AddSub	SingleEq	Strategy	Date	AVG
<i>Reasoning Paths = 10</i>					
Zero-Shot-CoT-SC	49.4	56.7	59.2	33.3	49.7
Few-Shot-CoT-SC	57.0	58.7	63.3	53.9	58.2
Auto-CoT-SC	69.0	74.9	57.3	51.3	63.1
RoSE (Ours)	77.2	76.6	67.8	63.7	71.3
<i>Reasoning Paths = 15</i>					
Zero-Shot-CoT-SC	51.1	57.7	61.8	35.8	51.6
Few-Shot-CoT-SC	59.5	60.0	66.2	52.6	59.6
Auto-CoT-SC	73.9	76.3	58.9	53.6	65.7
RoSE (Ours)	77.9	79.4	69.1	62.3	72.2

Table 5: The results on different numbers of reasoning paths.

The Impact of Different Uncertainty Thresholds

As shown in Table 3, we compare the performance of RoSE with different uncertainty thresholds. As introduced in the previous section, we adopt λ times the minimal value of uncertainty in a bucket as the uncertainty threshold of the bucket. We first compare the performance of RoSE when adopting different values for λ . We find that the value of lambda values should not be too large, or RoSE may retrieve ones with high uncertainty, resulting in lower performance. Moreover, we also evaluate the performance of RoSE with a fixed uncertainty threshold for each bucket. Using a fixed threshold leads to lower performance than RoSE with a dynamic uncertainty threshold. This represents selecting a suitable fixed threshold for different buckets is difficult and also proves that the adopted dynamic threshold is robust.

The Impact of Different Complexity Thresholds

As shown in Figure 4, we also compare the performance of selecting the questions with different complexity and find that the more complex the extracted questions, the more helpful they are. This is also consistent with our initial intuition mentioned in Sec 3.1, that the more complex a question, the more it includes the details of the reasoning that can better teach LLMs how to reason.

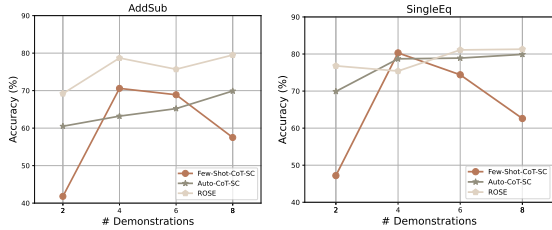


Figure 5: Results on different demonstration quantities.

Results on Different Temperature Values In this section, we evaluate RoSE under different temperature values. Table 4 shows the results. We observe that RoSE consistently outperforms baseline methods across different temperature values, which shows the stability of RoSE. Besides, RoSE performs worse when adopting a temperature of 0.8 than a temperature of 1.0 or 1.2. This is because lower temperatures result in less diversity of model-generating inference paths.

Results on Different Number of Reasoning Paths

Since RoSE needs to generate multiple reasoning paths for each question to estimate the uncertainty, we also evaluate RoSE under different numbers of reasoning paths. Table 5 shows the results and we can see that the performance of RoSE increases with the increase of the number of reasoning paths. Moreover, RoSE consistently outperforms baseline methods across different numbers of reasoning paths, which shows the stability of RoSE.

Results on Different Numbers of Demonstrations

We also evaluate RoSE under different numbers of demonstrations. According to the results in Figure 5, we see that RoSE consistently outperforms Few-Shot-CoT-SC and Auto-CoT-SC across different numbers of demonstrations, which shows the stability of RoSE. Besides, we can find that Few-Shot-CoT-SC is very unstable across different numbers of demonstrations, which also indicates that dynamically extracting demonstrations for each test question is more suitable than manual-crafting demonstrations.

Transferability on Different CoT methods

RoSE is a relatively general framework that can be adapted to many CoT prompting methods. To verify the versatility of RoSE, we evaluate the performance of RoSE on two additional advanced CoT prompting methods: Plan-and-Solve (Wang et al., 2023a) and ToT (Yao et al., 2023a). The detailed implementation settings are listed in Appendix C.

Results on four ablation datasets are shown in

Table 6. We observe that RoSE leads to consistent improvements, which shows its generality across various CoT methods. Moreover, when using the more advanced CoT methods, RoSE can get further performance improvements, which shows its potential in the future when the more powerful CoT method is proposed.

Method	AddSub	SingleEq	Strategy	Date	AVG
Zero-Shot-CoT	83.5	90.9	65.5	67.5	76.9
+ RoSE	90.9	92.2	71.3	88.6	85.8
Plan-and-Solve	85.6	91.8	65.9	68.6	78.0
+ RoSE	90.6	94.5	70.7	89.4	86.3
ToT	85.8	90.1	67.9	70.1	78.5
+ RoSE	91.5	93.9	71.7	88.9	86.5

Table 6: Comparison of various CoT methods on "gpt-3.5-turbo-16k-0613" model.

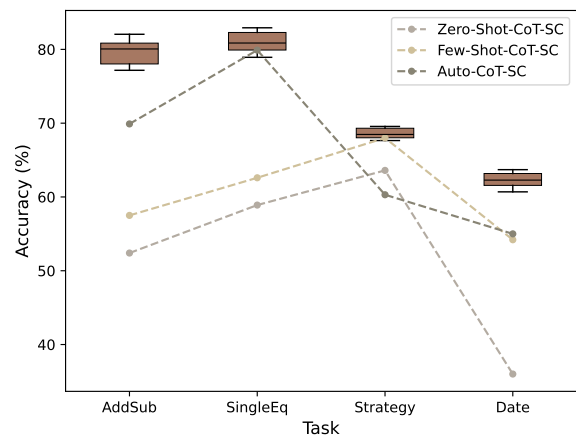


Figure 6: Results on different test orders.

Stability Analysis on Different Test Orders

The order of test questions will influence the performance because this can lead to different states of the experience pool. To verify the stability of RoSE, we conduct 10 evaluations on different test orders, and the distribution of results is shown in Figure 6. Performance fluctuates as the test order changes, but it is generally better than the baselines.

5 Conclusion

We present RoSE, a general framework for improving the performance of LLMs on reasoning tasks. RoSE can self-improve by constantly collecting questions into an experience pool and does not need other complex external help. To extract more helpful experience from the experience pool, we propose a systematic and novel experience orchestration mechanism that sequentially regards

diversity, uncertainty, and complexity of questions in the pool as important measures to filter experiences. The comprehensive experimental results on 9 reasoning tasks and 2 LLMs show that RoSE significantly improves the reasoning performance of LLMs. Moreover, we conduct extensive analysis experiments and verify the importance of each process and the stability of RoSE across various experimental settings.

Limitations

Since we estimate the complexity of a question using the number of reasoning steps and extract the most complex questions in the final filtering process, this may lead to a longer length of demonstrations and thus lead to slower efficiency.

Ethics Statement

In this paper, we let LLMs self-improve on reasoning tasks. only by the collected streaming experiences. All datasets used are reasoning type and have no unsafe samples. Moreover, the LLM cannot access the internet and control external tools. Hence we think the proposed method and all experiments are safe enough, which will not cause serious impact and unrecoverable consequences on society.

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References

Tom B. Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, Sandhini Agarwal, Ariel Herbert-Voss, Gretchen Krueger, Tom Henighan, Rewon Child, Aditya Ramesh, Daniel M. Ziegler, Jeffrey Wu, Clemens Winter, Christopher Hesse, Mark Chen, Eric Sigler, Mateusz Litwin, Scott Gray, Benjamin Chess, Jack Clark, Christopher Berner, Sam McCandlish, Alec Radford, Ilya Sutskever, and Dario Amodei. 2020. [Language models are few-shot learners](#). In *Advances in Neural Information Processing Systems 33: Annual Conference on Neural Information Processing Systems 2020, NeurIPS 2020, December 6-12, 2020, virtual*.

Wenhu Chen, Xueguang Ma, Xinyi Wang, and William W. Cohen. 2022. [Program of thoughts](#)

[prompting: Disentangling computation from reasoning for numerical reasoning tasks](#). *CoRR*, abs/2211.12588.

Aakanksha Chowdhery, Sharan Narang, Jacob Devlin, Maarten Bosma, Gaurav Mishra, Adam Roberts, Paul Barham, Hyung Won Chung, Charles Sutton, Sebastian Gehrmann, Parker Schuh, Kensen Shi, Sasha Tsvyashchenko, Joshua Maynez, Abhishek Rao, Parker Barnes, Yi Tay, Noam Shazeer, Vinodkumar Prabhakaran, Emily Reif, Nan Du, Ben Hutchinson, Reiner Pope, James Bradbury, Jacob Austin, Michael Isard, Guy Gur-Ari, Pengcheng Yin, Toju Duke, Anselm Levskaya, Sanjay Ghemawat, Sunipa Dev, Henryk Michalewski, Xavier Garcia, Vedant Misra, Kevin Robinson, Liam Fedus, Denny Zhou, Daphne Ippolito, David Luan, Hyeontaek Lim, Barret Zoph, Alexander Spiridonov, Ryan Sepassi, David Dohan, Shivani Agrawal, Mark Omernick, Andrew M. Dai, Thanumalayan Sankaranarayanan Pillai, Marie Pellat, Aitor Lewkowycz, Erica Moreira, Rewon Child, Aleksandr Polozov, Katherine Lee, Zongwei Zhou, Xuezhi Wang, Brennan Saeta, Mark Diaz, Orhan Firat, Michele Catasta, Jason Wei, Kathy Meier-Hellstern, Douglas Eck, Jeff Dean, Slav Petrov, and Noah Fiedel. 2022. [Palm: Scaling language modeling with pathways](#). *CoRR*, abs/2204.02311.

Karl Cobbe, Vineet Kosaraju, Mohammad Bavarian, Mark Chen, Heewoo Jun, Lukasz Kaiser, Matthias Plappert, Jerry Tworek, Jacob Hilton, Reiichiro Nakano, Christopher Hesse, and John Schulman. 2021. [Training verifiers to solve math word problems](#). *CoRR*, abs/2110.14168.

Yao Fu, Hao Peng, Ashish Sabharwal, Peter Clark, and Tushar Khot. 2023. [Complexity-based prompting for multi-step reasoning](#). In *The Eleventh International Conference on Learning Representations, ICLR 2023, Kigali, Rwanda, May 1-5, 2023*. OpenReview.net.

Mor Geva, Daniel Khashabi, Elad Segal, Tushar Khot, Dan Roth, and Jonathan Berant. 2021. [Did aristotle use a laptop? A question answering benchmark with implicit reasoning strategies](#). *Trans. Assoc. Comput. Linguistics*, 9:346–361.

Jordan Hoffmann, Sebastian Borgeaud, Arthur Mensch, Elena Buchatskaya, Trevor Cai, Eliza Rutherford, Diego de Las Casas, Lisa Anne Hendricks, Johannes Welbl, Aidan Clark, Tom Hennigan, Eric Noland, Katie Millican, George van den Driessche, Bogdan Damoc, Aurelia Guy, Simon Osindero, Karen Simonyan, Erich Elsen, Jack W. Rae, Oriol Vinyals, and Laurent Sifre. 2022. [Training compute-optimal large language models](#). *CoRR*, abs/2203.15556.

Mohammad Javad Hosseini, Hannaneh Hajishirzi, Oren Etzioni, and Nate Kushman. 2014. [Learning to solve arithmetic word problems with verb categorization](#). In *Proceedings of the 2014 Conference on Empirical Methods in Natural Language Processing, EMNLP 2014, October 25-29, 2014, Doha, Qatar, A meeting of SIGDAT, a Special Interest Group of the ACL*, pages 523–533. ACL.

- Tushar Khot, Harsh Trivedi, Matthew Finlayson, Yao Fu, Kyle Richardson, Peter Clark, and Ashish Sabharwal. 2023. [Decomposed prompting: A modular approach for solving complex tasks](#). In *The Eleventh International Conference on Learning Representations, ICLR 2023, Kigali, Rwanda, May 1-5, 2023*. OpenReview.net.
- Takeshi Kojima, Shixiang Shane Gu, Machel Reid, Yutaka Matsuo, and Yusuke Iwasawa. 2022. [Large language models are zero-shot reasoners](#). In *NeurIPS*.
- Rik Koncel-Kedziorski, Hannaneh Hajishirzi, Ashish Sabharwal, Oren Etzioni, and Siena Dumas Ang. 2015. [Parsing algebraic word problems into equations](#). *Trans. Assoc. Comput. Linguistics*, 3:585–597.
- Xiaonan Li and Xipeng Qiu. 2023. [Mot: Pre-thinking and recalling enable chatgpt to self-improve with memory-of-thoughts](#). *CoRR*, abs/2305.05181.
- Wang Ling, Dani Yogatama, Chris Dyer, and Phil Blunsom. 2017. [Program induction by rationale generation: Learning to solve and explain algebraic word problems](#). In *Proceedings of the 55th Annual Meeting of the Association for Computational Linguistics, ACL 2017, Vancouver, Canada, July 30 - August 4, Volume 1: Long Papers*, pages 158–167. Association for Computational Linguistics.
- Man Luo, Xin Xu, Yue Liu, Panupong Pasupat, and Mehran Kazemi. 2024. [In-context learning with retrieved demonstrations for language models: A survey](#). *CoRR*, abs/2401.11624.
- Xinxi Lyu, Sewon Min, Iz Beltagy, Luke Zettlemoyer, and Hannaneh Hajishirzi. 2023. [Z-ICL: zero-shot in-context learning with pseudo-demonstrations](#). In *Proceedings of the 61st Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers), ACL 2023, Toronto, Canada, July 9-14, 2023*, pages 2304–2317. Association for Computational Linguistics.
- Aman Madaan, Niket Tandon, Peter Clark, and Yiming Yang. 2022. [Memory-assisted prompt editing to improve GPT-3 after deployment](#). In *Proceedings of the 2022 Conference on Empirical Methods in Natural Language Processing, EMNLP 2022, Abu Dhabi, United Arab Emirates, December 7-11, 2022*, pages 2833–2861. Association for Computational Linguistics.
- OpenAI. 2023. [GPT-4 technical report](#). *CoRR*, abs/2303.08774.
- Long Ouyang, Jeffrey Wu, Xu Jiang, Diogo Almeida, Carroll L. Wainwright, Pamela Mishkin, Chong Zhang, Sandhini Agarwal, Katarina Slama, Alex Ray, John Schulman, Jacob Hilton, Fraser Kelton, Luke Miller, Maddie Simens, Amanda Askell, Peter Welinder, Paul F. Christiano, Jan Leike, and Ryan Lowe. 2022. [Training language models to follow instructions with human feedback](#). In *NeurIPS*.
- Arkil Patel, Satwik Bhattamishra, and Navin Goyal. 2021. [Are NLP models really able to solve simple math word problems?](#) In *Proceedings of the 2021 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, NAACL-HLT 2021, Online, June 6-11, 2021*, pages 2080–2094. Association for Computational Linguistics.
- Nils Reimers and Iryna Gurevych. 2019. [Sentence-bert: Sentence embeddings using siamese bert-networks](#). In *Proceedings of the 2019 Conference on Empirical Methods in Natural Language Processing and the 9th International Joint Conference on Natural Language Processing, EMNLP-IJCNLP 2019, Hong Kong, China, November 3-7, 2019*, pages 3980–3990. Association for Computational Linguistics.
- Subhro Roy, Tim Vieira, and Dan Roth. 2015. [Reasoning about quantities in natural language](#). *Trans. Assoc. Comput. Linguistics*, 3:1–13.
- Noah Shinn, Beck Labash, and Ashwin Gopinath. 2023. [Reflexion: an autonomous agent with dynamic memory and self-reflection](#). *CoRR*, abs/2303.11366.
- Aarohi Srivastava, Abhinav Rastogi, Abhishek Rao, Abu Awal Md Shoeb, Abubakar Abid, Adam Fisch, Adam R. Brown, Adam Santoro, Aditya Gupta, Adrià Garriga-Alonso, Agnieszka Kluska, Aitor Lewkowycz, Akshat Agarwal, Alethea Power, Alex Ray, Alex Warstadt, Alexander W. Kocurek, Ali Safaya, Ali Tazarv, Alice Xiang, Alicia Parrish, Allen Nie, Aman Hussain, Amanda Askell, Amanda Dsouza, Ameet Rahane, Anantharaman S. Iyer, Anders Andreassen, Andrea Santilli, Andreas Stuhlmüller, Andrew M. Dai, Andrew La, Andrew K. Lampinen, Andy Zou, Angela Jiang, Angelica Chen, Anh Vuong, Animesh Gupta, Anna Gottardi, Antonio Norelli, Anu Venkatesh, Arash Gholamidavoodi, Arfa Tabassum, Arul Menezes, Arun Kirubakaran, Asher Mullokandov, Ashish Sabharwal, Austin Herick, Avia Efrat, Aykut Erdem, Ayla Karakas, and et al. 2022. [Beyond the imitation game: Quantifying and extrapolating the capabilities of language models](#). *CoRR*, abs/2206.04615.
- Tianxiang Sun, Xiaotian Zhang, Zhengfu He, Peng Li, Qinyuan Cheng, Xiangyang Liu, Hang Yan, Yunfan Shao, Qiong Tang, Shiduo Zhang, Xingjian Zhao, Ke Chen, Yining Zheng, Zhejian Zhou, Ruixiao Li, Jun Zhan, Yunhua Zhou, Linyang Li, Xiaogui Yang, Lingling Wu, Zhangyue Yin, Xuanjing Huang, Yungang Jiang, and Xipeng Qiu. 2024. [MOSS: an open conversational large language model](#). *Mach. Intell. Res.*, 21(5):888–905.
- Alon Talmor, Jonathan Herzig, Nicholas Lourie, and Jonathan Berant. 2019. [Commonsenseqa: A question answering challenge targeting commonsense knowledge](#). In *Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, NAACL-HLT 2019, Minneapolis, MN, USA, June 2-7, 2019, Volume 1 (Long and Short Papers)*,

- pages 4149–4158. Association for Computational Linguistics.
- Romal Thoppilan, Daniel De Freitas, Jamie Hall, Noam Shazeer, Apoorv Kulshreshtha, Heng-Tze Cheng, Alicia Jin, Taylor Bos, Leslie Baker, Yu Du, YaGuang Li, Hongrae Lee, Huaixiu Steven Zheng, Amin Ghafouri, Marcelo Menegali, Yanping Huang, Maxim Krikun, Dmitry Lepikhin, James Qin, Dehao Chen, Yuanzhong Xu, Zhifeng Chen, Adam Roberts, Maarten Bosma, Yanqi Zhou, Chung-Ching Chang, Igor Krivokon, Will Rusch, Marc Pickett, Kathleen S. Meier-Hellstern, Meredith Ringel Morris, Tulsee Doshi, Renelito Delos Santos, Toju Duke, Johnny Soraker, Ben Zevenbergen, Vinodkumar Prabhakaran, Mark Diaz, Ben Hutchinson, Kristen Olson, Alejandra Molina, Erin Hoffman-John, Josh Lee, Lora Aroyo, Ravi Rajakumar, Alena Butryna, Matthew Lamm, Viktoriya Kuzmina, Joe Fenton, Aaron Cohen, Rachel Bernstein, Ray Kurzweil, Blaise Agüera y Arcas, Claire Cui, Marian Croak, Ed H. Chi, and Quoc Le. 2022. [Lamda: Language models for dialog applications](#). *CoRR*, abs/2201.08239.
- Hugo Touvron, Thibaut Lavril, Gautier Izacard, Xavier Martinet, Marie-Anne Lachaux, Timothée Lacroix, Baptiste Rozière, Naman Goyal, Eric Hambro, Faisal Azhar, Aurélien Rodriguez, Armand Joulin, Edouard Grave, and Guillaume Lample. 2023a. [Llama: Open and efficient foundation language models](#). *CoRR*, abs/2302.13971.
- Hugo Touvron, Louis Martin, Kevin Stone, Peter Albert, Amjad Almahairi, Yasmine Babaei, Nikolay Bashlykov, Soumya Batra, Prajwal Bhargava, Shrutu Bhosale, Dan Bikel, Lukas Blecher, Cristian Canton-Ferrer, Moya Chen, Guillem Cucurull, David Esiobu, Jude Fernandes, Jeremy Fu, Wenyin Fu, Brian Fuller, Cynthia Gao, Vedanuj Goswami, Naman Goyal, Anthony Hartshorn, Saghar Hosseini, Rui Hou, Hakan Inan, Marcin Kardas, Viktor Kerkez, Madian Khabsa, Isabel Kloumann, Artem Korenev, Punit Singh Koura, Marie-Anne Lachaux, Thibaut Lavril, Jenya Lee, Diana Liskovich, Yinghai Lu, Yuning Mao, Xavier Martinet, Todor Mihaylov, Pushkar Mishra, Igor Molybog, Yixin Nie, Andrew Poulton, Jeremy Reizenstein, Rashi Rungta, Kalyan Saladi, Alan Schelten, Ruan Silva, Eric Michael Smith, Ranjan Subramanian, Xiaoqing Ellen Tan, Binh Tang, Ross Taylor, Adina Williams, Jian Xiang Kuan, Puxin Xu, Zheng Yan, Iliyan Zarov, Yuchen Zhang, Angela Fan, Melanie Kambadur, Sharan Narang, Aurélien Rodriguez, Robert Stojnic, Sergey Edunov, and Thomas Scialom. 2023b. [Llama 2: Open foundation and fine-tuned chat models](#). *CoRR*, abs/2307.09288.
- Lei Wang, Wanyu Xu, Yihuai Lan, Zhiqiang Hu, Yunshi Lan, Roy Ka-Wei Lee, and Ee-Peng Lim. 2023a. [Plan-and-solve prompting: Improving zero-shot chain-of-thought reasoning by large language models](#). In *Proceedings of the 61st Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, ACL 2023, Toronto, Canada, July 9-14, 2023, pages 2609–2634. Association for Computational Linguistics.
- Xuezhi Wang, Jason Wei, Dale Schuurmans, Quoc V. Le, Ed H. Chi, Sharan Narang, Aakanksha Chowdhery, and Denny Zhou. 2023b. [Self-consistency improves chain of thought reasoning in language models](#). In *The Eleventh International Conference on Learning Representations, ICLR 2023, Kigali, Rwanda, May 1-5, 2023*. OpenReview.net.
- Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, Brian Ichter, Fei Xia, Ed H. Chi, Quoc V. Le, and Denny Zhou. 2022. [Chain-of-thought prompting elicits reasoning in large language models](#). In *NeurIPS*.
- Shunyu Yao, Dian Yu, Jeffrey Zhao, Izhak Shafran, Thomas L. Griffiths, Yuan Cao, and Karthik Narasimhan. 2023a. [Tree of thoughts: Deliberate problem solving with large language models](#). *CoRR*, abs/2305.10601.
- Shunyu Yao, Jeffrey Zhao, Dian Yu, Nan Du, Izhak Shafran, Karthik R. Narasimhan, and Yuan Cao. 2023b. [React: Synergizing reasoning and acting in language models](#). In *The Eleventh International Conference on Learning Representations, ICLR 2023, Kigali, Rwanda, May 1-5, 2023*. OpenReview.net.
- Aohan Zeng, Xiao Liu, Zhengxiao Du, Zihan Wang, Hanyu Lai, Ming Ding, Zhuoyi Yang, Yifan Xu, Wendi Zheng, Xiao Xia, Weng Lam Tam, Zixuan Ma, Yufei Xue, Jidong Zhai, Wenguang Chen, Zhiyuan Liu, Peng Zhang, Yuxiao Dong, and Jie Tang. 2023. [GLM-130B: an open bilingual pre-trained model](#). In *The Eleventh International Conference on Learning Representations, ICLR 2023, Kigali, Rwanda, May 1-5, 2023*. OpenReview.net.
- Danyang Zhang, Lu Chen, Situo Zhang, Hongshen Xu, Zihan Zhao, and Kai Yu. 2023a. [Large language model is semi-parametric reinforcement learning agent](#). *CoRR*, abs/2306.07929.
- Zhuosheng Zhang, Aston Zhang, Mu Li, and Alex Smola. 2023b. [Automatic chain of thought prompting in large language models](#). In *The Eleventh International Conference on Learning Representations, ICLR 2023, Kigali, Rwanda, May 1-5, 2023*. OpenReview.net.
- Andrew Zhao, Daniel Huang, Quentin Xu, Matthieu Lin, Yong-Jin Liu, and Gao Huang. 2023. [Expel: LLM agents are experiential learners](#). *CoRR*, abs/2308.10144.
- Denny Zhou, Nathanael Schärli, Le Hou, Jason Wei, Nathan Scales, Xuezhi Wang, Dale Schuurmans, Claire Cui, Olivier Bousquet, Quoc V. Le, and Ed H. Chi. 2023. [Least-to-most prompting enables complex reasoning in large language models](#). In *The Eleventh International Conference on Learning Representations, ICLR 2023, Kigali, Rwanda, May 1-5, 2023*. OpenReview.net.

A Dataset Details

We evaluate RoSE on the following reasoning tasks.

- **Arithmetic reasoning.** We consider 6 Math Word Problem datasets, including AddSub (Hosseini et al., 2014), AQuA (Ling et al., 2017), GSM8K (Cobbe et al., 2021), SingleEq (Koncel-Kedziorski et al., 2015), SingleOp (Roy et al., 2015), and SVAMP (Patel et al., 2021).
- **Commonsense reasoning.** We use CommonsenseQA (CSQA) (Talmor et al., 2019), StrategyQA (Strategy) (Geva et al., 2021), and one dataset from BIG-bench (Srivastava et al., 2022): Date Understanding (Date).

The detailed statistics of each task are shown in Table 7

B Examples of Few Shot Methods

For AddSub, AQuA, GSM8K, SingleEq, SVAMP, CommonsenseQA, and StrategyQA, we use the same few-shot demonstrations as Wei et al. (2022). We manual-crafted few-shot demonstrations for other datasets. We list all demonstrations of each task for Few-Shot-CoT and Few-Shot-CoT-SC methods in Table 8 - 16.

C Implementation Details of Different CoT Methods

We verify the versatility of RoSE on two other CoT prompting methods: Plan-and-Solve (Wang et al., 2023a) and ToT (Yao et al., 2023a). We also maintain a zero-shot setting for these two methods, i.e. there are no manual-crafted demonstrations. After combining the two methods with RoSE, we add each question and the corresponding thoughts into the streaming experience pool and orchestrate these collected experiences to assist in answering each new question. Although a zero-shot setting is adopted, these two methods have relatively more complex zero-shot prompts than traditional CoT methods. To take full advantage of these methods, we completed the analysis experiment on the gpt-3.5-turbo-16k-0613 model.

For the Plan-and-Solve method, we follow the prompts in the original paper and use the same uncertainty and complexity measures as the traditional CoT method.

For ToT methods, we implement a zero-shot ToT-BFS that samples multiple thoughts using a CoT prompt and makes a vote for the best one among all thoughts. We set the step limit T to 2 and generate 5 thoughts every step. To combine with our RoSE framework, we sum the percentage of the total votes for each best thought as the uncertainty measure and sum the number of steps in each best thought as the complexity measure. The prompt template for ToT is listed in Table 17

Dataset	Reasoning Type	Answer Type	# Demonstration	# Test	License
AddSub	Arithmetic	Number	8	395	Unspecified
AQuA	Arithmetic	Multi-choice	4	254	Apache-2.0
GSM8K	Arithmetic	Number	8	1319	MIT License
SingleEq	Arithmetic	Number	8	508	Unspecified
SingleOp	Arithmetic	Number	8	562	Unspecified
SVAMP	Arithmetic	Number	8	1000	MIT License
CommonsenseQA	Commonsense	Multi-choice	7	1221	Unspecified
StrategyQA	Commonsense	yes / no	6	800	MIT license
Date Understanding	Commonsense	Multi-choice	6	369	MIT license

Table 7: Detailed statistics of the datasets utilized in our experiment.

Q: There are 15 trees in the grove. Grove workers will plant trees in the grove today. After they are done, there will be 21 trees. How many trees did the grove workers plant today?
A: Let's think step by step. There are originally 3 cars. 2 more cars arrive. $3 + 2 = 5$. The answer is 5.
Q: If there are 3 cars in the parking lot and 2 more cars arrive, how many cars are in the parking lot?
A: Let's think step by step. There are 15 trees originally. Then there were 21 trees after some more were planted. So there must have been $21 - 15 = 6$. The answer is 6.
Q: Leah had 32 chocolates and her sister had 42. If they ate 35, how many pieces do they have left in total?
A: Let's think step by step. Originally, Leah had 32 chocolates. Her sister had 42. So in total, they had $32 + 42 = 74$. After eating 35, they had $74 - 35 = 39$. The answer is 39.
Q: Jason had 20 lollipops. He gave Denny some lollipops. Now Jason has 12 lollipops. How many lollipops did Jason give to Denny?
A: Let's think step by step. Jason started with 20 lollipops. Then he had 12 after giving some to Denny. So he gave Denny $20 - 12 = 8$. The answer is 8.
Q: Shawn has five toys. For Christmas, he got two toys each from his mom and dad. How many toys does he have now?
A: Let's think step by step. There are 15 trees originally. Shawn started with 5 toys. If he got 2 toys each from his mom and dad, then that is 4 more toys. $5 + 4 = 9$. The answer is 9.
Q: There were nine computers in the server room. Five more computers were installed each day, from Monday to Thursday. How many computers are now in the server room?
A: Let's think step by step. There were originally 9 computers. For each of 4 days, 5 more computers were added. So $5 * 4 = 20$ computers were added. $9 + 20$ is 29. The answer is 29.
Q: Michael had 58 golf balls. On Tuesday, he lost 23 golf balls. On Wednesday, he lost 2 more. How many golf balls did he have at the end of Wednesday?
A: Let's think step by step. There are 15 trees originally. Michael started with 58 golf balls. After losing 23 on Tuesday, he had $58 - 23 = 35$. After losing 2 more, he had $35 - 2 = 33$ golf balls. The answer is 33.
Q: Olivia has \$23. She bought five bagels for \$3 each. How much money does she have left?
A: Let's think step by step. Olivia had 23 dollars. 5 bagels for 3 dollars each will be $5 * 3 = 15$ dollars. So she has $23 - 15$ dollars left. $23 - 15$ is 8. The answer is 8.

Table 8: Few-Shot Demonstrations for AddSub.

Q: John found that the average of 15 numbers is 40. If 10 is added to each number then the mean of the numbers is?

Answer Choices: (A) 50 (B) 45 (C) 65 (D) 78 (E) 64

A: Let's think step by step. If 10 is added to each number, then the mean of the numbers also increases by 10. So the new mean would be 50. The answer is A.

Q: If $a/b = 3/4$ and $8a + 5b = 22$, then find the value of a.

Answer Choices: (A) $1/2$ (B) $3/2$ (C) $5/2$ (S) $4/2$ (E) $7/2$

A: Let's think step by step. If $a/b = 3/4$, then $b = 4a/3$. So $8a + 5(4a/3) = 22$. This simplifies to $8a + 20a/3 = 22$, which means $44a/3 = 22$. So a is equal to $3/2$. The answer is B.

Q: A person is traveling at 20 km/hr and reached his destiny in 2.5 hr then find the distance?

Answer Choices: (A) 53 km (B) 55 km (C) 52 km (D) 60 km (E) 50 km

A: Let's think step by step. The distance that the person traveled would have been $20 \text{ km/hr} * 2.5 \text{ hrs} = 50 \text{ km}$. The answer is E.

Q: How many keystrokes are needed to type the numbers from 1 to 500?

Answer Choices: (A) 1156 (B) 1392 (C) 1480 (D) 1562 (E) 1788

A: Let's think step by step. There are 9 one-digit numbers from 1 to 9. There are 90 two-digit numbers from 10 to 99. There are 401 three-digit numbers from 100 to 500. $9 + 90(2) + 401(3) = 1392$. The answer is B.

Table 9: Few-Shot Demonstrations for AQuA.

Q: There are 15 trees in the grove. Grove workers will plant trees in the grove today. After they are done, there will be 21 trees. How many trees did the grove workers plant today?

A: Let's think step by step. There are originally 3 cars. 2 more cars arrive. $3 + 2 = 5$. The answer is 5.

Q: If there are 3 cars in the parking lot and 2 more cars arrive, how many cars are in the parking lot?

A: Let's think step by step. There are 15 trees originally. Then there were 21 trees after some more were planted. So there must have been $21 - 15 = 6$. The answer is 6.

Q: Leah had 32 chocolates and her sister had 42. If they ate 35, how many pieces do they have left in total?

A: Let's think step by step. Originally, Leah had 32 chocolates. Her sister had 42. So in total they had $32 + 42 = 74$. After eating 35, they had $74 - 35 = 39$. The answer is 39.

Q: Jason had 20 lollipops. He gave Denny some lollipops. Now Jason has 12 lollipops. How many lollipops did Jason give to Denny?

A: Let's think step by step. Jason started with 20 lollipops. Then he had 12 after giving some to Denny. So he gave Denny $20 - 12 = 8$. The answer is 8.

Q: Shawn has five toys. For Christmas, he got two toys each from his mom and dad. How many toys does he have now?

A: Let's think step by step. There are 15 trees originally. Shawn started with 5 toys. If he got 2 toys each from his mom and dad, then that is 4 more toys. $5 + 4 = 9$. The answer is 9.

Q: There were nine computers in the server room. Five more computers were installed each day, from monday to thursday. How many computers are now in the server room?

A: Let's think step by step. There were originally 9 computers. For each of 4 days, 5 more computers were added. So $5 * 4 = 20$ computers were added. $9 + 20$ is 29. The answer is 29.

Q: Michael had 58 golf balls. On tuesday, he lost 23 golf balls. On wednesday, he lost 2 more. How many golf balls did he have at the end of wednesday?

A: Let's think step by step. There are 15 trees originally. Michael started with 58 golf balls. After losing 23 on tuesday, he had $58 - 23 = 35$. After losing 2 more, he had $35 - 2 = 33$ golf balls. The answer is 33.

Q: Olivia has \$23. She bought five bagels for \$3 each. How much money does she have left?

A: Let's think step by step. Olivia had 23 dollars. 5 bagels for 3 dollars each will be $5 * 3 = 15$ dollars. So she has $23 - 15$ dollars left. $23 - 15$ is 8. The answer is 8.

Table 10: Few-Shot Demonstrations for GSM8K.

Q: There are 15 trees in the grove. Grove workers will plant trees in the grove today. After they are done, there will be 21 trees. How many trees did the grove workers plant today?

A: Let's think step by step. There are originally 15 trees. 6 more trees arrive. $15 + 6 = 21$. The answer is 6.

Q: If there are 3 cars in the parking lot and 2 more cars arrive, how many cars are in the parking lot?

A: Let's think step by step. There are 3 cars originally. Then there were 5 cars after some more were planted. So there must have been $5 - 2 = 3$. The answer is 3.

Q: Leah had 32 chocolates and her sister had 42. If they ate 35, how many pieces do they have left in total?

A: Let's think step by step. Originally, Leah had 32 chocolates. Her sister had 42. So in total they had $32 + 42 = 74$. After eating 35, they had $74 - 35 = 39$. The answer is 39.

Q: Jason had 20 lollipops. He gave Denny some lollipops. Now Jason has 12 lollipops. How many lollipops did Jason give to Denny?

A: Let's think step by step. Jason started with 20 lollipops. Then he had 12 after giving some to Denny. So he gave Denny $20 - 12 = 8$. The answer is 8.

Q: Shawn has five toys. For Christmas, he got two toys each from his mom and dad. How many toys does he have now?

A: Let's think step by step. There are 5 toys originally. Shawn started with 5 toys. If he got 2 toys each from his mom and dad, then that is 4 more toys. $5 + 4 = 9$. The answer is 9.

Q: There were nine computers in the server room. Five more computers were installed each day, from monday to thursday. How many computers are now in the server room?

A: Let's think step by step. There were originally 9 computers. For each of 4 days, 5 more computers were added. So $5 * 4 = 20$ computers were added. $9 + 20$ is 29. The answer is 29.

Q: Michael had 58 golf balls. On tuesday, he lost 23 golf balls. On wednesday, he lost 2 more. How many golf balls did he have at the end of wednesday?

A: Let's think step by step. There are 58 trees originally. Michael started with 58 golf balls. After losing 23 on tuesday, he had $58 - 23 = 35$. After losing 2 more, he had $35 - 2 = 33$ golf balls. The answer is 33.

Q: Olivia has \$23. She bought five bagels for \$3 each. How much money does she have left?

A: Let's think step by step. Olivia had 23 dollars. 5 bagels for 3 dollars each will be $5 * 3 = 15$ dollars. So she has $23 - 15$ dollars left. $23 - 15$ is 8. The answer is 8.

Table 11: Few-Shot Demonstrations for SingleEq.

Q: There are 15 trees in the grove. Grove workers will plant trees in the grove today. After they are done, there will be 21 trees. How many trees did the grove workers plant today?

A: Let's think step by step. There are originally 15 trees. 2 more trees arrive. $15 + 2 = 17$. The answer is 17.

Q: If there are 15 trees in the parking lot and 2 more cars arrive, how many cars are in the parking lot?

A: Let's think step by step. There are 15 trees originally. Then there were 21 trees after some more were planted. So there must have been $21 - 15 = 6$. The answer is 6.

Q: Leah had 32 chocolates and her sister had 42. If they ate 35, how many pieces do they have left in total?

A: Let's think step by step. Originally, Leah had 32 chocolates. Her sister had 42. So in total they had $32 + 42 = 74$. After eating 35, they had $74 - 35 = 39$. The answer is 39.

Q: Jason had 20 lollipops. He gave Denny some lollipops. Now Jason has 12 lollipops. How many lollipops did Jason give to Denny?

A: Let's think step by step. Jason started with 20 lollipops. Then he had 12 after giving some to Denny. So he gave Denny $20 - 12 = 8$. The answer is 8.

Q: Shawn has five toys. For Christmas, he got two toys each from his mom and dad. How many toys does he have now?

A: Let's think step by step. There are 5 toys originally. Shawn started with 5 toys. If he got 2 toys each from his mom and dad, then that is 4 more toys. $5 + 4 = 9$. The answer is 9.

Q: There were nine computers in the server room. Five more computers were installed each day, from monday to thursday. How many computers are now in the server room?

A: Let's think step by step. There were originally 9 computers. For each of 4 days, 5 more computers were added. So $5 * 4 = 20$ computers were added. $9 + 20$ is 29. The answer is 29.

Q: Michael had 58 golf balls. On tuesday, he lost 23 golf balls. On wednesday, he lost 2 more. How many golf balls did he have at the end of wednesday?

A: Let's think step by step. There are 58 trees originally. Michael started with 58 golf balls. After losing 23 on tuesday, he had $58 - 23 = 35$. After losing 2 more, he had $35 - 2 = 33$ golf balls. The answer is 33.

Q: Olivia has \$23. She bought five bagels for \$3 each. How much money does she have left?

A: Let's think step by step. Olivia had 23 dollars. 5 bagels for 3 dollars each will be $5 * 3 = 15$ dollars. So she has $23 - 15$ dollars left. $23 - 15$ is 8. The answer is 8.

Table 12: Few-Shot Demonstrations for SingleOp.

Q: There are 15 trees in the grove. Grove workers will plant trees in the grove today. After they are done, there will be 21 trees. How many trees did the grove workers plant today?

A: Let's think step by step. There are originally 15 trees. 2 more trees arrive. $15 + 2 = 17$. The answer is 17.

Q: If there are 3 cars in the parking lot and 2 more cars arrive, how many cars are in the parking lot?

A: Let's think step by step. There are 3 cars originally. Then there were 5 cars after some more were planted. So there must have been $5 - 2 = 3$. The answer is 3.

Q: Leah had 32 chocolates and her sister had 42. If they ate 35, how many pieces do they have left in total?

A: Let's think step by step. Originally, Leah had 32 chocolates. Her sister had 42. So in total they had $32 + 42 = 74$. After eating 35, they had $74 - 35 = 39$. The answer is 39.

Q: Jason had 20 lollipops. He gave Denny some lollipops. Now Jason has 12 lollipops. How many lollipops did Jason give to Denny?

A: Let's think step by step. Jason started with 20 lollipops. Then he had 12 after giving some to Denny. So he gave Denny $20 - 12 = 8$. The answer is 8.

Q: Shawn has five toys. For Christmas, he got two toys each from his mom and dad. How many toys does he have now?

A: Let's think step by step. There are 5 toys originally. Shawn started with 5 toys. If he got 2 toys each from his mom and dad, then that is 4 more toys. $5 + 4 = 9$. The answer is 9.

Q: There were nine computers in the server room. Five more computers were installed each day, from monday to thursday. How many computers are now in the server room?

A: Let's think step by step. There were originally 9 computers. For each of 4 days, 5 more computers were added. So $5 * 4 = 20$ computers were added. $9 + 20$ is 29. The answer is 29.

Q: Michael had 58 golf balls. On tuesday, he lost 23 golf balls. On wednesday, he lost 2 more. How many golf balls did he have at the end of wednesday?

A: Let's think step by step. There are 58 trees originally. Michael started with 58 golf balls. After losing 23 on tuesday, he had $58 - 23 = 35$. After losing 2 more, he had $35 - 2 = 33$ golf balls. The answer is 33.

Q: Olivia has \$23. She bought five bagels for \$3 each. How much money does she have left?

A: Let's think step by step. Olivia had 23 dollars. 5 bagels for 3 dollars each will be $5 * 3 = 15$ dollars. So she has $23 - 15$ dollars left. $23 - 15$ is 8. The answer is 8.

Table 13: Few-Shot Demonstrations for SVAMP.

Q: What do people use to absorb extra ink from a fountain pen?
Answer Choices: (A) shirt pocket (B) calligrapher's hand (C) inkwell (D) desk drawer (E) blotter
A: Let's think step by step. The answer must be an item that can absorb ink. Of the above choices, only blotters are used to absorb ink. The answer is E.

Q: What home entertainment equipment requires cable?
Answer Choices: (A) radio shack (B) substation (C) television (D) cabinet
A: Let's think step by step. The answer must require cable. Of the above choices, only television requires cable. The answer is C.

Q: The fox walked from the city into the forest, what was it looking for?
Answer Choices: (A) pretty flowers (B) hen house (C) natural habitat (D) storybook
A: Let's think step by step. The answer must be something in the forest. Of the above choices, only natural habitat is in the forest. The answer is C.

Q: Sammy wanted to go to where the people were. Where might he go?
Answer Choices: (A) populated areas (B) race track (C) desert (D) apartment (E) roadblock
A: Let's think step by step. The answer must be a place with a lot of people. Of the above choices, only populated areas have a lot of people. The answer is A.

Q: Where do you put your grapes just before checking out?
Answer Choices: (A) mouth (B) grocery cart (C) super market (D) fruit basket (E) fruit market
A: Let's think step by step. The answer should be the place where grocery items are placed before checking out. Of the above choices, grocery cart makes the most sense for holding grocery items. The answer is B.

Q: Google Maps and other highway and street GPS services have replaced what?
Answer Choices: (A) united states (B) mexico (C) countryside (D) atlas
A: Let's think step by step. The answer must be something that used to do what Google Maps and GPS services do, which is to give directions. Of the above choices, only atlases are used to give directions. The answer is D.

Q: Before getting a divorce, what did the wife feel who was doing all the work?
Answer Choices: (A) harder (B) anguish (C) bitterness (D) tears (E) sadness
A: Let's think step by step. The answer should be the feeling of someone getting divorced who was doing all the work. Of the above choices, the closest feeling is bitterness. The answer is C.

Table 14: Few-Shot Demonstrations for CommonsenseQA.

Q: Do hamsters provide food for any animals?

A: Let's think step by step. Hamsters are prey animals. Prey are food for predators. Thus, hamsters provide food for some animals. The answer is yes.

Q: Could Brooke Shields succeed at University of Pennsylvania?

A: Let's think step by step. Brooke Shields went to Princeton University. Princeton University is about as academically rigorous as the University of Pennsylvania. Thus, Brooke Shields could also succeed at the University of Pennsylvania. The answer is yes.

Q: Yes or no: Hydrogen's atomic number squared exceeds number of Spice Girls?

A: Let's think step by step. Hydrogen has an atomic number of 1. 1 squared is 1. There are 5 Spice Girls. Thus, Hydrogen's atomic number squared is less than 5. The answer is no.

Q: Yes or no: Is it common to see frost during some college commencements?

A: Let's think step by step. College commencement ceremonies can happen in December, May, and June. December is in the winter, so there can be frost. Thus, there could be frost at some commencements. The answer is yes.

Q: Yes or no: Could a llama birth twice during War in Vietnam (1945-46)?

A: Let's think step by step. The War in Vietnam was 6 months. The gestation period for a llama is 11 months, which is more than 6 months. Thus, a llama could not give birth twice during the War in Vietnam. The answer is no.

Q: Yes or no: Would a pear sink in water?

A: Let's think step by step. The density of a pear is about 0.6g/cm³, which is less than water. Objects less dense than water float. Thus, a pear would float. The answer is no.

Table 15: Few-Shot Demonstrations for StrategyQA.

<p>Q: 2015 is coming in 36 hours. What is the date one week from today in MM/DD/YYYY?</p> <p>Answer Choices: (A) 01/05/2015 (B) 01/06/2015 (C) 01/04/2015 (D) 02/05/2015 (E) 12/05/2015 (F) 01/05/2016</p> <p>A: Let's think step by step. If 2015 is coming in 36 hours, then it is coming in 2 days. 2 days before 01/01/2015 is 12/30/2014, so today is 12/30/2014. So one week from today will be 01/05/2015. The answer is A.</p>
<p>Q: The first day of 2019 is a Tuesday, and today is the first Monday of 2019. What is the date today in MM/DD/YYYY?</p> <p>Answer Choices: (A) 01/08/2019 (B) 01/07/2019 (C) 01/06/2019 (D) 02/07/2019 (E) 12/07/2019 (F) 01/07/2018</p> <p>A: Let's think step by step. If the first day of 2019 was Tuesday, then 01/01/2019 was a Tuesday. Today is the first monday, would be six days later. So today is 01/07/2019. The answer is B.</p>
<p>Q: The concert was scheduled to be on 06/01/1943, but was delayed by one day to today. What is the date 10 days ago in MM/DD/YYYY?</p> <p>Answer Choices: (A) 05/22/1943 (B) 05/23/1943 (C) 05/24/1943 (D) 05/25/1943 (E) 05/26/1943 (F) 05/27/1943</p> <p>A: Let's think step by step. One day after 06/01/1943 is 06/02/1943, so today is 06/02/1943. 10 days before today is 05/23/1943. The answer is B.</p>
<p>Q: It is 4/19/1969 today. What is the date 24 hours later in MM/DD/YYYY?</p> <p>Answer Choices: (A) 04/23/1969 (B) 04/21/1969 (C) 04/22/1969 (D) 04/20/1969 (E) 04/24/1969 (F) 04/25/1969</p> <p>A: Let's think step by step. Today is 04/19/1969. 24 hours later is one day after today, which would be 04/20/1969. The answer is D.</p>
<p>Q: Jane thought today is 3/11/2002, but today is in fact Mar 12, which is 1 day later. What is the date 24 hours later in MM/DD/YYYY?</p> <p>Answer Choices: (A) 03/17/2002 (B) 03/14/2002 (C) 03/15/2002 (D) 03/16/2002 (E) 03/13/2002 (F) 03/18/2002</p> <p>A: Let's think step by step. Today is 03/12/2002. So the date 24 hours later will be 03/13/2002. The answer is E.</p>
<p>Q: Jane was born on the last day of Feburary in 2001. Today is her 16-year-old birthday. What is the date yesterday in MM/DD/YYYY?</p> <p>Answer Choices: (A) 03/04/2017 (B) 02/28/2017 (C) 03/01/2017 (D) 03/02/2017 (E) 03/03/2017 (F) 02/27/2017</p> <p>A: Let's think step by step. The last day of February is the 28th, so Jane was born on 02/28/2001. Today is her 16-year old birthday. So yesterday was 02/27/2017. The answer is F.</p>

Table 16: Few-Shot Demonstrations for Date Understanding.

Answer Format

addsub_format = "the answer is n" where n is a number'

single_format = "the answer is n" where n is a number'

strategy_format = 'either "the answer is yes" or "the answer is no"'

date_format = "the answer is n" where n is one of "A, B, C, D, E, F"'

Thought Format

Answer the following question: {input}

Make a strategy then write. Your output should be of the following format:

Strategy:

Your strategy about how to answer the question.

Answer:

Your answer to the question. It should end with {format}.

Voting Prompt

Given an instruction and several choices, decide which choice is most promising.

Analyze each choice in detail, then conclude in the last line

"The best choice is {s}", where s is the integer id of the choice.

Table 17: Prompt template for ToT methods.