

# Individuals and Modality

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## Abstract

In this paper, I argue that the modality contained in declarative sentences is heterogeneous by nature, and its indiscernible treatment evoked paradoxical behaviors of individuals in modal contexts. The difficulty is avoided by classifying the modality in two kinds from a normative point of view: 1) modality in a traditional sense, which is a freedom under a certain convention, and 2) multi-modality which stems from the difference of conventions. The former is treated in the traditional modal logic, but the latter kind of modality which typically appears in belief sentences is captured in another framework called multi-model. This solution deeply concerns the ontology of individuals, therefore the existential presupposition of individuals, too. So, I further propose its treatment by means of a kind of three-valued semantics.

## 1 Introduction

The modality of natural languages has always been one of the main objects of formal semantics. Because declarative sentences of natural languages are analyzed as a predication to individuals, and normally appear with modality, behaviors of individuals in modal contexts inevitably come into question.

In fact, the effort to clarify them has been continuing hundred years since Frege[8, 9]. And in the modal logic, researchers succeeded in it to a certain extent, proposed some possible solutions to the problems in the philosophy and ordinary languages. However, such a method is not almighty, suffers from difficulties, e.g. in the interpretation of belief sentences.

The reason for the difficulty lies in treating heterogeneous modal phenomena in a homogeneous manner. Therefore, it's necessary to analyze the concept of modality into its parts.

They consist of the modality in the narrow sense covered by the traditional modal logic and the modality found in belief sentences. The former is the freedom under the same ontological convention of language, the latter is the freedom under several different conventions. They are called (mere) 'modality' and 'multi-modality' respectively. The former can be treated by the traditional modal logic or the possible world semantics, but for the treatment of the latter, I introduce the method called 'multi-model'.

## 2 Origin of Modality

The introduction of possible worlds is motivated by the intuition: "This and this could have been otherwise". E.g., suppose that the following sentence

(1) Yamada is a student of Hiroshima City University.

is actually true. But Yamada could be a student of the Kyushu University under another circumstance. Or the prime minister of Japan (at the time point July 1995) could be Ozawa, although he's actually

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Murayama. Such intuitions can be explained by assuming possible worlds where the set of students of Hiroshima City University or the prime minister of Japan are different from the actual ones.

The substitutability of two terms which actually designate the same thing, and the existential generalization are used to determine if the context is intensional.<sup>1</sup> E.g., from

(2) It is necessary that 9 is greater than 7 [18]

and

(3) the number of planets = 7.

(4) It is necessary that the number of planets is greater than 7

cannot be inferred.

Likewise, from

(5) There is no such thing as Pegasus (*Ibid.*).

(6) There is something which there is no such thing as

cannot be inferred.

From the viewpoint of the possible world semantics, (2)-(4) can be explained by assuming the situation where the number of planets is not 9, and (5), (6) by the fact that 'Pegasus' doesn't designate anything, although it has a designation in the myth.

### 3 Modality in Belief Sentences

But the semantic behavior of individuals in the context of belief sentences evoked much annoyance among semanticists.

E.g., if Bill mistakes Mary for Anna, then

(7)  $\neg(\text{Mary} = \text{Anna})$

and

(8) Bill believes that Mary is (called) Anna

are both true. But according to the Kripkian thesis that the definite description determines the *dictum*, and the proper name the *res*, such a mistake cannot happen. (Cf. [14].)

Further, suppose that Tom believes that the orator Cicero and Tully, the author of *De Senectute*, are different, although they are actually the same. Then, both

(9) Tom believes that Cicero denounced Catiline,

and

(10) Tom believes that Tully didn't denounce Catiline

can be true [17, §44.]. But insofar as the above thesis is correct, and Tom is rational, (9) and (10) cannot be true at the same time.

In this case, there is another problem. 'Cicero' and 'Tully' denote the same individual, i.e. one and the same unseparable thing. Nevertheless, it seems to split up to two individuals in Tom's belief.

On the other hand, two individuals can merge together to one individual. E.g., if Bill thinks that Mary and Anna are the same person called Anna, and says that

(11) Anna is a high schooler,

the entity whose existence Bill believes is something like a chimera of Mary and Anna. But according to the Kripkian thesis, this cannot be the case either.<sup>2</sup>

## 4 Normative View of Modality

To sum up, the above problems are formulated as follows:

- (12) I) As to individuals:  
a) Does the individual domain change w.r.t. possible worlds or not? (5,6)  
b) Do individuals split up and merge together w.r.t. possible worlds or not? (9)–(11)  
II) As to the relation between expressions and their denotations:  
a) Do denotations of definite descriptions change w.r.t. possible worlds or not? (2)–(4)  
b) Do the denotations of the proper names change w.r.t. possible worlds or not? (7, 8)

But in a sense, the answers to them are clear from the beginning. As above, the individual domain actually changes, individuals split up and merge together, and denotations of definite descriptions and proper names change. The problem concerns if the phenomena should be seen w.r.t. “possible worlds”.

Formally, all the above examples seem to be treated within the possible world semantic framework. E.g., as to (7) and (8), in the possible world which represents Bill’s epistemic state, the individual Mary is designated by the proper name ‘Anna’. And as to (9) and (10), the individual Cicero in the actual world has two counterparts which are designated by the proper names ‘Cicero’ and ‘Tully’ in the possible world which represents Tom’s epistemic state.

But there is the following difficulty with this method.

- (13) Indexicals:  
O’Leary: “Are you going to the party?”  
Miller: “Yes, I’m going”. [22, p.278]
- (14) Proper names: in the situation of (8):  
Bill: “Is Anna going to the party?”  
Observer: “Yea, she is going”. (In reality: Not Anna but Mary is going to the party.)
- (15) Definite descriptions:  
Bill: “The man who is drinking martini is a movie director”.  
Observer: “Ah so”. (In reality: He isn’t drinking martini but water.) [5, p.86]

Intuitively, in (13), O’Leary and Miller talk about the same propositional content. But if we assume that the denotations of ‘I’ and ‘you’ are determined w.r.t. possible worlds (including contexts), their mutual propositional content cannot be the intension in this sense. Likewise in (14) and (15), in order for the observer to share the propositional content with Bill, he must have the following belief in secret:

(14’) Mary is going.

(15’) The man who is drinking water is a movie director.

But their intensions cannot be the same as those of Bill’s assertions.

But the main reason for the difficulty is found in the ontological, metaphysical view concerning the possible worlds.

For this, let’s see the following examples. Today, 1m is defined as the length of standard meter in Paris. And suppose that, actually, there is a boy called Bill. Then, insofar as the modality in

(16) 1m would have been the height of Napoleon,

(17) Bill would have been called David

is interpreted in a possible world semantic manner, it deviates from the traditional interpretation of proper names where they are rigid designators, because the denotations of the proper names in (16) and (17) – 1m and Bill – change w.r.t. possible worlds. But this suggests that the modality in (16) and (17) is different from the traditional possible world semantic modality. The difference between them is the following: the former concerns the freedom in determining the denotation, the latter the freedom after it. Normally, the determination of the denotations of proper names is empirical and contingent. 1m is defined using the physical, contingent attribute, 1/40000 of the circumference of the earth. So is the dubbing of Bill. But

once their denotation are determined, they necessarily designate the same thing among all possible worlds. I.e., they are rigid designators, because they are defined so. In other words, the necessity in the traditional possible world semantics is a normative necessity, and the contingency is the freedom in this normative framework.

The necessity is often contrasted with a prioricity and analyticity. The former is antonymous to a posteriority, the latter to syntheticity. A prioricity means "before the experience", i.e. something gained without experience, and set, number, logical laws etc. are examples for it. Analyticity means that something implicitly contained in a concept is inferred by analyzing it.

(18) Every bachelor is unmarried

is an example of analytic statements. In (18), the attribute 'unmarried' is contained in 'bachelor' from the beginning.

As to the relation between them, there are tough philosophical discussions, which normally think of them as different concepts. But I think that both a prioricity and analyticity are certain kinds of normative laws. So, insofar as the necessity is understood as a normative necessity, they should also be included to the category of necessity.

Each epistemic subject determines the norm, or precisely, the epistemic subject under a particular situation, because the same proper name 'Bill' can be used to denote another person in another situation.

As it's clear from (13), the necessity with indexicals 'I', 'now', 'you' as in

(19) I'm here now

is different from possible world semantic necessity, because we can say

(20) I might not be here now. [12, p.71]

This kind of necessity concerns the determination of a denotation. As in (16) and (17), the determination of a denotation is normally contingent. But there are some expressions accompanied with linguistic rules determining them. And indexicals are examples for them. Kaplan calls such a regularity 'validity' in contrast with the necessity in the traditional sense [12, 13]. Further, he calls the latter kind of modality 'content', the former 'character'.

There are also 'extensional' expressions which designate the same thing independent of their circumstances – in the sense of content as well as character. They are found among others in scientific terminology. They are a common asset in a linguistic community. Therefore, also philosophers of language started from extensional expressions, and even when the modality of natural language came into question, they implicitly assumed a common concept concerning the denotations of linguistic expressions, and failed to deeply recognize their relativity to epistemic subjects, from which the Russell's assertion results that indexicals are not necessary to scientific purpose [19, 20].

But as to natural languages, the elasticity of determining the denotation of an expression depending on contexts should be positively interpreted as efficiency of language (s. [3, 2]). We can easily understand that, if the partner of a conversation uses 'I', it doesn't mean me but herself, we can denote another person with the same proper name in another situation, and enjoy a myth or a science fiction assuming Pegasus or the Knight of Jedi which do not actually exist.

To determine a norm concerning denotation is to determine a model of interpreting a language, and it differs according to epistemic subjects in particular situations. In this sense, the framework of interpretation is a so-called 'point language', and the possible world semantic modality appears inside it. In contrast, the modality in choosing a point language, i.e. a model, is a multi-model semantic modality. In this paper, we call the traditional possible world semantic modality the (mere) modality, and the multi-model semantic modality the multi-modality.<sup>3</sup>

Now, the answer to (12) looks like as follows:

(21) I) As to individuals:

- a) The individual domain is constant w.r.t. possible worlds.
- b) Individuals don't split up nor merge together.

II) As to the relation between expressions and their denotations:

- a) Denotations of definite descriptions change w.r.t. possible worlds.
- b) Denotations of proper names are constant w.r.t. possible worlds.

In contrast, the answer with 'models' instead of 'possible worlds' is formulated as follows:

(22) I) As to individuals:

a) The individual domain changes w.r.t models.

b) Individuals split up and merge together w.r.t. models.

II) As to relation between expressions and their denotations:

a) Denotations of definite descriptions change w.r.t. models.

b) Denotations of proper names change w.r.t. models.

(21) covers the modality, and (22) the multi-modality. But it should be noticed that, according to the above formulation, less phenomena than before can be treated by the possible world semantic method, and those problems which cause much trouble to it are interpreted in the multi-modality. E.g., Pegasus in (5) and (6) is normally treated in the traditional modality, but insofar as the epistemic subject doesn't commit itself to its existence, it should be treated in a multi-modal manner.

Further, (21) and (22) imply the following consequence about the ontology of individuals.

First, (21a) justifies the following meaning postulate in [16]:

(23)  $\exists x \Box (\ulcorner x = \text{John} \urcorner)$

But it means that, assuming that John was born 1920 and died 1990, there exists the same individual John not only in his lifetime but also 1900 or 2000. There are some researchers (such as Carlson[4]) who attempts to formulate the total amount of John as a partial function which assigns a point of reference (i.e. the ordered pair of a possible world and a time point) to John at that point. But this is not a widely accepted strategy. By an individual, we understand an abstract object which is given by a single blow and to which an epistemic subject commits itself; the individual John exists before his birth and after his death in this sense. Dowty[7] calls the individual in Carlson's sense 'stage of individual' and the one in the normal sense 'individual'.

The above individual John exists at least at a time point in the actual world. But we can assume those individuals which don't exist in the actual world at all. E.g., in the following sentence

(24) If Nagoya had been more skillful in diplomacy, the Nagoya Olympic Games would have taken place in the year 1988.

the Nagoya Olympic Games which don't actually exist, exist in other worlds. So, the existence of individuals in (23) and (24) means that they have a physical appearance.

An example of (22Ia) is found in (5). I.e., if the speaker doesn't commit herself to the existence of Pegasus, then it means that an entity in the model of the myth doesn't exist in her model.

Further, the denotation of a definite description such as 'the round square' doesn't exist in any model at all.

Such kinds of existence represent a distance of existence. They would be distinguished as follows:

(25) a) John doesn't exist now,

b) The Nagoya Olympic Games do not actually exist,

c) Pegasus doesn't exist,

d) The round square cannot exist at all.<sup>4</sup>

In order to formulate (22I), I adopt the following machinery:

(22Ia): Even if the speaker of (5) does not assume the existence of Pegasus, she can understand what it is. And even if the speaker of (9) and (10) does not think that Cicero and Tully are different persons, she understands that Tom believes that they are different.

Therefore, the speaker of the above sentences can understand - besides individuals in her own individual domain - those individuals proper to the ontology of the myth or Tom. We call the former domain the real individual domain, the latter the unreal individual domain. The speaker of the principal clause holds the ontology of the speaker of the subordinate clause of a belief sentence and the myth. If we think of the ontology of the speaker of the principal clause, the speaker of the subordinate clause, or the myth as a model, we can say from the ontological viewpoint that the model of the speaker of the principal clause is a supermodel of the speaker of the subordinate clause or the myth; conversely; the models of the latter are submodels of the former.

(22Ib): ‘split up’ and ‘merge together’ of individuals in (7), (8) and (9), (10) are treated by means of counterpart relation between them. But the individual denoted with ‘Cicero’ and ‘Tully’ by the speaker of the principal clause in (9), (10) and the two individuals denoted with them by Tom belong to different kinds of ontology. So, the individuals in the counterpart relation - insofar as they aren’t the same - are interpreted to be different from each other.

## 5 Formal System

The above discussion is formalized as the following syntax and semantics of the language  $L$ .

### 5.1 Syntax of $L$

#### Vocabulary

- i)  $a, b, c, \dots \in Const$ : set of individual constants.
- ii)  $I, you, he, she, \dots \in Ind$ : set of indexicals.
- iii)  $x, y, z, \dots; x_1, x_2, \dots \in Var$ : set of individual variables.
- iv)  $pred_i^j \in Pred^j$  ( $i, j \geq 0$ ): set of  $j$ -ary predicates.  
 $\bigcup_j Pred^j = Pred$ : set of predicates.

**Formation Rules** The set of terms and the set of formulae in  $L$  are defined in the following simultaneous inductive manner.

**Set of Terms** The minimal set  $Term$  which satisfies the following conditions is called the set of terms of  $L$ .

- i) If  $t \in Const \cup Ind \cup Var$ , then  $t \in Term$ .
- ii) If  $x \in Var$ , and  $A \in Form$ , then  $\iota x A \in Term$ .

$IT \doteq \{\iota x A \mid x \in Var, \text{ and } A \in Form\}$  is called the set of iota terms.

**Set of Formulae** The minimal set  $Form$  which satisfies the following conditions is called the set of formulae of  $L$ .

- i)  $pred_i^j(t_1, \dots, t_j) \in Form$ .
- ii) If  $A, B \in Form$ , then  $(\neg A), (A \wedge B), (A \vee B), (A \supset B), (A \equiv B) \in Form$ .
- iii) If  $x \in Var, A \in Form$ , then  $(\forall x A), (\exists x A), (\bigwedge x A), (\bigvee x A) \in Form$ .
- iv) If  $A \in Form$ ,  $(\Box A), (\Diamond A) \in Form$ .
- v) If  $A \in Form$ ,  $Bel(a, A) \in Form$ .
- vi) If  $t, u \in Term$ ,  $G(t, u) \in Form$ .
- vii) If  $A \in Form$ ,  $(\oplus A), (\odot A) \in Form$ .

The set of vocabulary, terms, and formulae of  $L$  is called the set of expressions of  $L$ .

In ii),iii),iv),vii), the parentheses can be omitted in obvious cases. On the other hand, ‘[ ]’ instead of ‘( )’ can be used in order to facilitate the understanding.

Proper names are expressed as individual constants in  $L$ .

$Bel(\alpha, p)$  is read: “ $\alpha$  believes that  $p$ ”.  $G(t, u)$  represents the counterpart relation between  $t$  and  $u$ .

### 5.2 Semantics of $L$

#### 5.2.1 Model Structure

$MS = \langle D, W, R \rangle$

is called a model structure.

- i)  $D$ : individual domain.  
 $D$  is divided into the domain of real individuals  $RD$  and the domain of unreal individuals  $UD$ .
- ii)  $W$ : set of points of reference.
- iii)  $R$ : reachability relation between possible worlds.

$W$  is ordinarily interpreted as the set  $PW$  of possible worlds. But if necessary, we interpret it as the set  $PW \times T \times P \times \dots$  of ordered pairs consisting of possible worlds, time points, places and so on.

### 5.2.2 Model

$M = \langle MS, c, f \rangle$

is called a model of  $L$ .

- i)  $MS$ : model structure in 5.2.1.
- ii)  $c$ : context.  
 $c \rightarrow D$   
 $c(I) = cI, c(\text{you}) = c\text{you}, c(\text{he}) = c\text{he}, c(\text{she}) = c\text{she}, \dots$
- iii)  $f$ : value assignment to predicates and individual constants.  
 $W \times Pred^n \rightarrow D^n$   
 $Const \rightarrow D$

### 5.2.3 Model Constellation

$MC = \langle \{M_i\}_{i \in I}, \Pi, \Gamma \rangle$

is called a model constellation of  $L$ .

- i)  $M_i$ : a model.  
The index set  $I$  is the minimal set which satisfies the following conditions:
  - i)  $\epsilon \in I$ . ( $\epsilon$  represents the empty string. So,  $M_\epsilon$  equals  $M$ .)
  - ii) If  $M_i = \langle \langle D_i, W_i, R_i \rangle, c_i, f_i \rangle$ ,  $d_i \in D_i, w_i \in W_i$ , then  $iw_id_i \in I$ .
- ii)  $\Pi$  determines  $c_{iw_id_i}$  in  $M_{iw_id_i}$  from  $w_i \in W_i, d_i \in D_i$  in  $M_i$  such that  $\Pi(w_i, d_i) = c_{iw_id_i}$ .
- iii)  $\Gamma$ : a relation on  $RD_i \times RD_{iw_id_i}$ .  $\Gamma$  satisfies the transitivity.

If  $i, is \in I$  ( $s$  is not empty),  $M_{is}$  is called a submodel of  $M_i$ ,  $M_i$  a supermodel of  $M_{is}$ . If  $s$  is  $i_w i_d$ ,  $M_{is}$  is called a direct submodel of  $M_i$ ,  $M_i$  a direct supermodel of  $M_{is}$ . The individual domain of a supermodel includes the individual domains of its direct submodel, i.e.,  $D_i \supseteq D_{iw_id_i}$ .

### 5.2.4 Interpretation

Expressions of  $L$  are interpreted w.r.t. the interpretation function  $M_i(c_i, w_i, s_i, \cdot)$  as follows.

The partial function  $s_i: Var \rightarrow D_i$  is called the variable assignment of  $M_i$ .  $s_i \supseteq s_{is}$ .

- i) If  $a \in Const$ ,  $M_i(c_i, w_i, s_i, a) = f_i(w_i, a) \in D_i$ .
- ii) If  $ind \in Ind$ ,  $M_i(c_i, w_i, s_i, ind) = c_i(ind) \in D_i$ .
- iii) If  $x \in Var$ ,  $M_i(c_i, w_i, s_i, x) = s_i(x)$ .
- iv) If  $\iota x A \in IT$ ,
  - a) if for an unique  $d \in D_i$ ,  $M_i(c_i, w_i, s_i, x, A) = 1$ , then  $M_i(c_i, w_i, s_i, \iota x A) = d$ ,
  - b) otherwise undefined.
- v) If  $pred_i^j \in Pred^j$ ,  $M_i(c_i, w_i, s_i, pred_i^j) = f_i(w_i, pred_i^j) \in 2^{D^n}$ .  
 $M_i(c_i, w_i, s_i, \cdot) \upharpoonright Form: Form \rightarrow \{1, 0\}$ .  
('1' and '0' mean true and false respectively.)
- vi)  $M_i(c_i, w_i, s_i, pred_i^j(t_1, \dots, t_j)) = M_i(c_i, w_i, s_i, pred_i^j)(M_i(c_i, w_i, s_i, t_1), \dots, M_i(c_i, w_i, s_i, t_j))$ .
- vii)  $\neg p, p \wedge q, p \vee q, p \supset q, p \equiv q$  are interpreted in a normal manner.
- viii)  $M_i(c_i, w_i, s_i, \forall x A) = 1 \Leftrightarrow$  for all  $d \in RD_i$   $M_i(c_i, w_i, s_i, x, A) = 1$ .
- ix)  $M_i(c_i, w_i, s_i, \exists x A) = 1 \Leftrightarrow$  for some  $d \in RD_i$   $M_i(c_i, w_i, s_i, x, A) = 1$ .
- x)  $M_i(c_i, w_i, s_i, \bigwedge x A) = 1 \Leftrightarrow$  for all  $d \in D_i$   $M_i(c_i, w_i, s_i, x, A) = 1$ .
- xi)  $M_i(c_i, w_i, s_i, \bigvee x A) = 1 \Leftrightarrow$  for some  $d \in D_i$   $M_i(c_i, w_i, s_i, x, A) = 1$ .
- xii)  $M_i(c_i, w_i, s_i, \square A) = 1 \Leftrightarrow$  for all  $w \in W_i$   $M_i(c_i, w, s_i, A) = 1$ .
- xiii)  $M_i(c_i, w_i, s_i, \diamond A) = 1 \Leftrightarrow$  for some  $w \in W_i$   $M_i(c_i, w, s_i, A) = 1$ .
- xiv)  $M_i(c_i, w_i, s_i, Bel(a, A)) = 1 \Leftrightarrow M_{iw_i\alpha}(c_{iw_i\alpha}, w_{iw_i\alpha}, s_{iw_i\alpha}, A) = 1$ .  
 $(M_i(c_i, w_i, s_i, a) = \alpha.)$
- xv)  $M_i(c_i, w_i, s_i, G(t, u)) = 1 \Leftrightarrow M_i(c_i, w_i, s_i, t) \Gamma M_i(c_i, w_i, s_i, u)$ .

xvi)  $M_i(c_i, w_i, s_i, \oplus A) = 1 \Leftrightarrow$  for an arbitrary  $is \in I$  beginning with  $i$   $M_{is}(c_{is}, w_{is}, s_{is}, A) = 1$

xvii)  $M_i(c_i, w_i, s_i, \odot A) = 1 \Leftrightarrow$  for some  $is \in I$  beginning with  $i$   $M_{is}(c_{is}, w_{is}, s_{is}, A) = 1$

For  $form \in Form$ ,  $M_i(c_i, w_i, s_i, form) = 1$ ,  $M_i(c_i, w_i, s_i, form) = 0$  are also represented by  $M_i, c_i, w_i, s_i \models form$  and  $M_i, c_i, w_i, s_i \not\models form$  respectively.

## 6 Solution of Problems

Now, we discuss some problems in the previous sections using the theoretical framework in 5.

(2)-(4) can be solved using the traditional modal logic.

There are several semantically equivalent methods to treat the existential problems in (5), (6) and (25).

The ordinary method is to use the existential quantifier, but in order to distinguish the meaning of existence in (25), with restricted ranges of quantification.<sup>5</sup> E.g. (25c) is formulated as follows:

(25c')  $\neg \exists x(x = Pegasus)$ .

In this case, the range of quantification is restricted to  $RD_i$ , and there is no  $x$  which satisfies  $x = Pegasus$ , so that (5') is true as a whole.

(25d) is formulated using  $\bigvee$  ranging over  $D_i$  as follows:

(25d')  $\neg \bigvee x[\bigwedge y[y = Pegasus \equiv x = y]]$ .

Likewise, (25a,b) with some appropriately restricted quantifies.

But here, I adopt a formulation close to surface syntactic expressions using existential predicates instead of quantifiers:  $exist_{AN}$  for (25a),  $exist_A$  for (25b),  $exist$  for (25c), and  $designate$  for (25d) respectively. Further, the definite description is represented by  $\iota$ -term. Then (25a-d) are formulated as follows:

- (25) a')  $\neg exist_{AN}(John)$ ,  
 b')  $\neg exist_A(NO\!G)$ ,  
 c')  $\neg exist(Pegasus)$ ,  
 d')  $\neg designate(\iota x [round(x) \wedge square(x)])$ .

Here,  $M(c, w, s, exist_{AN})$  is the set of individuals which exists at the time point and the world of  $w$ . Likewise,  $M(c, w, s, exist_A)$  is the set of individuals which exists in the world  $w$ ,  $M(c, w, s, exist)$  is  $RD$ , and  $M(c, w, s, designate)$  is  $D$  respectively. According to this interpretation, the sentences in (25) are all true.

For normal quantified sentences, I use '∃' and '∀'. E.g.,

(26) Every Japanese is honest

is formulated as

(26')  $\forall x(Japanese(x) \supset honest(x))$ .

In this case, the subject noun restricts the range of quantification.

(8) is formulated as

(8')  $\bigvee x(G(Mary, x) \wedge Bel(Bill, x = Anna))$ .

In this case, the quantification of  $\bigvee$  ranges over  $D$ .  $G(Mary, x)$  singles out the counterpart in Bill's belief of the 'actual' Mary. And it is identified with the individual denoted by Bill with the proper name 'Anna'.

(9) and (10) are formulated as

(9')  $Bel(Tom, denounce(Cicero, Catiline))$ ,

(10')  $Bel(Tom, \neg denounce(Tully, Catiline))$

respectively. As to the proper names 'Cicero' and 'Tully', they represent the *de dicto* reading, i.e., their subordinate clauses directly express Tom's *dictum*.

But (9) and (10) can be arranged as follows:

- (9'') Tom believes of Cicero that he denounced Catiline,  
 (10'') Tom believes of Cicero that he didn't denounce Catiline.

They are formulated as

- (9''')  $\forall x[G(\text{Cicero}, x) \wedge \text{Bel}(\text{Tom}, \text{denounce}(x, \text{Catiline}))]$ ,  
 (10''')  $\forall x[G(\text{Cicero}, x) \wedge \text{Bel}(\text{Tom}, \neg \text{denounce}(x, \text{Catiline}))]$ .

I.e., 'Cicero' in the ontology of the speaker of the principal clause corresponds to 'Cicero' and 'Tully' – two different persons – in Tom's ontology, and the former denounced Catiline, but the latter not.

(17) is formulated as

- (17')  $\forall x(G(\text{Bill}, x) \wedge \odot(x = \text{David}))$ .

The necessity in (19) and (20) are expressed as follows:

- (19')  $\oplus \text{At}(\text{here})(\text{now})(I)$ ,  
 (20')  $\square \text{At}(\text{here})(\text{now})(I)$ .

Here, (19') is true, but (20') is false.

## 7 Existential Presupposition and Three-valued Logic

In (25), we saw the failure of existence. But it evokes some problems with the failure of existential presupposition.

Suppose that John died last year. Then, how about the truth value of the following sentences?

- (27) a) John is a student,  
 b) John is not a student.

The most plausible way seems to assign the truth value 'neutral' to both sentences, because the existential presupposition of John fails in them. But this interpretation cannot be done in the framework of the two-valued semantics: If we interpret the negation in (27b) as the sentential negation, (27a) is false, and (27b) is true. And even if we interpret it as the predicate negation which means

- (28) John is a non-student,

both (27a) and (27b) are false.<sup>6</sup> Then, in order to appropriately formulate the above-mentioned intuition, we must construct a three-valued semantics. And this is done in the present semantic framework as follows:

First, we consider that the predicate  $pred$  and its predicate negation  $\overline{pred}$  satisfy the following conditions:

- (29) a)  $M_i(c_i, w_i, s_i, pred) \cap M_i(c_i, w_i, s_i, \overline{pred}) = \phi$ ,  
 b)  $M_i(c_i, w_i, s_i, pred) \cup M_i(c_i, w_i, s_i, \overline{pred}) \subseteq D_i$ .

Then, consider the combination of truth values of the formula  $pred(a)$  and its negation. If the negation is the sentential negation  $\neg$ , then a) and d) in

- (30) a)  $M_i, c_i, w_i, s_i \models pred(a)$ ,  
            $M_i, c_i, w_i, s_i \models \neg pred(a)$ ;  
 b)  $M_i, c_i, w_i, s_i \models pred(a)$ ,  
            $M_i, c_i, w_i, s_i \not\models \neg pred(a)$ ;  
 c)  $M_i, c_i, w_i, s_i \not\models pred(a)$ ,  
            $M_i, c_i, w_i, s_i \models \neg pred(a)$ ;  
 d)  $M_i, c_i, w_i, s_i \not\models pred(a)$ ,  
            $M_i, c_i, w_i, s_i \not\models \neg pred(a)$

are not possible *per definitionem*. But if the negation is the predicate negation  $\overline{pred}$ , then only a) in

- (31) a)  $M_i, c_i, w_i, s_i \models \overline{pred}(a)$ ,  
 $M_i, c_i, w_i, s_i \models pred(a)$ ;  
 b)  $M_i, c_i, w_i, s_i \models \overline{pred}(a)$ ,  
 $M_i, c_i, w_i, s_i \not\models pred(a)$ ;  
 c)  $M_i, c_i, w_i, s_i \not\models \overline{pred}(a)$ ,  
 $M_i, c_i, w_i, s_i \models pred(a)$ ;  
 d)  $M_i, c_i, w_i, s_i \not\models \overline{pred}(a)$ ,  
 $M_i, c_i, w_i, s_i \not\models pred(a)$

is not possible, and we can interpret d) as the case where the truth value 'neutral' is assigned to  $pred(a)$  as well as  $\overline{pred}(a)$ . Formally, the truth value of this three-valued semantics is defined as follows:

- (32)  $pred(pred(a))$  and  $\overline{pred}(\overline{pred}(a))$  are called conjugate of each other. Then  
 a)  $pred(a)$  ( $\overline{pred}(a)$ ) is true w.r.t.  $M_i, c_i, w_i, s_i$  iff  $M_i(c_i, w_i, s_i, pred(a)) = 1$   
 ( $M_i(c_i, w_i, s_i, \overline{pred}(a)) = 1$ ).  
 b)  $pred(a)$  ( $\overline{pred}(a)$ ) is false w.r.t.  $M_i, c_i, w_i, s_i$  iff its conjugate is true.  
 c) Otherwise,  $pred(a)$  ( $\overline{pred}(a)$ ) is neutral w.r.t.  $M_i, c_i, w_i, s_i$ .

According to this definition, (27a,b) are interpreted as follows. First, they are formulated as

- (27) a')  $\overline{student}(John)$ ,  
 b')  $student(John)$ .

But here, the following conditions are assumed:

- (33) a)  $M_i(c_i, w_i, s_i, student) \cup M_i(c_i, w_i, s_i, \overline{student}) = M_i(c_i, w_i, s_i, exist_{AN})$ ,  
 b)  $M_i(c_i, w_i, s_i, John) \notin M_i(c_i, w_i, s_i, exist_{AN})$ .

Then, both (27a) and (27b) are neutral.<sup>7</sup>

Apropos, we could also assume

- (34) a)  $M_i(c_i, w_i, s_i, student) \cup M_i(c_i, w_i, s_i, \overline{student}) \subsetneq M_i(c_i, w_i, s_i, exist_{AN})$ ,  
 b)  $M_i(c_i, w_i, s_i, John) \in M_i(c_i, w_i, s_i, exist_{AN})$ ,  
 c)  $M_i(c_i, w_i, s_i, John) \notin M_i(c_i, w_i, s_i, student) \cup M_i(c_i, w_i, s_i, \overline{student})$

in order for them to be neutral. This would represent the case where it's not certain if John is a student, even if it's certain that John is alive. But it's different from the case in (27); the former occurs due to the lack of information, whereas the latter is a genuine indefiniteness under complete information. I'm of the opinion that the model like (34) should not be allowed, and the incomplete information should be treated by means of Situation Semantics[3] or Data Semantics[23].

## 8 Conclusion

In this paper, I argued that, besides the normal modality, there is another kind of modality as it appears in belief sentences, i.e. multi-modality. But it's difficult for the traditional method to treat the behavior of individuals in the latter kind of modality. So, I proposed a new method based on multi-model, and argued that it's effectively applicable to the multi-modal phenomena.

This paper is developed by means of traditional two-valued possible world semantics, and in fact, it's enough to treat the problem entitled "Individuals and Modality". But in this framework, the problems in 7 cannot be fully treated. There, only the three-valuedness of atomic sentences is treated in an implicate form. We need to extend the three-valued interpretation to arbitrary sentences. Further, in order to treat the problem with incomplete information pointed out there, and to set up a semantics which captures the human cognitive state more naturally and elastically, we need to replace the possible world semantic framework by the situation semantic one. This is the next very interesting theme.

## Notes

- 1 Here, 'intensional' and 'modal' are used in the same meaning. And the context means here 'co-text' such as 'it's possible ...' or 'He believes that ...'.
- 2 A formalization of such phenomena can be found e.g. in [15].
- 3 In this sense, the modal interpretation as a point language is a rigorous semantic framework excluding pragmatic aspects, contrasting the normal situation semantic view including "many of the issues that would normally be classified as pragmatics"[6, p.1]. But in the present framework, they are treated in a multi-model semantic manner.
- 4 We can interpret that such individual terms represent metaphysical concepts which occupy an intermediate stage between linguistic expressions and physical individuals. They are kind of pegs which are introduced by proper names or definite descriptions, and to which various attributes are appended. But they would lack physical appearance, be unreal, and not designate at all. In this regard, they resemble discourse markers of DRT (Cf. [10, 11].)
- 5 This is a method which linguists are fond of. E.g., see [24].
- 6 As to Japanese, the particle 'nai' of negation is used for predicate negation only. And for sentential negation, an artificial expression corresponding to 'it's not the case that ...' must be used. (Cf. Sakai[21]. He also advocates the opinion that 'nai' denies the nuclear sentence (i.e. predicate) only.)
- 7 The following sentences
  - i) The round square is  $10\text{cm}^2$ ,
  - ii) The round square is not  $10\text{cm}^2$

can be formulated in two forms depending on the formulation of the definite noun phrase:

Using definite description:

- i')  $\forall x[\wedge y[\text{round}(y) \wedge \text{square}(y) \equiv x = y] \wedge \underline{10\text{cm}^2}(x)]$ ,
- ii')  $\forall x[\wedge y[\text{round}(y) \wedge \text{square}(y) \equiv x = y] \wedge \overline{10\text{cm}^2}(x)]$ ,

Using  $\iota$ -operator:

- i'')  $\underline{10\text{cm}^2}(x)(\iota(\text{round}(y) \wedge \text{square}(y)))$ ,
- ii'')  $\overline{10\text{cm}^2}(x)(\iota(\text{round}(y) \wedge \text{square}(y)))$ .

In the two-valued framework, they are all false. But in the three-valued framework, i') and ii') are false, whereas i'') and ii'') are neutral. The former are false, because they commit themselves to the existence of the round square by the reading "there is one and only one round square, and ...". In sum, i'') and ii'') can grasp the semantic intuition about i) and ii) more appropriately than i') and ii'), which is a reason why I prefer the formulation using  $\iota$ -operator to definite description. (See also (27d').)

## References

- [1] Almog, J./ Perry, J./ Wettstein, H.(eds.) (1989), *Themes from Kaplan*, Oxford(Oxford Univ. Pr.).
- [2] Barwise, J.(1989), "On the Model Theory of Common Knowledge", in *The Situations in Logic*, 201-220, Stanford: CSLI.
- [3] Barwise, J./ Perry, J.(1983), *Situations and Attitudes*, Cambridge(MIT Pr.).
- [4] Carlson, Gregory N.(1977), "Reference to Kinds in English", doctoral dissertation, University of Massachusetts.
- [5] Castañeda, Hector-Neri(1982), *Sprache und Erfahrung*, Frankfurt am Main(Suhrkamp).
- [6] Devlin, K.(1995), "Lectures on Situation Theory: Notes from a lecture given at the European Summer School for Language, Logic, and Information, Barcelona, Spain, 13-26 August, 1995".
- [7] Dowty, D.R.(1979), *Word Meaning and Montague Grammar*, Dordrecht(Reidel).
- [8] Frege, Gottlob(1892), "Über Sinn und Bedeutung", *Zeitschrift für Philosophie und philosophische Kritik*, 26-50.

- [9] Frege, Gottlob(1918-19), "Logische Untersuchungen, erster Teil: Der Gedanke", *Beiträge zur Philosophie des deutschen Idealismus*, 1. 58-77.
- [10] Kamp, Hans(1981), "A Theory of truth and semantic representation". in Groenendijk et al. (eds.), *Formal Methods in the Study of Language*, 277-322, Amsterdam(Mathematisch Centrum Tracts).
- [11] Kamp, Hans/ Ryle, Uwe(1993), *From Discourse to Logic*, Dordrecht(Kluwer).
- [12] Kaplan, D.(1977), "Demonstratives", unpublished ms., UCLA.
- [13] Kaplan, D.(1989), "Afterthoughts", in Almog, J./ Perry, J./ Wettstein, H.(eds.)(1989), pp.565-614.
- [14] Kripke, Saul A.(1972), "Naming and Necessity". in Davidson, D./ Harman, G., *Semantics of Natural Language*, Dordrecht(Reidel).
- [15] Lewis, David K.(1968), "Counterpart Theory and Quantified Modal Logic", *The Journal of Philosophy*, 65, 113-126. New York(Columbia Univ).
- [16] Montague, R.(1973), "The Proper Treatment of Quantification in Ordinary English", in Hintikka, J./ Moravcsik, J. Suppes, P. *Approaches to Natural Language*, Dordrecht(Reidel).
- [17] Quine, W.V.O.(1960), *Word and Object*, Cambridge(MIT Pr.).
- [18] Quine, W.V.O.(1961), *From a Logical Point of View*, New York(Harper and Row).
- [19] Russell, Bertrand(1940), *An Inquiry into Meaning and Truth*, London.
- [20] Russell, Bertrand(1948), *Das menschliche Wissen*, Darmstadt(?).
- [21] Sakai, Hidehisa(1979), *Nihongo no Bunpo to Ronri (Grammar and Logic of Japanese)*, Tokyo(Keiso).
- [22] Stalnaker, R.(1970), "Pragmatics", *Synthese*, 22, 272-289, Dordrecht(Reidel).
- [23] Veltman, Fr.(1985), "Logics for Conditionals", dissertation, Univ. of Amsterdam, 1985.
- [24] Zaefferer, Dietmar(1984), *Frageausdrücke und Fragen im Deutschen*, Fink(München).