Model-Theoretic Incremental Interpretation Based on Discourse Representation Theory

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Abstract

This paper proposes a model-theoretic approach to incremental interpretation where all sentence prefixes have semantic values. The proposed semantics is based on Discourse Representation Theory (DRT), where semantic representations (called DRSs) are interpreted as assignment updates. In our semantics, a partial DRS of a sentence prefix is interpreted as two sets which stipulate the assignment updates. One denotes possible updates and the other denotes necessary updates. With the proposed semantics, we can assign truth values to sentence prefixes.

1 Introduction

Incremental semantic parsers construct semantic representations for each sentence prefix, and are useful for incremental dialogue systems (Allen et al., 2001; Aist et al., 2007). While most research on incremental semantic parsers has focused on how to construct such representations incrementally (Pulman, 1985; Milward, 1995; Poesio and Rieser, 2010; Purver et al., 2011; Peldszus and Schlangen, 2012; Sayeed and Demberg, 2012; Kato and Matsubara, 2015), there has been little work on how to formally interpret them.

An important issue with incremental interpretation, from a formal semantic viewpoint, is that sentence prefixes do not have propositional interpretations (Chater et al., 1995). In other words, standard formal semantics cannot be applied to incremental interpretation directly.

This paper proposes a model-theoretic approach to incremental interpretation where each sentence prefix has semantic values. The proposed semantics is an extension of Discourse Representation Theory (DRT) (Kamp and Reyle, 1993). In DRT, semantic representations are called discourse representation structures (DRSs) and are interpreted in terms of (non-deterministic) assignment updates (An assignment is a function that maps discourse referents to entities.) In this paper, we define two types of interpretations of partial DRSs. One denotes possible assignment updates and the other denotes necessary updates. The proposed semantics monotonically specifies the semantic values of a sentence on a word-by-word basis, and finally assigns the same value to the sentence in terms of DRT's semantics. In addition, it can assign truth values to sentence prefixes that are not sentential clauses. To the best of our knowledge, this is the first attempt to interpret underspecified semantic representations of sentence prefixes.

This paper is organized as follows. Section 2 introduces DRT. Section 3 gives an overview of incremental semantic parsers that construct a partial DRS for each sentence prefix. Then, Section 4 proposes our incremental interpretation method based on DRT. Finally, Section 5 compares our work with previous studies, and Section 6 presents our conclusions.

2 Discourse Representation Theory

This section provides a brief introduction to Discourse Representation Theory (DRT).

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2.1 Discourse Representation Structure

In DRT, semantic content is represented as a *discourse representation structure* (*DRS*). A DRS consists of a set of *discourse referents* and a set of *conditions*. A discourse referent denotes an entity, which is introduced by a sentence. A condition denotes a constraint imposed on discourse referents. DRSs are written as follows:

$$[x_1, \dots x_n \mid c_1, \cdots c_m]$$

Here, $x_1, \dots, x_n$ are discourse referents, and $c_1, \dots, c_m$ are conditions. For example, the following DRS intuitively represents a situation, where there is a student $\mathtt{x_1}$ and a laptop $\mathtt{x_2}$, and $\mathtt{x_1}$ uses $\mathtt{x_2}$:

$$[\mathtt{x_1}, \mathtt{x_2} \mid \mathtt{stu}(\mathtt{x_1}), \mathtt{laptop}(\mathtt{x_2}), \mathtt{use}(\mathtt{x_1}, \mathtt{x_2})]$$

Below, we define a DRT language based on that of Bos (2009), which adopts type theory to define expressions. The *basic types* are $\mathtt{e}$ (entities) and $\mathtt{t}$ (propositions). If α and β are types, then $\langle\alpha\beta\rangle$ is the type of a function from α to β. The language is defined as follows.

1. A variable of type α is an expression of type α.
2. A discourse referent is an expression of type $\mathtt{e}$.
3. If P is an n-place predicate symbol and $x_1, \dots, x_n$ are expressions of type $\mathtt{e}$, then $P(x_1, \dots, x_n)$ is a basic condition.
4. If x_1 and x_2 are expressions of type $\mathtt{e}$, then $x_1 = x_2$ is a basic condition.
5. A basic condition is a condition.
6. If $\mathcal{X}$ is a set of discourse referents and $\mathcal{C}$ is a set of conditions, then $[\mathcal{X} \mid \mathcal{C}]$ is an expression of type $\mathtt{t}$.
7. If E_1 and E_2 are expressions of type $\mathtt{t}$, then $(E_1; E_2)$ is an expression of type $\mathtt{t}$.
8. If E is an expression of type $\mathtt{t}$, then $\neg E$ is a condition.
9. If E_1 and E_2 are expressions of type $\mathtt{t}$, then $E_1 \vee E_2$ and $E_1 \Rightarrow E_2$ are conditions.
10. If X is a variable of type α and E is an expression of type β, then $(\lambda X.E)$ is an expression of type $\langle\alpha\beta\rangle$.
11. If E_1 is an expression of type $\langle\alpha\beta\rangle$ and E_2 is an expression of type α, then $(E_1 E_2)$ is an expression of type β.

Below, we impose the following constraints on all expressions.

- All discourse referents are declared at most once. Here, we say that x is declared in E if E takes the form $[\dots, x, \dots \mid \mathcal{C}]$.
- For all function types $\langle\alpha\beta\rangle$, $\beta \neq \mathtt{e}$.

Let E be an expression in β-normal form.[1] We say that E is *complete* if E does not include any variables. Otherwise, E is *partial*. If E is of type $\mathtt{t}$, we call E a *DRS*.

We can compositionally build up a DRS for a sentence in bottom-up fashion. In this paper, we adopt the approach of Bos (2008), which is based on Combinatory Categorial Grammar (Steedman, 2000). As an example, let us consider constructing a semantic expression for the noun phrase "a student" using the lexicon shown in Table 1. The categories of "a" and "student" are $\mathtt{NP/N}$ and $\mathtt{N}$, respectively. We can combine these, since $\mathtt{NP/N}$ means that it receives an expression of category $\mathtt{N}$ from the right and returns one of category $\mathtt{NP}$. The corresponding semantic expression can be obtained by function application as follows[2]:

$$\Big(\lambda PQ.(([\mathtt{x_1} \mid\ ]; P\mathtt{x_1}); Q\mathtt{x_1})\Big)(\lambda X.[\ \mid \mathtt{stu}(X)])$$
$$\twoheadrightarrow_\beta \lambda Q.([\mathtt{x_1} \mid \mathtt{stu}(\mathtt{x_1})]; Q\mathtt{x_1})$$

Here, $\twoheadrightarrow_\beta$ is the reflexive transitive closure of β-reduction. Figure 1 shows the derivation of the DRS for the following example sentence:

$$\text{A student uses every red laptop.} \qquad (1)$$

[1] If an expression E does not contain a β-redex, namely an expression of the form $(\lambda X.E_1)E_2$, we say that E is in β-normal form.

[2] To simplify the notation, we allow *merge operations* that substitute $[\mathcal{X}_1 \cup \mathcal{X}_2 \mid \mathcal{C}_1 \cup \mathcal{C}_2]$ for $([\mathcal{X}_1 \mid \mathcal{C}_1]; [\mathcal{X}_2 \mid \mathcal{C}_2])$ in any expression. This does not affect the following discussion.

Word	Expression	Type	Category
a	$\lambda PQ.(([x \mid\ ]; Px); Qx)$	$\langle \mathtt{p}\langle \mathtt{pt}\rangle\rangle$	NP/N
every	$\lambda PQ.[\ \mid ([x \mid\ ]; Px) \Rightarrow Qx]$	$\langle \mathtt{p}\langle \mathtt{pt}\rangle\rangle$	NP/N
student	$\lambda X.[\ \mid \mathtt{stu}(X)]$	$\mathtt{p}$	N
laptop	$\lambda X.[\ \mid \mathtt{laptop}(X)]$	$\mathtt{p}$	N
blue	$\lambda PX.([\ \mid \mathtt{blue}(X)]; PX)$	$\langle \mathtt{pp}\rangle$	N/N
red	$\lambda PX.([\ \mid \mathtt{red}(X)]; PX)$	$\langle \mathtt{pp}\rangle$	N/N
use	$\lambda PQ.Q(\lambda X.P(\lambda Y.[\ \mid \mathtt{use}(X,Y)]))$	$\langle\langle \mathtt{pt}\rangle\langle\langle \mathtt{pt}\rangle \mathtt{t}\rangle\rangle$	(S\NP)/NP
don't	$\lambda PQ.[\ \mid \neg PQ]$	$\langle\langle\langle \mathtt{pt}\rangle \mathtt{t}\rangle\langle\langle \mathtt{pt}\rangle \mathtt{t}\rangle\rangle$	(S/NP)/(S/NP)

Here, we abbreviate the type $\langle \mathtt{et}\rangle$ to $\mathtt{p}$.

Table 1: Semantic expressions for words.

2.2 Interpretation of DRSs

This section explains how DRSs are interpreted based on Muskens (1996). A DRS is interpreted with respect to a model M, which is defined as a pair (D, I). Here, D is a (non-empty) set of entities called a *domain* and I is a function that maps n-place predicate symbols to sets of n-tuples of entities. $\langle d_1, \ldots, d_n\rangle \in I(P)$ means that the entities $d_1, \ldots, d_n$ stand in the relation P. The interpretation of a DRS is a function that takes an *assignment* and returns a set of (updated) assignments. An assignment a is a partial function that maps discourse referents to entities, and we write an assignment a such that $a(x) = d$ as $[x \mapsto d, \ldots]$. The interpretation functions $[\![\cdot]\!]$ for DRSs and conditions[3] are defined with respect to a model M as follows:

1. If $E \equiv P(x_1, \ldots, x_n)$, then
$$[\![E]\!]_M = \{a \mid \langle a(x_1), \ldots, a(x_n)\rangle \in I(P)\}.$$
2. If $E \equiv x_1 = x_2$, then
$$[\![E]\!]_M = \{a \mid a(x_1) = a(x_2)\}.$$
3. If $E \equiv [\mathcal{X} \mid \mathcal{C}]$, then
$$[\![E]\!]_M(a) = \{a' \mid a \subseteq_{\mathcal{X}} a' \wedge a' \in \bigcap_{c \in \mathcal{C}} [\![c]\!]_M\}.$$
Here, $a \subseteq_{\mathcal{X}} a'$ means that $\mathtt{Dom}(a') = \mathtt{Dom}(a) \cup \mathcal{X}$ ($\mathtt{Dom}(a)$ is the domain of a. In addition, $\mathtt{Dom}(a) \cap \mathcal{X} = \emptyset$) and $a(x) = a'(x)$ for all $x \in \mathtt{Dom}(a)$. If there is a set $\mathcal{X}$ such that $a \subseteq_{\mathcal{X}} a'$, we call a' an *extension* of a and write $a \subseteq a'$.
4. If $E \equiv (E_1; E_2)$, then
$$[\![E]\!]_M(a) = \{a'' \mid \exists a' \in [\![E_1]\!]_M(a)\big(a'' \in [\![E_2]\!]_M(a')\big)\}.$$
5. If $E \equiv \neg E_1$, then
$$[\![E]\!]_M = \{a \mid [\![E_1]\!]_M(a) = \emptyset\}.$$
6. If $E \equiv E_1 \vee E_2$, then
$$[\![E]\!]_M = \{a \mid [\![E_1]\!]_M(a) \cup [\![E_2]\!]_M(a) \neq \emptyset\}.$$
7. If $E \equiv E_1 \Rightarrow E_2$, then
$$[\![E]\!]_M = \{a \mid \forall a' \in [\![E_1]\!]_M(a)\big([\![E_2]\!]_M(a') \neq \emptyset\big)\}.$$

A DRS E is defined to be *true* in a model M if and only if $[\![E]\!]_M(\phi) \neq \emptyset$, where ϕ is the empty assignment ($\mathtt{Dom}(\phi) = \emptyset$). Otherwise, E is defined to be *false* in M.

As an example, consider the interpretation of the DRS S_6 for sentence (1), shown in Figure 1. S_6 is true in $M_{\mathtt{ex}}$ shown in Figure 2, since the following holds:

$$[\![S_6]\!]_{M_{\mathtt{ex}}}(\phi) = \{[\mathtt{x_1} \mapsto d_1]\}.$$

Intuitively, this interpretation means that there is a situation which satisfies the conditions of S_6, and that the discourse referent $\mathtt{x_1}$ introduced by the word "a" denotes the entity d_1 in the situation.

[3]The interpretation of a condition is the set of assignments that satisfy the condition.

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Word	Partial DRS
A	$S_1 \equiv U_2^{(\mathtt{S\backslash NP})}(\lambda Q.(([\mathtt{x}_1 \mid\ ]; U_1^{(\mathtt{N})}\mathtt{x}_1); Q\mathtt{x}_1))$
student	$S_2 \equiv U_2^{(\mathtt{S\backslash NP})}(\lambda Q.([\mathtt{x}_1 \mid \mathtt{stu}(\mathtt{x}_1)]; Q\mathtt{x}_1))$
uses	$S_3 \equiv [\mathtt{x}_1 \mid \mathtt{stu}(\mathtt{x}_1)]; (U_3^{(\mathtt{NP})}(\lambda Y.[\ \mid \mathtt{use}(\mathtt{x}_1, \mathtt{Y})]))$
every	$S_4 \equiv [\mathtt{x}_1 \mid \mathtt{stu}(\mathtt{x}_1), ([\mathtt{x}_4 \mid\ ]; U_4^{(\mathtt{N})}\mathtt{x}_4) \Rightarrow [\ \mid \mathtt{use}(\mathtt{x}_1, \mathtt{x}_4)]]$
red	$S_5 \equiv [\mathtt{x}_1 \mid \mathtt{stu}(\mathtt{x}_1), ([\mathtt{x}_4 \mid\ ]; ([\ \mid \mathtt{red}(\mathtt{x}_4)]; U_5^{(\mathtt{N})}\mathtt{x}_4)) \Rightarrow [\ \mid \mathtt{use}(\mathtt{x}_1, \mathtt{x}_4)]]$
laptop	$S_6 \equiv [\mathtt{x}_1 \mid \mathtt{stu}(\mathtt{x}_1), [\mathtt{x}_4 \mid \mathtt{red}(\mathtt{x}_4), \mathtt{laptop}(\mathtt{x}_4)] \Rightarrow [\ \mid \mathtt{use}(\mathtt{x}_1, \mathtt{x}_4)]]$

Table 2: Incremental semantic constructions for "A student uses every red laptop."

3 Incremental Construction of DRSs

We can construct a partial DRS for any sentence prefix by assigning variables to the underspecified parts of the sentence. Here, we will not discuss how to assign these variables, and instead refer the interested reader to the literature (Peldszus and Schlangen, 2012; Kato and Matsubara, 2015).

As an example, Table 2 shows the incremental construction of partial DRSs for the sentence (1). Here, the bracketed superscripts indicate the categories to which the variables correspond. To see how partial DRSs are constructed, let us consider the following sentence prefix, which will be followed by a category $\mathtt{N}$:

$$\text{A student uses every} \ldots \tag{2}$$

We can obtain the partial DRS S_4 by assigning a variable $U_4^{(\mathtt{N})}$ of type $\langle \mathtt{et} \rangle$ to the underspecified part of the sentence. Figure 3 shows the derivations of the partial DRSs.

We treat incremental semantic construction as the process of substituting concrete expressions for variables in semantic representations, and formalize it as follows.

- Let E_1 and E_2 be expressions in β-normal form, and let U be a free variable that occurs in E_1. If there exists an E' such that $E_1[U := E'] \twoheadrightarrow_\beta E_2$, then we write $E_1 \rhd E_2$. Here, $E[U := E']$ is the capture-avoiding substitution of E' for U in E. When $E_1 \rhd^* E_2$, we say that E_2 is derived from E_1.

The relation $\rhd$ represents the incremental semantic construction process. For example, $S_2 \rhd S_3$, because $S_2[U_2^{(\mathtt{S\backslash NP})} := E^{(\mathtt{S\backslash NP})}] \twoheadrightarrow_\beta S_3$ where $E^{(\mathtt{S\backslash NP})}$ is obtained by combining the expression for "use" and a variable $U_3^{(\mathtt{NP})}$ of type $\langle\langle \mathtt{et} \rangle \mathtt{t}\rangle$:

$$E^{(\mathtt{S\backslash NP})} \equiv \lambda R.R(\lambda X.U_3^{(\mathtt{NP})}(\lambda Y.[\ \mid \mathtt{use}(\mathtt{X}, \mathtt{Y})]))$$

4 Interpretation of Partial DRSs

In this section, we propose a method of semantically interpreting partial DRSs. Since sentence prefixes may not have any propositional content (Chater et al., 1995), we give alternative semantic values to partial DRSs instead. The essential idea is to consider two types of interpretation: one stipulates which updates will necessarily be included by the complete DRS derived from the partial DRS ($[\![\cdot]\!]_M^\Box$), and the other stipulates which updates may be included ($[\![\cdot]\!]_M^\Diamond$). We call these $\Box$-*interpretations* and $\Diamond$-*interpretations*, respectively. At the end of this section, we will show that these interpretations can assign truth values to sentence prefixes in a consistent manner.

The interpretations are defined as follows:

1. If E is a basic condition, then
$$[\![E]\!]_M^\Box = [\![E]\!]_M^\Diamond = [\![E]\!]_M.$$
2. If E is a variable of type $\mathtt{t}$ or an expression of the form $(E_1 E_2)$ of type $\mathtt{t}$, then
$$[\![E]\!]_M^\Box(a) = \emptyset,$$
$$[\![E]\!]_M^\Diamond(a) = \{a\}.$$

Below, we only give the inductive clauses of $[\![\cdot]\!]^\Box$, but those of $[\![\cdot]\!]^\Diamond$ can be obtained by swapping $\Box$ and $\Diamond$.

3. If $E \equiv [\mathcal{X} \mid \mathcal{C}]$, then
$$[\![E]\!]_M^\Box(a) = \{a' \mid a \subseteq_\mathcal{X} a' \wedge a' \in \bigcap_{c \in \mathcal{C}} [\![c]\!]_M^\Box\}.$$

$$\dfrac{\dfrac{\mathtt{NP/N} : \lambda PQ.(([\mathtt{x}_1 \mid];P\mathtt{x}_1);Q\mathtt{x}_1) \qquad \mathtt{N} : U_1^{(\mathtt{N})}}{\mathtt{NP} : \lambda Q.(([\mathtt{x}_1 \mid]; U_1^{(\mathtt{N})}\mathtt{x}_1); Q\mathtt{x}_1)} \qquad \mathtt{S\backslash NP} : U_2^{(\mathtt{S\backslash NP})}}{\mathtt{S} : S_1 \equiv U_2^{(\mathtt{S\backslash NP})}(\lambda Q.(([\mathtt{x}_1 \mid]; U_1^{(\mathtt{N})}\mathtt{x}_1); Q\mathtt{x}_1))}$$

$$\dfrac{\dfrac{\mathtt{NP/N} : \lambda PQ.(([\mathtt{x}_1 \mid];P\mathtt{x}_1);Q\mathtt{x}_1) \qquad \mathtt{N} : \lambda X.[\ \mid \mathtt{stu}(X)]}{\mathtt{NP} : \lambda Q.([\mathtt{x}_1 \mid \mathtt{stu}(\mathtt{x}_1)];Q\mathtt{x}_1)} \qquad \dfrac{\mathtt{(S\backslash NP)/NP} : \lambda PQ.Q(\lambda X.P(\lambda Y.[\ \mid \mathtt{use}(X,Y)])) \qquad \dfrac{\mathtt{NP/N} : \lambda PQ.[\ \mid ([\mathtt{x}_4 \mid];P\mathtt{x}_4) \Rightarrow Q\mathtt{x}_4] \qquad \mathtt{N} : U_4^{(\mathtt{N})}}{\mathtt{NP} : \lambda Q.[\ \mid ([\mathtt{x}_4 \mid];U_4^{(\mathtt{N})}\mathtt{x}_4) \Rightarrow Q\mathtt{x}_4]}}{\mathtt{S\backslash NP} : \lambda Q.Q(\lambda X.[\ \mid ([\mathtt{x}_4 \mid];U_4^{(\mathtt{N})}\mathtt{x}_4) \Rightarrow [\ \mid \mathtt{use}(X,\mathtt{x}_4)]])}}{\mathtt{S} : S_4 \equiv [\mathtt{x}_1 \mid \mathtt{stu}(\mathtt{x}_1), ([\mathtt{x}_4 \mid];U_4^{(\mathtt{N})}\mathtt{x}_4) \Rightarrow [\ \mid \mathtt{use}(\mathtt{x}_1,\mathtt{x}_4)]]}$$

Figure 3: Derivations of partial DRSs.

4. If $E \equiv (E_1; E_2)$, then

$$\begin{aligned}&[\![E]\!]^{\Box}_M(a)\\ &\quad = \{a'' \mid \exists a' \in [\![E_1]\!]^{\Box}_M(a)\big(a'' \in [\![E_2]\!]^{\Box}_M(a')\big)\}.\end{aligned}$$

5. If $E \equiv \neg E_1$, then

$$[\![E]\!]^{\Box}_M = \{a \mid [\![E_1]\!]^{\Diamond}_M(a) = \emptyset\}.$$

6. If $E \equiv E_1 \vee E_2$, then

$$[\![E]\!]^{\Box}_M = \{a \mid [\![E_1]\!]^{\Box}_M(a) \cup [\![E_2]\!]^{\Box}_M(a) \neq \emptyset\}.$$

7. If $E \equiv E_1 \Rightarrow E_2$, then

$$\begin{aligned}&[\![E]\!]^{\Box}_M\\ &\quad = \{a \mid \forall a' \in [\![E_1]\!]^{\Diamond}_M(a)\big([\![E_2]\!]^{\Box}_M(a') \neq \emptyset\big)\}.\end{aligned}$$

The key point in this definition is Clause 2. Since any DRS E' can be derived from E,[4] there are no necessary updates ($[\![E]\!]^{\Box}_M(a) = \emptyset$), and any extension of a has a possibility of becoming an update ($[\![E]\!]^{\Diamond}_M(a) = \{a\}$). Furthermore, it is remarkable that the definitions of $[\![\cdot]\!]^{\Box}_M$ and $[\![\cdot]\!]^{\Diamond}_M$ are mutually recursive (Clauses 5 and 7). This indicates that $[\![\cdot]\!]^{\Box}_M$ and $[\![\cdot]\!]^{\Diamond}_M$ are complementary to each other, and that we must consider the two types of interpretations simultaneously.

The interpretation functions $[\![\cdot]\!]^{\Box}_M$ and $[\![\cdot]\!]^{\Diamond}_M$ have several interesting properties.

[4]If E is a variable of type α, any expression E' of type α can be derived from E, because $E[E := E'] \twoheadrightarrow_\beta E'$. If E is of the form E_1E_2, E_1 is a variable or an expression of the form E_3E_4 or $\lambda P.E_5$. If E_1 is a variable, then any expression E' of type α can be derived from E_1E_2, since $(E_1E_2)[E_1 := \lambda P.E'] \twoheadrightarrow_\beta E'$ (where P is a fresh variable). If E_1 is of the form E_3E_4, we can prove that $\lambda P.E'$ can be derived from E_3E_4 by mathematical induction. Any expression E' of type α therefore can be derived from E_1E_2. The third case $\lambda P.E_5$ is not allowed, since $E \equiv (\lambda P.E_5)E_2$ is a β-redex, i.e., E is not in β-normal form.

Theorem 1 (upper and lower bounds)**.** For any partial DRS E, model M, and assignment a, the following statements hold:

$$[\![E]\!]^{\Box}_M(a) \subseteq \{a' \mid a \subseteq_{\mathtt{DR}(E)} a'\} \cap \left(\bigcap_{E' \in \mathtt{Comp}(E)} \{a' \mid a' \sqsubseteq [\![E']\!]_M(a)\}\right) \tag{3}$$

$$[\![E]\!]^{\Diamond}_M(a) \supseteq \{a' \mid a \subseteq_{\mathtt{DR}(E)} a'\} \cap \left(\bigcup_{E' \in \mathtt{Comp}(E)} \{a' \mid a' \sqsubseteq [\![E']\!]_M(a)\}\right) \tag{4}$$

Here, $\mathtt{Comp}(E)$ is the set of complete expressions derived from E, and $a \sqsubseteq A$ means that $a \subseteq a'$ holds for some assignment $a' \in A$. $\mathtt{DR}(E)$ is a set of discourse referents and defined as follows.

$$\mathtt{DR}(E) = \begin{cases} \mathcal{X} & (E \equiv [\mathcal{X} \mid \mathcal{C}]) \\ \mathtt{DR}(E_1) \cup \mathtt{DR}(E_2) & (E \equiv (E_1; E_2)) \\ \emptyset & (\text{otherwise}) \end{cases}$$

The right-hand sides of the set inclusion relations (3) and (4) represent necessary and possible updates, respectively. We have equality in (3) and (4) when all free variables in E occur at most once.[5]

We can obtain Theorem 2, which concerns the truth values of sentence prefixes by the following lemmas.

Lemma 1 (monotonicity)**.** Let E_1 and E_2 be expressions of type $\mathtt{t}$ such that $E_1 \rhd^* E_2$. For any model M and assignment a, the following statements hold:

[5]When a free variable occurs more than once, our interpretation functions treat each occurrence of the variable independently, which is why the equality does not always hold. We plan to solve this problem in future work.

$$\texttt{N/N} : \lambda PX.([\,|\,\texttt{red}(X)];PX) \qquad \texttt{N} : \lambda X.[\,|\,\texttt{laptop}(X)]$$

$$\texttt{NP/N} : \lambda PQ.[|\ ([\texttt{x}_4\,|\,];P\texttt{x}_4) \Rightarrow Q\texttt{x}_4] \qquad \texttt{N} : \lambda X.[\,|\,\texttt{red}(X),\texttt{laptop}(X)]$$

$$\texttt{(S\textbackslash NP)/NP} : \lambda PQ.Q(\lambda X.P(\lambda Y.[\,|\,\texttt{use}(X,Y)])) \qquad \texttt{NP} : \lambda Q.[\,|\ [\texttt{x}_4\,|\,\texttt{red}(\texttt{x}_4),\texttt{laptop}(\texttt{x}_4)] \Rightarrow Q\texttt{x}_4]$$

$$\texttt{NP/N} : \lambda PQ.(([\texttt{x}_1\,|\,];P\texttt{x}_1);Q\texttt{x}_1) \qquad \texttt{N} : \lambda X.[\,|\,\texttt{stu}(X)]$$

$$\texttt{NP} : \lambda Q.([\texttt{x}_1\,|\,\texttt{stu}(\texttt{x}_1)];Q\texttt{x}_1) \qquad \texttt{S\textbackslash NP} : \lambda Q.Q(\lambda X.[\,|\ [\texttt{x}_4\,|\,\texttt{red}(\texttt{x}_4),\texttt{laptop}(\texttt{x}_4)] \Rightarrow [\,|\,\texttt{use}(X,\texttt{x}_4)]])$$

$$\texttt{S} : S_6 \equiv [\texttt{x}_1\,|\,\texttt{stu}(\texttt{x}_1), [\texttt{x}_4\,|\,\texttt{red}(\texttt{x}_4),\texttt{laptop}(\texttt{x}_4)] \Rightarrow [\,|\,\texttt{use}(\texttt{x}_1,\texttt{x}_4)]]$$

Figure 1: Derivation for sentence (1).

$D = \{d_1, d_2, d_3, d_4, d_5, d_6, d_7\}$
$I(\texttt{stu}) = \{\langle d_1\rangle, \langle d_2\rangle\}$
$I(\texttt{laptop}) = \{\langle d_3\rangle, \langle d_4\rangle\}$
$I(\texttt{tablet}) = \{\langle d_5\rangle, \langle d_6\rangle, \langle d_7\rangle\}$
$I(\texttt{red}) = \{\langle d_3\rangle, \langle d_5\rangle, \langle d_6\rangle\}$
$I(\texttt{blue}) = \{\langle d_4\rangle, \langle d_7\rangle\}$
$I(\texttt{use}) = \{\langle d_1, d_3\rangle, \langle d_1, d_4\rangle, \langle d_1, d_5\rangle, \langle d_1, d_6\rangle, \langle d_2, d_7\rangle\}$

Figure 2: Model $M_{\texttt{ex}}$.

- For any assignment $a_1 \in [\![E_1]\!]^{\square}_M(a)$, there is an assignment $a_2 \in [\![E_2]\!]^{\square}_M(a)$ such that $a_1 \subseteq a_2$.
- For any assignment $a_2 \in [\![E_2]\!]^{\Diamond}_M(a)$, there is an assignment $a_1 \in [\![E_1]\!]^{\Diamond}_M(a)$ such that $a_1 \subseteq a_2$.

Lemma 2 (consistency)**.** For any complete DRS E, model M, and assignment a, the following holds:

$$[\![E]\!]^{\square}_M(a) = [\![E]\!]^{\Diamond}_M(a) = [\![E]\!]_M(a)$$

Theorem 2 (truth values of partial DRSs)**.** Let E be a partial DRS. For any model M and DRS $E' \in \texttt{Comp}(E)$, the following statements hold.

- E' is true in M if $[\![E]\!]^{\square}_M(\phi) \neq \emptyset$.
- E' is false in M if $[\![E]\!]^{\Diamond}_M(\phi) = \emptyset$.

Below, we say that a partial DRS E is true in M if $[\![E]\!]^{\square}_M(\phi) \neq \emptyset$, and false in M if $[\![E]\!]^{\Diamond}_M(\phi) = \emptyset$.

4.1 Example

Table 3 shows interpretations of partial DRSs for example sentence (1). Below, we clarify our proposed semantics using this example.

4.1.1 Truth value of a partial DRS

As an example, let us consider the interpretation of partial DRS S_4 (Table 2) in the model $M_{\texttt{ex}}$ (Figure 2). For any entity $d_i (1 \leq i \leq 7)$, we have the following:

$$\begin{aligned} &[\![[\texttt{x}_4\ |\ ]]\!]^{\Diamond}_{M_{\texttt{ex}}}([\texttt{x}_1 \mapsto d_i]) \\ &\quad = \{[\texttt{x}_1 \mapsto d_i, \texttt{x}_4 \mapsto d_j] \mid 1 \leq j \leq 7\} \end{aligned} \tag{5}$$

For any assignment a, Clause 2 gives us the following:

$$[\![U_4^{(\texttt{N})}\texttt{x}_4]\!]^{\Diamond}_{M_{\texttt{ex}}}(a) = \{a\} \tag{6}$$

Word	$[\![\cdot]\!]^{\Box}$	$[\![\cdot]\!]^{\Diamond}$
A	$[\![S_1]\!]^{\Box}_{M_{\mathsf{ex}}}(\phi)=\emptyset$	$[\![S_1]\!]^{\Diamond}_{M_{\mathsf{ex}}}(\phi)=\{\phi\}$
student	$[\![S_2]\!]^{\Box}_{M_{\mathsf{ex}}}(\phi)=\emptyset$	$[\![S_2]\!]^{\Diamond}_{M_{\mathsf{ex}}}(\phi)=\{\phi\}$
uses	$[\![S_3]\!]^{\Box}_{M_{\mathsf{ex}}}(\phi)=\emptyset$	$[\![S_3]\!]^{\Diamond}_{M_{\mathsf{ex}}}(\phi)=\{[\mathtt{x_1}\mapsto d_1],[\mathtt{x_1}\mapsto d_2]\}$
every	$[\![S_4]\!]^{\Box}_{M_{\mathsf{ex}}}(\phi)=\emptyset$	$[\![S_4]\!]^{\Diamond}_{M_{\mathsf{ex}}}(\phi)=\{[\mathtt{x_1}\mapsto d_1],[\mathtt{x_1}\mapsto d_2]\}$
red	$[\![S_5]\!]^{\Box}_{M_{\mathsf{ex}}}(\phi)=\{[\mathtt{x_1}\mapsto d_1]\}$	$[\![S_5]\!]^{\Diamond}_{M_{\mathsf{ex}}}(\phi)=\{[\mathtt{x_1}\mapsto d_1],[\mathtt{x_1}\mapsto d_2]\}$
laptop	$[\![S_6]\!]^{\Box}_{M_{\mathsf{ex}}}(\phi)=\{[\mathtt{x_1}\mapsto d_1]\}$	$[\![S_6]\!]^{\Diamond}_{M_{\mathsf{ex}}}(\phi)=\{[\mathtt{x_1}\mapsto d_1]\}$

Table 3: Incremental interpretations of “A student uses every red laptop.”

From equations (5) and (6), and Clause 4, we have

$$[\![([\mathtt{x_4}\mid\ ];U^{(\mathtt{N})}_4\mathtt{x_4})]\!]^{\Diamond}_{M_{\mathsf{ex}}}([\mathtt{x_1}\mapsto d_i]) \\ =\{[\mathtt{x_1}\mapsto d_i,\mathtt{x_4}\mapsto d_j]\mid 1\le j\le 7\} \quad (7)$$

On the other hand, $\langle d_i, d_1\rangle \notin I(\mathtt{use})$ in the model $M_{\mathtt{ex}}$ gives us the following equation:

$$[\![[\ \mid \mathtt{use}(\mathtt{x_1},\mathtt{x_4})]]\!]^{\Box}_{M_{\mathsf{ex}}}([\mathtt{x_1}\mapsto d_i,\mathtt{x_4}\mapsto d_1])=\emptyset \quad (8)$$

By equations (7) and (8), and Clause 7, we obtain the following:

$$[\mathtt{x_1}\mapsto d_i]\notin[\![([\mathtt{x_4}\mid];U^{(\mathtt{N})}_4\mathtt{x_4})\Rightarrow[\mid\mathtt{use}(\mathtt{x_1},\mathtt{x_4})]]\!]^{\Box}_{M_{\mathsf{ex}}}$$

This means any assignment a such that $\phi \subseteq_{\{\mathtt{x_1}\}} a$ does not belong to the $\Box$-interpretaton of the second condition of S_4, so we find the following:

$$[\![S_4]\!]^{\Box}_{M_{\mathsf{ex}}}(\phi)=\emptyset \quad (9)$$

In other words, there are no assignments that satisfy the conditions of any DRS derived from S_4.

Next, let us consider the $\Diamond$-interpretation of S_4. From $[\![U^{(\mathtt{N})}_4\mathtt{x_4}]\!]^{\Box}_{M_{\mathsf{ex}}}=\emptyset$, we obtain the following:

$$[\![([\mathtt{x_4}\mid\ ];U^{(\mathtt{N})}_4\mathtt{x_4})]\!]^{\Box}_{M_{\mathsf{ex}}}([\mathtt{x_1}\mapsto d_i])=\emptyset \quad (10)$$

Therefore, for any assignment a, we have

$$a\in[\![([\mathtt{x_4}\mid];U^{(\mathtt{N})}_4\mathtt{x_4})\Rightarrow[\mid\mathtt{use}(\mathtt{x_1},\mathtt{x_4})]]\!]^{\Diamond}_{M_{\mathsf{ex}}} \quad (11)$$

Since the $\Diamond$-interpretation of the first condition of S_4 has members $[\mathtt{x_1}\mapsto d_1]$ and $[\mathtt{x_1}\mapsto d_2]$, and that of the second condition also has the same members, we obtain the following:

$$[\![S_4]\!]^{\Diamond}_{M_{\mathsf{ex}}}(\phi)=\{[\mathtt{x_1}\mapsto d_1],[\mathtt{x_1}\mapsto d_2]\}$$

By Theorem 2, we can therefore conclude that the partial DRS S_4 is neither true nor false in the model $M_{\mathtt{ex}}$.

Let us now consider another example, where the word “red” follows (2):

$$\text{A student uses every red } \ldots \quad (12)$$

The partial DRS of the prefix (12) is S_5, and its interpretations are as follows:

$$[\![S_5]\!]^{\Box}_{M_{\mathsf{ex}}}(\phi)=\{[\mathtt{x_1}\mapsto d_1]\},$$

$$[\![S_5]\!]^{\Diamond}_{M_{\mathsf{ex}}}(\phi)=\{[\mathtt{x_1}\mapsto d_1],[\mathtt{x_1}\mapsto d_2]\}$$

The important point here is that any DRS derived from S_5 is guaranteed to be true, independently of any input that follows (12), due to Theorem 2. These examples demonstrate that our framework can discuss the propositional contents of sentence prefixes, even if they are not sentential clauses.

4.1.2 Negation

Here, let us consider another sentence prefix, namely the following:

$$\text{A student doesn't use every} \ldots \quad (13)$$

The partial DRS of this sentence prefix is as follows:

$$[\ \mid\neg S_4] \quad (14)$$

Since $[\![S_4]\!]^{\Box}_{M_{\mathsf{ex}}}(\phi)=\emptyset$ and $[\![S_4]\!]^{\Diamond}_{M_{\mathsf{ex}}}(\phi)\neq\emptyset$, we obtain $\phi\in[\![\neg S_4]\!]^{\Diamond}_{M_{\mathsf{ex}}}$ and $\phi\notin[\![\neg S_4]\!]^{\Box}_{M_{\mathsf{ex}}}$, respectively. We therefore have

$$[\![(14)]\!]^{\Diamond}_{M_{\mathsf{ex}}}(\phi)=\{\phi\} \quad (15)$$

and

$$[\![(14)]\!]^{\Box}_{M_{\mathsf{ex}}}(\phi)=\emptyset \quad (16)$$

At this point, we cannot determine the truth value of (14).

Word	Partial DRS	Sub-expression including $\mathtt{x}_1$	$[\![\cdot]\!]^{\Diamond}_{M_{\mathrm{ex}}}(\phi)$
A	$U_2^{(\mathtt{S\backslash NP})}(\lambda Q.(([\mathtt{x}_1 \mid\ ]; U_1^{(\mathtt{N})}\mathtt{x}_1); Q\mathtt{x}_1))$	$(([\mathtt{x}_1 \mid\ ]; U_1^{(\mathtt{N})}\mathtt{x}_1); Q\mathtt{x}_1)$	$\cup_{i=1,\ldots,7}\{[\mathtt{x}_1 \mapsto d_i]\}$
red	$U_2^{(\mathtt{S\backslash NP})}(\lambda Q.(([\mathtt{x}_1 \mid\ ]; ([\ \mid \mathtt{red}(\mathtt{x}_1)]; U_3^{(\mathtt{N})})); Q\mathtt{x}_1))$	$(([\mathtt{x}_1 \mid\ ]; ([\ \mid \mathtt{red}(\mathtt{x}_1)]; U_3^{(\mathtt{N})})); Q\mathtt{x}_1)$	$\cup_{i=3,5,6}\{[\mathtt{x}_1 \mapsto d_i]\}$
laptop	$U_2^{(\mathtt{S\backslash NP})}(\lambda Q.([\mathtt{x}_1 \mid \mathtt{red}(\mathtt{x}_1), \mathtt{laptop}(\mathtt{x}_1)]; Q\mathtt{x}_1))$	$([\mathtt{x}_1 \mid \mathtt{red}(\mathtt{x}_1), \mathtt{laptop}(\mathtt{x}_1)]; Q\mathtt{x}_1)$	$\{[\mathtt{x}_1 \mapsto d_3]\}$

Table 4: Incremental reference resolution of “A red laptop…”

Next, we consider the following sentence prefix:

$$\text{A student doesn't use every red} \ldots \qquad (17)$$

Now, its partial DRS is $[\ \mid \neg S_5]$. Since $[\![S_5]\!]^{\Box}_{M_{\mathrm{ex}}}(\phi) \neq \emptyset$, i.e., $\phi \notin [\![\neg S_5]\!]^{\Diamond}_{M_{\mathrm{ex}}}$, we have $[\![[\ \mid \neg S_5]]\!]^{\Diamond}_{M_{\mathrm{ex}}}(\phi) = \emptyset$. By Theorem 2, we can therefore conclude that $[\ \mid \neg S_5]$ is false in M_{ex}.

4.1.3 Referential interpretation

By applying the $\Diamond$-interpretation function to DRS's sub-expressions that include discourse referents, we can identify the entities to which the discourse referents refer. Here, let us consider the sentence prefix

$$\text{A red laptop} \ldots \qquad (18)$$

Table 4 shows the partial DRSs, the sub-expressions, and their $\Diamond$-interpretations. In $\Diamond$-interpretations, the entities to which discourse referents can refer are incrementally specified. This example demonstrates that our semantics has a potential to be useful for incremental reference resolution (Schlangen et al., 2009).

5 Comparisons with Previous Work

Unlike incremental semantic construction, there has been little work on how to interpret partial semantic representations incrementally, with two exceptions: (Schuler et al., 2009) and (Hough and Purver, 2014). These papers proposed an incremental referential interpretation where noun phrase prefixes are interpreted as entities to which the noun phrase derived from the prefix can refer. In their interpretation process, such entities are incrementally specified. Our $\Diamond$-interpretaion provides a similar mechanism, as shown in Section 4.1.3. In addition, our approach provides a method of determining the truth values of sentence prefixes (Theorem 2), whereas that of the previous studies has no way to deal with truth values, and thus cannot offer sentential interpretations. Furthermore, their semantics cannot treat quantifiers, while ours provides interpretations of both existential and universal quantifiers.

Chater et al. (1995) adopted another approach to incremental interpretation called *two-level incremental interpretation*. One level carries out incremental semantic construction, while the other serves as interpretation. At the first level, a first-order formula with λ-operators is constructed incrementally. At the second level, this formula is then converted into a first-order formula without λ-operators by an existential closure-like mechanism (replacing the λ-operators with existential quantifiers). Since the second level representation is a proper first-order formula, it can be interpreted by standard semantics. However, this approach has the drawback that the truth value of the formula may be inconsistent with that of the final formula obtained from the whole sentence, even when there is no misanalysis at the first level. This issue is inevitable as long as the principle of bivalence is adopted, because there are cases where truth value of sentence prefix cannot be determined. In contrast, this is not the case for our proposed incremental interpretation, because it allows sentence prefixes to have truth-value gaps, i.e., to be neither true nor false. This is achieved by using the two types of interpretations proposed above.

6 Conclusion

This paper has proposed a model-theoretic incremental interpretation framework that can treat the propositional contents of sentence prefixes, including phenomena such as negation and quantification. We believe that this framework will help to clarify the roles semantic representations can play in incremental processing.

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