

Nonparametric Forest-Structured Neural Topic Modeling

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Abstract

Neural topic models have been widely used in discovering the latent semantics from a corpus. Recently, there are several researches on hierarchical neural topic models since the relationships among topics are valuable for data analysis and exploration. However, the existing hierarchical neural topic models are limited to generate a single topic tree. In this study, we present a nonparametric forest-structured neural topic model by firstly applying the self-attention mechanism to capture parent-child topic relationships, and then build a sparse directed acyclic graph to form a topic forest. Experiments indicate that our model can automatically learn a forest-structured topic hierarchy with indefinite numbers of trees and leaves, and significantly outperforms the baseline models on topic hierarchical rationality and affinity.

1 Introduction

Topic model has been widely used in modeling a collection of documents and encoding the text content to a low dimensional feature space. Traditional topic models can be divided into probabilistic graphical models and matrix factorization based methods. Probabilistic graphical models, such as Latent Dirichlet Allocation (LDA) (Blei et al., 2003b), rely on approximate approaches (e.g., variational inference and Gibbs sampling) with complex derivation or high computational costs to estimate parameters (Srivastava and Sutton, 2017). Matrix factorization based methods (Lee and Seung, 1999) can effectively decompose the document-word representation into two sub-matrices but are subject to a low stability (Chen et al., 2021b). Recently, neural topic models based on Neural Variational Inference (NVI) (Srivastava and Sutton, 2017; Miao et al., 2017; Chen et al., 2021b) have attracted great attention owing to the advantages of fast parameter inference and flexibility.

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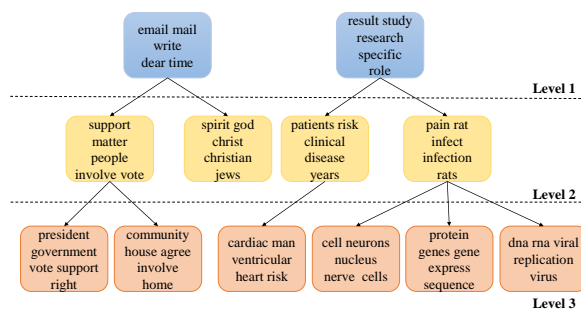


Figure 1: An example of forest-structured topics. Each topic is represented by 5 top words.

Despite some notable successes for neural topic models, most of the existing methods can only extract topics at the same level. This may cause confusions on identifying the hierarchical structure about the relationships among topics, which is valuable for data analysis and exploration in various domains (Paisley et al., 2015). To address this, a few neural topic models have been developed to model the hierarchical structure of topics (Isonuma et al., 2020; Chen et al., 2021b,a). The above methods, however, all assume that the hierarchical structure is a tree with a single root node. This is a significant limitation because for a real-world corpus, topics can be organized into several trees, where the structure of each tree is independent. As an illustration, Figure 1 shows two root topics on email and study. Topics at level 2 include email contents (i.e., politics and religion) and those describing the mode of study (i.e., patients and rats). Some of the topics at level 2 have several children topics which are very specific to distinguish the scope of politics (e.g., country and community) or focus of biology (e.g., cell, protein and genes), and others have only one child topic or none. It indicates that the real-world hierarchical topic structure is more likely to be a forest rather than a tree.

The forest-structured topic models still face challenges since the hierarchical structure of topics

needs to be (1) rational—root topics are general and children topics are specific to their corresponding parent topics (Viegas et al., 2020); (2) affinitive—each topic is more similar to their children topics than topics from other parents (Kim et al., 2012); (3) diverse—the topic-word distributions associated with parents and children are distinguishable (Blei et al., 2003a); (4) flexible—children of each topic are automatically assigned (Kim et al., 2012) and topic numbers at each level are unbounded (i.e., nonparametric) (Chen et al., 2021b). Besides, the number of root topics, depth, and width of a forest structure are hard to be pre-defined.

In this work, we propose a nonparametric Forest-structured Neural Topic Model (nFNTM) to tackle these challenges, which firstly captures parent-child topic relationships based on the self-attention mechanism (Vaswani et al., 2017)¹, and then learns a sparse Directed Acyclic Graph (DAG) to build a topic forest by federating document-topic distributions and parent-child relationships based on NVI. To our best knowledge, the current topic models with a DAG structure are based on Bayesian learning (Li and McCallum, 2006; Mimno et al., 2007) or Non-negative Matrix Factorization (NMF) (Liu et al., 2018; Viegas et al., 2020), and there is no work under the NVI framework. To sum up, the main contributions are summarized as follows:

- We are the first to introduce the sparse DAG into neural topic modeling with the aim of learning a forest-structured topic hierarchy.
- We develop a self-attention mechanism to capture the relationships among topics.
- We evaluate nFNTM on three benchmark datasets. Empirical results indicate that our model significantly outperforms baselines.

2 Related Work

Traditional topic models, such as LDA (Blei et al., 2003b), are powerful tools for modeling text in an unsupervised fashion, while they lack the exploration of the relationship among topics. To overcome this issue, a tree-structured topic model named hLDA (Blei et al., 2003a) was first proposed. In hLDA, the nested Chinese Restaurant Process

¹We use the self-attention mechanism as it is effective to draw the global dependencies between input tokens with little reliance on the external information (Yao et al., 2021). Furthermore, it allows efficient computation by parallelization.

(nCRP) was used to generate a topic tree. To alleviate the single-path constraint assumed by nCRP, a nested Hierarchical Dirichlet Process (HDP), i.e., nHDP (Paisley et al., 2015) was developed, which provided the ability of cross-thematic borrowing while keeping general topic areas in separate subtrees. The nested Chinese Restaurant Franchise (nCRF) process developed in (Ahmed et al., 2013) combined the advantages of HDP (Teh et al., 2004) and nCRP. In (Kim et al., 2012), the recursive Chinese Restaurant Process (rCRP) was proposed to discover a hierarchical topic structure with unbounded depth and width.

These models based on the Chinese restaurant process can be effectively employed to discover the hierarchical topic structure by Bayesian learning, but the posterior inference method requires a high computational cost. The scalability of NMF-based methods (Liu et al., 2018; Viegas et al., 2020) is also quite limited. There is a new direction to build tree-structured topic models based on NVI due to its advantages of fast parameter inference and flexibility. A tree-structured neural topic model (TSNTM) (Isonuma et al., 2020) was proposed, which applied doubly-recurrent neural networks to parameterize topic distributions over a tree. But it lacked the ability of learning appropriate semantic embeddings for topics and relied on heuristic rules to update the tree structure. A nonparametric tree-structured neural topic model (nTSNTM) (Chen et al., 2021b) tackled these weaknesses by directly sampling the leaf topics and generating the paths from bottom up automatically. nTSNTM used a common stick-breaking construction to infer topic distributions from the leaf nodes to the root node and applied dependency matrices to keep track of the affiliations among topics. However, the dependency matrices which determine the topic hierarchy are neural weights between the network layers. It results in the structure (i.e., depth and width) of the tree can only be set in advance.

3 Methodology

3.1 Model Architecture

Our nFNTM consists of an encoder, a topic attention, and a decoder, as shown in Figure 2.

3.1.1 Encoder

Given a collection of documents, each document $d \in \mathbb{R}^V$ is represented by Bag-of-Words (BoW), where V is the vocabulary size. For the encoder,

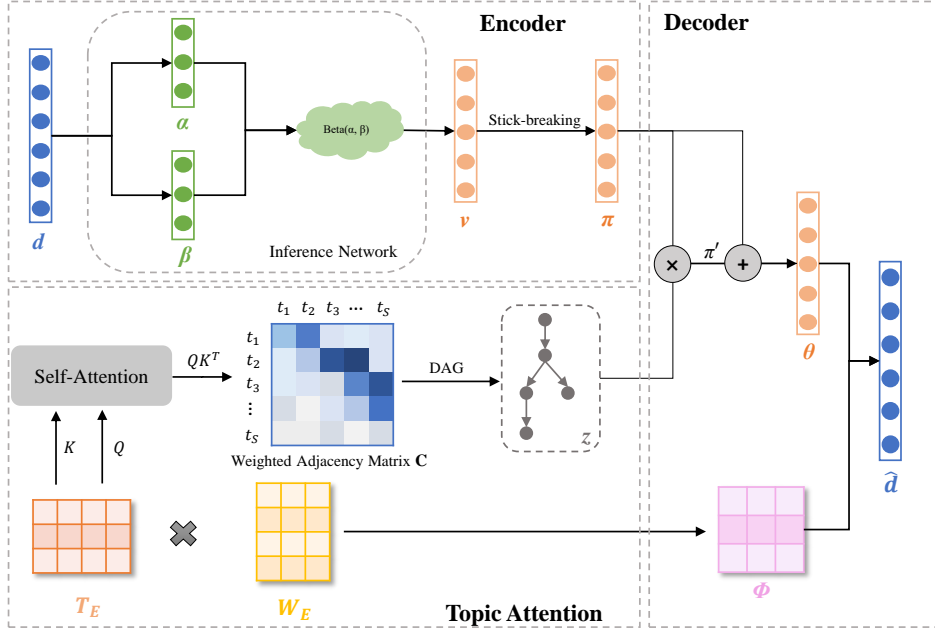


Figure 2: Structure of nFNTM.

we transform \mathbf{d} into document-topic distribution π by the Stick-Breaking Process (SBP) (Ishwaran and James, 2001), which provides a solution to define atomic measures associated Bayesian non-parametric methods. Any almost sure (a.s.) discrete probability measure \mathcal{P} is an SBP if it can be represented by:

$$\mathcal{P} = \sum_{i=1}^{\infty} \pi_i \delta_{x_i}, \quad \pi_i = \begin{cases} v_1 & i = 1, \\ v_i \prod_{t < i} (1 - v_t) & i > 1, \end{cases} \quad (1)$$

where $x_i \sim \mathcal{H}$, \mathcal{H} is the base probability measure, δ_{x_i} is a discrete measure concentrated at x_i , $\mathbf{v} \sim \text{Beta}(\boldsymbol{\alpha}, \boldsymbol{\beta})$ with $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ being the prior parameters, $\{\pi_i\}$ are random weights independent of \mathcal{H} and satisfy $0 \leq \pi_i \leq 1$ and $\sum_{i=1}^{\infty} \pi_i = 1$. SBP specifies to the Dirichlet process if $\mathbf{v} \sim \text{Beta}(1, \hat{\boldsymbol{\beta}})$, then the joint distribution over the infinite sequence of stick-breaking weights with concentration parameter $\hat{\boldsymbol{\beta}}$ is $\{\pi_i\} \sim \text{GEM}(\hat{\boldsymbol{\beta}})$ (Teh et al., 2004).

Note that the existing nTSNTM (Chen et al., 2021b) chooses the Kumaraswamy distribution (Kumaraswamy, 1980) to approximate the Beta distribution since it does not have a differentiable non-centered parametrization. However, the Beta distribution is a one-parameter subfamily of symmetric distributions and has more ways of generating the distribution via physical processes (Jones, 2009). To estimate the Beta distribution unbiasedly, we inference it through computing

implicit reparameterization gradients and the details will be introduced in Section 3.3. As shown in Figure 2, we introduce an inference network to build the Beta distribution. We obtain $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ by $\boldsymbol{\alpha} = l_{\alpha}(\boldsymbol{\eta})$ and $\boldsymbol{\beta} = l_{\beta}(\boldsymbol{\eta})$, where $\boldsymbol{\eta} = \text{MLP}(\mathbf{d})$ is the low-dimensional representation of \mathbf{d} , l_{α} and l_{β} are the single layer of linear connection, and MLP denotes a multilayer perceptron.

3.1.2 Topic Attention

We intend the model to attend on relationships among topics in a manner that the resultant attention is distributed according to the topics generated from the corpus. We employ topic embeddings $\mathbf{T}_E \in \mathbb{R}^{S \times d_t}$ to perform attention on topics, where $S \rightarrow \infty$ is the breaking number in SBP, i.e., the number of topics learned by SBP, and d_t is the dimension of topic embeddings. Considering that the relevance of each topic should be computed and learned independently, we regard each topic embedding as a subspace and project the input hidden representations to different topic embedding subspaces as follows:

$$\mathbf{Q} = \mathbf{T}_E \mathbf{W}_Q, \quad \mathbf{K} = \mathbf{T}_E \mathbf{W}_K, \quad (2)$$

where $\mathbf{W}_Q \in \mathbb{R}^{d_t \times d_r}$ and $\mathbf{W}_K \in \mathbb{R}^{d_t \times d_r}$ are trainable parameters, d_r is the dimension of each topic embedding subspace, $\mathbf{Q} \in \mathbb{R}^{S \times d_r}$ and $\mathbf{K} \in \mathbb{R}^{S \times d_r}$ denote the matrices of queries and keys. Based on queries and keys, we calculate a weighted adjacency matrix $\mathbf{C} \in \mathbb{R}^{S \times S}$, where each element

$C_{i,j}$ denotes the parent-child correlation degree of relevance between topics t_i and t_j ². To ensure that each element $C_{i,j}$ is discrete, a softmax function with temperature τ is applied (Hinton et al., 2015):

$$C_{i,j} = \frac{\exp(\frac{\hat{C}_{i,j}}{\tau})}{\sum_{k=1}^S \exp(\frac{\hat{C}_{i,k}}{\tau})}, \quad (3)$$

where $\hat{\mathbf{C}} = \mathbf{Q}\mathbf{K}^T$.

To further incorporate the parent-child correlations into document-topic distributions meanwhile maintaining the nonparametric characteristic from SBP, we firstly calculate the parent document-topic distribution by integrating the weighted adjacency matrix, i.e., $\pi' = \pi \times \mathbf{C}$. Particularly, as the weighted adjacency matrix has not captured parent-child correlations at the beginning, the document-topic distribution is initially represented by π . With the learning of our self-attention module, the quality of the weighted adjacency matrix \mathbf{C} is steadily enhanced, and π' becomes a valuable supplement of π due to strong parent-child topic relationships. Then, we generate the consolidated document-topic distribution by exploiting both π' and π as $\theta = (1-\gamma)\pi' + \gamma\pi$, where γ is a decay coefficient.

3.1.3 Decoder

As the BoW document representations lack of the word relatedness information, we incorporate pre-trained word embeddings \mathbf{W}_E (Viegas et al., 2020; Wu et al., 2020) into the network. For the decoder, we firstly obtain the topic-word distribution $\Phi = \text{softmax}(\mathbf{T}_E \times \mathbf{W}_E)$, where $\mathbf{W}_E \in \mathbb{R}^{d_t \times V}$ and $\Phi \in \mathbb{R}^{S \times V}$. Then, we reconstruct document $\hat{\mathbf{d}}$ by combining document-topic distribution θ with topic-word distribution Φ .

3.2 Topic Forest Hierarchy

To construct the topic forest, we build a forest hierarchy of topics from the weighted adjacency matrix \mathbf{C} which contains the parent-child relationships among topics. However, the structure of weighted adjacency matrix \mathbf{C} may not be a reasonable hierarchical structure, i.e., it may have loops if without constraints. To tackle this challenge, we propose to construct the structure of topics as a sparse DAG. Zheng et al. (2018) have proved that

²Different from (Vaswani et al., 2017), we capture the relationships among topics without using the value matrix. This is because we found experimentally that topics trained with value matrix tend to be the same, which might result from concentrating on several important topics.

a weighted adjacency matrix \mathbf{C} is a DAG if and only if $h(\mathbf{C}) = \text{tr}(e^{\mathbf{C} \circ \mathbf{C}}) - d = 0$, where \circ is the Hadamard product and $e^{\mathbf{C}}$ is the matrix exponential of \mathbf{C} . We employ $h(\mathbf{C}) = 0$ with an augmented Lagrangian method to ensure the acyclicity of the weighted adjacency matrix.

Considering that the topic hierarchy is reasonable and children topics can be represented by their parent topics, we assume that the sum of children topics' document-topic distributions is similar to their parent topics' document-topic distributions. Accordingly, we learn the forest-structured topic hierarchy by minimizing the difference between documents reconstructed by $\pi \times \Phi$ and those reconstructed by their parent document-topic distributions (i.e., $\pi' \times \Phi$) under the constraint of $h(\mathbf{C}) = 0$ using the augmented Lagrangian method, as follows:

$$\min_{\mathbf{C} \in \mathbb{R}^{S \times S}} \mathcal{L}_C = \frac{1}{2} \|(\pi - \pi') \times \Phi\|_F^2 + \frac{\rho}{2} |h(\mathbf{C})|^2 + \epsilon h(\mathbf{C}), \quad (4)$$

where ρ is a penalty parameter, and ϵ is the Lagrange multiplier. We update parameters ρ and ϵ by following (Zheng et al., 2018), as follows: $\begin{cases} \rho_i = 2\rho_{i-1} \\ \epsilon_i = \epsilon_{i-1} + \rho h_{i-1} \end{cases}$, where $\rho_0 = 1$, $\epsilon_0 = 0$, i is a training epoch, and h is a constraint value.

The generative process of nFNTM is described as follows:

1. For each document d :
 - (a) Draw a topic proportion $\pi \sim \text{GEM}(\hat{\beta})$;
 - (b) Get the weighted adjacency matrix \mathbf{C} ;
 - (c) Get the correlational topic distribution θ .
2. For each word $w_{d,n} \in d$:
 - (a) Draw a topic $z_{d,n} \sim \text{Mult}(\theta)$;
 - (b) Obtain the topic-word distribution Φ ;
 - (c) Draw a word $w_{d,n} \sim \text{Mult}(\Phi_{z_{d,n}})$.

3.3 Parameter Inference

We apply NVI to inference network parameters, which is proven to be efficient and flexible (Srivastava and Sutton, 2017; Miao et al., 2017; Isonuma et al., 2020; Chen et al., 2021b). The likelihood of each reconstructed document $\hat{\mathbf{d}}$ is estimated by $p(\hat{\mathbf{d}} | \theta, \Phi) = \sum_z p(\hat{\mathbf{d}} | \Phi_z) p(z | \theta)$, where z is the topic assigned for each word in $\hat{\mathbf{d}}$. To maximize the log-likelihood, we derive the lower bound

as follows:

$$\mathcal{L}_B = \mathbb{E}_{q(\boldsymbol{\theta}, \Phi | \mathbf{d})} \left[\log p(\hat{\mathbf{d}} | \boldsymbol{\theta}, \Phi) \right] - D_{KL} [q(\boldsymbol{\theta} | \mathbf{d}) \| p(\boldsymbol{\theta})] - \mathcal{L}_C, \quad (5)$$

where $\mathbb{E}_{q(\boldsymbol{\theta}, \Phi | \mathbf{d})} \left[\log p(\hat{\mathbf{d}} | \boldsymbol{\theta}, \Phi) \right]$ is the reconstruction loss, $D_{KL} [q(\boldsymbol{\theta} | \mathbf{d}) \| p(\boldsymbol{\theta})]$ is the Kullback-Leibler (KL) divergence between the prior Beta distribution $p(\boldsymbol{\theta})$ and the posterior Beta distribution $q(\boldsymbol{\theta} | \mathbf{d})$. The KL divergence of two Beta distributions is given below:

$$\begin{aligned} KL(\text{Beta}(\alpha_1, \beta_1) \| \text{Beta}(\alpha_2, \beta_2)) = & \\ & \ln \Gamma(\alpha_2) + \ln \Gamma(\beta_2) + \ln \Gamma(\alpha_1 + \beta_1) \\ & - (\ln \Gamma(\alpha_1) + \ln \Gamma(\beta_1) + \ln \Gamma(\alpha_2 + \beta_2)) \\ & + (\alpha_1 - \alpha_2)F(\alpha_1) \\ & + (\beta_1 - \beta_2)F(\beta_1) \\ & + (\alpha_2 + \beta_2 - \alpha_1 - \beta_1)F(\alpha_1 + \beta_1), \end{aligned} \quad (6)$$

where $F(x) = \frac{d}{dx} \ln \Gamma(x) = \frac{\Gamma'(x)}{\Gamma(x)}$.

Note that we transform \mathbf{d} to the variational Beta distribution, thus $q(\boldsymbol{\theta} | \mathbf{d})$ is derived by:

$$\begin{aligned} q(\boldsymbol{\theta} | \mathbf{d}) = & \text{Beta}(\boldsymbol{\theta} | \boldsymbol{\alpha}, \boldsymbol{\beta}) \\ = & \text{Beta}(\boldsymbol{\theta} | l_\alpha(\text{MLP}(\mathbf{d})), l_\beta(\text{MLP}(\mathbf{d}))). \end{aligned} \quad (7)$$

To compute reparameterization gradients, the Beta samples are obtained from Gamma samples since the latter do not require inverting the standardization function (Figurnov et al., 2018): for $z_1 \sim \text{Gamma}(\boldsymbol{\alpha}, 1)$ and $z_2 \sim \text{Gamma}(\boldsymbol{\beta}, 1)$, it has $\frac{z_1}{z_1 + z_2} \sim \text{Beta}(\boldsymbol{\alpha}, \boldsymbol{\beta})$. The lower bound \mathcal{L}_B is used to calculate gradients and parameters are updated by Adam (Kingma and Ba, 2015).

4 Experiment

4.1 Experimental Setup

Datasets To evaluate our nFNTM³ comprehensively, we conduct experiments on three datasets which are observed to be hierarchical (Chen et al., 2021b): 20News (Miao et al., 2017), Reuters (Wu et al., 2020), and Wikitext-103 (Nan et al., 2019). According to (Liu et al., 2018; Viegas et al., 2020), a topic often comprises of some sub-topics and owns a hierarchical structure in Web directory of news (e.g., 20News and Reuters) and encyclopedia (e.g., Wikitext-103 which is extracted from Wikipedia). For these datasets, each document

³<https://github.com/Angr4Mainyu/nFNTM>

is associated with a manually-curated hierarchy of categories. Take 20News as an example, “rec.autos” and “rec.motor.cycles” are sub categories of “rec”. Instead of relying on the prior coarse-grained categories, we build the hierarchical structure from a corpus at the fine-grained topic level, by automatically mining a set of representative words for each topic in a forest structure to help a user comprehend her/his interested topics.

All datasets have undergone a preprocessing of removing stop words and deleting low-frequency words. Table 1 shows the numbers of training and test documents, as well as the vocabulary size.

Dataset	#Docs(Train)	#Docs(Test)	Vocabulary size
20News	11,314	7,531	1,995
Reuters	7,769	3,019	2,000
Wikitext-103	28,472	120	20,000

Table 1: Statistics of datasets.

Baselines We employ four tree-structured topic models, including hLDA⁴ (Blei et al., 2003a), rCRP⁵ (Kim et al., 2012), TSNTM⁶ (Isonuma et al., 2020), and nTSNTM⁷ (Chen et al., 2021b), and three DAG-structured topic models, including hPAM (Mimno et al., 2007), HSOC (Liu et al., 2018), and CluHTM (Viegas et al., 2020) as baselines. For tree-structured baselines, the max-depth of topic tree is set to 3 by following (Isonuma et al., 2020). In addition, we adopt seven flat topic models for comparison, including parametric models of GSM⁸ (Miao et al., 2017), GSB (Miao et al., 2017), NB-NTM⁹ (Wu et al., 2020) and GNB-NTM⁹ (Wu et al., 2020), and nonparametric models of HDP¹⁰ (Teh et al., 2004), iTM-VAE¹¹ (Ning et al., 2020), and HiTM-VAE¹¹ (Ning et al., 2020). Except for GSB, we use the open source codes for all other baseline models.

Hyper-parameter Settings To ensure fair comparisons, we follow (Chen et al., 2021b) to set topic numbers to 50 and 200 for all parametric models. For non-parametric models based on SBP (i.e., iTM-VAE and nTSNTM), the maximum number of topics is set to 200, and the concentration

⁴<https://github.com/joewandy/hlda>

⁵<https://github.com/uilab-github/rCRP>

⁶<https://github.com/misonuma/tsntm>

⁷<https://github.com/hostnlp/nTSNTM>

⁸<https://github.com/linkstrife/NVDM-GSM>

⁹<https://github.com/mxiny/NB-NTM>

¹⁰<https://github.com/arnim/HDP>

¹¹https://github.com/walkerning/itmvae_public

parameter $\hat{\beta}$ of the GEM distribution is set to 20. According to (Chen et al., 2021b), we select the topic with a total probability exceeding 95% as an active topic. Besides, the penalty parameter ρ is updated by $\rho = 2^x$, where x denotes the number of training epochs. The temperature τ changes exponentially from 5 to 1×10^{-4} to ensure the adjacency matrix to be sparse. The hidden layer size of the encoder is set to 256, which is consistent with other models. In the construction of the weighted adjacency matrix, we use an exponential change strategy to dynamically adjust the decay coefficient. This is because the quality of such a weighted adjacency matrix is gradually enhanced when learning the self-attention module. Finally, the decay coefficient γ changes exponentially from 1 to 0.5. We implement our model by pytorch and run it on a computer with NVIDIA 1080Ti and 128GB RAM.

4.2 Topic Hierarchy Analysis

To evaluate the rationality, affinity, and diversity of the topic hierarchy generated by different models, we adopt four metrics: topic specialization (Kim et al., 2012), Cross-Level Normalized Point-wise Mutual Information (CLNPMI) (Chen et al., 2021b), hierarchical affinity (Kim et al., 2012), and Topic Uniqueness (TU) (Nan et al., 2019). Key words of each topic are ranked by the topic-word matrix Φ (Blei et al., 2003b).

Topic Hierarchical Rationality For the tree-structured topics, the topics closer to the root node will be more general, while topics closer to the leaf node will be more specific. Topic specialization score is to quantify this feature by computing the cosine similarity of the word distribution between each topic and the entire corpus. Since our forest-structured model generates several trees, we pick out all three-layer topic trees to obtain the average score for comparison. Figure 3 shows the topic specialization results, from which we can observe that our model outperforms baselines except for HSOC and CluHTM. Although the topic specializations of HSOC and CluHTM at different levels are close to 1, they still lack rationality since the root topic in a tree should be more general than others.

To measure the relationship between two connected topics, a metric of CLNPMI (Chen et al., 2021b) was proposed by calculating the average Normalized Point-wise Mutual Information (NPMI) score of every parent and its children topics, as follows: $CLNPMI(W_p, W_c) =$

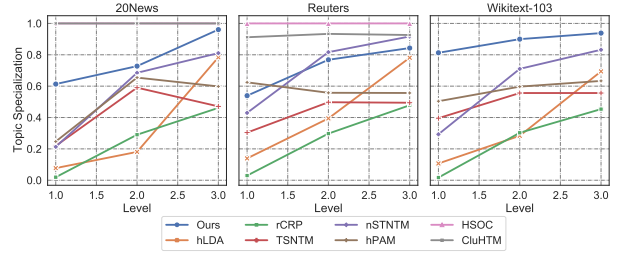


Figure 3: Topic specialization of different topics trees generated from the three datasets. A higher score with a growing trend means better performance.

Models	20News		Reuters		Wikitext-103	
	CLNPMI	TU	CLNPMI	TU	CLNPMI	TU
hLDA	0.065	0.051	0.050	0.447	0.063	0.597
rCRP	0.098	0.285	0.072	0.227	0.088	0.355
TSNTM	0.086	0.430	0.027	0.370	0.065	0.615
nSTNTM	0.109	0.745	0.102	0.708	0.113	0.730
hPAM	0.046	0.606	0.011	0.470	0.047	0.713
HSOC	0.128	0.231	0.047	0.211	-	-
CluHTM	0.123	0.116	0.016	0.117	-	-
Ours	0.152	0.757	0.125	0.798	0.118	0.766

Table 2: The average CLNPMI and TU scores of hierarchical topic models with top 5, 10, and 15 words for each topic. A higher score means better performance and the best scores are highlighted by boldface.

$\sum_{w_i \in W'_p} \sum_{w_j \in W'_c} \frac{NPMI(w_i, w_j)}{|W'_p| |W'_c|}$, where $W'_p = W_p - W_c$ and $W'_c = W_c - W_p$, in which W_p and W_c denote the top N words of a parent topic its child topic, respectively. A higher CLNPMI indicates that children topics are more coherent with their corresponding parent topics. We compare our nFNTM with hierarchical topic models mentioned above, and the CLNPMI results of all models are shown in Table 2. Note that the two NMF-based models (i.e., HSOC and CluHTM) had not converged after running for more than 48 hours on Wikitext-103, thus we did not include their results in the table. As shown in Table 2, our nFNTM achieves the best performance on all datasets.

Topic Uniqueness To evaluate the diversity of hierarchical topics, we calculate the topic uniqueness by: $TU = \frac{Count(Set(W_{topN}))}{N \times S}$, where $Count(Set(W_{topN}))$ is the number of distinct words in top N words of all topics. A higher TU means that the generated topics are more diverse. Table 2 shows the TU results for all models, from which we can observe that our model generates more diverse topics than others. The baselines of HSOC and CluHTM perform quite poor since they need to preset up to 600 topics for convergence.

Topic Hierarchical Affinity A reasonable assumption for topic hierarchy is that topics with parent-child relationships show larger similarities in their topic-word distributions than topics without any parent-child relationship (Kim et al., 2012). According to (Kim et al., 2012), we firstly evaluate the similarity between parent-child topics and non-parent-child topics by computing the cosine similarity between their topic-word distributions. Then, the topic hierarchical affinity is measured according to the difference of those similarities.

Let $\Phi(k)$ be a topic at level k , $\lambda(k)$ be children topics of $\Phi(k)$, and $\bar{\lambda}(k)$ be non-children topics of $\Phi(k)$. The topic hierarchical affinity metric compares the average cosine similarity $S_{\lambda}(k)$ between $\Phi(k)$ and all topics in $\lambda(k)$ against the average cosine similarity $S_{\bar{\lambda}}(k)$ between $\Phi(k)$ and all topics in $\bar{\lambda}(k)$. A large difference between $S_{\lambda}(k)$ and $S_{\bar{\lambda}}(k)$ indicates a good topic hierarchical affinity.

Since all topics between paired levels in hPAM are fully connected, it is impossible to clearly distinguish topics with parent-child and non-parent-child relationships. Thus, we exclude hPAM in this part. As shown in Figure 4, our model achieves high similarities between parent-child topics and low similarities between non-parent-child topics, indicating a good hierarchical affinity. Note that both HSOC and CluHTM are based on NMF, which rely on a predefined number of topic trees. To avoid missing potential topic sub-structures, these models need to set a large number of topic trees (e.g., 100) according to their default settings. Given superabundant topic trees, the similarity between non-parent-child topics will be underestimated due to the huge number of non-parent-child topics, resulting in a competitive hierarchical affinity. However, it leads to a very poor TU score for HSOC or CluHTM, as already shown in Table 2.

4.3 Topic Interpretability

We employ the NPMI score (Lau et al., 2014) to evaluate the interpretability of topics since this metric is shown to be close to human judgments (Lau et al., 2014). We extract top 5, 10, and 15 words for each topic and compute the average NPMI scores of all models over the three datasets. The higher the value of NPMI score, the more interpretable the generated topics is. The corresponding numbers of topics automatically determined by our nonparametric model on 20News, Reuters, and Wikitext-103 are 61, 76, and 121, respectively.

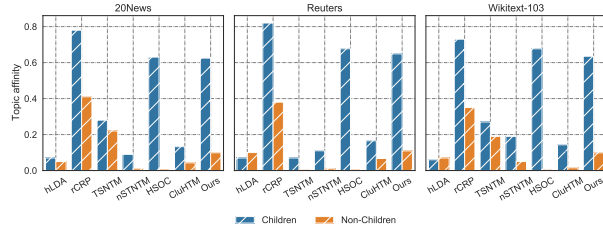


Figure 4: Topic hierarchical affinity results. For each associated topic, a larger difference of word distributions between children and non-children topics is better.

Table 3 shows the NPMI scores of topics generated by different models. Although our nFNTM outperforms hierarchical topic models by a large margin, it performs slightly worse than the top-performing flat topic model. The reason is that NPMI is the average score of all topics, which may be unfair for hierarchical topic models. Flat topic models treat topics independently and result in specific topics, while higher-level topics in hierarchical topic models are general with a lower NPMI score. Figure 5 shows two examples of topic trees generated by nFNTM on Wikitext-103 (left) and 20News (right), which focus on different topics, i.e., *politics* and *war* in the left tree, and *play*, *religion*, and *computer* in the right tree. Take the left tree as an illustration, the NPMI score of the root topic [*day hour people home left*] is obviously lower than that of the leaf topic [*force army battle attack war*], which affects the overall result. We can also observe that the topics closer to root are more general and those closer to leaves are more specific. Besides, children topics are related to parent topics, e.g., encryption is a child of computer, and law is a child of governance. These examples of topics validate the advantage of a forest structure when compared with a tree structure: the former can not only learn reasonable parent-child relationships among topics, but also generate a flexible topic hierarchy with unbounded depth and width.

4.4 Ablation Study

We perform ablation experiments on our model to validate the effectiveness of each module. Table 4 shows the ablation results of our nFNTM without three modules, where “Ours w/o SBP” denotes a parametric model with Gaussian distribution instead of SBP, “Ours w/o self-attention” means that the weighted adjacency matrix is randomly set rather than learned by the proposed self-attention mechanism, and “Ours w/o self-attention + DAG”

Datasets Model	20News		Reuters		Wikitext-103	
	50	200	50	200	50	200
GSM	0.211	0.165	0.198	0.155	0.214	0.217
GSB	0.231	0.191	0.152	0.136	0.229	0.131
NB-NTM	0.188	0.223	0.248	0.245	0.127	0.125
GNB-NTM	0.240	0.228	0.237	0.255	0.127	0.093
HDP	0.192		0.266		0.157	
iTM-VAE	0.195		0.201		0.184	
HiTM-VAE	0.237		0.269		0.233	
rCRP	0.186		0.206		0.201	
hLDA	0.221		0.185		0.186	
TSNTM	0.212		0.206		0.213	
nTSNTM	0.219		0.234		0.237	
hPAM	0.213		0.229		0.223	
HSOC	0.223		0.210		-	
CluHTM	0.219		0.161		-	
Ours	0.235		0.251		0.240	

Table 3: The average NPMI scores of different models using top 5, 10, and 15 words for each topic. A higher score means better performance and the best scores are highlighted by boldface.



Figure 5: Two examples of topic trees generated by nFNTM on Wikitext-103 (left) and 20News (right). Each node represents a topic with top 5 words, and the arrow direction is from parent to children.

denotes reconstructing documents by their topic distributions generated from SBP. Note that there is a flat topic hierarchy without DAG, thus the CLNPMI metric can not be calculated. As shown in Table 4, nonparametric and self-attention modules are beneficial to generate a good topic hierarchy and achieve improvements in topic interpretability.

4.5 Concentration Parameter Evaluation

Here, we vary the values of concentration parameter $\hat{\beta}$ to validate the nonparametric property of our model on 20News. Figure 6 shows that the topic numbers of all nonparametric models are promoted by increasing the value of $\hat{\beta}$, which is reasonable since a larger $\hat{\beta}$ leads to a smoother distribution of SBP, and the smoother distribution generates more topic numbers than a denser distribution. It also demonstrates that our model generates more topics for a larger $\hat{\beta}$ since these topics are dispersed (Wu

Model	#Topics	NPMI	CLNPMI
Ours	61	0.235	0.152
Ours w/o SBP	50	0.223	0.135
Ours w/o self-attention	57	0.209	0.097
Ours w/o self-attention+DAG	48	0.203	-

Table 4: Ablation evaluation on 20News.

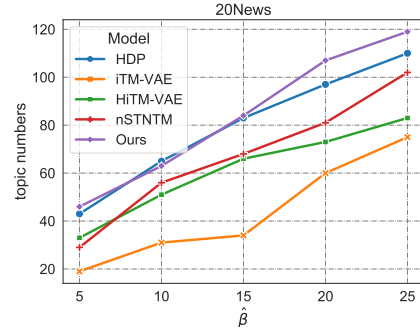


Figure 6: Topic numbers derived by different nonparametric models with various values of $\hat{\beta}$.

et al., 2020), and our model performs better than other models on approximating the nonparametric property of HDP.

4.6 Forest-Structured Topic Visualization

In this part, we qualitatively analyze the rationality of the topic forest generated by our model. We determine parent-child topic relationships by the value of the weighted adjacency matrix \mathbf{C} . $C_{i,j} \approx 1$ means that child topic t_i connects to parent topic t_j . For clarity, we show an example of \mathbf{C} and how to build a topic forest from it in Figure 7. Such a topic hierarchy is based on the assumption that a document can be decomposed into a weighted sum of multiple topics, where a topic can also be decomposed into a weighted sum of multiple sub-topics (Chen et al., 2021b).

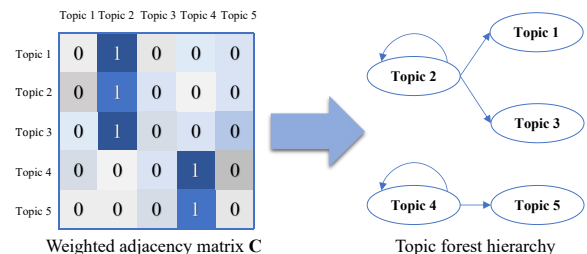


Figure 7: An example of building a topic forest from a weighted adjacency matrix.

Figure 8(a) shows the visualization of the weighted adjacency matrix generated from Reuters. Except for the dots on the diagonal, the rest of

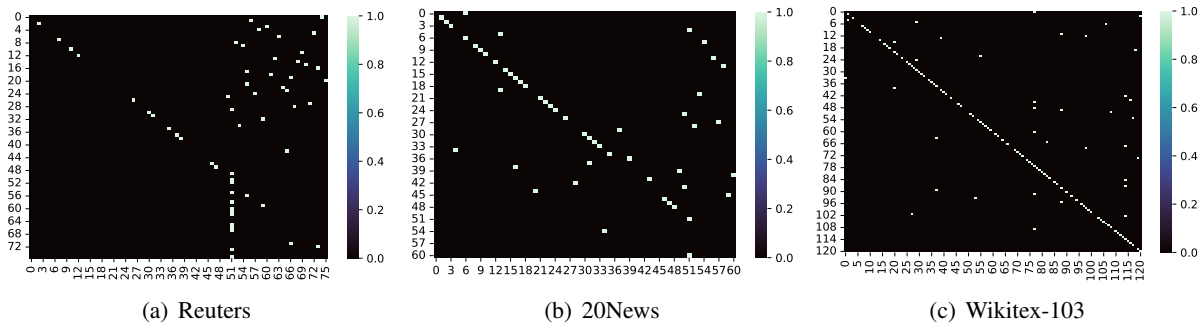


Figure 8: Visualization of the weighted adjacency matrix generated from (a) Reuters, (b) 20News, and (c) Wikitext-103, respectively. The white dot means that there is a correlation between two topics indexed in the row and column.

dots present a discrete distribution to meet the constraints of DAG. In addition, we observe that topic 51 is connected to various topics and becomes the parent topic of multiple topics, which is a general root topic of the whole corpus. Figures 8(b) and 8(c) present the weighted adjacency matrices generated by our model on 20News and Wikitext-103, respectively. The results also indicate that except for the dots on the diagonal, the rest of dots present a discrete distribution on both datasets, i.e., they meet the constraints of DAG.

Besides, we use Gephi¹² to visualize the topic forests generated by our model over 20News, Reuters, and Wikitext-103 in Figures 9, 10, and 11, where each node represents a topic with top 5 words and the arrow direction is from parent to children. The color of each node is determined by indegree. It indicates that the topic forest generated from Reuters contains a master tree whose root topic is about interviews. Furthermore, the topic forests generated from 20News and Wikitext-103 both consist of multiple trees, where the root topics mainly involve news and daily, respectively. All results indicate a rational hierarchical structure of topics obtained by our model, i.e., root topics are general and children topics are specific to their corresponding parents.

4.7 Speed Comparison

For speed comparison, we record the running time by taking the average cost of training each model for 5 times over 10k documents sampled from 20News. The running time for baseline methods are: hLDA - 16.74s, HSOC - 50.97s, CluHTM - 49.72s, and nTSNTM - 1.57s. Our model takes 1.35s, which is significantly faster than models based on Bayesian learning and NMF. Although

¹²<http://gephi.org/>

nTSNTM achieves a competitive result, it applies Kumaraswamy distribution in parameter inference which slows down the model speed. Besides, such a tree-structured method is limited to local information and data sparsity along the hierarchy of topics (Viegas et al., 2020). In particular, the deeper the topic tree, the less coincident data remains. This deteriorates the quality of tree-structured methods inevitably. Compared with a single deep tree, the forest structure with several shallower trees could circumvent the above problem theoretically. Furthermore, with varied numbers of trees, a topic forest has a more flexible hierarchical structure and better adaptability than a fixed tree structure.

5 Conclusion

In this work, we present a forest-structured neural topic model named nFNTM. Particularly, the self-attention mechanism is applied to capture topic correlation and an augmented Lagrangian method is employed to build the topic forest under DAG constraints. By computing reparameterization of Gamma distribution, the SBP is adopted to obtain a nonparametric model. We empirically demonstrate that our nFNTM generates more rational, affinitive, and diverse topics than state-of-the-art hierarchical topic models, meanwhile achieves better topic interpretability than various baselines. Although flat topic models can generate distinguishable topics with larger NPMI scores than hierarchical ones, they treat all topics independently without explaining the corpus well to a user.

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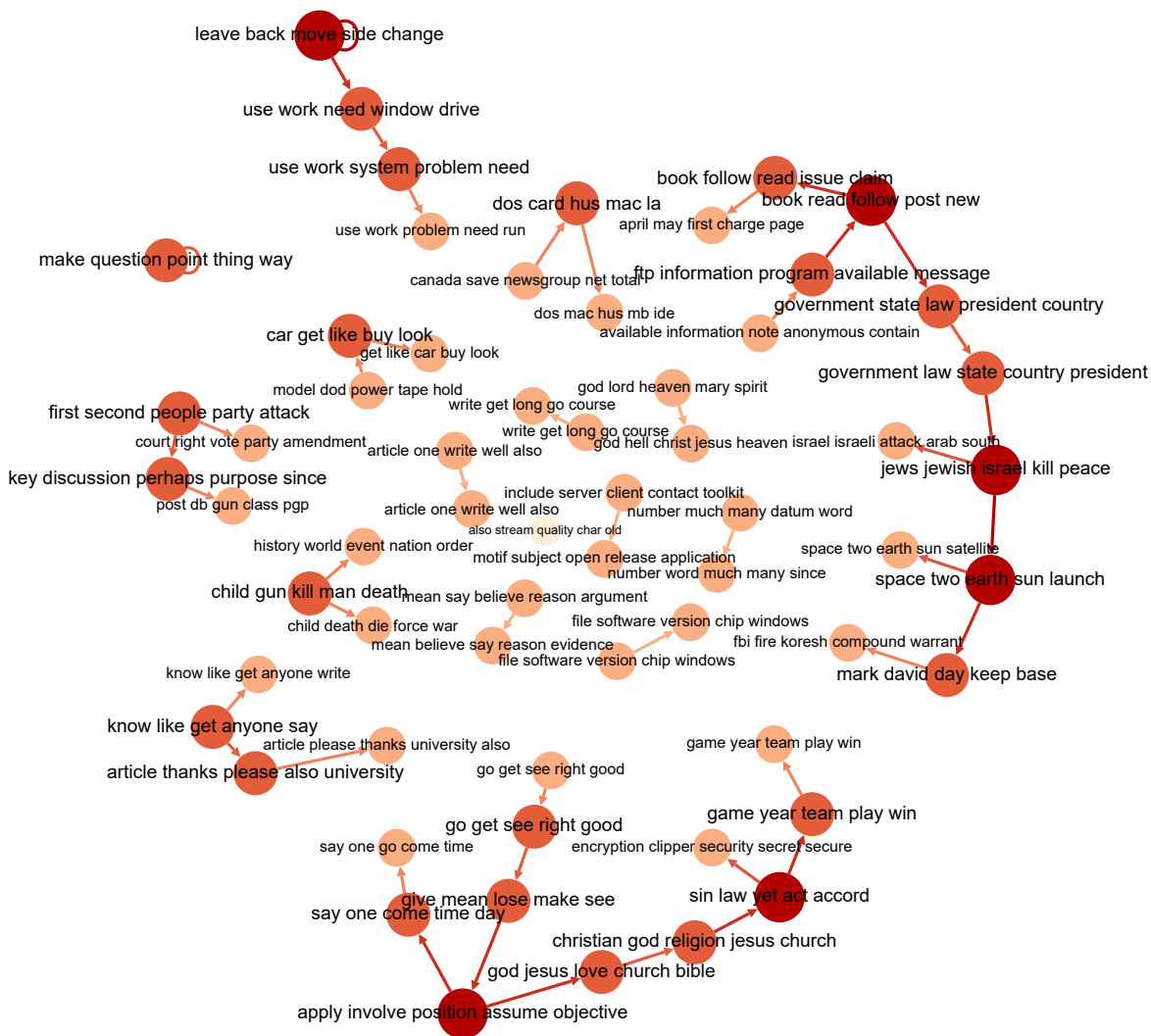


Figure 9: Visualization of the topic forest generated from 20News. Each node represents a topic and the arrow direction is from parent to children. There are about 8 trees in the forest, two of which have complex levels and structures, and two trees have only one root node. Take three of these trees as an illustration, the top 5 words of root topics and their children topics are (1) [know, like, get, anyone, say]—>[article, thanks, please, also, university]; (2) [apply, involve, position, assume, objective]—>[god, jesus, love, church, bible] and [game, year, team, play, win]; (3) [available, information, note, anonymous, contain]—>[ftp, information, program, available, message]—>[book, read, follow, post, new]. The root topics are general while children topics are specific.

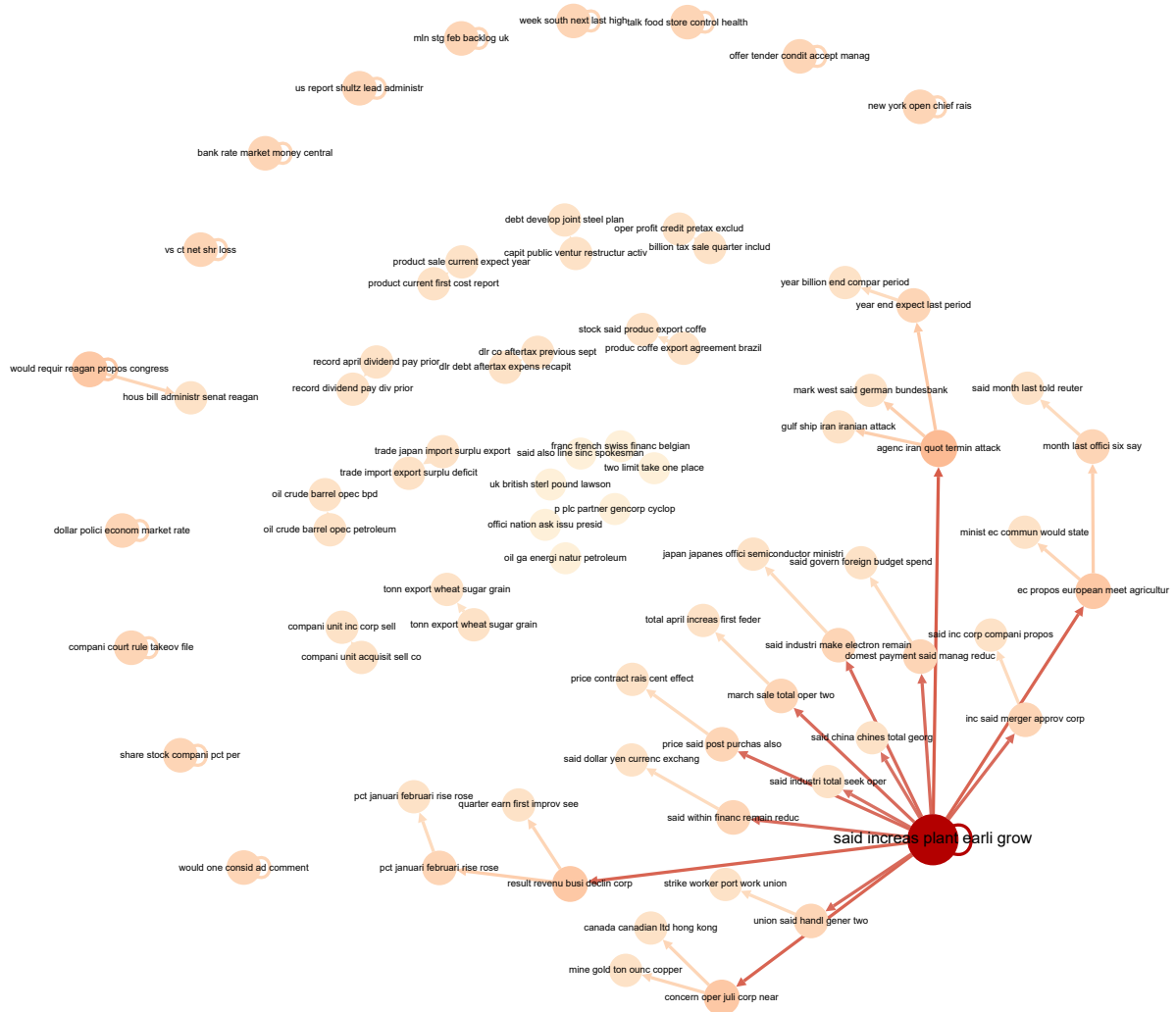


Figure 10: Visualization of the topic forest generated from Reuters, where a huge tree is surrounded by several trees with only two levels. The root topic of the largest tree contains words: [said, increase, plan, earli, grow], which are commonly used during interviews. The surrounding areas are related to the topics of various categories, including [mine, gold, ton, ounce, copper], [strike, worker, port, work, union], and [year, end, expect, last, period].



Figure 11: Visualization of the topic forest generated from Wikitext-103. Possibly due to a large variety of topics in this corpus, the result presents a more dispersed and shallower structure than others. The top words of two root topics and their children topics are (1) [day, hour, people, home, left]→[court, law, case, right, act] and [party, government, support, right, general]; (2) [main, instead, rest, provides, included]→[christmas, discuss, holiday, tom, expectation] and [gun, inch, class, battery, battleship].