Squibs and Discussions

Sethood and Situations

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In *Situations and Attitudes* (Barwise and Perry 1983) Barwise and Perry decide that meaning should be taken to be a triadic relation not a dyadic one. While there may be good reasons for deciding to procede on this basis, one of the reasons they give is definitely a bad one, and this is the principal subject of this note.

On pages 222–23 they remark that if meaning is to be a dyadic relation it is necessary that the complement of a situation should—at least sometimes—be another situation. In the set theory that is the basis for their development—KPU—it is elementary that the complement of a set is never a set. This compels the designers of situation semantics to make meaning a triadic relation as we will now explain.

Barwise and Perry take **individuals**, **properties**, **relations**, and **locations** as primitives. A **situation-type** is a partial function from n-ary relations and n individuals to the set $\{0,1\}$ (p. 8). In modern situation-semantic parlance, this is often referred to as an **infon**, or (more precisely) a **basic infon**. An **event**, or **course-of-events** (*coe*), is a function from locations to situation-types. For example, the situation-type corresponding to a (real-world) situation in which a dog named Molly barks would be:

$$\langle \text{at } l, \text{barks}, \text{Molly}, 1\rangle$$

and one related *coe* might be:

$$\begin{aligned} e = \{ & \langle \text{at } l, \text{barks}, \text{Molly}, 1\rangle, \\ & \langle \text{at } l', \text{shouts at}, \text{Mr. Levine}, \text{Molly}, 1\rangle, \\ & \langle \text{at } l'', \text{barks}, \text{Molly}, 0\rangle \} \end{aligned}$$

Consider the predicate *SO* (**seeing option**) on *coe*s. In a given event, s, an individual, a, classifies events according to what s/he sees and knows. That is:

$$\langle SO, a, e_1, 1\rangle \in s \text{ if } e_1 \text{ is compatible with what } a \text{ sees and knows;}$$

$$\langle SO, a, e_2, 0\rangle \in s \text{ if } e_2 \text{ is incompatible with what } a \text{ sees and knows.}$$

This is a partial classification of events; i.e., some events may be neither *SO*-yes nor *SO*-no.

Further to this:

- **Definition**
  In a given situation, s, an event, e, is a **visual option** for agent a if $\langle SO, a, e, 1\rangle \in s$
- **Definition**
  Similarly, e is a **visual alternative** for a if it is not the case that $\langle SO, a, e, 0\rangle \in s$

Given s as above, let:

$$\begin{aligned} X_{VO} &= \{e : \langle SO, a, e, 1\rangle \in s\} \\ &= \text{collection of events that are visual/seeing options for } a \text{ in } s. \end{aligned}$$

Also, let:

$$X_{NVO} = \{e : \langle SO, a, e, 0\rangle \in s\}$$
$$= \text{collection of events that } a \text{ classifies as } \textit{not} \text{ being visual options.}$$

Then:

$$X_{VA} = \text{collection of visual alternatives for } a = \overline{X_{NVO}}.$$

(In general, we cannot assume $\overline{X_{NVO}} = X_{VO}$.)

An utterance, ϕ, determines a triple $\Phi = \langle d, c, \phi\rangle$ composed of a **discourse situation**, d, a **speaker connection function**, c, and the utterance, ϕ.

Furthermore, our **interpretation** relation (a function from utterances of the above form to collections of events) is given as:

$$[\![\Phi]\!] = \text{interpretation of } \phi \text{ according to } d \text{ and } c = \{e : d, c[\![\phi]\!]e \textit{ holds}\}.$$

The speaker connection function, c, (or **anchor**) grounds the individuals, relations, and locations mentioned in ϕ to actual entities participating in the discourse situation, d. $[\![*]\!]$ is thus a binary relation, relating the utterance triple to the described situation, $[\![\Phi]\!]$. Note that the discourse situation, d, is the situation in which ϕ is uttered and thus is usually distinct from the described situation, $[\![\Phi]\!]$, except in cases of self-reflexive discourse.

For example, if $\phi =$ FIDO RAN, c(FIDO) ["FIDO" is mentioned] = *Fido* ["Fido" is used], and c(RAN) = l [a location], then if:

$$\langle l, ran, Fido, 1\rangle \in e$$

we have $e \in [\![\Phi]\!]$.

There is a problem with this analysis that leads Barwise and Perry to seek a representation of mental states and events with which to augment the interpretation relation. The problem involves a distinction Barwise and Perry make between **epistemic** and **non-epistemic** perception. Attitudinal reports involving the phrase "see that" followed by a finite complement involve epistemic perception—that is, they yield information about the inference an agent has performed after seeing a given *coe* or situation (p. 207). The problem comes about when Barwise and Perry attempt to characterize attitude reports involving "see that" in terms of the relation SO.

On pages 209–11 Barwise and Perry claim:

$$\text{a sees that } \phi \Longrightarrow \{e : \textit{not } d, c[\![\phi]\!]e\} \subseteq X_{NVO}(\star)$$

i.e., those events not in the interpretation of ϕ must be classified as SO-no. They give the following proof, on page 211.

Proof

"A situation e is one where a sees that ϕ if ϕ holds in each of a's visual alternatives at the appropriate location, l." That is:

$$a \text{ sees that } \phi \Longrightarrow X_{VA} \subseteq [\![\Phi]\!]$$

and since $X_{VA} \subseteq [\![\Phi]\!] \Leftrightarrow \overline{[\![\Phi]\!]} \subseteq X_{NVO}$ (taking complements), we obtain the result.

This yields the following situational analysis of attitudinal reports involving epistemic perception. Given an utterance:

$$\sigma := a \text{ SAW THAT } \phi$$

in order for σ to describe an event, e, we use ($\star$) to obtain that we must have at $l = c(SAW)$, for every event, e_1, either:

$$e_1 \in [\![\Phi]\!]$$

or:

$$\langle l, SO, a, e_1, 0 \rangle \in e.$$

The fact that any SO-no event must be classified as such by the event e (the event corresponding to the attitude report) means that if we view e as a collection of infons, we will have:

$$|e| \geq |X_{NVO}|$$

and by a result above, $|X_{NVO}| \geq \left|\overline{[\![\Phi]\!]}\right|$.

It is Barwise and Perry's contention (p. 222) that $\overline{[\![\Phi]\!]}$ is a proper class, therefore e is as well. Consider the utterance:

$$\sigma_1 := \text{JOE SAW THAT JACKIE WAS BITING MOLLY.}$$

Barwise and Perry argue that

> there is a proper class of events e_1 in which Jackie was not biting Molly, events that must be classified with SO-no. But then [the event] e required to classify Joe's visual state must be a proper class. (p. 222)

Thus such events cannot, for example, be constituents of other situations. In particular, **iterated** (or **embedded**) attitude reports cannot be handled in this framework. A report such as:

$$\sigma_2 := \text{JOHN SAW THAT JOE SAW THAT JACKIE WAS BITING MOLLY}$$

would require that the event, e, classifying Joe's visual state be a constituent of σ_2's interpretation, $[\![\Sigma_2]\!]$. This is because the interpretation relation, $d, c[\![\Sigma_2]\!]e$ holds: intuitively, the putative event corresponding to the situation described in σ_2 would have to include e since Joe's visual state in fact comprises the complement of the outer "see that" clause. Yet e is a proper class and so we cannot have $e \in [\![\Sigma_2]\!]$ as we require.

This can be rectified by adopting as a set-theoretic basis a set theory in which the complement of a set is always a set. In this case, the analysis proceeds as before, saving that the collection $\overline{[\![\Phi]\!]}$ (as above) is now a set. With the collection X_{NVO} no longer formally constrained to being a class, arguments of the type rife throughout (Barwise and Perry 1983) can be lodged to illustrate X_{NVO}'s "set-ness," as well as that of the interpretations of utterances such as σ_1. Thus the situational analysis of attitudinal reports extends to iterated reports such as σ_2 without violation of set membership dicta.

Whatever reasons caused Barwise and Perry to desire a set theory with ur-elements should presumably still be respected, so if we can find a consistent set theory with ur-elements and a universal set, the outlook will be a lot brighter. Fortunately there is such a system, the Jensen-Quine system of set theory known as *NFU*. For more on this see Holmes (1994, 1996). Of course, an easy consequence of an axiom of complementation such as we have in NFU is the negation of the axiom of foundation. Barwise has elsewhere (1984) argued that we should not regard the axiom of foundation as essential.

References

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