ıodology and Verifiability in Montague Grammar

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Abstract
Methodological problems in Montague Grammar are discussed. Our observations show that a model-theoretic approach to natural language semantics is inadequate with respect to its verifiability from logical point of view. But, the formal attitudes seem to be of use for the development in computational linguistics.

0. Introduction

In this paper we discuss the methodology of verifiability taken by researchers on model-theoretic semantics for natural language such as Montague grammar. Though Montague grammar (hereafter MG) has been developing since the publication of Montague[10], there has been few serious studies of its 'sense' and methodology.

We take the purpose of semantics to be as follows.

(a) To define a 'meaning'.
(b) To define a 'meaning' of certain linguistic expressions.
(c) To generalize a 'meaning' referred as (b) in connection with internal world (human) and external world.

Here (a) is so abstract that it must be dicussed in general linguistic terms rather than in MG. But it is no doubt that the methodologies in MG are based on the assumption (c). The problem (c) is central to MG. In MG semantic structure corresponding to syntactic structure of natural language is realized by means of its methodologies.

The problem (c) is closely related with pragmatics and epistemology thus MG includes parts of them. As Chomsky's early transformational grammar was obliged to changes of the system for the sake of autonomous syntax hypothesis, the problem is important in MG. Intensional logic and possible-world semantics could solve parts of the problems. But it is difficult to say that MG is a system facilitating (c). And methodological problems of MG including (c) are mainly ascribed to model theory underlying MG. We shall focus on the point and discuss MG's methodology. Especially following problems are investigated.

(1) Is intensional logic necessary?
(2) Can modal (tense) logic express a modality (tense) in natural language?
(3) Is first-order logic necessary?
(4) Is there a possibility of natural logic?
(5) Are there appropriate methods for the interpretation of logical form?
(6) Is there a distinction between logical words and content words in natural language?

1. MG and Model Theory

The purpose of model theory is to investigate the relationships between a true statement in some formal language and its model. Namely, it is to define a concept of 'truth' in some mathematical structures. In mathematical logic Tarski[14] first attempted to study the idea of model. In his paper Tarski mainly concerned himself with the definition of truth (the correct definition of a true sentence). He confined his discussions to the object in the framework of predicate logic in the sense modern logic. He despaired to define a true sentence in natural language. Since we are obliged to face to paradoxes for the sake of universality of natural language. But he suggested that there exists a possibility of application of the results for model theory, which he gave to the language he called 'formalized language', to natural language.

About forty years after the publication, Montague, who is a disciple of his, could give a model theory for natural language. Montague regarded intensional logic as a basis of his theory so as to overcome complexities of natural language. He was able to solve paradoxes, that Frege and others faced, by means of intensional logic.

First we consider the problems of intensional logic. The model of intensional logc comes to be more complicated because it has a greater descriptive power than predicate logic in general. As Gallin[3] pointed out, valid formulas in intensional logic fail to constitute a recursively enumerable set since it is essentially based on type theory. Thus we have no axiomatization for this logic. For this reason, we must restrict the scope of sentences in natural language capable of being treated by intensional logic. But the notation of intensional logic used in PTQ such as '^' and 'v' work efficiently for analysis. For example, consider the following sentences.

Every man loves a man. (1-1)

We have two interpretations for the sentences, namely,

(every man)(loves a man.) (1-2)
(every man loves)(a man.) (1-3)

In general we call (1-2) de dicto reading, (1-3) de re reading, and obtain the following translations respectively.

$$\forall x(man'(x) \rightarrow love'(x, \hat{}\lambda Q \exists y(woman'(y) \wedge Q(y)))). \quad (1\text{-}4)$$

$$\exists y(woman'(y) \wedge \forall x(man'(x) \rightarrow love'(x, \hat{}\lambda P P(y)))). \quad (1\text{-}5)$$

Seen from the above formulas, in (1-2) that every man loves is not an individual 'woman' but a property of property of a individual 'woman'. That is, the meaning of individuals (intension) is considered as a mapping from possible-worlds to a reference (extension). If we define a possible-world as a set of indices, and determine the value for each index, then some extension is defined. But we doubt that an intension defined in intensional logic properly represents a meaning.

In MG individuals and truth values are assumed as semantic primitives. Using possible-world semantics we can extend predicate logic. This extension causes the structure of model to be more complex, and produces lots of contradictions as natural language semantics. Above all the problems of logical equivalence is serious. For example, assume a and b for logically equivalent formulas, that is, a and b are true in same indices. Then it is a valid inference from doubt(p, ^a) to doubt(p, ^b). If we doubt a, we would doubt b logically equivalent to a from the standpoint of logically equivalence thus for p, a and b have differernt meanings. To put it more correctly, the meaning of 'doubt' in a and b is different unless p knows the correct sense of logically equivalence between a and b. Such a statement fails to be explained in traditonal logic. This is nothing but a limitation of ordinary model theory. Researchers such as Keenan[8], Thomason[15] and Turner[16] tried to extend intensional logic from various viewpoints. Thomason added intensional logic to third type of propositions, which is a denotation of a sentence. Thus we clearly need a domain containing at least two propositions of a model for intensional logic. Keenan introduced the concept of ontologically perfection, that is, the element of the ontology are possible denotatons for extensions for expressions, by means of Boolean algebra. His motivation is to restrict a domain of intensional logic. Thus the set of possible world is defined in terms of maximally consistent set of propositions, sentence denotations.

Turner[16] extended intensional logic in the sense

of type-free theory in which a self-application is permitted for the treatment of nominalizations. We are very interesting in such strategies since in Scott-type denotational semantics we have no intermediate language as in PTQ. Thus we can obtain semantic interpretation of a sentence directly. We have an idea for types of natural language, namely, polymorphic types, which can have various types. These types are essentially considered as a subset of type-free theory.

Above mentioned trials are restrictions to a model for intensional logic. But such perplexed constructions must cause us more difficulties in reality. Must we give up this logic? It is certain though intensional logic has the sides against our intuitions, it can provide a powerful model for some phenomena. For example, consider the following sentences referred to as Partee's paradox.

(1) The temperature is ninety.
(2) The temperarure rises.
(3) Ninety rises.

These are translated into formulas in intensional logic as:

(1) $\exists y(\forall x(\text{temerature}'(x) \leftrightarrow x=y) \wedge {}^{\vee}y=n)$
(2) $\exists y(\forall x(\text{temerature}'(x) \leftrightarrow x=y) \wedge \text{rise}'(y))$
(3) $\text{rise}'({}^{\wedge}n)$ (1-7)

As seen from (7) Montague dealt with noun phrases as objects which have intensions and extensions. In the examples, intensions are represented as functions that denote some number at each index, and extensions are rendered as particular number such as 90 at certain index. Namely, the truth value of sentence (2) in (1-6) depends not on extension but on intension. For this reason verbs such as 'rise' referred to as intensional verbs. But such formalisms seem to be recaputulated in the framework of predicate logic. If so, it is effective from not only intuitive but also computational point of views. Such formalisms are divided into two approaches. One is an approach that is an extension of predicate logic to intensional logic using some devices as in Schubert and Pelletier[13]. Another is an approach that intensional logic is interpreted as a programming language such as LISP as in Hobbs and Rosenschein[6]. Schubert and Pelletier stated that predicate logic is suitable from the viewpoint of AI systems. According to them, the expressions in intensional logic are not comprehensive to human being. For example, it is better understandable to capture definite noun phrases as individuals than a set of properties. Slot representations conquest gaps to intensional. In this formulation a proper name is represented as a constant, a common noun as a monadic pedicate and a transitive verb as a binary predicate.

'Mary' ==> Mary 1
'boy' ==> (#1 boy) (1-8)
'loves' ==> (#1 loves #2)

Here #n is called argument slot that is filled from higher number in turn. The sentence (1-2) and (1-3) are translated as follows.

de dicto:

for all(#1 man)((#1 loves #2)(for some(#2 woman)))
==> $\forall x(x \text{ man}) \rightarrow (x \text{ loves} \wedge \exists y(y \text{ woman}))$ (1-9)

de re:

for some(#2 woman)(#1 loves #2)(for all(#1 man))
==> $\exists y((y \text{ woman}) \wedge (\forall x(x \text{ man}) \rightarrow x \text{ loves } y))$ (1-10)

These translations are similar to the formulas in predicate logic. Here slot representations enable us to operate a scoping of noun phreses. This device seems to have some simulating with combinators in combinatory logic.

Hobbs and Rosenschein tried to convert intensional logic to S-expressions in LISP. The lambda expressions are considered as the pure LISP thus the conversion is plausible. Such expressions are exemplified as follows.

α(constant) ==> (QUOTE α)
α(a variable of type <s,b> for any b) ==> (QUOTE α)
${}^{\wedge}\alpha$ ==> (LIST(QUOTE QUOTE)α) (1-11)
${}^{\vee}\alpha$ ==> EVAL α

The sentence (1) in (1-7) is translated in

ninety ==> (LAMBDA(Q)((Q*)(INT* 90))
be ==> (LAMBDA(P)(LAMBDA(x)((P*)(INT*(LAMBDA(Y)
(EQUAL(X*)(Y*)))))))
the temperature ==> (LAMBDA (P)(FOR SOME
ENTITY-CONCEPTS(LAMBDA(Y)(FOR ALL
ENTITY-CONCEPTS(LAMBDA(X)
(AND(IFF(TEMP X)(EQUAL X Y))((P*)Y)))))))
the temperature is ninety
==> ((THE(FUNCTION TEMP))
(INT*(BE(INT*(FUNCTION NINETY))))
INT = (LAMBDA(G)(LAMBDA(*)G)) (1-12)

Here we may assume there is a variable named * to the value of which are applied to produce the corresponding extensions. Above two trials are for approximating the functions of intensional logic by means of simpler system in order to reduce inherited complexities in this logic. In any case deficiencies of intensional logic are ascribed to modal theory, and even if we take it off, it is doubtful that intension formulated in intensional logic corresponds to the meaning of linguistic expressions.

Next we consider tense logic and modal logic. As both logics are based on possible-world semantics we come to face the same problems in general. Here we discuss the problems involved in direct applications to natural language. In tense logic the operators F and T are able to apply infinitely in principle but in practice the scope of tense has some boundary. Thus it is not easy to solve tense in natural language only by these two operators. Bäuerle[2] introduced third operator T (it is the case on ... that ...) so as to overcome shortcomings of traditional tense logic as in the axiomatization by Prior[13]. In tense logic the following relations hold.

$FF\phi \rightarrow F\phi$ (1-13)
$PP\phi \rightarrow P\phi$ (1-14)

These formulas are proved by means of the transitivity of <. Such relations assume all forms of the past (future) tense as quantification over times past (future). But to avoid the infinite application of tense operators we must take a strategy that tense can be considered as a point of reference by Reichenbach. That is, we can regard past tense as direct reference to some particular past time, not universal quantification.

Similarly in modal logic it is doubt that the two operators enable us to explain the modality of natural language. First of all modalities are divided into the objective and the subjective. And modal logic can manage only objective modality. Suppose the following examples.

John cannot be a Japanese. (1-15)
It is impossible John is a Japanese. (1-16)

If we translate these sentences into formulas in MG we obtain the one in only (1-16).

$\sim\Diamond\text{Japanese}'(j)$ $(=\Box\sim\text{Japanese}'(j))$ (1-17)

In other words the sentence in (1-15) belong to the category of subjective modality thus it is impossible that the subject is a logical connection of the function to each constituent (namely content word) in the statement rather than some kind of operation to the statement (namely truth value). Unfortunately, most of the modalities in natural language belong to objective modality. We can state that semantics in logic is not always linguistically valid. Chomsky[3] called MG a type of descriptive semantics except that he thinks it is not semantics really, in the sense that it does not deal with the classical questions of semantics such as the relation between language and the world.

The situations do not change even if we restrict logic to predicate logic. And if we want predicate logic to be psychologically real, though we will discuss this in section 2 in detail, we will reply in negative due to Lowenheim-Skolem's theorem.

When we interpret the so-called logical forms, if we depend on the idea of intensional logic, it happens a lot of irrationalities. Namely, the interpretation is nothing but a decision procedure of truth condition. Since MG is based on Fregean principle, the truth value

of a sentence is a function of the one of parts and it is difficult to add interpretation of linguistic constraints to the system of formal logic. Thus Natural Logic was proposed. Lakoff[9] said that the semantic structure of natural language corresponds to the grammartical structure and that Natural Logic must be able to explain logical inferences in natural language. Thus it is possible to consider that Natural Logic possesses similar framework to TG rather than MG. From the standpoint of GB theory in TG, Hornstain[7] pursueted logical forms. He claimed that semantics should also be explained from the same hypotheses (innateness) as syntax. We think that his approach is more realistic and rational theory if such theories are to be formalized in view of psychology. We can find a similar approach, though it may be more ambitious, in Johnson-Laird[8]. Necessity of Natural Logic seems to be derived from the drawbacks of formal logic owing to its artificality. As we take up the sixth problem before, there is a clear distinction between logical words and content words, and we faced strict type constraints. Most inferences in natural language are executed by means of logical words. In an extreme terms, we can infer only if we know inference rules. But our daily inferences seem to depend on the property of content words.

We therefore need the counterpart of inference rules in logic for inferences depended on content words. The abuse of meaning postulates at lexical level provide no solution. Since Natural Logic is based on the principle of universal grammar in grammartical theory. But if Natural Logic adopts predicate logic as a device for logical forms, it is impossible that the logic overcome its difficulties.

2. MG and Linguistic Theory

Finally we shall investigate into philosophical aspects of MG. We can find few research involved in the issues of methodology and philosophy in MG. The exception is Partee[11]. She tried to justify theoretical validity of MG in connection with psychological reality. Montague himself apprared to reconstruct linguistics on the basis of the same methodology in mathematics, thus there exists no psychological factor here. Dowty[3] also stands in the position that semantics is a field handling the relationships between linguistic exprssions and external worlds. Are there hypotheses in MG in different place from our mind? We hard to receive such radical opinions. Even if we discover reality in MG, it is doubtful whether theoretical validity of MG is verified. For example, we have the assumption that individuals and truth values are semantic primitives in MG. What is an individual? At a first glance individuals are grasped at ease, but we can never clarify what it is. The assumption of model theory says that a set of individuals is never empty in some structure. Suppose a possible-world that consists of only humans as its elements. Even if this set has countably infinite power, it will be empty someday because humans are mortal. This contradicts the assumption. More doubtful fact is how individuals corresponding to dead humans are represented in a model. And, by Lowenheim-Skolem's thorem there exists a countable model if a model exists. This implies that we have difficulties to identify a set of individuals in its model. Can we find verifiability and reality in such concepts?

Now we cannot deny a human semantic competence. Partee devided level of semantics into two parts and insisted that semantics in lexical level is a mental part. The claim shows that it is improper to advance model-theoretic approaches in MG to linguistic level. Here we recognize many problems in her insistence. According to her argument, it is realistic to choose appropriate individuals and possible-worlds in models of intensional logic and Montague's attempt is to define not a unique intensional model but a family of models. We believe human can never recognize such models in his mind. She said that human need not know all possible-worlds and choose optimal world by means of the mechanisms as induction. This idea is very suspicious but we do not know how to verify it now. That is, the specification of a particular actual model, which she called, cannot be 'realistic' if we use model theoretic semantics as intensional (or predicate) logic.

From above considerations, we will conclude the following. Linguistics is a part of philosophy rather than psychology. Since psychology has not complete systems, we do not intend to say psychology is an incomplete study, the object of semantics is both humans ourselves and external worlds. Of course we can mention that methodology in MG is a small part of our internal world. We want to insist that we ought to unify pragmatics as MG provided the way unifying syntax and semantics. Methodology in MG must be a foothold of it. At that time it does not matter whether there exists a reality in the methodology. The important thing is that such a methodology can constitute a part of realistic linguistic theory. In other words, logical forms may be interpreted both more logically and psychologically.

After all we can only see the worlds through tinted glasses, namely our language. To make matters worse, we never take off our glasses. Living things such as bees and birds may look the worlds in more effective ways. And we want to know about the worlds more. To do so, we come to set down our tinted lense. In the case of MG its settings are performed by model theory. If the degree of lense slip down we will look at the world in strayed eyes. If we fall into the case, we should reflect on ourselves again. This reflection will cause us to find the way how to know natural language better.

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