The Rhetorical Structure of Modus Tollens: An Exploration in Logic-Mining

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Abstract

A general method for mining discourse for occurrences of the rules of inference would be useful in a variety of natural language processing applications. The method described here has its roots in Rhetorical Structure Theory (RST). An RST analysis of a rule of inference can be used as an exemplar to produce a relational complex in the form of a nested relational proposition. This relational complex can be transformed into a logical expression using the logic of relational propositions. The expression can then be generalized as a logical signature for use in logic-mining discourse for instances of the rule. Generalized logical signatures reached in this manner appear to be grounded in identifiable logical relationships with their respective rules of inference. Thus, from a text, it is possible to identify a rhetorical structure, and from the structure, a relational proposition, and from the relational proposition, a generalized logical signature, and from the signature, the rule of inference residing within the text. The focus in this paper is on modus tollens and its variants, but the method is extensible to other rules as well.

1 Introduction

Recognizing occurrences of rules of inference in discourse is difficult for humans and computers alike. A method for doing so would be valuable for natural language processing, discourse analysis, and studies in logic and argumentation. Potter (2018) showed that some standard rules, including modus ponens, disjunctive syllogism, and some basic logical operations are directly accessible using Rhetorical Structure Theory (RST). This arises as a result of direct mappings between RST relations, corresponding relational propositions, and the rules of inference. For others there is no direct correspondence. This is because the rules of inference rules tend to manifest, not as individual relations, but as relational complexes, which may be embedded within deeply nested relational propositions.

This paper provides a description of a method for using RST to discover occurrences of modus tollens in natural discourse. The paper will extend this method to biconditional elimination, particularly as it relates to valid forms of denying the antecedent. Identifying relational complexes associated with these rules will support the specification of generalized logical signatures that can be used in logic-mining texts. While the method defined here is limited to modus tollens and its variants, it provides guidance for investigating other rules of inference, such as hypothetical syllogism and dilemma, and may lead to a general methodology for signature-based logic mining. This also suggests the possibility of discovering rules of inference present in discourse but not recognized in the literature of classical logic.

The approach described here presumes the availability of RST analyses, created, either interactively using tools such as O'Donnell's (1997) RSTTool or Zeldes' (2016) rstWeb, or computationally (e.g., Corston-Oliver, 1998; Hernault, Prendinger, duVerle, & Ishizuka, 2010; Pardo, Nunes, & Rino, 2004; Soricut & Marcu, 2003). These RST analyses may be restated as nested relational propositions, and these propositions can be used to generate the underlying logical organization of the text (Potter, 2018). Discovery of inference rule instantiations within this logical expression proceeds by aligning logical signatures with structural constituents of the comprehensive expression.

Lest there be any confusion as to the scope of this study, note that the objective here is not to develop a system of reasoning based on linguistic form, as in natural logic (MacCartney & Manning, 2009; Van Benthem, 1986), nor is it concerned with the logical forms of imperatives, questions, and statements, nor with the relationship between grammar and reasoning (Lakoff, 1970). The scope of this study concerns the discovery of occurrences of rules of inference as presented in discourse, as manifested in rhetorical structures, and with particular focus on modus tollens. Consistent with the fundamentals of RST, it is a logic of intended effect.

The remaining sections of this paper are as follows. First, a brief review of RST is presented using an analysis of a relevant example. This is followed by an overview of the logic of relational propositions, showing how complexes of nested relational propositions provide the basis for logical signatures useful in logic mining. Four generalized signatures for modus tollens are discussed, consisting of canonical, evidential, biconditional, and antithetical signatures. This includes a brief analysis concerning inference rule identification for incomplete relational complexes. Following this analysis is an explanation for how the logical signatures derived from discourse can be used to validate the rules of inference they serve to instantiate. The paper concludes with a discussion of the results and directions for future study. Relevant literature will be cited in passim.

2 RST Analysis of a Relevant Example

Rhetorical Structure Theory (RST) is an account of textual coherence (Mann & Thompson, 1988). It is used for describing texts in terms of the relations that hold among the text spans comprising the text. An RST relation consists of three parts: a satellite, a nucleus, and a relation. The satellite and nucleus are text spans, which are either elementary discourse units or subordinate RST relations. The distinction between satellite and nucleus arises as a result of the asymmetry of the relations. Within a relation, the nucleus is more salient than the satellite. A key consideration in defining nuclearity is the concept of locus of intended effect. The locus of intended effect may be in the nucleus, the satellite, or shared between the two. Locating the effect is important for the logical analysis of RST relations, particularly in implicative relations where the locus of intended effect will usually be the implicand (Potter, 2018).

Figure 1 shows an example of an RST analysis. The text is a short passage from J. L. Austin's translation of Frege's *Foundations of Arithmetic* (1884/1980, p. 37). The text presents an argument against the claim that numbers are merely ideas without objective reality. Frege begins by stating that he disagrees with a claim made by the mathematician Oskar Schlömilch, that numbers are ideas, not things. Frege supports his statement first by conceding that if numbers were merely ideas, then mathematics would be part of psychology. The CONDITION relation is used to indicate the dependency of the nucleus on the satellite. But this conditional is rejected using a comparison of mathematics with astronomy. This analogy is used as EVIDENCE for rejecting Schlömilch's position. That Frege's argument is an application of modus tollens

$$(((p \rightarrow q) \land \neg q) \rightarrow \neg p)$$

and that the RST structure presented here maps to the rule of inference may be intuitively apparent. However, as will be developed in this paper, this need not, and in most cases cannot, be merely a matter of intuition.

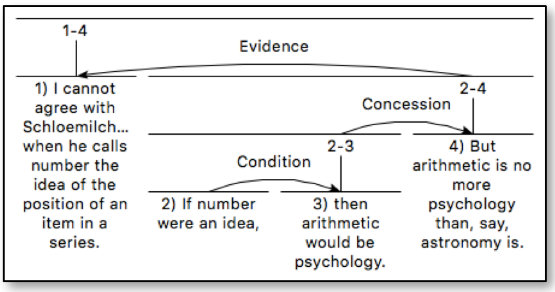

Figure 1: RST Analysis of Frege's Argument Against Psychologism

3 The Logic of Relational Propositions

It has been argued that Rhetorical Structure Theory is incapable of representing inferential patterns, because argumentative and rhetorical relations are said to be orthogonal to one another, and because RST relations provide little or no indication of alignment with the rules of inference (Budzynska, Janier, Reed, & Saint-Dizier, 2016). However, the structure of an RST analysis reflects the structure of its argument. EVIDENCE is evidential, MOTIVATION is motivational, and ENABLEMENT is enabling. This would suggest the logic and reasoning are not too far below the surface. As shown by Potter (2018), for any RST relation there is a corresponding logical form, and these forms combine to construct logical expressions that map to RST tree structures and serve as logical interpretations of the organization of a text. The approach used for deriving these interpretations is based on discourse entities known as relational propositions. Relational propositions are implicit assertions that arise between clauses within a text and are essential to the effective functioning of the text (Mann & Thompson, 1986a, 1986b, 2000). RST and relational propositions provide parallel accounts of discourse coherence. While RST identifies structures of coherence relations among the spans within a text, relational propositions treat these relations as implicit relational acts that account for how the text functions (Mann & Thompson, 1986b).

A relational proposition consists of a predicate and a pair of discourse units. The predicate corresponds to the RST relation, and the units correspond to the satellite and nucleus. In this paper relational propositions are specified using a functional notation. This permits concise representation of nested relational propositions. For example, the relational proposition for the RST analysis of the Frege argument shown in Figure 1 is as follows:

evidence(concession(condition(2,3),4),1)

where each elementary discourse unit is identified numerically in order of appearance in the text. Each relational predicate is associated with a logical form. In the above relational proposition, the *condition* predicate is defined as material implication, $(s \rightarrow n)$. The satellite materially implies the nucleus. Granted, there are persuasive arguments in favor of treating *condition* as biconditional (e.g., Geis & Zwicky, 1971; Horn, 2000; Karttunen, 1971; Moeschler, 2018; van der Auwera, 1997a, 1997b); however, for the purpose of logic mining the biconditional interpretation of *condition* will frequently be unnecessary, and preserving the distinction conditional and biconditional can be a useful.

With the CONCESSION predicate, the writer acknowledges a perceived incompatibility between the situations presented in the satellite and nucleus and uses this acknowledgement to forestall objections that might otherwise have arisen as a result of the perceived incompatibility. By pre-empting the objection, the writer smooths the way to increasing the reader's positive regard for the situation presented in the nucleus. Logically then, we can say that it is not the case that the satellite provides grounds for rejecting the nucleus: $\neg(s \rightarrow \neg n)$. Upon neutralizing this objection, the writer further invites the reader to infer from this the claim presented by the nucleus. The reasoning thus becomes an instance of modus ponens in which the condition of the major premise is a negated conditional statement:

$$(((\neg(s \rightarrow \neg n) \rightarrow n) \wedge \neg(s \rightarrow \neg n)) \rightarrow n)$$

With the EVIDENCE predicate, the satellite provides evidence in support of the nucleus. For the relation to achieve its intended effect, the reader must accept the satellite and recognize its implicative relationship with the nucleus. If the antecedent is believable, the consequent will also be believable. To achieve its effect, EVIDENCE requires that the antecedent (i.e. the satellite) be asserted. Hence the logical form of EVIDENCE is modus ponens:

$$(((s \rightarrow n) \wedge s) \rightarrow n)$$

The three logical forms (*condition*, *concession*, and *evidence*), corresponding to the relations used in the Frege analysis, can be used to construct the logical expression of the nested relational proposition, which expands to the following valid argument:

$$((((((\neg((2 \rightarrow 3) \rightarrow \neg 4) \rightarrow 4) \wedge \neg((2 \rightarrow 3) \rightarrow \neg 4)) \rightarrow 4) \rightarrow 1) \wedge (((\neg((2 \rightarrow 3) \rightarrow \neg 4) \rightarrow 4) \wedge \neg((2 \rightarrow 3) \rightarrow \neg 4)) \rightarrow 4)) \rightarrow 1)$$

Using this technique, it is possible to generate logical expressions for any RST analysis. While the resulting expressions can be complex, they are constructed from the simple logical forms defined for each of the relational predicates. As will be detailed in Section 4, these forms are generalizable as logical signatures that may be used in mining texts for occurrences of rules of inference.

Note that discourse units used in relational propositions need not be truth-functional in the restrictive sense of the term. Although it is common practice present logic in terms of truth values and truth functions, these semantics are arbitrary, and we could just as well speak of on and off, + and -, 1 and 0, yes and no, open and closed, satisfiability and unsatisfiability, or any other bivalent conceptualization, including belief and disbelief, positive and negative regard, desire and indifference, interest and disinterest, understanding and misunderstanding, or ability and inability. To the extent that the primitives of RST can be understood in terms of bivalent values, they are amenable to logical treatment.

4 Relational Complexes

As noted earlier, some inference rules manifest as single relational predicate, but this is not always the case. Modus tollens requires multiple predicates, and these predicates may be combined in various ways. Each of these combinations, for any given instance, is a relational complex. A relational complex may then be generalized and normalized to create a signature, or logical pattern that may then be used to locate other instances of the rule in discourse.

The generalization process consists in replacing the numeric unit identifiers with normalized alphabetic variables. Normalization consists in identifying discourse units that are sufficiently similar semantically to indicate material equivalence or negation. This paper makes no attempt to define a technology for measuring semantic textual similarity. There are already numerous research efforts in that area. For example, Finch, Hwang, and Sumita (2005) repurposed machine translation evaluation methods to determine sentence-level semantic equivalence, Tsatsaronis, Varlamis, and Vazirgiannis (2010) developed a measure of semantic relatedness which capitalizes on a word-to-word semantic relatedness measure and extended it to measure the relatedness between texts, and Sultan, Bethard, and Sumner (2015) developed supervised and unsupervised systems for measuring sentence similarity. Addressing negation detection, Basile, Bos, Evang, and Venhuizen (2012) used discourse representation structures for negation detection, and Harabagiu, Hickl, and Lacatusu (2006) interpreted negation using a combination of overt and indirectly licensed negation. For the present study, normalizations are hand-crafted. Thus, for the generalized signature

$$((((((\neg((p \to q) \to \neg r) \to r) \wedge \neg((p \to q) \to \neg r)) \to r) \to s) \wedge (((\neg((p \to q) \to \neg r) \to r) \wedge \neg((p \to q) \to \neg r)) \to r)) \to s)$$

the normalized logical form, with double negations removed is:

$$((((((\neg((p \to q) \to q) \to \neg q) \wedge \neg((p \to q) \to q)) \to \neg q) \to \neg p) \wedge (((\neg((p \to q) \to q) \to \neg q) \wedge \neg((p \to q) \to q)) \to \neg q)) \to \neg p)$$

We can use this logical form as a template for identifying comparable relational propositions within texts, keeping in mind that any of the elements of the expression may refer recursively to lower level complex expressions. To the extent that the comparisons align, the logical expressions for each relational proposition will comprise the sought-after relational complexes, which provide the basis for the logical signature.

5 Canonical Modus Tollens

Modus tollens is a valid argument of the form:

$$(((p \to q) \wedge \neg q) \to \neg p)$$

The categorical premise $(\neg q)$ denies the consequent of the conditional premise, implying the negation of the antecedent $(\neg p)$. Figure 2 shows an RST analysis of a *Wikipedia* example of modus tollens. As shown, the writer concedes that the conditional relationship between Rex as a chicken and Rex as a bird holds, but rejects the proposition that he is a bird. From this, we may reason, Rex is no chicken. The relational proposition for this structure is

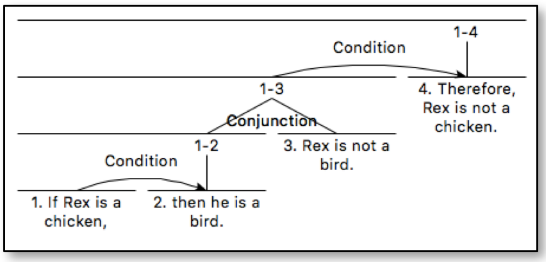

Figure 2: Rhetorical Structure of Modus Tollens

condition(conjunction(condition(1,2),3),4)

And the relational complex for this proposition therefore is:

$$(((1 \to 2) \wedge 3) \to 4)$$

This may be generalized and normalized to

$$(((p \to q) \wedge \neg q) \to \neg p)$$

which is modus tollens. Stated canonically, the RST relations are subject matter, rather than presentational, because there is no intent to influence an inclination in the reader. In practice, however, modus tollens is commonly used as an act of persuasion. This leads to the evidential and antithetical signatures for modus tollens.

6 Evidential Modus Tollens

When the writer uses modus tollens with the intent to influence the reader's beliefs, the EVIDENCE relation may be employed. This intended effect adds to the complexity of the logical structure of the argument. This occurs in Frege's argument against the claim that numbers are merely ideas without objective reality, introduced earlier. Frege's argument, shown in Figure 1, relies on modus tollens for its validity. EVIDENCE is used to link the argument's premises to the conclusion. As specified by the definition of modus tollens, the argument starts with a conditional premise:

> *If number were an idea, then arithmetic would be psychology,*

followed by a categorical premise that denies the consequent of the conditional premise,

> *But arithmetic is no more psychology than, say, astronomy is,*

and a conclusion that infers the denial of the antecedent of the conditional premise:

> *I cannot agree with Schloemilch…when he calls number the idea of the position of an item in a series.*

The relational proposition for the Frege analysis,

> *evidence(concession(condition(p,q),r),s)*

generalizes to the logical expression:

$$((((((\neg((p \to q) \to q) \to \neg q) \wedge \neg((p \to q) \to q)) \to \neg q) \to \neg p) \wedge (((\neg((p \to q) \to q) \to \neg q) \wedge \neg((p \to q) \to q)) \to \neg q)) \to \neg p)$$

Any analysis that matches this generalized signature will be an instantiation of the modus tollens rule of inference. That this is so is supported in part by the signature's derivation from an exemplar of modus tollens, and is further supported, as will be discussed in detail in Section 9, by the realization that the rule of inference is deducible from the signature. That is to say, for any such argument, the canonical rule is logically implicit within the RST analysis, and therefore within the text.

7 Biconditional Modus Tollens

The CONDITION relation sometimes represents a biconditional logical relation. This is apparent in part from the definition of the relation as specified by Mann and Thompson (1987), that realization of the situation presented in the nucleus (the consequent) *depends* upon the realization of the situation presented in the satellite (the antecedent), and it is also observable in the text they used as their example of the relation:

> *N: Employees are urged to complete new beneficiary designation forms for retirement or life insurance benefits*
>
> *S: whenever there is a change in marital or family status.*

A *change in marital or family status* is the condition under which employees are urged to complete new beneficiary designation forms. The reader recognizes that the realization of the nucleus

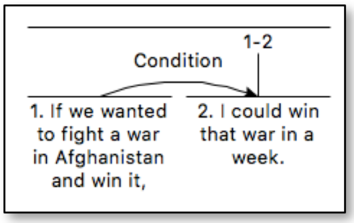

Figure 3: Counterfactual Modus Tollens

depends on the realization of satellite. If there is no change in status, there is no need to complete new forms. If the satellite remains unrealized, so will the nucleus. Thus, the relation is biconditional ($s \leftrightarrow n$).

Occurrences of the biconditional as modus tollens may employ the counterfactual in the antecedent. The counterfactual contains the denial of the antecedent within the antecedent itself. In the example shown in Figure 3, Donald Trump argues that if he wanted to win the war in Afghanistan, he could do so within a week. The counterfactuality of the antecedent indicates that he does not wish to do so, with the implication that we therefore cannot do so. This interpretation leads to a relational proposition defined not only on the basis of the explicit rhetorical structure, but the implicit relations as well:

> *condition(conjunction(condition(1,2),[3]),[4])*

which normalizes to the biconditional modus tollens: $(((p \leftrightarrow q) \wedge \neg p) \to \neg q)$. When the normalization process indicates denial of the antecedent, the charitable interpretation will be that the CONDITION relation is being used as biconditionally. Not only may the denial of the antecedent be implicit, the consequent itself may be implicit. Incomplete conditionals such as

1. *If only Miss Hawkins would get a job…*

have an implicit implicative potentiality. While this example leaves much to the reader's imagination, with assistance from context provided by the writer, or from the reader's world knowledge (Elder & Savva, 2018), a pragmatic conjecture such as

2. *[then surely her situation would be improved.]*
3. *[But, alas, she has not gotten a job.]*
4. *[And so her situation remains unimproved.]*

seems plausible, and results in the relational complex:

cause(concession(condition(1,[2]),[3]),[4])

As constructed, the inference relies on denying the antecedent. Hence it is another example of biconditional modus tollens. However, the logic differs from the previous example, due to the use of the *cause* predicate instead of *condition*. The *cause* predicate has the same logical form as *evidence*, and as such is used to link the argument's premises to the conclusion. Clearly, however, the more fragmentary the information, the greater the risks of conjecture, and the greater risk of false positives.

8 Antithetical Modus Tollens

ANTITHESIS is used as part of a modus tollens relational complex in a manner rhetorically similar to CONCESSION. This is perhaps owing to the

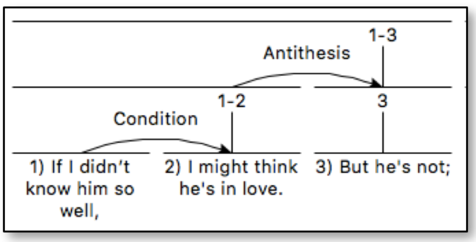

Figure 4: ANTITHESIS as Modus Tollens

similarity of the two relations (Stede, 2008). In the example shown in Figure 4, the structure follows the familiar pattern of modus tollens, but now the CONDITION is a satellite of the ANTITHESIS rather than of CONCESSION. The logical form, and hence the signature, is disjunctive syllogism,

$$((((p \rightarrow q) \lor \neg q) \land \neg(p \rightarrow q)) \rightarrow \neg q)$$

Thus ANTITHESIS, when the satellite is conditional, is modus tollens. Alternatively, the CAUSE relation may be used as satellite to the ANTITHESIS relation, as shown in Figure 5. This text is interesting in

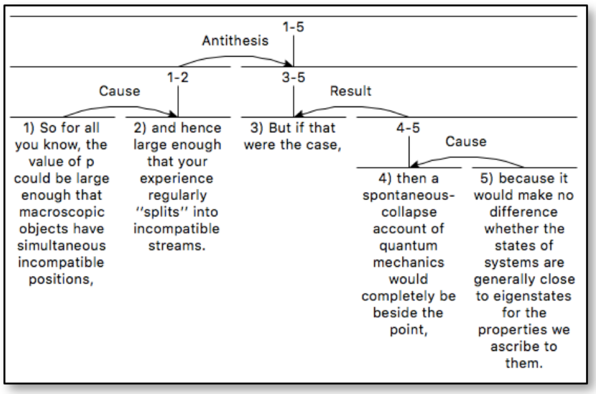

Figure 5: The Cause-Antithesis Modus Tollens

several respects. From the logical perspective, there are arguments within arguments such that the consequences of one become the condition of another. And counterfactual conditionality is used to implement a strategy of reductio ad absurdum, such that the conclusion of the text indicates its own negation. Logic mining is useful in sorting this out. The text divides conveniently into two parts. Units 1-3 implement the causal variety of antithetical modus tollens:

$$((((((p \rightarrow q) \land p) \rightarrow q) \lor r) \land \neg(((p \rightarrow q) \land p) \rightarrow q)) \rightarrow r)$$

That this is an occurrence of antithetical modus tollens can be realized by evaluating the causal argument to obtain its result, q, so that the expression becomes

$$(((q \lor r) \land \neg q) \rightarrow r)$$

which when normalized becomes a signature for antithetical modus tollens:

$$(((p \lor q) \land \neg p) \rightarrow q)$$

As discussed below in Section 9, modus tollens is provable using disjunctive syllogism. An alternative approach would be to realize that if $((p \rightarrow q) \land p)$, as indicated by CAUSE, then the CONDITION $(p \rightarrow q)$ holds as well. The same approach can be used for segments 3-5. The if-then statement of 3-4 is coded as a RESULT, because it is the antecedent of the condition that is salient in this text. Segment 3, or r, situated conditionally within the argument, is the negation of "that is not the case." The consequent, provided in 4-5, provides the reductio ad absurdum. That is, if "that were the case," untenable results would follow.

9 The Significance of Signatures

The question will arise as to the significance of logical signatures. Are they grounded in identifiable logical relationships with their respective rules of inference, or is the correspondence between signatures and rules simply a happy coincidence? Both signatures and rules are valid arguments, both share the same elementary propositions, and both reach the same conclusion. It would therefore be useful to determine whether the rules of inference are deducible from the signatures, and if not, what the nature of the relationship is. So now we can examine each the signatures introduced above and determine their relationship to modus tollens. The signatures to be considered include canonical, evidential, biconditional, and antithetical modus tollens. For *canonical modus tollens*, the signature maps directly to the inference rule; it is indeed simply a statement of the rule, $(((p \rightarrow q) \wedge \neg q) \rightarrow \neg p)$. *Evidential modus tollens* is a more interesting case. It has already been shown that the logical signature for

evidence(concession(condition(p,q),r),s)

is

$$((((((\neg((p \rightarrow q) \rightarrow q) \rightarrow \neg q) \wedge \neg((p \rightarrow q) \rightarrow q)) \rightarrow \neg q) \rightarrow \neg p) \wedge (((\neg((p \rightarrow q) \rightarrow q) \rightarrow \neg q) \wedge \neg((p \rightarrow q) \rightarrow q)) \rightarrow \neg q)) \rightarrow \neg p)$$

This expression contains two occurrences of the valid argument

$$(\neg((p \rightarrow q) \rightarrow q) \rightarrow \neg q)$$

We evaluate and replace those occurrences with their consequent, $\neg q$, resulting in

$$(((((\neg q \wedge \neg((p \rightarrow q) \rightarrow q)) \rightarrow \neg q) \rightarrow \neg p) \wedge ((\neg q \wedge \neg((p \rightarrow q) \rightarrow q)) \rightarrow \neg q)) \rightarrow \neg p)$$

which contains two occurrences of the valid argument

$$((\neg q \wedge \neg((p \rightarrow q) \rightarrow q)) \rightarrow \neg q)$$

for which we also substitute the consequent, $\neg q$, resulting in the valid argument

$$(((\neg q \rightarrow \neg p) \wedge \neg q) \rightarrow \neg p)$$

for which the implicant

$$((\neg q \rightarrow \neg p) \wedge \neg q)$$

is materially equivalent to the implicant of modus tollens:

$$((\neg q \rightarrow \neg p) \wedge \neg q) \leftrightarrow ((p \rightarrow q) \wedge \neg q)$$

Thus the evidential interpretation effectively reduces to modus tollens. This is applicable to the logical forms of each of the modus ponens presentational relations, including BACKGROUND, ENABLEMENT, EVIDENCE, JUSTIFY, MOTIVATION, and PREPARATION, as well as the causal relations.

The presentational version of *biconditional modus tollens* operates similarly. The relational proposition

evidence(concession(condition(p,q),r),s)

normalizes to

$$((((((\neg((p \leftrightarrow q) \rightarrow p) \rightarrow \neg p) \wedge \neg((p \leftrightarrow q) \rightarrow p)) \rightarrow \neg p) \rightarrow \neg q) \wedge (((\neg((p \leftrightarrow q) \rightarrow p) \rightarrow \neg p) \wedge \neg((p \leftrightarrow q) \rightarrow p)) \rightarrow \neg p)) \rightarrow \neg q)$$

The modus ponens

$$(((\neg((p \leftrightarrow q) \rightarrow p) \rightarrow \neg p) \wedge \neg((p \leftrightarrow q) \rightarrow p)) \rightarrow \neg p)$$

occurs twice within this expression. Replacing this with its consequent, $\neg p$, yields

$$(((\neg p \rightarrow \neg q) \wedge \neg p) \rightarrow \neg q)$$

which is modus tollens. This is applicable to biconditional occurrences of the same RST relations as evidential modus tollens, except that the categorical premise normalizes to the negation of the antecedent of the conditional premise, rather than the consequent. It is by this means that this biconditional modus tollens can be distinguished from evidential modus tollens.

The relational proposition of *antithetical modus tollens* is *antithesis(condition(p,q),r)* for which the generalized signature is

$$((((p \rightarrow q) \vee \neg q) \wedge \neg(p \rightarrow q)) \rightarrow \neg q)$$

Since one of the proofs of modus tollens is based on disjunctive syllogism, it can be shown that modus tollens follows from the normalized expression. The major premise of the disjunctive syllogism, $((\neg p \lor q) \land \neg q)$, implies $(p \rightarrow q)$, so that if it is the case that

$$(((\neg p \lor q) \land \neg q) \rightarrow (p \rightarrow q))$$

it follows that both the premise and the conclusion hold,

$$(((\neg p \lor q) \land \neg q) \land (p \rightarrow q))$$

and it is a tautology that

$$(((\neg p \lor q) \land \neg q) \land (p \rightarrow q)) \leftrightarrow ((p \rightarrow q) \land \neg q)$$

Thus, modus tollens may be inferred from the logical signature for *antithesis(condition(p,q),r)*. And thus, the evidential, biconditional, and antithetical signatures can be used, not only to discover instances of modus tollens in discourse, they are grounded in the rule of inference they are designed to detect.

10 Conclusion

This exploration of modus tollens has shown how relational propositions can be used to support discourse logic-mining using logical signatures as a means for discovering occurrences of standard rules of inference in discourse. In addition to modus tollens, several other signatures that serve as indicators of rules of inference have been noted. EVIDENCE and other pragmatic and causal relations map directly to modus ponens, and ANTITHESIS implements disjunctive syllogism. Further research is needed to determine what additional signatures can be identified. These would provide a rich set of resources for logic-mining discourse and reduce the need for ad hoc procedures for inference rule identification and would eventually support a greater capability for automated analysis.

Automated identification of inference rules within discourse would require development and integration of several capabilities. Although there has been significant work in automated detection of RST relations (e.g., Corston-Oliver, 1998; Hernault et al., 2010; Pardo et al., 2004; Soricut & Marcu, 2003), such a capability would need to generate output as nested relational propositions of complex structures. Prototype software already exists for generating logical expressions from nested relational propositions of arbitrary size and complexity (Potter, 2018). A unification algorithm could be used for identifying instantiations of inference rules in nested relational propositions. The generalized signatures would subsume instances of inference rules in a relational proposition. Subsumption would succeed when the proposition contains a logical structure isomorphic with the signature. The signature would need to match both simple and composite spans, so that instantiation could occur at any level within the structure.

Using RST as the starting point for inference rule discovery simplifies the task, but also delimits it. These delimitations arise not so much the result of well-known concerns about the validity of RST (e.g., Asher & Lascarides, 2003; Budzynska et al., 2016; Grosz & Sidner, 1986; Knott, Oberlander, O'Donnell, & Mellish, 2001; Moore & Pollack, 1992; Sanders, Spooren, & Noordman, 1992; Webber, Stone, Joshi, & Knott, 2003; Wiebe, 1993; Wolf & Gibson, 2005), but out of a fundamental feature of the theory—namely that it is a theory of coherence relations. Perhaps this delimitation is an asset. By basing the concept of logic-mining on a theory of coherence relations, it is by definition constrained to discursive inferences discoverable within a text. The granularity of analysis being at the clausal level, the inferences discoverable among these clauses are propositional. A benefit of this is that many problems in natural language inferencing, such as those described by Lakoff (1970), van Benthem (2008), MacCartney (2009) and Karttunen (2015), e.g., determining logical relationships among arbitrarily selected assertions, are avoided. They are avoided not because they do not exist, for indeed they do, but because they need not come to the surface. A practical solution for logic-mining texts for rules of inference should be both useful and interesting, and perhaps the techniques arising from this work will contribute to solving grander challenges. For now, the essence of logic-mining is that from a text, it is possible to identify a rhetorical structure, and from the structure, a relational proposition, and from the relational proposition, a generalized logical signature, and from the signature, the rule of inference residing within the text.

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