On the Maximalization of the Witness sets in Independent Set readings

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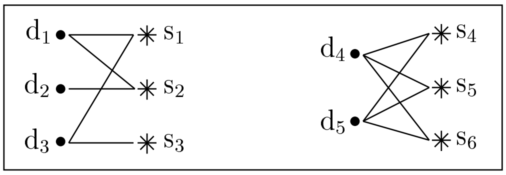
1 Pre - Introduction

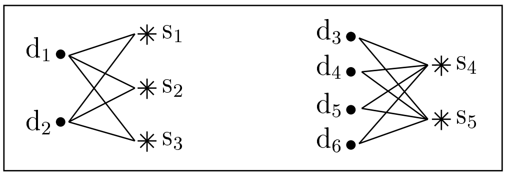
Before starting, I would like to ask reader's opinion about the truth/falsity of certain NL statements. The statements are about figures depicting dots connected to stars. In the figures, we distinguish between dots and stars that are connected, i.e. such that every dot is connected with at least one star and every star is connected with at least one dot, and dots and stars that are *totally* connected, i.e. such that every dot is connected to every star. For instance, in (1), the dots d_1, d_2, and d_3 are connected with the stars s_1, s_2, and s_3 (on the left) while d_4 and d_5 are *totally* connected with s_4, s_5, and s_6 (on the right).

(1)



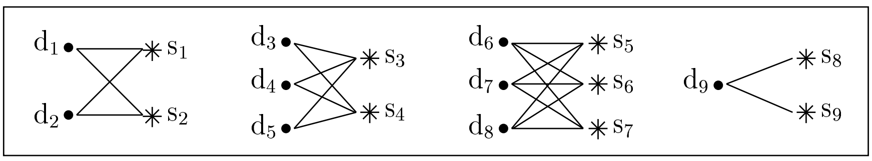
given these premises, is it true that in the next figure *Less than half of the dots are totally connected with exactly three stars*? (do not read below before answering)

(2)



I do think that the answer is yes. The same answer has been given by several friends/colleagues that were asked to judge the example. In fact, the figure does contain two dots d_1 and d_2, which are less than half of all the dots in the figure, and they are both connected with three same stars s_1, s_2, and s_3.
Now, is it true in (3) that *Few dots are totally connected with few stars?*

(3)



It is somehow harder to provide an answer to this second question. At first sight, it seems the sentence is false, or at least 'strange': no English speaker would ever utter that sentence in that context, whatever he wants to describe.

We are ready now to explore the proposals that aimed at formally defining the truth conditions of sentences as the two ones above. In the literature, most logical approaches to the problem state that the two sentences are both false in contexts (2) and (3). In (Robaldo, 2009a), drawing from (Sher, 1997), I proposed a new alternative where they are both evaluated as true. It seems then that neither proposals is completely satisfatory. The present paper proposes a "pragmatic" revision of (Robaldo, 2009a) that achieves – what are claimed to be – the proper truth values of such sentences.

2 Introduction

In the Pre-Introduction, it has been asked to judge the truth values of two NL sentences according to their 'Scopeless interpretation', termed in (Robaldo, 2009a) as 'Independent Set (IS) reading'. In constrast, in a linear reading one of the sets may vary on the entities in the other one. An example is *Each boy ate two apples*, whose preferred reading is a linear reading where *Each* outscopes *Two*, i.e. where each boy ate two *different* apples. Four kinds of IS readings have been identified in the literature, from (Scha, 1981).

(4) a. **Branching Quantifier readings**, e.g. *Two students of mine have seen three drug-dealers in front of the school*. (Robaldo, 2009a)

b. **Collective readings**, e.g. *Three boys made a chair yesterday*. (Nakanishi, 2007)

c. **Cumulative readings**, e.g. *Three boys invited four girls*. (Landman, 2000)

d. **Cover readings**, e.g. *Twenty children ate ten pizzas*. (Kratzer, 2007)

The preferred reading of (4.a) is the one where there are exactly two[1] students and exacly three drug-dealers and each of the students saw each of the drug-dealers. Note that these are the truth values assigned to (1)-(3) when dots and stars are asked to be *totally* connected. (4.b) may be true in case three boys cooperated in the construction of a single chair. In the preferred reading of (4.c), there are three boys and four girls such that each of the boys invited at least one girl, and each of the girls was invited by at least one boy. These are the truth values assigned to (1) when dots and stars are asked to be connected, possibly not totally. Finally, (4.d) allows for any sharing of ten pizzas between twenty children. In Cumulative readings, the single actions are carried out by *atomic*[2] individuals only, while in (4.d) it is likely that the pizzas are shared among subgroups of children. For instance, *Three children ate five pizzas* is satisfied by the following extension of ate' ('$\oplus$' is the standard sum operator (Link, 1983)):

(5) $$\|ate'\|^M \equiv \{\langle c_1 \oplus c_2 \oplus c_3, p_1 \oplus p_2\rangle, \langle c_2 \oplus c_3, p_3 \oplus p_4\rangle, \langle c_3, p_5\rangle\}$$

In (5), children c_1, c_2, and c_3 (cut into slices and) share pizzas p_1 and p_2, c_2 and c_3 (cut into slices and) share p_3 and p_4, and c_3 also ate pizza p_5 on his own.

Branching Quantifier readings have been the more controversial (cf. (Beghelli et al., 1997) and (Gierasimczuk and Szymanik, 2009)). Many authors claim that those readings are always subcases of Cumulative readings, and they often co-occur with certain adverbs (May, 1989), (Schein, 1993). In fact, in the Pre-Introduction, in order to force such a reading on (1)-(3), it was necessary to add the adverb *totally* to the verb *connected*. Collective and Cumulative readings have been largely studied; see (Scha, 1981), (Link, 1983), (Beck and Sauerland, 2000), and (Ben-Avi and Winter, 2003).

However, the focus here is on Cover readings. This paper assumes – following (van der Does, 1993), (van der Does and Verkuyl, 1996), (Schwarzschild, 1996), (Kratzer, 2007) – that they are *the* IS readings, of which the three kinds exemplified in (4.a-c) are merely special cases. The name "Cover readings" comes from the fact that their truth values are traditionally captured in terms of Covers. A Cover is a mathematical structure defined with respect to one or more sets. With respect to two sets S_1 and S_2, a Cover Cov is formally defined as:

[1] In (4.a-d) "two/three/ten/etc." are interpreted as "*exactly* two/three/ten/etc." as in (Scha, 1981). That is actually a pragmatic implicature, as noted in (Landman, 2000), pp.224-238.

[2] In line with (Landman, 2000), pp.129, and (Beck and Sauerland, 2000), def.(3), that explicitly define Cumulative readings as statements among atomic individuals only.

(6) A Cover Cov is a subset of $Cov_1 \times Cov_2$, where $Cov_1 \subseteq \wp(S_1)$ and $Cov_2 \subseteq \wp(S_2)$ s.t.

a. $\forall s_1 \in S_1, \exists cov_1 \in Cov_1$ s.t. $s_1 \in cov_1$, and $\forall s_2 \in S_2, \exists cov_2 \in Cov_2$ s.t. $s_2 \in cov_2$.

b. $\forall cov_1 \in Cov_1, \exists cov_2 \in Cov_2$ s.t. $\langle cov_1, cov_2 \rangle \in Cov$.

c. $\forall cov_2 \in Cov_2, \exists cov_1 \in Cov_1$ s.t. $\langle cov_1, cov_2 \rangle \in Cov$.

Covers may be denoted by 2-order variables called "Cover variables". We may then define a meta-predicate $Cover$ that, taken a Cover variable C and two unary predicates P_1 and P_2, asserts that the extension of the former is a Cover of the extensions of the latter:

(7) $$Cover(C, P_1, P_2) \Leftrightarrow$$
$$\forall_{X_1 X_2}[C(X_1, X_2) \rightarrow \forall_{x_1 x_2}[((x_1 \subset X_1) \wedge (x_2 \subset X_2)) \rightarrow (P_1(x_1) \wedge P_2(x_2))]] \wedge$$
$$\forall_{x_1}[\, P_1(x_1) \rightarrow \exists_{X_1 X_2}[\, (x_1 \subset X_1) \wedge C(X_1, X_2)\,]\,] \wedge$$
$$\forall_{x_2}[\, P_2(x_2) \rightarrow \exists_{X_1 X_2}[\, (x_2 \subset X_2) \wedge C(X_1, X_2)\,]\,]$$

Thus, it is possible to decouple the quantifications from the predications. This is done by introducing two relational variables whose extensions include the *atomic* individuals involved. Another relational variable that covers them describes how the actions are actually done. For instance, in (5), in order to evaluate as true the variant of (4.d), we may introduce three variables P_1, P_2, and C such that:

$$\|P_1\|^M = \{c_1, c_2, c_3\} \qquad \|P_2\|^M = \{p_1, p_2, p_3, p_4, p_5\}$$
$$\|C\|^M = \{\, \langle c_1 \oplus c_2 \oplus c_3,\ p_1 \oplus p_2 \rangle,\ \langle c_2 \oplus c_3,\ p_3 \oplus p_4 \rangle,\ \langle c_3,\ p_5 \rangle\, \}$$

The above extensions of P_1, P_2, and C satisfy $Cover(C, P_1, P_2)$.

Among the Cover approaches mentioned above, an interesting one is (Schwarzschild, 1996). Schwarzschild discusses numerous NL sentences where the identification of Covers appears to be pragmatically determined, rather than existentially quantified. In other words, in the formulae the value of the Cover variables ought to be provided by an assignment g. One of the examples mostly discussed in (Schwarzschild, 1996) is:

(8) a. The cows and the pigs were separated.

b. The cows and the pigs were separated *according to color*.

The preferred reading of (8.a) is the one where the cows were separated from the pigs. However, that is actually an implicature that may be rewritten as in (8.b), where the separation is not done by race. Examples like (8) are used by (Schwarzschild, 1996) in order to argue against the existence of groups and the overgeneration of readings, extensively advocated by (Landman, 2000). Schwarzschild claims that the NP in (8.a) must correspond to a unary predicate whose extension is the set of *individual* cows and pigs, while the precise separation is described by a contextually-dependent Cover variable. Similarly, in (4.c) the Cumulative interpretation is preferred as in real contexts invitations are usually thought as actions among pairs of persons. But it may be the case that two or more boys *collectively* invited two or more girls. On the other hand, in (4.a) the fact that each student saw each drug-dealer seems to be favoured by the low value of the numerals. If the sentence were *Almost all of my students have seen several drug-dealers in front of the school*, the preferred reading appears to be Cumulative.

The next section illustrates a final component needed to build whole formulae for representing Cover readings. This is the requirement of Maximal participancy of the witness sets, e.g. the Maximal participancy of P_1 and P_2's extension in the formula representing the meaning of the variant of (4.d). It will be also shown that there are two possible ways to maximize the witness sets: *Locally* and *Globally*. The former predicts that both examples in (2) and (3) are true, while the latter predicts that they are both false.

3 The Maximality requirement

The previous section showed that, for representing IS readings, it is necessary to reify the witness sets into relational variables as P_1 and P_2. Separately, the elements of these sets are combined as described by the Cover variables, in order to assert the predicates on the correct pairs of (possibly plural) individuals. Conversely, it is not possible to represent an IS reading by nesting quantifiers into the scope of other quantifiers, as it is done in the standard Generalized Quantifier (GQ) approach (Keenan and Westerståhl, 1997), because the set of entities quantified by the narrow-scope quantifier would vary on each entity quantified by the wide-scope one.
As argued by (van Benthem, 1986), (Kadmon, 1987), (Sher, 1990), (Sher, 1997), (Spaan, 1996), (Steedman, 2007), (Robaldo, 2009a), and (Robaldo, 2009b) the relational variables must, however, be *Maximized* in order to achieve the proper truth values with any quantifier, regardless to its monotonicity. To see why, let us consider sentences in (9), taken from (Robaldo, 2009a), that involve a single quantifier.

(9) a. At least two men walk.

b. At most two men walk.

c. Exactly two men walk.

In terms of reified relational variables, it seems that the meaning of (9.a-c) may represented via (10.a-c), where $^{\geq}2$, $^{\leq}2$, and $^{=}2$ are, respectively, an M↑, an M↓, and a non-M Generalized Quantifier.

(10) a. $\exists P[\ ^{\geq}2_x(\text{man'}(x), P(x)) \wedge \forall_x[P(x) \rightarrow \text{walk'}(x)]\ ]$

b. $\exists P[\ ^{\leq}2_x(\text{man'}(x), P(x)) \wedge \forall_x[P(x) \rightarrow \text{walk'}(x)]\ ]$

c. $\exists P[\ ^{=}2_x(\text{man'}(x), P(x)) \wedge \forall_x[P(x) \rightarrow \text{walk'}(x)]\ ]$

Only (10.a) correctly yields the truth values of the corresponding sentence. To see why, consider a model in which three men walk. In such a model, (10.a) is true, while (10.b-c) are false. Conversely, all formulae in (10) evaluate to true, as all of them allow to choose P such that $\|P\|^M$ is a set of two walking men. Therefore, we cannot allow a free choice of P. Instead, P must denote the Maximal set of individuals satisfying the predicates, i.e. the Maximal set of walking men, in (10). This is achieved by changing (10.b-c) to (11.a-b) respectively.

(11) a. $\exists P[\ ^{\leq}2_x(\text{man'}(x), P(x)) \wedge \forall_x[P(x) \rightarrow \text{walk'}(x)] \wedge$
$\forall'_P[(\forall_x[P(x) \rightarrow P'(x)] \wedge \forall_x[P'(x) \rightarrow \text{walk'}(x)]) \rightarrow \forall_x[P'(x) \rightarrow P(x)]\ ]\ ]$

b. $\exists P[\ ^{=}2_x(\text{man'}(x), P(x)) \wedge \forall_x[P(x) \rightarrow \text{walk'}(x)] \wedge$
$\forall'_P[(\forall_x[P(x) \rightarrow P'(x)] \wedge \forall_x[P'(x) \rightarrow \text{walk'}(x)]) \rightarrow \forall_x[P'(x) \rightarrow P(x)]\ ]\ ]$

The clauses $\forall'_P[\ \ldots\ ]$ in the second rows are Maximality Conditions asserting the non-existence of a superset P' of P that also satisfies the predication. There is a single choice for P in (11.a-b): it must denote the set of *all* walking men. Note that, for the sake of uniformity, the Maximality condition may be added in (10.a) as well: in case of M↑ quantifiers, it does not affect the truth values.

3.1 Local Maximalization

Let me term the kind of Maximalization done in (11) as *Local* Maximalization. The Maximality conditions in (11) require the non-existence of a set $\|P'\|^M$ of walkers *that includes* $\|P\|^M$. In (Robaldo, 2009a) and (Robaldo, 2009b), I proposed a logical framework for representing Branching Quantifier based on Local Maximalization. For instance, in (Robaldo, 2009a), the *two* witness sets of students and drug-dealers in (4.a) are respectively reified into two variables P_1 and P_2, and the Maximality condition requires the non-existence of a *Cartesian Product* $\|P'_1\|^M \times \|P'_2\|^M$, that also satisfies the main predication and *that includes* $\|P_1\|^M \times \|P_2\|^M$:

(12) $\exists P_1 P_2[\,^{=}2_x(\text{stud'}(x), P_1(x)) \wedge\, ^{=}3_x(\text{drugD'}(y), P_2(y)) \wedge$
$\forall_{xy}[(P_1(x) \wedge P_2(y)) \rightarrow \text{saw'}(x, y)] \wedge$
$\forall_{P'_1 P'_2}[\,(\,\forall_{xy}[(P_1(x) \wedge P_2(y)) \rightarrow (P'_1(x) \wedge P'_2(y))] \wedge$
$\forall_{xy}[(P'_1(x) \wedge P'_2(y)) \rightarrow \text{saw'}(x, y)]\,) \rightarrow$
$\forall_{xy}[(P'_1(x) \wedge P'_2(y)) \rightarrow (P_1(x) \wedge P_2(y))]\,]\,]$

In order to extend (Robaldo, 2009a) to Cover readings, which are assumed to be the most general cases of IS readings, we cannot simply require the inclusion of $\|P_1\|^M \times \|P_2\|^M$ into the main predicate's extension. Rather, we require the inclusion therein of a pragmatically-determined Cover $\|C\|^{M,g}$ of $\|P_1\|^M$ and $\|P_2\|^M$. Furthermore, the (local) Maximality condition must require the non-existence of a superset of either $\|P_1\|^M$ or $\|P_2\|^M$ whose corresponding Cover is a superset of $\|C\|^{M,g}$ that is also included in the main predicate's extension. Thus, (4.d) is represented as[3]:

(13) $\exists P_1 P_2[\,^{=}20_x(\text{child'}(x), P_1(x)) \wedge\, ^{=}10_y(\text{pizza'}(y), P_2(y)) \wedge$
$Cover(C, P_1, P_2) \wedge \forall_{xy}[C(x, y) \rightarrow \text{ate'}(x, y)] \wedge$
$\forall_{P'_1}[(\forall_x[P_1(x) \rightarrow P'_1(x)] \wedge \exists_{C'}[Cover(C', P'_1, P_2) \wedge \forall_{xy}[C(x, y) \rightarrow C'(x, y)] \wedge$
$\forall_{xy}[C'(x, y) \rightarrow \text{ate'}(x, y)]]) \rightarrow \forall_x[P'_1(x) \rightarrow P_1(x)]\,]\,] \wedge$
$\forall_{P'_2}[(\forall_y[P_2(y) \rightarrow P'_2(y)] \wedge \exists_{C'}[Cover(C', P_1, P'_2) \wedge \forall_{xy}[C(x, y) \rightarrow C'(x, y)] \wedge$
$\forall_{xy}[C'(x, y) \rightarrow \text{ate'}(x, y)]]) \rightarrow \forall_y[P'_2(y) \rightarrow P_2(y)]\,]\,]\,]$

Note that there are two Maximality conditions: $\forall_{P'_1}[\ \dots\ ]$ and $\forall_{P'_2}[\ \dots\ ]$. In fact, contrary to what is done with Cartesian Products, in Cover readings P_1 and P_2 must be Maximized independently, as it is no longer required that *every* member of the former is related with *every* member of the latter. Note also that the inner Cover variable C' is existentially quantified. Of course, it would make no sense to pragmatically interpret it as it is done with C.

3.2 Global Maximalization

The other kind of Maximalization of the witness sets, termed here as 'Global Maximalization' has been advocated by (Schein, 1993), and formalized in most formal theories of Cumulativity, e.g. (Landman, 2000), (Hackl, 2000), and (Ben-Avi and Winter, 2003). With respect to IS readings involving two witness sets $\|P_1\|^M$ and $\|P_2\|^M$, Global Maximalization requires the non-existence of other two witness sets that also satisfy the predication but *that do not necessarily include* $\|P_1\|^M$ *and* $\|P_2\|^M$. For instance, the event-based logic defined by (Landman, 2000) represents the Cumulative reading of (4.c) as:

(14) $\exists e \in {}^*\text{INVITE: } \exists x \in {}^*\text{BOY: } |x|{=}3 \wedge {}^*\text{Ag}(e){=}x \wedge \exists y \in {}^*\text{GIRL: } |y|{=}4 \wedge {}^*\text{Th}(e){=}y \wedge$
$|{}^*\mathbf{Ag}(\bigcup\{e \in \mathbf{INVITE: Ag}(e) \in \mathbf{BOY} \wedge \mathbf{Th}(e) \in \mathbf{GIRL}\})| \mathbf{= 3} \wedge$
$|{}^*\mathbf{Th}(\bigcup\{e \in \mathbf{INVITE: Ag}(e) \in \mathbf{BOY} \wedge \mathbf{Th}(e) \in \mathbf{GIRL}\})| \mathbf{= 4}$

Formula in (14) asserts the existence of a plural event e whose Agent is a plural individual made up of three boys and whose Theme is a plural individual made up of four girls. The two final conjuncts, in boldface, are Maximality conditions *asserted on pragmatic grounds* (see footnote 1 above). Taken e_x as the plural sum of all inviting events having a boy as agent and a girl as theme, i.e.

$$e_x{=}\bigcup\{e \in \text{INVITE: Ag}(e) \in \text{BOY} \wedge \text{Th}(e) \in \text{GIRL}\}$$

the cardinality of its agent ${}^*\text{Ag}(e_x)$ is exactly three while the one of its theme ${}^*\text{Th}(e_x)$ is exactly four. Therefore, Landman's Maximality conditions in (14) do not refer to the same events and actors quantified in the first row. Rather, they require that the number of the boys who invited a girl *in the whole model* is exactly three and the number of girls who were invited by a boy *in the whole model* is exactly four.

[3]Without going down into further details, I simply stipulate that the GQs used in the article are Conservative (Barwise and Cooper, 1981), (Keenan and Stavi, 1986). In other words, for every quantifier Q_x, we require $\|P^B_x\|^M \subseteq \|P^R_x\|^M$.

4 Local Maximalization VS Global Maximalization

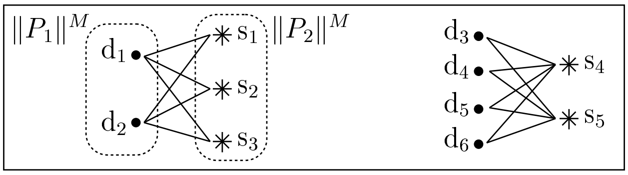
We are ready now to compare the two kinds of Maximalization. Global Maximalization appears to be more problematic than Local one. Since Branching Quantifier readings are special cases of Cumulative readings, and it has been discussed above that many authors, e.g. (Beghelli et al., 1997), argue that this is even a good reason to avoid an explicit representation of them, sentence (15.a) entails (15.b).

(15) a. Less than half of the dots are totally connected with exactly three stars.

b. Less than half of the dots are connected with exactly three stars.

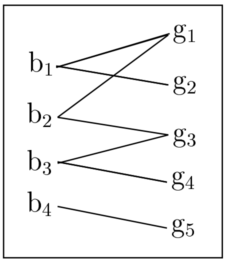
Nevertheless, Global Maximalization predicts that (15.b) is false in figure (2). The number of all dots in the model connected to a star is six, while the number of all stars in the model connected to a dot is five, not exactly three. On the contrary, once the witness sets have been identified as in (16), Local Maximalization predicts (15.b) as true, in that no other star is connected to a dot *occurring in* $\|P_1\|^M$, and no other dot is connected to a star *occurring in* $\|P_2\|^M$.

(16)



Another scenario where Global Maximalization predicts presumably wrong truth values, with respect to formula (14) and sentence (4.c), is shown in (17):

(17)



In (17), the Cumulative readings of all (18.a-c) appear to be true provided that numerals N are still interpreted as exactly-N.

(18) a. Three boys invited four girls.

b. One boy invited one girl.

c. Four boys invited five girls.

Global Maximalization states that only (18.c) is true in (17). Local Maximalization evaluates all (18.a-c) as true; the witness sets are obviously identified.

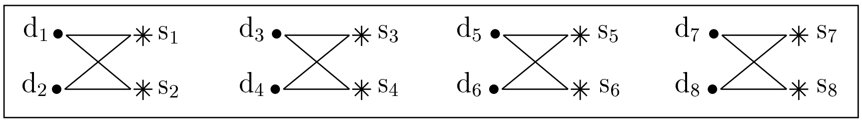
Landman does not discuss the evaluation of his formulae in contexts like (17). This is done instead by (Ferreira, 2007) and (Brasoveanu, 2009). However, the latter do not provide strong linguistic motivations: they simply claim that (18.a-b) are false in (17), as the present paper claims they are not. A comparison between Local and Global Maximalization is found in (Schein, 1993), even if no formalization is presented. (Schein, 1993), §12, reasonably argues, contra (Sher, 1997), that (19.a-b) are false in contexts like (20) (or (3)), while (19.c) is true. Local Maximalization predicts all (19.a-c) as true.

(19) a. Few dots are totally connected with few stars.

b. Exactly two dots are totally connected with exactly two stars.

c. At least two dots are totally connected with at least two stars.

(20)



From these observations, Schein concludes that (Sher, 1997)'s Local Maximalization, which is defined for any kind of quantifier, with any monotonicity, is incorrect. A proper semantics for NL quantification should instead stipulate two *different* semantics depending on the monotonicity: one for M↑ quantifiers, e.g. *At least two*, and one for M↓ quantifiers, e.g. *Few*, and non-M quantifiers, e.g. *Exactly two*. The truth conditions of the former should be defined in terms of Local Maximalization, while those of the latter in terms of Global Maximalization.

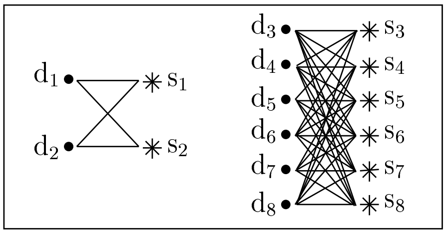
While I accept the truth values attested by Schein for sentences (19.a-c) in (20), I do not share his conclusions. On the one hand, there are several cases, particularly mixed cases, that are quite hard to reconcile in Schein's view. An example is the sentence evaluated in (2), which include a M↓ quantifier (*Less than half*) and a non-M one (*Exactly three*). Global Maximalization, contrary to Local Maximalization, evaluates the sentence as false in (2), as pointed out above. Also (21.a), which includes an M↓ quantifier and an M↑ one (*More than half*), and sentence (21.b), which is not a mixed case as it includes two M↓ quantifiers, seems to be true in (2), contra Schein's predictions.

(21) a. Less than half of the dots are connected with more than half of the stars.

b. Less than half of the dots are connected with less than five stars.

On the other hand, all sentences in (19.a-c) seems to be true in (22), while in Schein's view they should have the same truth values they have in (20).

(22)



These considerations lead to conclude that the oddity of sentences (19) in contexts (20) or (3) does not depend on the monotonicity of the quantifiers involved.
The present paper suggests instead that such an oddity stems from Pragmatics. No English speaker would ever utter those sentences in those contexts, as they would not be informative enough, and so they would violate a Gricean Maxim. From the examples above, it seems that sentences involving non-M↑ quantifiers sound odd in contexts where more pairs of witness sets are available. For instance, the reader gets confused when he tries to evaluate (19.a) in (20), as multiple pairs of (witness) sets of dots and stars are available, i.e. $\langle\{d_1, d_2\}, \{s_1, s_2\}\rangle$, $\langle\{d_3, d_4\}, \{s_3, s_4\}\rangle$, etc., and he does not have enough information to prefer one of them upon the others. This does not arises in (3) or (22), where the witness sets are immediatly and uncontroversially identified.
The multiple availability of witness sets does not seem to confuse the reader for sentences involving M↑ quantifiers, perhaps because they are simpler to interpret (cf. (Geurts and van der Silk, 2005)). However, several cognitive experimental results showed that many other factors besides monotonicity, e.g. expressivity/computability, fuzzyness, the fact that quantifiers are cardinal rather than proportional, etc., may affect the accuracy and reaction time of the interpretation of IS readings (cf. (Sanford and Paterson, 1994), (Bott and Radó, 2009), (Musolino, 2009), and (Szymanik and Zajenkowski, 2009)).
As it is clear to understand, however, extra-linguistic factors seem the ones that mainly affect the interpretation of quantifiers. For instance, in (17), if the boys b_1, b_2, b_3 are friends who decided to go to a party with some girls, and b_4 wants to go there with his girlfriend (g_5) only, the witness sets are most

likely identified for (18.a-b) rather than for (18.c), as the two groups of persons are not related.
Conversely, if the four boys belong to the same group of friends hanging out together, the identification of the witness sets most likely fails in (18.a-b). That is probably the assumption done by (Ferreira, 2007) and (Brasoveanu, 2009) for claiming that sentences like (18.a-b) are false in contexts like (17). Analogously, in the children-pizza example in (4.d), the arrangement of the children among the tables of the pizzeria, their mutual friendship, and so on, may affect the identification of the witness sets. Similar discussions may be found in (Fintel, 1994) and (Winter, 2000).

Of course, an exhaustive study of all factors involved in the pragmatic identification of the witness sets goes much beyond the goal of the present paper. The aim of this paper is to argue that, once witness sets are identified, Local Maximalization applies to them. In order to formally obtain this result, a final modification of the formulae is needed: it is necessary to pragmatically interpret the relational variables denoting the witness sets, besides those denoting the Covers. Formula (13) is then revised as in (23).

(23)
$$
\begin{aligned}
&{}^{=}20_x(\text{child'}(x), P_1(x)) \wedge {}^{=}10_y(\text{pizza'}(y), P_2(y)) \wedge \\
&Cover(C, P_1, P_2) \wedge \forall_{xy}[C(x, y)) \rightarrow \text{ate'}(x, y)] \wedge \\
&\forall_{P'_1}[(\forall_x[P_1(x) \rightarrow P'_1(x)] \wedge \exists_{C'}[Cover(C', P'_1, P_2) \wedge \forall_{xy}[C(x, y) \rightarrow C'(x, y)] \wedge \\
&\quad \forall_{xy}[C'(x, y) \rightarrow \text{ate'}(x, y)]]) \rightarrow \forall_x[P'_1(x) \rightarrow P_1(x)]\ ]\ ] \wedge \\
&\forall_{P'_2}[(\forall_y[P_2(y) \rightarrow P'_2(y)] \wedge \exists_{C'}[Cover(C', P_1, P'_2) \wedge \forall_{xy}[C(x, y) \rightarrow C'(x, y)] \wedge \\
&\quad \forall_{xy}[C'(x, y) \rightarrow \text{ate'}(x, y)]]) \rightarrow \forall_y[P'_2(y) \rightarrow P_2(y)]\ ]\ ]
\end{aligned}
$$

The only difference between (23) and (13) is that the value of P_1 and P_2 is provided by an assignment g, as it is done for the Cover variable C. g must obey to all (extra-)linguistic pragmatic constraints briefly listed above. The reader could start thinking that, in the new version of the formulae, we may avoid Maximality conditions, either Local or Global. In fact, Maximalization could be simply implemented as a constraint on the assignment function g. In other words, we could simply impose g to select only Maximal witness sets. If g is unable to do so, the intepretation fails as in the cases discussed above. Such a solution has been actually proposed in (Steedman, 2007) and (Brasoveanu, 2009). Conversely, in (Robaldo, 2009b) I explained that we do need to explicitly represent the Maximality conditions. In other words, those are not only seen as necessary conditions needed to determine if a sentence is true or false in a certain context. Rather, in (Robaldo, 2009b), it is extensively argued that they are part of the knowledge needed to draw the appropriate inferences from the sentences' meaning.

5 Conclusions

This paper compared the two kind of Maximalization proposed in the literature for handling the proper truth values of Independent Set readings. They have been termed as Local and Global Maximalization. The former requires the non-existence of any tuple of supersets of the witness sets that also satisfy the predication. The latter requires the witness sets to be the only tuple of sets that satisfy the predication.
The present paper argues in favour of Local Maximalization, and claims that the motivations that led to the definition of Global Maximalitation, and its incorporation within most current formal approaches to NL quantification, do not appear to be justified enough. These claims are supported by showing that, for many NL sentences, Global Maximalization predicts counter-intuitive truth conditions.
Also several examples are hard to reconcile in a logical framework based on Local Maximalization. It seems, however, that the oddity of such examples depends upon pragmatic grounds.

Based on these assumptions, the solution presented here still adopts Local Maximalization, but advocates a pragmatic interpretation of all relational variables. Drawing from (Schwarzschild, 1996), the present paper evolves the formulae in (Robaldo, 2009a) and (Robaldo, 2009b), making them able to handle Cover readings, which are assumed to be the more general cases of Independent Set readings.

In the resulting formulae, the witness sets are firstly pragmatically identified, as it is done with Cover variables, then they are locally Maximized. In other words, Pragmatics is responsible for identifying both the (atomic) individuals involved, and the way they sub-combine to carry out the singular actions.

The result is able to predict the suitable truth values of Cover readings in all examples considered, and seems to mirror the correct interplay between the Semantics and the Pragmatics of NL quantifiers.

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