

TEST-SCORE SEMANTICS  
FOR NATURAL LANGUAGES

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Test-score semantics is based on the premise that almost everything that relates to natural languages is a matter of degree. Viewed from this perspective, any semantic entity in a natural language, e.g., a predicate, predicate-modifier, proposition, quantifier, command, question, etc. may be represented as a system of elastic constraints on a collection of objects or derived objects in a universe of discourse. In this sense, test-score semantics may be viewed as a generalization of truth-conditional, possible-world and model-theoretic semantics, but its expressive power is substantially greater.

INTRODUCTION

Test-score semantics represents a break with the traditional approaches to semantics in that it is based on the premise that almost everything that relates to natural languages is a matter of degree. The acceptance of this premise entails an abandonment of bivalent logical systems as a basis for the analysis of natural languages and suggests the adoption of fuzzy logic (Zadeh (1975), Bellman and Zadeh (1977), Zadeh (1979)) as the basic conceptual framework for dealing with natural languages.

In fuzzy logic, as in natural languages, almost everything is a matter of degree. To put it metaphorically, the use of fuzzy logic may be likened to writing with a spray-can, rather than with a ball-point pen. The spray-can, however, has an adjustable orifice, so that one may write, if need be, as finely as with a ball-point pen. Thus, a commitment to fuzzy logic does not preclude the use of a bivalent logic when it is appropriate to do so. In effect, such a commitment merely provides a language theorist with a much more flexible framework for dealing with natural languages and, especially, for representing meaning, knowledge and strength of belief.

An acid test of the effectiveness of a meaning-representation system is its ability to provide a basis for inference from premises expressed in a natural language. In this regard, an indication of the capability of test-score semantics is provided by the following examples, in which the premises appear above the line and the question which may be answered is stated below it.

- (a) During much of the past decade Pat earned far more than all of his close friends put together  

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How much did Pat earn during the past decade?
- (b) Most tall men are not fat  
Many fat men are bald  
Big is tall and fat

How many big men are bald?

- (c) If X is large then it is not likely that Y is small  
 If X is not very large then it is very likely that Y is large  
 X is not large

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How likely is it that Y is more or less small?

In fuzzy logic, the answer to a question is, in general, a possibility distribution (Zadeh (1978)). For example, in the case of (a) the answer would be a possibility distribution in the universe of real numbers which associates with each number  $u$  the possibility,  $\pi(u)$ ,  $0 \leq \pi(u) \leq 1$ , that  $u$  could be the cumulative income of Pat given (i) the premise, and (ii) the information resident in a database.

In test-score semantics, a semantic entity such as a proposition, predicate, predicate-modifier, quantifier, qualifier, command, question, etc., is represented as a system of elastic constraints on a collection of objects or derived objects in a universe of discourse. Simple examples of semantic entities whose meaning can be represented in this manner are the following:

1. Anca has a young son. (Proposition.)
2. When Dan is tired or tense, he smokes a lot. (Conditional proposition.)
3. It is not quite true that John has very few close friends. (Truth-qualified proposition.)
4. It is very likely that Marie will become well-known. (Probability-qualified proposition.)
5. It is almost impossible for Manuel to be unkind. (Possibility-qualified proposition.)
6. Expensive car. (Fuzzy predicate.)
7. Very. (Modifier)
8. Several large apples. (Second-order fuzzy predicate.)
9. More or less. (Modifier/Fuzzifier.)
10. Not very true. (Qualifier.)
11. Very unlikely. (Qualifier)
12. Much taller than most. (Fuzzy predicate.)
13. Bring me several large apples. (Fuzzy command.)
14. Who are Edie's close friends. (Question.)

Although test-score semantics has a much greater expressive power than the meaning-representation systems based on predicate, modal and intensional logics, its expressiveness is attained at the cost of downplaying, if not entirely severing, the connection between syntax and semantics. In particular, the homomorphic connection between syntax and semantics which plays a central role in Montague semantics (Montague (1974), Partee (1976) and attributed grammars for programming languages (Knuth (1968))), plays a much lesser role in test-score semantics—a role represented in the main by a collection of local translation rules governing the use of modifiers, qualifiers, quantifiers and connectives. In effect, the downplaying of the connection between syntax and semantics in test-score semantics reflects our belief that, in the case of natural languages, the connection is far too complex and far too fuzzy to be amenable to an elegant mathematical formulation in the style of Montague semantics, except for very small fragments of natural languages in which the connection can be formulated and exploited.

The conceptual framework of test-score semantics is closely related to that of PRUF (Zadeh (1978)), which is a meaning-representation system in which an essential use is made of possibility theory (Zadeh (1978))—a theory which is distinct from the bivalent theories of possibility related to modal logic and possible-world semantics (Cresswell (1973), Rescher (1975)).

In effect, the basic idea underlying both PRUF and test-score semantics is that most of the imprecision and lack of specificity which is intrinsic in natural languages is possibilistic rather than probabilistic in nature, and hence that possi-

bility theory and fuzzy logic provide a more appropriate framework for dealing with natural languages than the traditional logical systems in which there are no gradations for truth, membership and belief, and no tools for coming to grips with vagueness, fuzziness and randomness.

In what follows, we shall sketch some of the main ideas underlying test-score semantics and illustrate them with simple examples. A more detailed exposition and additional examples may be found in Zadeh (1981).

#### BASIC ASPECTS OF TEST-SCORE SEMANTICS

As was stated earlier, the point of departure in test-score semantics is the assumption that any semantic entity may be represented as a system of elastic constraints on a collection of objects or derived objects in a universe of discourse.

Assuming that each object may be characterized by one or more fuzzy relations, the collection of objects in a universe of discourse may be identified with a collection of relations which constitute a fuzzy relational database or, equivalently, a state description (Carnap (1952)). In this database, then, a derived object would be characterized by one or more fuzzy relations which are derived from other relations in the database by operations expressed in an appropriate relation-manipulating language.

In more concrete terms, let SE denote a semantic entity, e.g., the proposition

$p \triangleq$  During much of the past decade Pat earned far more than  
all of his close friends put together,

whose meaning we wish to represent. To this end, we must (a) identify the constraints which are implicit or explicit in SE; (b) describe the tests which must be performed to ascertain the degree to which each constraint is satisfied; and (c) specify the manner in which the degrees in question or, equivalently, the partial test scores are to be aggregated to yield an overall test score. In general, the overall test score would be represented as a vector whose components are numbers in the unit interval or, more generally, possibility/probability distributions over this interval.

Spelled out in greater detail, the process of meaning-representation in test-score semantics involves three distinct phases. In Phase 1, an explanatory database frame or EDF, for short, is constructed. EDF consists of a collection of relational frames each of which specifies the name of a relation, the names of its attributes and their respective domains, with the understanding that the meaning of each relation in EDF is known to the addressee of the meaning-representation process. Thus, the choice of EDF is not unique and is strongly influenced by the desideratum of explanatory effectiveness as well as by the assumption made regarding the knowledge profile of the addressee of the meaning-representation process. For example, in the case of the proposition  $p \triangleq$  During much of the past decade Pat earned far more than all of his close friends put together, the EDF might consist of the relational frames FRIEND [Name1; Name2;  $\mu$ ], where  $\mu$  is the degree to which Name1 is a friend of Name2; INCOME [Name; Income; Year], where Income is the income of Name in year Year, counting backward from the present; MUCH [Proportion;  $\mu$ ], where  $\mu$  is the degree to which a numerical value of Proportion fits the meaning of much in the context of  $p$ ; and FAR.MORE [Number1; Number2;  $\mu$ ], in which  $\mu$  is the degree to which Number1 fits the description far more in relation to Number2. In effect, the composition of EDF is determined by the information that is needed for an assessment of the compatibility of the given SE with any designated object or, more generally, a specified state of affairs in the universe of discourse.

In Phase 2, a test procedure is constructed which upon application to an explanatory database -that is, an instantiation of EDF - yields the test scores,

$\tau_1, \dots, \tau_n$ , which represent the degrees to which the elastic constraints induced by the constituents of SE are satisfied. For example, in the case of  $p$ ; the test procedure would yield the test scores for the constraints induced by close friend, much, far more, etc.

In Phase 3, the partial test scores obtained in Phase 2 are aggregated into an overall test score,  $\tau$ , which serves as a measure of the compatibility of SE with ED, the explanatory database. As was stated earlier, the components of  $\tau$  are numbers in the unit interval or, more generally, possibility/probability distributions over this interval. In particular, when the semantic entity is a proposition,  $p$ , and the overall test score,  $\tau$ , is a scalar,  $\tau$  may be interpreted as the truth of  $p$  relative to ED or, equivalently, as the possibility of ED given  $p$ . In this interpretation, then, the classical truth-conditional semantics may be viewed as a special case of test-score semantics which results when the constraints induced by  $p$  are inelastic and the overall test score is allowed to be only pass or fail. The test procedure which yields the overall test score  $\tau$  is interpreted as the meaning of SE.

To illustrate the phases in question, we shall consider a few simple examples

(a) SE  $\hat{=}$  Ellen resides in a small city near Oslo.

In this case, EDF is assumed to comprise the following relational frames ( + stands for union ):

EDF  $\hat{=}$  RESIDENCE [Name; City.Name]+  
 POPULATION [City.Name; Population]+  
 SMALL [Population;  $\mu$ ]+  
 NEAR [City.Name1; City.Name2;  $\mu$ ]

In RESIDENCE, City.Name is the name of the city in which Name resides; in POPULATION, Population is the number of residents in City.Name; in SMALL,  $\mu$  is the degree to which a city with a population equal to the value of Population is small; and in NEAR,  $\mu$  is the degree to which City.Name1 is near City.Name2.

The test procedure which leads to the overall test score  $\tau$  -- and thus represents the meaning of SE -- is described below. In this procedure, Steps 1 and 2 involve the determination of the value of an attribute given the values of other attributes; Steps 3 and 4 involve the testing of constraints; and Step 5 involves an aggregation of the partial test scores into the overall test score  $\tau$ .

1. Find the name of the residence of Ellen:

RE  $\hat{=}$   $\underset{\text{City.Name}}{\Delta}$  RESIDENCE[Name=Ellen]

which means that the value of Name is set to Ellen and the value of City.Name is read, yielding RE, the residence of Ellen.

2. Find the population of the residence of Ellen:

PRE  $\hat{=}$   $\underset{\text{Population}}{\Delta}$  POPULATION[City.Name=RE]

3. Test the constraint induced by SMALL:

$\tau_1 \hat{=}$   $\underset{\mu}{\Delta}$  SMALL[Population=RE]

where  $\tau_1$  denotes the resulting test score.

4. Test the constraint induced by NEAR:

$\tau_2 =$   $\underset{\mu}{\Delta}$  NEAR[City.Name=Oslo; City.Name2=RE]

5. Aggregate  $\tau_1$  and  $\tau_2$ :

$\tau = \tau_1 \wedge \tau_2$

where  $\wedge$  stands for min in infix position, and  $\tau$  is the overall test score. This

mode of aggregation implies that, in SE, the denotation of conjunction is taken to be the cartesian product of the denotations of the conjuncts (Zadeh (1981)).

- (b) SE  $\Delta$  During much of the past decade Pat earned far more than all of his close friends put together.

In this case, we shall employ the EDF described earlier, that is:

$$\begin{aligned} \text{EDF} \triangleq & \text{INCOME}[\text{Name}; \text{Year}; \text{Amount}] + \\ & \text{FRIEND}[\text{Name}_1; \text{Name}_2; \mu] + \\ & \text{FAR.MORE}[\text{Number}_1; \text{Number}_2; \mu] + \\ & \text{MUCH}[\text{Proportion}; \mu] \end{aligned}$$

The test procedure comprises the following steps:

1. Find the fuzzy set of Pat's friends:

$$\text{FP} \triangleq_{\text{Name}_1 \times \mu} \text{FRIEND} [\text{Name}_2 = \text{Pat}]$$

in which the left subscript  $\text{Name}_1 \times \mu$  signifies that the relation  $\text{FRIEND} [\text{Name}_2 = \text{Pat}]$  is projected on the domain of the attributes  $\text{Name}_1$  and  $\mu$ , yielding the fuzzy set of friends of Pat.

2. Intensify FP to account for the modifier close:

$$\text{CFP} \triangleq \text{FP}^2$$

in which  $\text{FP}^2$  denotes the fuzzy set which results from squaring the grade of membership of each component of FP. The assumption underlying this step is that the fuzzy set of close friends of Pat may be derived from that of friends of Pat by intensification.

3. Find the fuzzy multiset of incomes of close friends of Pat in year  $\text{Year}_i$ ,  $i=1, \dots, 10$ :

$$\text{ICFP}_i \triangleq_{\text{Amount}} \text{INCOME}[\text{Name} = \text{CFP}; \text{Year} = \text{Year}_i]$$

In stipulating that the right-hand member be treated as a fuzzy multiset, we imply that the identical elements should not be combined, as they would be in the case of a fuzzy set. With this understanding,  $\text{ICFP}_i$  will be of the general form

$$\text{ICFP}_i = \delta_1/e_1 + \delta_2/e_2 + \dots + \delta_m/e_m$$

where  $e_1, \dots, e_m$  are the incomes of  $\text{Name}_1, \dots, \text{Name}_m$ ,

respectively, in  $\text{Year}_i$ , and  $\delta_1, \dots, \delta_m$  are the grades of membership of  $\text{Name}_1, \dots, \text{Name}_m$  in the fuzzy set of close friends of Pat.

4. Find the total income of close friends of Pat in  $\text{Year}_i$ ,  $i=1, \dots, 10$ :

$$\text{TICFP}_i = \delta_1 e_1 + \dots + \delta_m e_m$$

which represents a weighted arithmetic sum of the incomes of close friends of Pat in  $\text{Year}_i$ .

5. Find Pat's income in  $\text{Year}_i$ :

$$\text{IP}_i \triangleq_{\text{Amount}} \text{INCOME}[\text{Name} = \text{Pat}; \text{Year} = \text{Year}_i].$$

6. Test the constraint induced by FAR.MORE:

$$\tau_i \triangleq_{\mu} \text{FAR.MORE}[\text{Number}_1 = \text{IP}_i; \text{Number}_2 = \text{TICFP}_i]$$

7. Find the sigma-count (Zadeh (1981)) of years during which Pat's income was far greater than the total income of all of his close friends:

$$C \triangleq \sum_i \tau_i$$

8. Test the constraint induced by MUCH:

$$\tau \stackrel{\Delta}{=} \frac{\text{MUCH}[\text{Proportion}=\frac{C}{10}]}{\mu}$$

where  $\tau$  represents the overall test score.

The two examples described above are intended merely to provide a rough outline of the meaning-representation process in test-score semantics. A more detailed exposition of some of the related issues may be found in Zadeh (1978) and Zadeh (1981)

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