# SAAS: Solving Ability Amplification Strategy for Enhanced Mathematical Reasoning in Large Language Models

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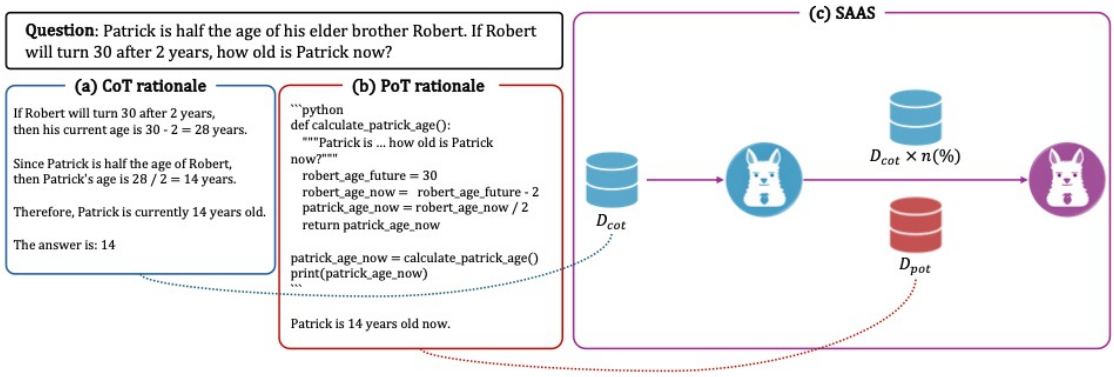
## Motivation

❖ Need to address the limitations of LLMs in mathematical reasoning to ensure robust logical thinking and problem-solving capabilities

❖ While Chain-of-Thought (CoT) methods enhance reasoning, they often introduce computational errors, whereas Program-of-Thought (PoT) methods improve execution but lack reasoning depth

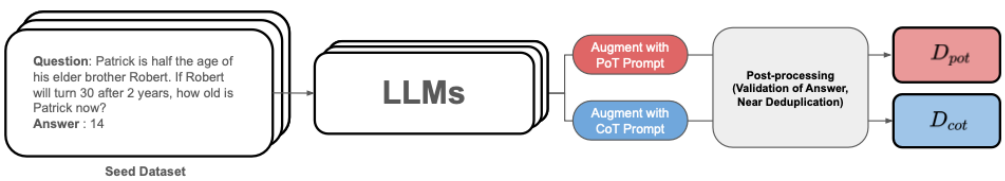
## SAAS (Solving Ability Amplification Strategy)




❖ SAAS is a sequential learning strategy aimed at first enhancing logical reasoning skills through CoT learning, followed by bolstering problem-solving capabilities with PoT learning

❖The cognitive retention strategy mitigates information loss by incorporating CoT rationales into the PoT phase

## Generate trainset




| Seed Dataset | Rationale | Models | Size |
|---|---|---|---|
| MetaMathQA | CoT | GPT, WizardMath | 465K |
| MATH, GSM8K | CoT | WizardMath | 300K |
| QANDA | CoT | WizardMath | 120K |
| MetaMathQA | PoT | ToRA | 60K |
| MATH, GSM8K | PoT | ToRA | 226K |
| MathInstruct | PoT | ToRA | 38K |
| QANDA | PoT | ToRA | 12K |

❖ The data is augmented with models like GPT and validated through post-processing to ensure correctness and quality

## Experiments

| Model | Size | GSM8K | MATH | GSM-Hard | SVAMP | TabMWP | ASDiv | MAWPS | Avg. |
|---|---|---|---|---|---|---|---|---|---|
| Mathematics Domain-Specific Models | | | | | | | | | |
| WizardMath | 7B | 54.9 | 10.7 | 20.6 | 57.3 | 38.1 | 59.1 | 73.7 | 44.9 |
| MetaMath | 7B | 66.5 | 19.8 | - | - | - | - | - | - |
| MuggleMATH | 7B | 68.4 | - | - | - | - | - | - | - |
| Toolformer | 7B | - | - | - | 29.4 | - | 40.4 | 44.0 | - |
| MathCoder | 7B | 64.2 | 23.3 | - | - | - | - | - | - |
| MathCoder-CODE | 7B | 67.8 | 30.2 | - | - | - | - | - | - |
| MAmmoTH | 7B | 53.6 | 31.5 | - | - | - | - | - | - |
| MAmmoTH-CODE | 7B | 59.4 | 33.4 | - | - | - | - | - | - |
| ToRA | 7B | 68.8 | 40.1 | 54.6 | 68.2 | 42.4 | 73.9 | 88.8 | 62.4 |
| SAAS | 7B | 74.3 | 43.2 | **58.3** | **74.3** | 49.6 | 77.3 | 93.6 | 67.2 |
| ToRA-CODE | 7B | 72.6 | 44.6 | 56.0 | 70.4 | 51.6 | 78.7 | 91.3 | 66.5 |
| SAAS-CODE | 7B | **74.8** | **45.2** | 58.1 | 73.6 | **64.0** | **80.4** | **93.8** | **70.0** |
| SAAS | 10.7B | **82.0** | 50.1 | **64.9** | **85.0** | **72.5** | **87.5** | **95.7** | **76.8** |
| WizardMath | 13B | 63.9 | 14.0 | 28.4 | 64.3 | 46.7 | 65.8 | 79.7 | 51.8 |
| MetaMath | 13B | 72.3 | 22.4 | - | - | - | - | - | - |
| MuggleMATH | 13B | 74.0 | - | - | - | - | - | - | - |
| MathCoder | 13B | 72.6 | 29.9 | - | - | - | - | - | - |
| MathCoder-CODE | 13B | 74.1 | 35.9 | - | - | - | - | - | - |
| MAmmoTH | 13B | 62.0 | 34.2 | - | - | - | - | - | - |
| MAmmoTH-CODE | 13B | 64.7 | 36.3 | - | - | - | - | - | - |
| ToRA | 13B | 72.7 | 43.0 | 57.3 | 72.9 | 47.2 | 77.2 | 91.3 | 65.9 |
| SAAS | 13B | 76.6 | 46.2 | 61.6 | 77.8 | 58.2 | 80.5 | 94.3 | 70.7 |
| ToRA-CODE | 13B | 75.8 | 48.1 | 60.5 | 75.7 | 65.4 | 81.4 | 92.5 | 71.3 |
| SAAS-CODE | 13B | 79.4 | **50.6** | 61.6 | 80.6 | 68.2 | 84.5 | 95.4 | 74.3 |
| MathCoder-CODE | 34B | 81.7 | 45.2 | - | - | - | - | - | - |
| MAmmoTH-CODE | 34B | 72.7 | 43.6 | - | - | - | - | - | - |
| ToRA-CODE | 34B | 80.7 | 50.8 | 63.7 | 80.5 | 70.5 | 84.2 | 93.3 | 74.8 |
| SAAS-CODE | 34B | 82.9 | 52.3 | 64.1 | 82.8 | 73.9 | 85.4 | 95.2 | 76.6 |
| SAAS-LLEMA | 34B | **85.4** | **54.7** | **67.0** | **85.2** | **80.2** | **87.6** | **96.6** | **79.5** |

| Strategy | GSM8K | MATH |
|---|---|---|
| Chain-of-Thought (CoT) | 69.7 | 26.9 |
| Program-of-Thought (PoT) | 76.8 | 47.7 |
| Combination of CoT and PoT | 79.0 | 49.2 |
| **SAAS** | **79.4** | **50.6** |
| without cognitive retention strategy | 79.0 | 49.6 |
| Reverse **SAAS** | 76.8 | 47.1 |
| without cognitive retention strategy | 69.4 | 27.6 |

❖ SAAS consistently and significantly outperformed all competitors across various model sizes, achieving state-of-the-art performance on reputable mathematical reasoning benchmarks

❖ Ablation studies demonstrated the effectiveness of SAAS's key components: the sequential learning strategy (transitioning from CoT to PoT) and the cognitive retention strategy both contributed to improved performance compared to alternative approaches

## Conclusion

❖ SAAS, which transitions from CoT to PoT learning with a cognitive retention strategy, achieved state-of-the-art performance on mathematical reasoning benchmarks, demonstrating the effectiveness of sequential learning in enhancing both reasoning and problem-solving abilities in LLMs.

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