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Epistemic Model and Three-valued Interpretation

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Abstract

In this paper, I propose a general framework called *Epistemic Logic* (EL) which captures a full range of epistemic phenomena, and apply it to the three-valued interpretion of natural language sentences. Further, I mention the ability which EL implies, i.e. treatment of indexicals, relationship to Data Semantics etc.

1 Epistemic Logic

Epistemic Logic consists of the syntax and semantics of EL with respect to which natural language sentences are interpreted.

1.1 Syntax of Epistemic Logic

The syntax of Epistemic Logic consists of a normal version of first-order predicate logic with modal operators 'may' and 'must' which is defined as follows:

(1) Vocabulary of EL
- i) individual constants: a, b, c, ... $\in$ *Const*.
- ii) individual variables: x, y, z, ... $\in$ *Var*.
- iii) $j(\geq 0)$-ary predicate symbols: $\mathrm{pred}_i^j \in Pred^j$. ($i \in I(j)$. $I(j)$ is a segment of the set of natural numbers.)
- iv) logical connectives: $\neg, \wedge, \vee, \supset, \equiv \in LC$.
- v) quantification symbols: $\forall, \exists \in Q$.
- vi) epistemic modal symbols: must, may$\in EM$.
- vii) auxiliary symbols: (,).

$Term = Const \cup Var$ is called the set of terms of EL.
$Pred = \bigcup_j Pred^j$ is called the set of predicate symbols of EL.
$Voc = Term \cup Pred \cup LC \cup Q \cup EM \cup \{(,)\}$ is called the set of the vocabulary of EL.
$Exp = Voc^{\circledast 1}$ is called the set of expressions of EL.

(2) Formation Rules of EL
Formulas: The set *Form* of EL is the least set which satisfies the following conditions:
- i) If $\alpha_1, \cdots, \alpha_j \in Term$, $\mathrm{pred}_i^j \in Pred^j$, then $\mathrm{pred}_i^j(\alpha_1, \cdots, \alpha_j) \in Form$.
- ii) If $\phi, \chi \in Form$, then $(\neg\phi), (\phi \wedge \chi), (\phi \vee \chi), (\phi \supset \chi), (\phi \equiv \chi) \in Form$.
- iii) If $\phi \in Form$, then $(\forall x\phi), (\exists x\phi) \in Form$.
- iv) If $\phi \in Form$, then $(\mathrm{must}\, \phi), (\mathrm{may}\, \phi) \in Form$.

$WExp = (Voc \setminus \{(,)\}) \cup Form$ is called the set of well-formed expressions of EL.
Auxiliary symbols of formulas are eliminated in obvious cases.

1.2 Semantics of Epistemic Logic

The Semantics of Epistemic Logic consists of the epistemic model which is constructed from extensional model structures via extensional models.

1.2.1 Extensional Model Structure

I start from the traditional well-known relational structure for the interpretation of first-order formulas which is found e.g. in Bridge(1977):

(3) $\mathfrak{A} = \langle A, \{R_i^j\}_{i\in I(j), j\in J}, \{F_k^l\}_{k\in K(l), l\in L}, \{c_m\}_{m\in M}\rangle$

Here, R_i^j is the i-th j-ary relation, F_k^l is the k-th l-ary function, and c_m is the m-th constant. But to simplify (3), I omit functions and constants so that (3) looks like as follows:

(4) $\mathfrak{A} = \langle A, \{R_i^j\}_{i\in I(j), j\in J}\rangle$

On the other hand, I complicate (4) by replacing relations with attributes and their extensional functions for reasons explained in 2, and call it an extensional model structure:

(5) Extensional Model Structure:
$MS = \langle Ind, \{Attr_i^j\}_{i\in I(j), j\in J}, Ext\rangle$
Ind: domain, i.e. (non-empty) set of individuals
$Attr_i^j$: the i-th j-ary attribute
$Attr^j \stackrel{\text{def}}{=} \bigcup_{i\in I(j)} Attr_i^j$: set of j-ary attributes
$Attr \stackrel{\text{def}}{=} \bigcup_{j\in J} Attr^j$: set of attributes
Ent: extensional function
$Attr^n \rightarrow Rel^n \subseteq Ind^n \ (n \geq 0)$

1.2.2 Extensional Model

An extensional model is defined as follows:

(6) Extensional Model:
$M = \langle MS, f\rangle$
Here,
MS: extensional model structure in (5)
f (value assignment) is the following partial function:
$Const \rightarrow Ind$,
$Pred^n \rightarrow Attr^n$

1.2.3 Epistemic Model

The Epistemic Model is defined as follows:

(7) Epistemic Model:
$\mathfrak{M} = \mathcal{P}ow(\mathcal{M}) \setminus \phi$
Here, $\mathcal{M}$ is the set of extensional models with the same domain and set of attributes.

The elements of $\mathcal{M}$ are called situations.

1.3 Entity of EL

The set *Ent* of entities of EL is defined parallel to *Exp* as follows:

(8) Atomic Entity of EL
i) individuals: $a, b, c, \ldots \in Ind$.
ii) $Par = \{x \mid x \in Var\}$: set of parameters of EL.
iii) $Attr^n$: set of n-ary attributes.
iv) $LO = LC$: set of logical operators of EL.
v) $Quant = Q$: set of quantifiers of EL.
vi) epistemic modal operators: $must$, $may \in EpMod$

vii) (,): parentheses.

$GInd = Ind \cup Par$ is called the set of general individuals of EL.
$Attr = \bigcup_n Attr^n$: is called the set of attributes.
$AEnt = GInd \cup Attr \cup LO \cup Quant \cup EpMod \cup \{(,)\}$ is called the set of atomic entities of EL.
$Ent = AEnt^{\circledast}$ is called the set of entities of EL.

(9) Infons

The set $Inf \subset Ent$ of infons of EL is the least set which satisfies the following conditions:
i) $\dagger \in Inf$ ($\dagger$ means the 'undefined' infon.)
ii) If $t_1, \cdots, t_n \in GInd$, $Attr_i^n \in Attr^n$, then $Attr_i^n(t_1, \cdots, t_n) \in Inf$.
iii) If $inf_1, inf_2 \in Inf$, $\{\neg, \wedge, \vee, \supset, \equiv\} = LO$, then $(\neg inf_1), (inf_1 \wedge inf_2), (inf_1 \vee inf_2), (inf_1 \supset inf_2), (inf_1 \equiv inf_2) \in Inf$.
iv) If $inf \in Inf$, $Q \in Quant$, $x \in Par$, then $(Qx\, inf) \in Inf$.
v) If $inf \in Inf$, $epmod \in EpMod$, then $(epmod\, inf) \in Inf$.

$WEnt = (AEnt \setminus \{(,)\}) \cup Inf$ is called the set of well-formed entities of EL.
Parentheses of infons are eliminated in obvious cases.

1.4 Interpretation

Natural language sentences are translated to the corresponding formulas of EL. However, the latter are not directly interpreted, but translated to the intermediate entity of EL, i.e. infons, which are then interpreted w.r.t. a situation, and their semantic behaviors are explained.

1.4.1 Interpretation of infons

(10) The truth value $s(c, inf)$ of $inf \in Inf$ w.r.t. a situation s and a context situation c is defined as follows:
i) $s(c, inf) = 1 \Leftrightarrow$ for all $M \in s$, $M(c, inf) = 1$,
ii) $s(c, inf) = 0 \Leftrightarrow$ for all $M \in s$, $M(c, inf) = 0$.[2]

(1 and 0 mean 'true' and 'false' respectively.)

(11) $$\pi : Par \to Ind$$

is called a parameter assignment.

$d \in Ind$, and $x \in Par$. Then,
$$\pi_d^x = (\pi \setminus \{\langle x, \pi(x)\rangle\}) \cup \{\langle x, d\rangle\}$$
is called the (x, d)-variant of π.

(12) The truth value $M(c, inf)$ of $inf \in Inf$ w.r.t. an extensional model M and a context situation c is defined as follows:
i) $M(c, inf) = 1 \Leftrightarrow$ for all π, $M(c, \pi, inf) = 1$,
ii) $M(c, inf) = 0 \Leftrightarrow$ for all π, $M(c, \pi, inf) = 0$.

(13) Let $M = \langle MS, f\rangle$ ($MS = \langle Ind, \{Attr_i^j\}_{i \in I(j), j \in J}, Ext\rangle$) be an extensional model, π be a parameter assignment, and c be a context situation. Then, the function $M(c, \pi, .)$ with the domain $WEnt$ is defined as follows:
i) a) $M(c, \pi, .) \upharpoonright Ind$: constant function.
b) $M(c, \pi, .) \upharpoonright Par \doteq \pi$.
c) $M(c, \pi, .) \upharpoonright Attr \doteq Ext$.
d) $M(c, \pi, .) \upharpoonright LO$ is the function such that, for an arbitrary $lo \in LO$, $M(c, \pi, lo)$ is the correspondig truth function of lo upon $\{1, 0\}$ defined in the first-order logic in a familiar manner.
e) $M(c, \pi, .) \upharpoonright Quant \cup EpMod$ is a function such that $\forall \mapsto \bigwedge$, $\exists \mapsto \bigvee$, $must \mapsto \bigwedge$, and $may \mapsto \bigvee$. ($\bigwedge, \bigvee$ are supremum and infimum function respectively.)
ii) In the following, $inf, inf_1, inf_2 \in Inf$.
$M(c, \pi, .) \upharpoonright Inf$ is the partial function $Inf \to \{1, 0\}$ defined as follows:

a) $M(c, \pi, \dagger)$ is undefined.
b) If $t_1, \cdots, t_n \in GInd$, $Attr_i^n \in Attr^n$, then
$M(c, \pi, Attr_i^n(t_1, \cdots, t_n)) = M(c, \pi, Attr_i^n)(M(c, \pi, t_1), \cdots, M(c, \pi, t_n))$.
c) $M(c, \pi, (\neg inf)) = M(c, \pi, \neg)(M(c, \pi, inf))$.
d) $M(c, \pi, (inf_1 \wedge inf_2)) = M(c, \pi, inf_1)M(c, \pi, \wedge)M(c, \pi, inf_2)$.
e) $M(c, \pi, (inf_1 \vee inf_2)) = M(c, \pi, inf_1)M(c, \pi, \vee)M(c, \pi, inf_2)$.
f) $M(c, \pi, (inf_1 \supset inf_2)) = M(c, \pi, inf_1)M(c, \pi, \supset)M(c, \pi, inf_2)$.
g) $M(c, \pi, (inf_1 \equiv inf_2)) = M(c, \pi, inf_1)M(c, \pi, \equiv)M(c, \pi, inf_2)$.
In the following, $x \in Par$.
h) If $quant \in Quant$, then $M(c, \pi, quant\, x\, inf) = M(c, \pi, quant)\{M(c, \pi_d^x, inf) \mid d \in Ind\}$.
i) If $epmod \in EpMod$, then $M(c, \pi, epmod\, inf) = M(c, \pi, epmod)\{M(c, \pi', inf) \mid M \in c\}$.
(π' is an arbitrary parameter assignment.)

In (13iic), if $M(c, \pi, inf)$ is undefined, $M(c, \pi, \neg inf)$ is also undefined. Likewise for (13iid-i).
c in $M(c, \pi, .)$, $s(c, .)$ etc. is omitted in unnecessary cases.
The interpretation of $inf \in Inf$ always begins with $c(c, inf)$.

1.4.2 Interpretation of EL formulas

(14) Φ_M: $Exp \rightarrow Ent$

is called the translation function of Exp to Ent w.r.t. M, and defined as follows:
Let $\alpha_1, \cdots, \alpha_n \in Voc$. Then,

$$\Phi_M(\alpha_1 \cdots \alpha_n) = \begin{cases} \text{If } \Phi_M(\alpha_1), \cdots, \Phi_M(\alpha_n) \text{ are all defined: } \Phi_M(\alpha_1) \cdots \Phi_M(\alpha_n). \\ \text{otherwise : } \dagger \end{cases}$$

Here,
a) $\Phi_M \restriction Const = f \restriction Const$.
b) $\Phi_M \restriction Var$: the function such that, for an arbitrary x $\in Var$, x $\mapsto x \in Par$
c) $\Phi_M \restriction Pred = f \restriction Pred$.
d) $\Phi_M \restriction LC \cup Q \cup EM \cup \{(,)\}$: the function such that, for an arbitrary $\alpha \in LC \cup Q \cup EM \cup \{(,)\}$, $\alpha \mapsto \alpha \in LO \cup Quant \cup EpMod \cup \{(,)\}$.

According to the above definition of Φ_M, elements of $WExp$ and $WEnt$ correspond to each other as follows:

(15)

WExp	*WEnt*
individual constant	individual
individual variable	parameter
n-ary predicate symbol	n-ary attribute
logical connective	logical operator
quantification symbol	quantifier
auxiliary symbol	parentheses
formula	infon

We represent Φ_M(a) with a ($a \in Const$), and Φ_M(pred) with $pred$ ($pred \in Pred$). So, the notational difference between an expression of EL and the corresponding entity is that terms and predicates in the former are represented in roman characters, whereas the corresponding general individuals and attributes in the latter are represented in italicized characters. But the former is a string of symbols, the latter is an semantic entity.

$form \in Form$ is interpreted w.r.t. $s(c, .)$, $M(c, .)$ and $M(c, \pi, .)$ as follows:

(16) ia) $s(c, form) = 1 \Leftrightarrow$ for all $M \in s$, $M(c, form) = 1$,
ib) $s(c, form) = 0 \Leftrightarrow$ for all $M \in s$, $M(c, form) = 0$.
iia) $M(c, form) = 1 \Leftrightarrow$ for all π, $M(c, \pi, form) = 1$,
iib) $M(c, form) = 0 \Leftrightarrow$ for all π, $M(c, \pi, form) = 0$.
iiia) $M(c, \pi, form) = 1 \Leftrightarrow M(c, \pi, \Phi_M(form)) = 1$,
iiib) $M(c, \pi, form) = 0 \Leftrightarrow M(c, \pi, \Phi_M(form)) = 0$.

1.4.3 $\models$-notation

(17) Let A be a formula or an infon. Then, $M(c, \pi, A) = 1$, $M(c, A) = 1$, and $s(c, A) = 1$ are also represented by $M, \pi \models_c A$, $M \models_c A$, and $s \models_c A$ respectively.

(18) i) $M, \pi \not\models_c A \Leftrightarrow$ It is not the case that $M, \pi \models_c A$,
ii) $M \not\models_c A \Leftrightarrow$ It is not the case that $M \models_c A$,
iii) $s \not\models_c A \Leftrightarrow$ It is not the case that $s \models_c A$.[3]

c in $M(c, .)$, $\models_c$ etc. is omitted in unnecessary cases.

1.4.4 Translation of natural language sentences to formulas of EL

For the interpretation of natural language sentences, we only assume an intuitive translation function Tr which translates natural language sentences to their corresponding formulas of EL. I.e.,

(19) $Tr(sent) = form$,
($sent$ is a natural language sentence, and $form \in Form$.)

whose truth value is then determined by $c(c, Tr(sent))$ w.r.t. the context situation c, i.e. the situation where $sent$ is uttered.

2 Infon as an Intermediate Entity

The reason for assuming the intermediate entity "attribute" is the following: Let $M = \langle\langle Ind, Attr, Ext\rangle, f\rangle$ be an extensional model. f represents the rule of an epistemic subject with which semantic entities are assigned to expressions of EL. But which semantic entities are they? $f(pred_i^j)$ is an attribute which is then extensionalized by Ext , but $f(\alpha)$ (α is a term of EL.) is an individual. $\Phi_M(form)$, i.e. an infon, could be thought of as the intension of a sentence in the traditional sense. But there are following differences. First, the semantic correspondence of terms appears as individuals in infons, which represents the (alternative) opinion of Kripke(1972), Kaplan(1977,1978,1989) that proper names and indexicals directly denote individuals without intermediate entities such as intension. Second, attributes $Attr_i^j$ (which correspond to traditional intensions of predicates) are introduced as undefined elements, and their extensions are one of their properties induced by Ext. $\Phi_M(form)$ is an semantic entity written in a general concept language which can be compared and posessed by different epistemic subjects. So, we could interpret that $form$ expresses the Kaplanian character, $\Phi_M(form)$ the Kaplanian content, and $M(\pi, \Phi_M(form))$ represents the extension.

f and Ext enables the following distinction:

(20) i) $Ext_1(f_1(\text{red})) = \{a, b\}$
ii) $Ext_2(f_1(\text{red})) = \{c, d\}$
iii) $Ext_1(f_2(\text{red})) = \{c, d\}$

(20i) is the extension of 'red' in the actual world (Ext_1) by means of the 'correct' linguistic rule f_1 in a community. (20ii) is its extension in another world, but the linguistic rule as such is correct. But in (20iii), the linguistic rule is false. For example, the epistemic subject mistakes 'red' for yellow. And $\{c, d\}$ could be the set of yellow things.

Now, consider the following example:

(21) i) John: "I'm going to the party."
ii) Marry (to John): "You are going to the party."

In (21), the character is different between i) and ii):

(22) i) go-to-the-party(I)
ii) go-to-the-party(you)

But the content is the same between them:

(23) i) *go-to-the-party(john)*,

ii) *go-to-the-party(john)*,

assuming that $\{M_j\}$ represents John's context situation of (21i) with $f_j(I) = john$, and $\{M_m\}$ represents Marry's context situation of (21ii) with $f_m(you) = john$.

3 Three-valued Interpretation of Natural Language Sentences

3.1 Classification of Three-valuedness

Here, I distinguish the following three types of three-valuedness:

(24) Type I: category error, failure of existence presupposition,
Type II: undefined cases,
Type III: failure of information,

which are exemplified as follows:

(25) i) The capitalism is yellow.
ii) Naomi is XYZ.
iii) It's gold. (In the situation s where it's not certain if it's gold.)

Intuitively, the third truth value 'neutral' appears just in the case that an utterance is interpreted as neither true nor false. But the meaning of 'false' is not so clear. As to Type I, it should be understood as the predicate negation, so that the three-valuedness of (25i) w.r.t. M, π is formulated as

(26) i) $M, \pi \not\models yellow(capitalism)$
ii) $M, \pi \not\models \overline{yellow}(capitalism)$

Here, we assume

$$
\begin{aligned}
(27)\quad & M(\pi, yellow) = Ext(yellow) = yellow^{+} \subseteq Ind, \\
& M(\pi, \overline{yellow}) = Ext(\overline{yellow}) = yellow^{-} \subseteq Ind, \\
& Ext(yellow) \cap Ext(\overline{yellow}) = \phi, \\
& Ind \setminus (Ext(yellow) \cup Ext(\overline{yellow})) = yellow^{N}, \\
& \overline{\overline{yellow}} = yellow,
\end{aligned}
$$

and call $yellow^{+}, yellow^{-}, yellow^{N}$ the positive, negative, and neutral domain of *yellow* respectively. And the individual *capitalism* belongs to $yellow^{N}$ so that the truth value 'neutral' is assigned to (25i).

As to (25ii), we assume that f(XYZ) is undefined. Then, Φ_M(XYZ(naomi)) = †. But, according to (13iia), $M(c, \pi, \dagger)$ is undefined, i.e. neither true nor false.

As to (25iii), suppose that $s = \{M_1, M_2\}$ such that (25iii) is true w.r.t. M_1, but it's false w.r.t. M_2. Then,

(28) i) $s \not\models gold(it)$,
ii) $s \not\models \neg gold(it)$.

And, (25iii) is interpreted as neutral w.r.t. s.

There is a certain difference between the three-valuedness of Type I, II and Type III. The former appears w.r.t. an extensional model, but the latter appears w.r.t. a situation. As we see from (10), $s(c, inf)$ is interpreted in a modal fashion, i.e. in order to determine the truth value of inf w.r.t. s, we must refer to several extensional models. For this reason, we call the former types of three-valuedness the non-modal three-valuedness, the latter the modal three-valuedness. But this modality should not be understood in the traditional modal logical manner, because $s(c, inf)$ is not determined w.r.t. other "possible worlds", but w.r.t. extensional models in s. So, we call this kind of modality "epistemic modality".

In order to enable an three-valued interpretation, we must a bit modify (10), (12) and (13ii) in the following manner:

(29) I) To (10) and (12),
iii) Otherwise, $s(c, inf) = N$ (N means 'neutral'.) and

iii) Otherwise, $M(c, inf) = N$
are added respectively.

II) (13iia) is altered to '$M(c, \pi, \dagger) = N$'.

III) (13iib) is altered to:

1) $M(c, \pi, Attr_i^n(t_1, \cdots, t_n)) = 1$, if $M(c, \pi, \underline{Attr_i^n})(M(c, \pi, t_1), \cdots, M(c, \pi, t_n)) = 1$.
2) $M(c, \pi, Attr_i^n(t_1, \cdots, t_n)) = 0$, if $M(c, \pi, \overline{Attr_i^n})(M(c, \pi, t_1), \cdots, M(c, \pi, t_n)) = 1$.
3) $M(c, \pi, Attr_i^n(t_1, \cdots, t_n)) = N$, otherwise.

IV) An appropriate three-valued interpretation are given to $\neg, \wedge, \vee, \supset, \equiv, \bigwedge, \bigvee$. (See e.g. Prior(1968) etc.)

Now, we can summarize the above-mentioned three-valued interpretation as follows:

(30) $s \models^{TP} PP\,inf$

PP and TP represents the truth state of Type I, II, and Type III respectively, i.e., $TP, PP \in \{T, \overline{T}, N, \overline{N}, F, \overline{F}\}$. T, $\overline{T}$, N, $\overline{N}$, F, $\overline{F}$ mean 'true', 'not true', 'neutral', 'not neutral', 'false', 'not false' respectively.

The interpretation of inf which (30) presents is defined in the following manner:

(31) i) $M(\pi, PP\,inf) = 1 \Leftrightarrow M(\pi, inf) = PP$. (I.e., 'neutral' if $PP = N$, etc.)
ii) $M(\pi, PP\,inf) = 0 \Leftrightarrow M(\pi, inf) \neq PP$.

(32) i) $s \models^{T} PP\,inf \Leftrightarrow$ for all $M \in s$, $M \models PP\,inf$.
ii) $s \models^{\overline{T}} PP\,inf \Leftrightarrow$ for some M, π ($M \in s$), $M, \pi \not\models PP\,inf$.
iii) $s \models^{N} PP\,inf \Leftrightarrow$ for one M, π ($M \in s$), $M, \pi \models PP\,inf$, and for another M', π' ($M' \in s$), $M', \pi' \not\models PP\,inf$.
iv) $s \models^{\overline{N}} PP\,inf \Leftrightarrow$ for all $M \in s$, $M \models PP\,inf$, or for all $M \in s$, $M \not\models PP\,inf$.
v) $s \models^{F} PP\,inf \Leftrightarrow$ for all $M \in s$, $M \not\models PP\,inf$.
vi) $s \models^{\overline{F}} PP\,inf \Leftrightarrow$ for some M, π ($M \in s$), $M, \pi \models PP\,inf$.

3.2 Hierachy of Epistemic Modality

But (30) is not the complete form of three-valued interpretation yet. There remains a possibility to extend its epistemic modal part, which is recognized by the following comparison of (simplified two-valued) interpretations:

(33) Extensional Model: $M \not\models^{-} \neg p$ ($-$ means 'false'.)
Situation Semantics: $M \not\models \langle \neg, p; 0 \rangle$
Signed Formula of Hintikka: $M \not\models F \neg p$

Here, we find three kinds of negation: ($\not\models$), ($^{-}, 0, F$), and ($\neg$). And this corresponds to the tripartite distinction of neustic, tropic, and phrastic part of a sentence utterance discussed in Hare(1970) and Lyons(1977:749pp.). According to them, an utterance of a sentence can be analyzed as follows:

(34) I-say-so[it-is-so[prop]]

And I-say-so part, it-is-so part, and prop part (i.e. the propositional content of the sentence) are called neustic part, tropic part, and phrastic part respectively. The three kinds of negation are applied to these respective parts, i.e.:

(35)

Hare	neustic negation	tropic negation	phrastic negation
Epistemic Model	$\not\models$	$-$	$\neg$
Situation Semantics	$\not\models$	0	$\neg$
Signed Formula	$\not\models$	F	$\neg$

Then the extended form of (30) so as to include the neustic part looks like as follows:

(36) $NP(S \models^{TP} PP\,inf)$

Here, $NP, TP, PP \in \{T, \overline{T}, N, \overline{N}, F, \overline{F}\}$. NP, TP, and PP are truth values of neustic, tropic, and phrastic part respectively, and $S \in \mathcal{P}ow\,(\mathcal{P}ow(\mathcal{M}))$.

The neustic part of (36) is interpreted as follows:

(37) i) $T(S, \models^{TP} PP\,inf) \Leftrightarrow$ for all $s \in S$, $s \models^{TP} PP\,inf$.
ii) $\overline{T}(S, \models^{TP} PP\,inf) \Leftrightarrow$ for some $s \in S$, it is not the case that $s \models^{TP} PP\,inf$.
iii) $N(S, \models^{TP} PP\,inf) \Leftrightarrow$ for one $s \in S$, $s \models^{TP} PP\,inf$, and for another $s' \in S$, it is not the case that $s' \models^{TP} PP\,inf$.
iv) $\overline{N}(S, \models^{TP} PP\,inf) \Leftrightarrow$ for all $s \in S$, $s \models^{TP} PP\,inf$, or for all $s \in S$, it is not the case that $s \models^{TP} PP\,inf$.
v) $F(S, \models PP\,inf) \Leftrightarrow$ for all $s \in S$, it is not the case that $s \models^{TP} PP\,inf$.
vi) $\overline{F}(S, \models^{TP} PP\,inf) \Leftrightarrow$ for some $s \in S$, $s \models^{TP} PP\,inf$.

Theoretically, we can further relativize the neustic modality, and construct an infinite hierarchy of higher order neustic modalities:

(38) $\cdots NP2(\mathcal{S} \models NP(S \models^{TP} PP\,A)) \cdots$

(Here, $NP2$ is analogous to NP, TP, and PP. $\mathcal{S} = \mathcal{P}ow(\mathcal{P}ow(\mathcal{P}ow(\mathcal{M})))$.) And this would be the full-fledged form of three-valued interpretation of natural language sentences. But linguistically, I believe that it's enough with the epistemic modalities up to neustic part.

3.3 Contraction of Neustic and Phrastic Part to Two-valuedness

In (30), the three-valuedness proper to EL is the tropic part, i.e. the three-valuedness caused by epictemic modality. So, we focus on the tropic three-valuedness, omit higher-order neustic parts, and contract the neustic and phrastic part to two-valued interpretation. But then, the tree-valued interpretation of the tropic part is expressed using neustic and phrastic negation as follows:

The phrastic part is contracted to two-valued form as follows. First, we treat the neutral case in Type I and II in (29) as false. Next, we omit the phrastic $N, \overline{N}$, identify $T, \overline{F}$ and $F, \overline{T}$ on the right hand side of (31) with 1 and 0 respectively, further, $T, \overline{F}$ and $F, \overline{T}$ on the left hand side with ϵ (empty string) and $\neg$ respectively. Then we can totally eliminate the phrastic part.

The two-valued interpretation of neustic part consists of (37i,ii). Further, we assume that $S = \{s\}$. Then (37i) and ii) amount to

(39) i) $T(S, \models^{TP} inf) \Leftrightarrow s \models^{TP} inf$,
and
ii) $\overline{T}(S, \models^{TP} inf) \Leftrightarrow s \not\models^{TP} inf$.

respectively. I.e., the neustic T and $\overline{T}$ are identified with $\models$ and $\not\models$ respectively.

But then, considering (18), (32) and the two-valuedness of $M(\pi, inf)$, the interpretation of infons is expressed using $\models, \not\models$ and $\neg$ without tropic part as follows:

(40) i) $s \models^{T} A$: $s \models A$
ii) $s \models^{\overline{T}} A$: $s \not\models A$
iii) $s \models^{N} A$: $s \not\models A$ and $s \not\models \neg A$
iv) $s \models^{\overline{N}} A$: $s \models A$ or $s \models \neg A$
v) $s \models^{F} A$: $s \models \neg A$
vi) $s \models^{\overline{F}} A$: $s \not\models \neg A$

3.4 Data Semantic Modality and Epistemic Logic

In (25iii) and (28), we could neither say that it was gold, nor it wasn't gold. Here, let $s = c$ for simplicity. Then, according to the definition in 1.4.1, we certainly say that

(41) It may be gold,

w.r.t. s. Further, we can say that

(42) He must be a honest guy.

w.r.t. the situation s such that, for every $M \in s$, (42) without 'must' is true. Such 'may' and 'must' reduce the interpretation to two-valuedness, insofar as inf in $epmod\, inf$ is defined.

But, as (13.ii.i) indicates, they do not represent traditional possible world semantic modalities, but 'data semantic' modalities disucussed in Veltman(1981,1985), Landman(1986).

I shortly consider the relationship between Data Semantics and EL.

3.4.1 Disjunctive sentences

In the situation of (25iii), we certainly say

(43) It's gold, or it's not gold.

In general, we can have the following situation:

(44) i) $s \not\models inf$,
ii) $s \not\models \neg inf$,
iii) $s \models inf \vee \neg inf$.

But the interpretation of disjunctive sentences poses a problem on Data Semantics. Because, in Data Semantics, the disjunction is interpreted in the following manner:

(45) $s \models inf \vee \neg inf \;\Leftrightarrow\; s \models inf$ or $s \models \neg inf$.

But then, in the situation of (44i,ii), (44iii) cannot be the case.

In order to avoid this, Data Semantics assumes that the disjuncitive sentence in the above situation is interpreted with *may*. I.e.:

(46) $s \models_s may(inf \vee \neg inf)$.

Then (44i,ii) and (46) are all true, because the data semantic '*may*' is interpreted in the same manner as EL. On the other hand, Data Semantics interprets conjunctive sentences as '$inf_1 \wedge inf_2$' without modality. So, the treatment of disjunctive sentences remains unnatural.

But EL treats this problem in a straightforward manner. I.e., according to (10),

(47) $s \models inf \vee \neg inf \;\Leftrightarrow$ for all $M \in s$, $M \models inf \vee \neg inf$.

Because '$inf \vee \neg inf$' is a logical truth, $M \models inf \vee \neg inf$ for all $M \in s$. So, $s \models inf \vee \neg inf$. This is an advantage of constructing a situations out of extensional models.

The whole story is illustrated as Fig.1. Here, the whole figure represents the lattice structure of $\mathfrak{M}$ with the top element $\mathcal{M}$. And,

(48) $inf \in Inf$, $s \in \mathfrak{M}$. Then,
i) $sit(inf) = \{M \in \mathcal{M} \mid M \models inf\}$
ii) $PI(inf) = \{s \mid s \subseteq sit(inf)\}$
iii) $PI(s) = \{s' \mid s' \subseteq s\}$

$sit(inf)$ is called the situation generated by inf. $PI(inf)$ and $PI(s)$ are called the pseudo-ideal generated by inf and s respectively.

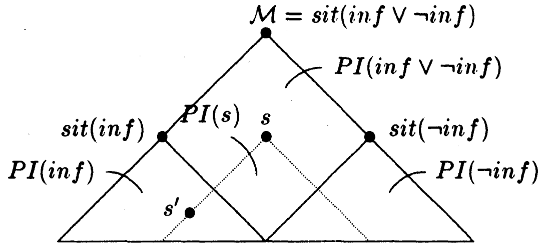

Fig.1

$PI(\alpha)$ is represented by the triangle with the top element $sit(\alpha)$ or α. Here, it is the case that

(49) i) '$s \models inf \vee \neg inf$' with Data Semantic '$\vee$' $\Leftrightarrow s \in PI(inf)$ or $s \in PI(\neg inf)$,
i) '$s \models inf \vee \neg inf$' with Epistemic Logical '$\vee$' $\Leftrightarrow s \in PI(inf \vee \neg inf)$.

The crucial defect of Data Semantics consists in the non-existence of 'disjunctive' situations such as $sit(inf \vee \neg inf)$ which exist in Epistemic Model that enable the latter to treat disjunctive sentences straightforwardly.

3.4.2 Data Semantic modality and tropic interpretation

Considering (13) and (40), it is clear that

(50) i) $s \models must\, inf \Leftrightarrow s \models^{T} inf$,

ii) $s \models may\, inf \Leftrightarrow s \models^{\overline{F}} inf$.

(Here, we assume that inf is defined.) I.e., 'may' and 'must' tranfer the tropic interpretation to the phrastic part. In this sense, Data Semanic modality is a kind of tropic modality.

4 Conclusive Remarks

In this paper, I proposed a treatment of three-valuedness in natural language sentences using EL. But, the present framework of EL is a small portion of its full-fledged form. In this paper, '*may*' and '*must*' are always interpreted w.r.t. the context situation. The context situation corresponds to the utterance situation in Situation Semantics. However, if we want to treat tense, indexicals, belief sentences etc., we need another index corresponding to described (or focal) situation in Situation Semantics w.r.t. which '$must\, inf$' and '$may\, inf$' are interpreted more naturally.

Further, '*must*' does not have the non-monotonicity which is captured by Data Semantics, although '*may*' has it in the present framework too. In order to capture the non-monotonicity, we must introduce e.g. 'background of speech' (Germ.: Redehintergrund, see Kratzer(1978)).

But they do not mean defects of EL, but are all solved in an extended version of it.

Notes

1 Generally, $A^{\circledast}$ represents the set of non-empty concatenation of elements of A.

2 A similar interpretation w.r.t. a situation is found in Fagin et al.(1995). But they apply it to the interpretation of the knowledge operator 'K'.

3 According to the present definition, $\mathfrak{A} \not\models \mathfrak{B}$ means $Not(\mathfrak{A} \models \mathfrak{B})$. So, $s \not\models inf$ does not mean that, for all $M \in s$, $M \not\models inf$, which is normally the case in elementary logic books. Likewise for $M \not\models inf$ etc.

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