Phonological processes with intersecting tier alphabets

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Abstract

Aksënova and Deshmukh (2018) conjecture that if the phonology of a language requires projection to multiple tiers, the tier alphabets of those tiers are either disjoint or stand in a subset/superset relation, but never form a non-trivial intersection. We provide three counterexamples to this claim.

1 Introduction

An important goal of computational phonology is to determine the complexity of the phonological patterns of natural language. A recent hypothesis is that these patterns are sub-regular and more specifically can be described as tier-based strictly local languages (Heinz et al. 2011, McMullin 2016 i.a.), or slight extensions thereof (Mayer and Major 2018, Graf and Mayer 2018, de Santo and Graf 2019). The general idea is that even non-local processes can be made local over appropriate representations, namely by masking out all irrelevant intervening elements or alternatively, projecting only elements participating in a process on a separate tier where they are adjacent again. Research in that area often focuses on a single pattern/process and a single tier. However, natural languages tend to have more than one phonotactic restriction or more than one phonological process; one might therefore expect that more than one tier is necessary to completely describe the phonology of a language. Moreover, it is the *interaction* of distinct processes that is of particular interest to phonologists. Aksënova and Deshmukh (2018), building on work by McMullin (2016), set out to investigate cases where more than a single tier is needed. They explore the possible relations that the sets of elements on different tiers can stand in: they can be disjoint ($\{a, b\}, \{c, d\}$), they can stand in a subset/superset relation ($\{a, b, c\}, \{b, c\}$) or they can non-trivially intersect ($\{a, b, c\}, \{c, d\}$), i.e. their intersection is neither empty nor the special case of a sub/superset relation (informally *intersection* for the rest of the paper). While being careful to point out the preliminary nature of their work, they claim that no natural language phonology requires a single element to be present on two tiers where each tier contains elements the other does not; in other words that there is no non-empty intersection of tier alphabets that do not stand in a sub/superset relation. They show that, as a function of the number of elements considered, the number of ways to create two sets with a non-empty intersection grows much faster than the respective ways to create true subsets or disjoint sets. As an example, when one considers all possible ways to create proper subsets, disjoint or intersecting sets for 10 elements, the number of intersecting sets already makes up more than 95% of all possibilities. If such a constellation were never to arise, a learner could discard the majority of combinatorially possible multiple TSL grammars. However, in this article we provide three counterexamples to this claim, showing that there are phenomena where one element plays a role in two processes that affect otherwise distinct elements. In Section 2 we provide the necessary background to the use of TSL in phonology and the claims about tier alphabet relationships from Aksënova and Deshmukh (2018). In Section 3, we provide the data for the three counterexamples (Sibe, Tsilhqút'hín, Koryak) that require a description involving overlapping tiers. We close with Section 4 where we discuss an alternative description of two of the three languages as Strictly Piecewise (SP, Rogers et al. 2010); Sibe, however, still resists a description with a single grammar, be it SP or TSL. It remains an open issue whether all existing intersecting TSL phenomena belong to a restricted subset of all possible intersections.

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2 Background

To familiarize the reader with the TSL-perspective and the type of data Aksënova and Deshmukh (2018) deal with, we provide a short summary of TSL grammars and the examples they give for processes that require disjoint and containing tiers. For more in-depth discussion, the reader is referred to the original paper.

Tier-based strictly local (TSL) grammars (Heinz et al. 2011) work by forbidding substrings of a finite length on a tier. They consist of a tier projection mechanism that scans the original string and projects every segment that is a member of a tier alphabet to a separate tier. There is a set of n-grams of finite size that is forbidden from occurring in the string on the projected tier.

Imagine a toy language with the three vowels *a*, *i* and *u* and an arbitrary consonant inventory. The language requires that all vowels in a word be either high (*i*,*u*) or low (*a*), i.e. we forbid the bigrams **ai,*ia,*au,*ua*. The tier alphabet is the set of all vowels $\{a,i,u\}$. The projection mechanism projects every vowel from that set it encounters to the tier. A word **blabliblu* thus would have the string *aiu* on its tier. While *iu* is an allowed substring, the combination **ai* is not since it is a forbidden bigram consisting of a high vowel followed by a low one.

Aksënova and Deshmukh (2018) provide an example of processes in a language that require two disjoint tiers, namely vowel harmony and nasal agreement in Kikongo. Vowels have to agree in height; the suffixes *-il/-el* and *-ol/-ul* have a different realization depending on their environment. Examples of different realizations of the former are *-leng-el-* or *-sik-il-*, for the latter *-tomb-ol-* or *-vil-ul-*. In nasal harmony, /*d*/ and /*l*/ become [*n*] if preceded by a nasal in the root, as can be seen for the suffix *-idi*: *-suk-idi-* but *-nik-ini-*. As a result, one needs a nasal harmony tier with the tier alphabet $\{n, m, d, l\}$, forbidding bigrams such as **nd,*nl,*md,*ml*. For vowel harmony, there is a tier with the vowels $\{e, i, o, u\}$, forbidding any bigram with mismatching height features. The tier alphabets of both tiers are disjoint.

(1) $\{n, m, d, l\} \cap \{e, i, o, u\} = \emptyset$

A sub/superset relation is instantiated in Imdlawn Tashlhiyt. Sibilants regressively harmonize in voicing and anteriority. The causative prefix /*s-*/ surfaces as [*s*] in *sː-uga*, [*ʃ*] in *ʃ-fiaʃr* or [*z*] in *z-bruːa*. There are blockers for voicing harmony, namely voiceless obstruents (*sː-ukz*, not **zː-ukz*); but they do not act as blockers for anteriority harmony (*ʃ-quʒːi*, not **s-quʒːi*). As a result, one needs a tier of all sibilants $\{s,z,ʃ,ʒ\}$, blocking any bigrams of mismatching anteriority (**ʃʒ,...*), and a second tier for all sibilants and voiceless obstruents $\{s,z,ʃ,ʒ,ħ,k,f,\chi,q\}$ to forbid any bigram of adjacent sibilants with distinct values for anteriority (**sz,*ʃz,...*) and forbidding any bigrams of voiced sibilants and voiceless obstruents to model their behaviour as blockers (**zk,*zq,...*). The tier alphabet of the second tier is a superset of the first one.

(2) $\{\boldsymbol{s,z,ʃ,ʒ}\} \subset \{\mathbf{s,z,ʃ,ʒ},ħ,k,f,\chi,q\}$

As mentioned above, Aksënova and Deshmukh conjecture that there are no phenomena whose tiers have a non-empty, non-containing intersection. We provide examples of processes that do require intersecting tier alphabets in the next section.

3 Counterexamples

3.1 Sibe

In Sibe (Tungusic, Xinjiang, China), rounding harmony affects all vowels. High back vowels are round if preceded by any round vowel, and non-high vowels agree in rounding with preceding non-high vowels. All the Sibe data is from Li (1996) via Nevins (2005). For the vowel inventory, see Table 1.

	–back		+back	
	–rd	+rd	–rd	+rd
+high	i	y	ɨ	u
–high	ɛ	ø	a	ɔ

Table 1: Sibe vowel inventory

The first effect of rounding harmony is a restriction on a non-round vowel. The high back non-round vowel is not licit following a round vowel (Nevins, 2005: 165):

(3) a. fulxu ‘root’, *fulxɨ
b. ɕøgu ‘vegetable’, *ɕøgɨ

The other high vowels, [*u*] and front high vowels are not restricted in this way and appear freely after vowels with the opposite value for round (Nevins, 2005: 166):

(4) a. χɔnin ‘sheep’
b. narχun ‘slim’

Secondly, non-high vowels must agree in rounding with preceding non-high vowels, as is shown in (5), (Nevins, 2005: 165-167).

(5) a. ɔmɔl 'grandson', *ɔmɛl, *ɔmal
b. tømχɔ 'nipple', *tømχɛ, *tømχa
c. χɛrχa 'pine tree', *χɛrχø, *χɛrχɔ
d. aχa 'rain', *aχø, *aχɔ

The latter process is restricted to roots, while the former extends to suffixes as well, as can be seen by the examples in (6) and (7).

Following Aksënova and Deshmukh we can establish a vowel tier with all vowels and the conditions on output forms on said tier (Table 2).

Vowel Tier T= {i,y,ɨ,u,ɛ,ø,a,ɔ}	
1.	*[+rd][+high, +back,–rd] H_{r1}{*yɨ, *uɨ, *øɨ, *ɔɨ }
2.	*[–high, α rd][–high, $-\alpha$ rd] H_{r2}{*ɛø, *ɛɔ, *øɛ, *øa, *aø, *aɔ, *ɔɛ, *ɔa }

Table 2: Tier and Filters for rounding harmony

The second relevant process in Sibe is uvularisation, a long distance vowel-consonant assimilation. Velars in affixes are turned into uvulars if they attach to a root containing a non-high vowel. In (6), no non-high vowel is present in the root, so the affixes surface with a velar. In (7), all root vowels are non-high and the affix consonant is uvularised (Nevins, 2005: 169-170):

(6) Velars with [+high] roots
a. ɕymi(n)-kɨn 'deep-DIM'
b. ulu-kun 'deep-DIM'
c. tyry-xu 'come-PST'
d. tɨ-xɨ 'sit-PST'

(7) Uvulars with [–high] roots
a. ɕa-qɨn 'good-DIM'
b. tɔndɔ-qun 'honest-DIM'
c. gø-χu 'hit-PST'
d. sav-χɨ 'see-PST'

In mixed roots, roots with both high and non-high vowels, the consonant is always uvular, whether it is adjacent to the [–high] vowel or not. Consider (8-b) and (8-d), where the low vowel triggers uvularisation across a high vowel.

(8) Uvulars with mixed roots
a. sul**a**-qɨn 'loose-DIM'
b. χ**ɔ**du-qun 'quick-DIM'
c. tyk**ɛ**-χɨ 'watch-PST'
d. **ø**mi-χɨ 'drink-PST'

The tier that is needed to check uvular assimilation includes velars[1] and [-high] vowels (Table 3).[2] Crucially, it must exclude [+high] vowels since they are transparent. If they were included, they would interfere with the locality on the tier and block uvularisation in mixed roots.

Tier of velars and [–high] vowels T= {k, g, x, ɣ, ɛ, ø, a, ɔ}	
1.	*[–high][+velar] H_{uv}{*ɛk, *ɛx, *ɛg, *ɛɣ,*øk, *øx, *øg, *øɣ, *ak, *ax, *ag, *aɣ, *ɔk *ɔx, *ɔg, *ɔɣ, }

Table 3: Tier and filters for uvularisation

We thus have intersecting tiers where [–high] vowels are both in the vowel tier as well as in the uvular assimilation tier but both tiers have elements that are not in the other tier, i.e. velars and [+high] vowels.

(9) {i,y,ɨ,u,**ɛ,ø,a,ɔ**} ∩ {**ɛ,ø,a,ɔ**,k,g,x,ɣ} ≠ ∅

Note that nothing changes about this fact if the vowel tier in Table 2 which handles two processes, rounding harmony for high and for non-high vowels, is split into two: both processes require non-high vowels on their tier, which are crucial for the intersection.

3.2 Tsilhqút'ín

In Tsilhqút'ín (Athabaskan, British Columbia, Canada; all data from Cook 1993, 2013 and Goad 1989), anterior sibilants come in pairs; they have a pharyngealised (or retracted), and a plain version. Anterior sibilants agree long-distance in pharyngealisation. The right-most sibilant functions as the trigger of sibilant harmony and determines the value for every other sibilant. The other sibilants are targets and agree in their retraction value with the rightmost one. Consider (10-a), where

[1] We remain agnostic about which feature distinguishes velar from uvular dorsals.

[2] We also need a third superset tier that includes all vowels and dorsals in order to derive the prohibition on more local [+high][uvular] sequences. Note, though, that it is not possible to describe all three processes on that same tier since high vowels would interfere with the locality of uvularisation and dorsals would interfere in rounding harmony.

the rightmost sibilant is a plain [z] and triggers depharyngealisation on the preceding sibilant. (10-b) shows the reverse pattern (Cook, 1993: 160-161):

(10) a. tɛ-**zˤ**-iː-ɬ-**ts**æː**z** → tɛ**z**iːɬ**ts**æː**z**
'I started to cook'
b. næː-**s**ɛ-næː-ɣĩː-l-**tsˤ**ɛ̃**sˤ** → naː**sˤ**ənaːɣə̃ĩl**tsˤ**ə̃**sˤ**
You are hitting me'

For this process, anterior sibilants must form a tier to the exclusion of everything else (Table 4).[3]

Tier of anterior sibilants T= {s, z, ts, dz, ts', sˤ, zˤ, tsˤ, dzˤ, ts'ˤ}	
1.	*[–R][+R] H_{sib1}{*ssˤ, *szˤ, ... *ts'ts'ˤ}
2.	*[+R][–R] H_{sib2}{*sˤs, *sˤz, ... *ts'ˤts' }

Table 4: Tier and filters for sibilant harmony

A second non-local process is retraction or 'flattening', where a vowel is retracted[4] in context of a pharyngealised sibilant or a uvular.[5] In (11-a) the uvular [q] triggers retraction of the vowels from /ɛ/ to schwa, and in (11-b) the pharyngealised sibilant acts as the trigger (Goad 1989: 23; Cook 1993: 161):

(11) a. sˤ**ɛ**-l-**qʷɛ**s → s**ə**l**qʷə**s
'he coughed'
b. χ**æː**t**æː**s-g**ɛ̃sˤ** → χ**aː**t**aː**sˤg**ə̃sˤ**
'I'll twist it out'

For retraction, therefore, a tier is needed that contains the triggers of the process, pharyngealised sibilants and (labialised) uvulars, and the target, vowels (Table 5).

We thus have intersecting tiers where pharyngealised sibilants are both in the sibilant harmony tier and retraction tier, but the former contains also non-pharyngealised sibilants and the latter vowels and uvulars.

[3]We use the more abstract feature R for both sibilant harmony and retraction, as is usual in the literature on Tsilhqút'ín.

[4]The featural changes a vowel partakes in under retraction are complex but irrelevant for this discussion.

[5]This is a gross simplification of the process. There are differences regarding the trigger – sibilant induced retraction is more long-distance than uvular induced retraction – and regarding directionality: leftward retraction is unblockable, whereas rightward retraction may be blocked by velars and long vowels function as icy targets. None of this affects the intersection that we discuss here. For a thorough discussion of the data and theoretical implications we refer to Goad (1989); Mullin (2011); Gleim (2021).

Tier of retraction participants T= {sˤ, zˤ, tsˤ, dzˤ, ts'ˤ, ɢ, ɢʷ, q, qʷ, q', qʷ', χ, χʷ, ʁ, ʁʷ, iː, ɪ, uː, ʊ, æː, ɛ}	
1.	*[–R][+R] R_1{*iːsˤ, *iːzˤ, ... *ɛʁʷ}
2.	*[+R][–R] R_2{*sˤiː, *sˤɪ, ... *ʁʷɛ }

Table 5: Tier and filters for retraction

(12) {s, z, ts, dz, ts', **sˤ, zˤ, tsˤ, dzˤ, ts'ˤ** } ∩ {**sˤ, zˤ, tsˤ, dzˤ, ts'ˤ,** ɢ, ɢʷ, q, qʷ, q', qʷ', χ, χʷ, ʁ, ʁʷ, iː, ɪ, uː, ʊ, æː, ɛ} ≠ ∅

3.3 Koryak

In Koryak (Chukcho-Kamchatkan, Kamchatka, Russia; all data is from Abramovitz 2021) vowels in a word must be from one of three sets. The recessive set {*i, u, e, ə*}, the so-called 'mixed' set {*i, u, a, ə*} or the dominant set {*e, o, a, ə*}. Some vowels are phonetically identical between sets, but need to be distinguished phonologically (for a justification we refer to Abramovitz 2021: ch. 3). A morpheme always has vowels belonging to one set only. If a morpheme with mixed vowels, i.e. a vowel or vowels taken from the 'mixed' set, such as the diminutive *-piʎ* or the root *maqmi* in (13), and a morpheme with recessive vowels are combined, recessive /*e*/ is lowered to [*a*]. The high vowels and schwa are not affected (Abramovitz, 2021: 60,58):

(13) *e*-lowering
a. ujetiki-piʎ → ujatikpiʎ
'little sled'
b. maqmi-te → maqmita
'with a bow'

If a morpheme with a dominant vowel and a morpheme with a recessive or mixed vowel are combined, recessive and mixed /*i*/ and /*u*/ are lowered to [*e*] and [*o*] respectively, and recessive /*e*/ is lowered to [*a*]. Nothing happens to (mixed or recessive) *a* or schwa. Consider (14), where the same mixed and recessive morphemes as in (13) are now put in a context with dominant vowels (Abramovitz, 2021: 61f):

(14) general lowering
a. ujetiki-piʎʎaq-ŋqo →

ojatekpeʎʎaqəŋqo
‘from the small sled’

b. qoja-te → qojata
‘by reindeer’

Vowel harmony in Koryak is obviously less phonetically grounded than the processes discussed above. We will implement it in a TSL grammar with diacritic features instead of the more usual phonological ones and leave any discussion of naturalness aside.

We will assume the diacritic features R, M and D which are part of the vowels’ specifications that derive these classes. This gives us the vowel inventory in (15). To reduce clutter, recessive vowels do not carry diacritics.

(15) $\{$e, i, u, ə, a^M, i^M, u^M, $ə^M$, e^D, $ə^D$, a^D, o^D $\}$

First, we will present a convenient tier for each process individually and show that the tiers do intersect. After that, we show that the tiers can neither be reconstructed as a single tier nor as tiers in a superset-subset relation. The tier that derives *e*-lowering (Table 6) must contain all vowels with the M-diacritic as well as recessive *e*.

Tier of e and M-vowels T= $\{$e, i^M, u^M, a^M, $ə^M\}$	
1.	*Me, *eM $eL\{$*ei^M, *eu^M, *ea^M, *e$ə^M$, *i^Me, *u^Me, *a^Me, *$ə^M$e $\}$

Table 6: Tier and filters for e-lowering

On the tier that derives general lowering (Table 7), all dominant vowels, all recessive vowels except recessive schwa, and *i* and *u* with the M-diacritic must be present, but crucially not *a* with the M-diacritic.

Both tiers share *e* and the high vowels with the M-diacritic, but only the first contains the non-high vowels with the M-diacritic; and only the second the recessive high vowels and the dominant vowels:

(16) $\{a^M$, $ə^M$, **e, $\mathbf{i^M}$, $\mathbf{u^M}$**$\} \cap \{$**e, $\mathbf{i^M}$, $\mathbf{u^M}$**, i, u, o^D, a^D, e^D $\} \neq \emptyset$

Now, let us consider alternatives with non-intersecting tiers. If we conflate the two tiers above into a single tier, which contains every vowel but recessive schwa, we run into problems with a sequence like the one in (17).

Tier of dominant vowels, high vowels and *e* T= $\{$e, i, u, i^M, u^M, e^D, $ə^D$, a^D, $o^D\}$	
1.	*DR, *RD $GL_1\{$*o^De, *o^Di, *o^Du, *a^De, *a^Di, *a^Du, *e^De, *e^Di, *e^Du, *$ə^D$e, *$ə^D$i, *$ə^D$u, *eo^D, *io^D, *uo^D, *ea^D, *ia^D, *ua^D, *ee^D, *ie^D, *ue^D, *e$ə^D$, *i$ə^D$, *u$ə^D\}$
2.	*DM, *MD $GL_2\{$ *$o^D i^M$, *$o^D u^M$, *$a^D i^M$, *$a^D u^M$, *$e^D i^M$, *$e^D u^M$, *$ə^D i^M$, *$ə^D u^M$, *$i^M o^D$, *$u^M o^D$, *$i^M a^D$, *$u^M a^D$, *$i^M e^D$, *$u^M e^D$, *$i^M ə^D$, *$u^M ə^D\}$

Table 7: Tier and filters for general lowering

(17) *e-i-i^M*

e-i and *i-i^M* are both perfectly fine bigrams, so the structure as a whole should be fine. However, in Koryak we actually get an output with a lowered */e/* in such a configuration. The second alternative for making the Koryak tiers compatible with Aksënova and Deshmukh’s hypothesis, is to project the elements that are uniquely in the *e*-lowering tier, *a^M* and *$ə^M$*, into the general lowering tier as well. This yields unwanted results in strings like (18).

(18) i-a^M-o^D

Again, both *i-a^M* and *a^M-o^D* are perfectly fine sequences on the general lowering tier. Only by banning *a^M* from the tier, we get the desired violation of **io^D*. To conclude, the tiers we proposed for each process individually do derive the data correctly and are necessarily intersecting.

4 Discussion

The absence of phonological processes that share a subset of their elements would have been computationally appealing since it would have eliminated a large share of logically possible tier alphabet relations. However, as we have demonstrated above, such processes do in fact exist. This raises the question if tiers of two interacting processes can form any possible intersection of their tier alphabets or if these intersections are subject to additional restrictions that at least somewhat narrow down the combinatorial possibilities.

One such possibility would be that all phenomena that require intersecting tiers in a multiple TSL description can be described by a single grammar from a class that is incomparable to TSL, i.e. a class that neither contains nor is contained by TSL. We want to mention one such class that has previously been used in the literature, Strictly Piecewise (Rogers et al. 2010), that works for two of the processes but unfortunately fails for Sibe. Strictly Piecewise (SP) is a class that is incomparable to TSL (see e.g. de Santo and Graf 2019 for an overview of containment relationships of classes). Due to the 'global' nature of vowel harmony in Koryak, it is possible to describe both phenomena discussed in 3.3 with a single SP grammar. Intuitively, Strictly Piecewise grammars forbid certain subsequences of strings, regardless of the number and nature of intervening elements. As an example, we can forbid that a dominant vowel is followed by a recessive vowel at any distance in a word by forbidding e.g. the subsequence $*o^D e$ (and the reverse for the equally forbidden co-occurrence). With this, one can simply list all impossible co-occurrences of vowels from different classes without worrying about interveners. As far as we can see, this derives the Koryak data just like the tier-based procedure above. The same goes for the (simplified version of) the Tsilhqút'ín data. As already mentioned in Rogers et al. (2010), sibilant harmony can be modelled as SP by forbidding subsequences of mismatching sibilants. This derives the process described by our first tier. To add vowel retraction in the context of pharyngealised sibilants and uvulars does not interfere with the first process; we can state further co-occurrence restrictions for vowel-sibilant/uvular combinations in the same SP grammar. The joint statement of such restrictions is not possible in a unified tier-based attempt where the additional vowels would interfere with the locality on the tier for sibilant harmony.

A potential conjecture that all processes that require intersecting tiers can be described by a single SP grammar unfortunately fails for Sibe: we know that the next vowel after [*y*] cannot be [*ɨ*] (**yɨ*), yet (8-c) *tykɛ-χɨ* is well-formed. This is because neither *yɛ* nor *ɛɨ* are problematic vowel sequences due to the opaque nature of *ɛ*. A simple SP-grammar cannot describe such blocking effects. One needs to simultaneously rule out **yɨ* and rule in *yɛɨ* subsequences. SP cannot distinguish both cases. Another option are classes that use more fine-grained projection mechanisms for their tiers such as input (and/or) output-TSL, I/O-TSL (de Santo and Graf 2019, Mayer and Major 2018, Graf and Mayer 2018). Intuitively, one can specify that a certain segment is only projected if it is preceded/followed by another specific segment in the input string (ITSL); or that a segment is only projected if it then precedes/follows a specific segment on the tier (OTSL); or a combination of both (IO-TSL). A reviewer asks whether a single IO-TSL can be used to describe the Sibe data. One option would be to project all vowels, but only project velars if they are then preceded at some distance by a [-high] vowel on the already existing tier. However, we have seen in (8) that the relation between the relevant dorsals and [-high] vowels is non-local. Whether a finite distance between a [-high] vowel and a dorsal is possible depends on whether recursive word formation processes (e.g. repeated affixation) are attested. We follow the practice of treating non-local processes as unbounded if they are only constrained by the maximal size of existing words (as is implicit e.g. in the treatment of the data from Aksënova and Deshmukh 2018).

Therefore it remains to be seen if the phonologies of natural languages allow all possible tier alphabet intersections or if there are hidden restrictions such that all intersecting tier alphabets can be described by a single class of languages incomparable to TSL.

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