

## A Properties of the duplication 2-morphism

In this section, we provide proofs of the properties of  $[//]$ , the duplication 2-morphism.

**Claim 1.** *If the merge operation is associative, then there is only one way to duplicate  $n$  wires.*

We can see this by considering the expansion of three wires in terms of a repeated application of  $[//]$  on two wires. Evaluating both sides will give a resulting diagram (Figure 1) that differs only in the order in which the spare wire enters the merge boxes; and in particular both diagrams will be equal if the merge operation is associative.

**Claim 2.** *If each marker has a unique colour and the merge operation is associative and commutative, then the duplication operation is associative in it's order of application.*

Consider an initial diagram containing two distinct marker boxes, each of which is duplicating exactly two wires. Evaluating the two orders of application gives two diagram that differ only up to the associativity and commutativity of the generated merging operations, as per the diagram in Figure 2. The alpha boxes have been labelled with the overall order in which they enter the merge boxes, showing that in this case boxes 2 and 3 appear swapped. This indeterminacy is similar to that obtained when expanding  $(a + b)(c + d)$ , where the ambiguity is resolved by the commutativity of  $+$ . Thus, imposing a commutative merge operation will ensure the diagrams are equal.

By Claim 1, this means that markers duplicating any more wires will also give equivalent diagrams. Thus,  $[//]$  is associative.

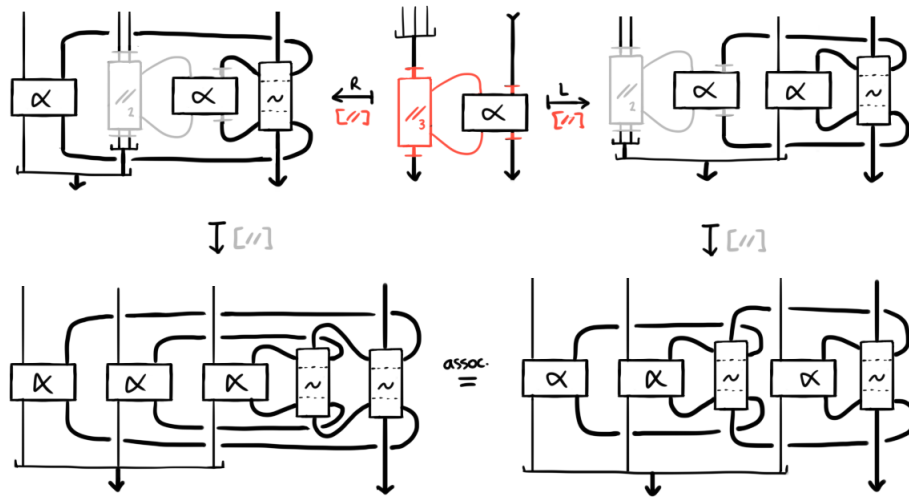


Figure 1:  $[//]$  on multi-wires

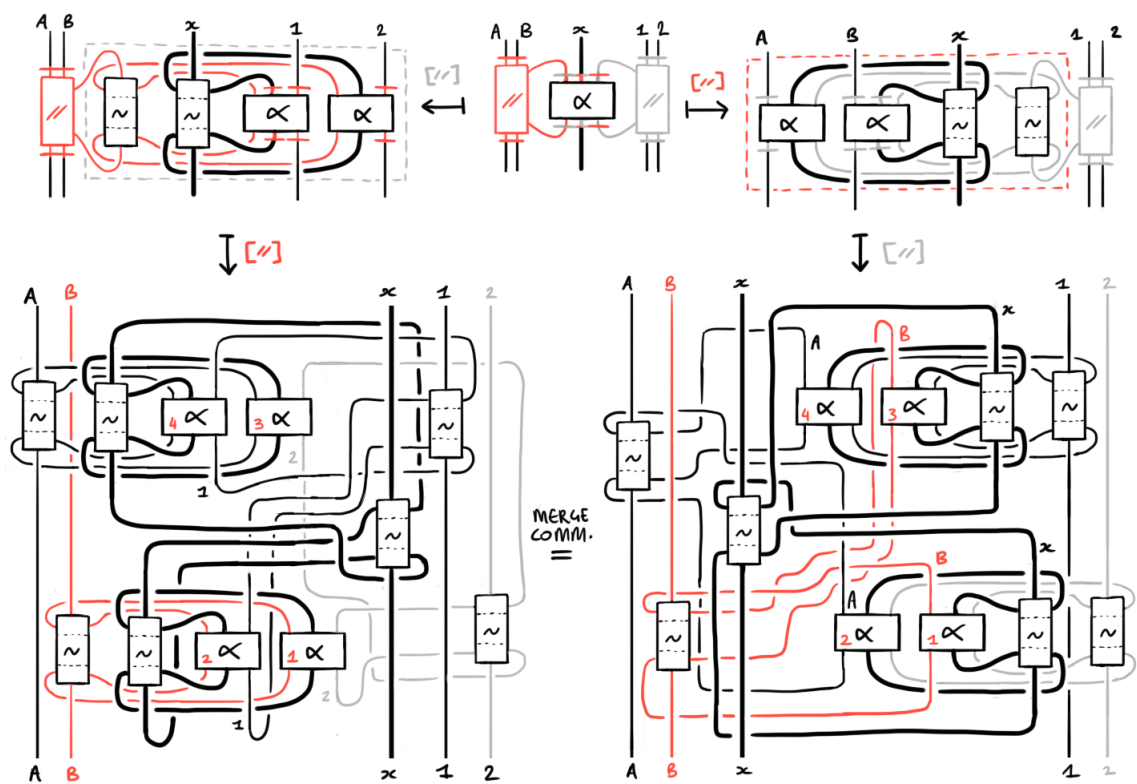


Figure 2: Associativity of  $[//]$