A MOVE TOWARDS A GENERAL SEMANTIC THEORY

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ABSTRACT

In this paper, I would like to propose an idea of semantics for discourse and pictorial representations. The central notion of this paper is *function*. A representation is a representation of something. A function determines the connection between representation and what is represented. This is true for both discourse and pictorial representations. To properly interpret a representation system means to find an intended function that connects representation and what is represented. By combining different semantic functions, we can express complex semantic relations. In the first part of this paper, I will present a general semantic theory; in the second part, this theory will be applied to several examples.

1. THEORY

1.1. Standard semantics

A standard semantics gives the meaning of a sentence by giving its truth condition. I modify the standard definition of *interpretation* slightly.

(1) Definition of interpretation function

- a) If c is a term, then I(c) is an element of a universe U, i.e. $I(c) \in U$.
- b) If R is a n-place relation symbol, then I(R) is a n-place relation based on U, i.e. $I(R) \subseteq U \times ... \times U$.
- c) An interpretation function I^* is a x-variant of I iff (if and only if) I^* is at most different from I in the interpretation of x.
- d) $I(R(c_1, ..., c_n)) = truth$ iff $\langle I(c_1), ..., I(c_n) \rangle \in I(R)$.
- e) $I(\neg p) = truth \text{ iff } I(p) = falsity.$
- f) $I(p \land q) = truth$ iff I(p) = truth and I(q) = truth.
- g) $I(\exists x \ p(x)) = truth$ iff there is a x-variant I^* of I such that $I^*(p(x)) = truth$.

An interpretation function I for a first order language L is a function that satisfies the above conditions. It is a function that correlates well-formed expressions in L with objects generated from U.

1.2. Discourse representation theory (DRT)

DRT organizes a discourse not as a sequence of sentences but as a structure (cf. [3]). In DRT, a new piece of information is continuously integrated into the given discourse structure. To make DRT more flexible, I define a *DRT-function f* in a similar way as in section 1.1.

- (2) Definition of DRT-function
- a) A discourse representation structure (DRS) is a pair that consists of a set V of variables and a set C of conditions.
- b) A DRT-function f is a function that correlates well-formed expressions with objects generated from a universe U.
- c) f verifies $R(c_1, ..., c_n)$ iff $\langle f(c_1), ..., f(c_n) \rangle \in f(R)$.
- d) f verifies $K_1 \Rightarrow K_2$ iff for every f's extension g that verifies K_1 , there is g's extension h that verifies K_2 .
- e) f verifies a DRS K iff f verifies all conditions in K.

A DRT-function determines extensions of all primitive symbols. This is the crucial difference to the classical definition; in the classical definition, a structure of a world is presupposed and only variables are interpreted by an assignment. A DRT-function determines not only values of variables but also a structure of a world.

1.3. A semantic theory for pictorial representations

A pictorial representation visually describes a world. Nakayama characterizes semantics of pictorial representations by using *projective functions* (cf. [9]). A projective function is a function that expresses information described in a pictorial representation. An interpretation of a pictorial representation A for B is explicated as an injection from parts of A into parts of B, where it preserves the part-whole relation. This injection is called a *projective function*. This theory by [9] is based on a mereological theory called *Natural Representation Language* (NRL) that is proposed by [8]. Usual maps and sketches also contain symbolic expressions that can be easily interpreted. Interpretations of symbols are considered as constraints on proper projective functions that are intended as correct readings. [9] defines several constraints on projective functions such as *standard-name*, *standard-attribution*, 1/n-contraction, proportion-preserving, [r%]-fuzzy-proportion, and so on.

As a semantic theory for maps, Pratt's proposal is well known. He attempts to define semantics for maps under the influence of traditional truth conditional semantics (cf. [10]). However, it is odd to speak of the truth of maps. Normally, we ask only whether a map is appropriate for a certain use. There are also different methods for the projection of the reality. The answer for the question, which projection should be used for a map, depends on its intended use. Exact projection is not always needed and in some cases not desirable, because readability is sometimes more preferable than exactness (cf. [1]). For pictorial representations, it is crucial to know what kind of projective functions are connected with them.

1.4. Fundamental operations

So far, we have discussed how to express semantic information by a function. Now, we would like to consider the problem how to combine different information. Combination of semantic information can be expressed by combining semantic functions. As fundamental operators, I propose unification and composition; extension can be defined as certain unification.

(3) Let f and g be functions.

- a) h is a unification of f and g iff $h = f \cup g$ and h is a function. The unification of f and g is denoted by $[f \cup g]$.
- b) *h* is a composition of *f* and *g* iff $h = \{\langle x, y \rangle : y = g(f(x))\}$. The composition of *f* and *g* is denoted by [g*f].
- c) f is an extension of g iff f is the unification of f and g, i.e. $f = [f \cup g]$. $g \subseteq f$ expresses that f is an extension of g.

We can combine these operations and build a new function such as $[f \cup [g*h]] \cup [g*k]$, where these

functions might be projective functions or DRT-functions. By using these operators, we can combine verbal and visual information and interpret them uniformly by using a combined function.

In the following applications, a single sentence is sometimes interpreted by different interpretation functions. For example, interpretations of figurative expressions and counterfactuals often require a flexible combination of different interpretations.

2. APPLICATIONS

2.1. Presuppositions and anchors in DRT

Suppose that a DRS K_1 presupposes the content of K_0 . Let f and g be intended DRT-functions for K_0 and K_1 . Then, the desired DRT-function h can be constructed as the unification of both functions, i.e. $h = [f \cup g]$. Thus, h has to verify not only K_1 but also K_0 . In this case, h is an extension of f, because it holds: $h = [h \cup f]$. An interpretation of sentences needs not start from the beginning; we may also start our interpretation from common background knowledge. The verifier f delivers such a starting point to interpret the assertion. Let us consider the following example.

(4) "Ivan has stopped beating his wife" presupposes that Ivan has beaten his wife.

 $K_0 = \langle \{u_1, u_2, u_3\}, \{u_1 = Ivan, u_2 = wife(u_3), u_3 = u_1, u_1 has beaten u_2, ...\} \rangle$, $K_1 = \langle \{x_1, x_2, x_3\}, \{x_1 = Ivan, x_2 = wife(x_3), x_3 = x_1, x_1 has stopped beating x_2, ...\} \rangle$. Let *f* be an intended DRT-functions for K_0 . Then, an intended DRT-function *h* for K_1 satisfies the following conditions: $f \subseteq h$, *h* verifies K_1 , $h(x_1) = f(u_1)$, $h(x_2) = f(u_2)$, and $h(x_3) = f(u_3)$.

Often, presuppositions are related to certain objects. When the discourse referent x_1 in K_1 presupposes characterizations of u_1 given by K_0 , we should accept $f(x_1) = g(u_1)$ as a constraint.

(5) "The king of France is dead."

Statement (5) presupposes some information on the person who is claimed to be a king of France. Suppose that this information is described by a DRS K_0 , where $K_0 = \langle \{u_1, ...\}, \{C(u_1), ...\} \rangle$. Let $K_1 = \langle \{x_1, x_2\}, \{x_2 = France, king-of(x_1, x_2), dead(x_1)\} \rangle$. Then, an intended DRT-function for K_1 is a DRT-function h such that $f \subseteq h$, h verifies K_1 , and $h(x_1) = f(u_1)$, where f is an intended DRT-function that verifies K_0 .

An external anchor in DRT connects a direct referential expression with a referent. When a speaker, pointing at Peter, says, "The man is a spy", the expression "the man" refers to Peter. This situation can be described as follows:

(6) $K_1 = \langle \{x_1\}, \{man(x_1), spy(x_1)\} \rangle$. Let $f = \{\langle x_1, \text{Peter} \rangle\}$. Then, an intended DRT-function for K_1 is a DRT-function h such that $f \subseteq h$ and h verifies K_1 . f corresponds to an external anchor in DRT.

Presuppositions and external and internal anchors are constraints on intended DRT-functions; these constraints are not explicitly expressed by statements. Usually, the hearer has to guess these constraints by using cues given in the context. In many cases, the hearer has to find the intended referents and the intended situation.

2.2. Combining discourse and pictorial representations

To explain a location of an object, we sometimes use a pictorial representation such as a map or a sketch and give additional information verbally. Think about this situation, in which a man points at the line on the map A and says,

(7) "This is a railway. There is a station on it. You can see a church near the station. Peter's house lies next to the church. This is a picture of the church."



Let f_1 be an appropriate projective function from parts of the map A into the world M. Let f_2 be an appropriate projective function from parts of the picture B into M. Let K be a DRS for (7), and g be a DRT-function from components of K into M. Then, $h = [f_1 \cup f_2 \cup g]$ is a function that unifies information from these three sources. From the map, we can get spatial information such as orientation and distance, when it is proportion-preserving. From the picture, the shape of the church can be seen. The following is a detailed formal description of this situation:

- (8) The projective function f_1 has the property of 1/20000-contraction.
 - $K = \langle \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}, \{railway(x_1), station(x_2), at(x_2, x_1), church(x_3), near(x_3, x_2), x_4 = Peter, x_5 belong to x_4, house(x_5), next-to(x_5, x_3), church(x_6), x_6 = x_3, picture-of(x_7, x_6)\} \rangle.$
 - g is an intended DRT-function that verifies K.

 $f_1(d_1) \in g(railway), f_1(d_2) \in g(station), f_1(d_2) = g(station Hoshi), f_1(d_3) \in g(church), f_2(c_1) \in g(church), f_2(c_2) \in g(house), g(x_7) = Picture B, and d_3 is associated with the icon <math>\dagger$.

To connect information from these three sources, we pose the following constraints:

 $h = [f_1 \cup f_2 \cup g], f_1(d_1) = g(x_1), f_1(d_2) = g(x_2), f_1(d_3) = f_2(c_1) = g(x_3), \text{ and } f_2(c_2) = g(x_5).$

Verbal and pictorial information can be unified by using a function that combines both.

2.3. Combination of several evaluations

Fauconnier proposes a theory of mental spaces and shows how to generally interpret complicated expressions in a natural language (cf. [2]). In this section, I will demonstrate how to deal with his complicated examples. Many examples in [2] are related to different evaluation areas; this is why he uses mental spaces. Example (9) can be considered as an ellipsis of (9') and (9') can be analyzed as (9"):

- (9) In Len's painting, the girl with blue eyes has green eyes.
- (9') In Len's painting, the girl with blue eyes in the actual world has green eyes.
- (9") [[The girl with blue eyes]_(actual world) has green eyes]_(world of Len's painting)

To describe the meaning of (9), we can use functions, f and g, with the following properties:

(10) f is an interpretation function of the actual world and g is an interpretation function of a possible world described by Len's painting. Furthermore:

$$f(d_1) \in f(girl), f(d_1) \in f(have \ blue \ eyes), \text{ and } f(d_1) \in g(have \ green \ eyes).$$

Len's picture describes a possible world. This is the reason why the girl can have green eyes. She has blue eyes in the actual world but has green eyes in the world described by Len's picture.

Many of examples in [2] can be analyzed in this way. Let us consider example (11) that can be analyzed as (11'):

- (11) There is a play about Golda Meir played by Ingrid Bergman. In the play, Golda Meir meets Ingrid Bergman who is played by another actress. Someone claims, "In this play, Ingrid Bergman meets Ingrid Bergman".
- (11') [[Ingrid Bergman]_(actual world) meets Ingrid Bergman]_(world described in this play).

Thus, (11') claims: $\langle f(Ingrid Bergman), g(Ingrid Bergman) \rangle \in g(meet)$, where f is an interpretation function of the actual world and g is an interpretation function of a possible world described by the play.

These examples show that many uses of mental spaces can be reinterpreted as marks of ranges of interpretation functions; they mark the world with respect to which an expression should be evaluated.

2.4. Requirement of an appropriate interpretation

The following example cannot be explained by the method proposed in the former section:

(12) Plato is on the top shelf. It is bound in leather. He is a very interesting author.

In (12), "Plato" stands both for a philosopher and for a book that he wrote. In this case, we need a new interpretation of "Plato". The truth condition of (12) can be described as follows:

(13) Let f be an interpretation function of the actual world and g be a function that is created in order to properly interpret (12). Then, the meaning of (12) can be given as follows:

 $g(Plato) \in f(book), \langle f(Plato), g(Plato) \rangle \in f(wrote), g(Plato) \in f(on the top shelf),$ $f(it) \in f(bound in leather), f(it) = g(Plato),$ $f(he) \in f(very interesting author), and f(he) = f(Plato).$

The condition, $\langle f(Plato), g(Plato) \rangle \in f(wrote)$, connects two different interpretations of the name "Plato". Also in the example "The mushroom omelet paid only for himself", it holds $\langle f(omelet), g(omelet) \rangle \in f(eaten)$. In many of Fauconnier's examples, we can find the connection that can be expressed in form $\langle f(c), g(c) \rangle \in f(R)$, where R expresses a popular relation. In his notions, f(c) is a trigger and g(c) is a target; a connector is a function F such that g(c) = F(f(c)).

2.5. Metaphor

Metaphors can be described by using functions. For example, to understand the statement "Anger is fire", you would try to find an interpretation function g such that $f(anger) \subseteq g(fire)$ and $f(fire) \subseteq g(fire)$, where f is a standard interpretation function. In this case, it holds: $f(anger) \cup f(fire) \subseteq g(fire)$. Certain common properties between anger and fire are properties that the speaker wanted to point out as properties of anger. The hearer has to use his imagination to find out what these common properties are.

Lakoff and Johnson distinguish target and source category (cf. [5]). In the metaphor "Time is money", *money* is the source category and *time* is the target category. We obtain: $f(money) \cup f(time) \subseteq g(money)$. Generally, when we have $f(A) \cup f(B) \subseteq g(A)$, then A is the source category and B is the target category, where f is a standard interpretation function.

However, not every metaphor has the form "A is B" with common nouns "A" and "B". In the example "Jeremy is a lion", "Jeremy" refers to an object and does not express any concept. Generally, to understand a metaphor, we replace the standard interpretation of A by a weaker interpretation of A, so that the statement becomes true. As a result, metaphors are open to a number of different interpretations; there is rarely one correct meaning.

(14) Definition of metaphorical interpretation:

Let f be a standard interpretation function. Let β be an expression in the sentence γ .

Then, g verifies γ by metaphorically interpreting β iff

 $f(\beta) \subset g(\beta), f(\beta) \neq g(\beta)$, [for all expressions α in γ , if $\alpha \neq \beta$, then $g(\alpha) = f(\alpha)$], and $g(\gamma) = truth$.

(15) Examples of metaphorical interpretation:

a) "Anger is fire".

If g(anger) = f(anger) and $f(anger) \cup f(fire) \subseteq g(fire)$, then g verifies "Anger is fire" by metaphorically interpreting "fire", because $[g(anger) \cup f(fire) \subseteq g(fire)] \Rightarrow [g(anger) \subseteq g(fire)] \Rightarrow$ [g(Anger is fire) = truth]. A possible interpretation for g(fire) is the class of objects that are dangerous.

b) "Time is money".

If g(time) = f(time) and $f(time) \cup f(money) \subseteq g(money)$, then g verifies "Time is money" by metaphorically interpreting "money", because $[g(time) \cup f(money) \subseteq g(money)] \Rightarrow [g(Time \ is money) = truth]$. A possible interpretation for g(money) is the class of objects that are valuable.

c) "Jeremy is a lion".

If g(Jeremy) = f(Jeremy), $f(Jeremy) \in g(lion)$, and $f(lion) \subseteq g(lion)$, then g verifies "Jeremy is a lion" by metaphorically interpreting "lion", because $[g(Jeremy) \in g(lion)] \Rightarrow [g(Jeremy is a lion) = truth]$. A possible interpretation for g(lion) is the class of creatures that are brave.

Sperber and Wilson claim that there is no discontinuity between loose talk and metaphor (cf. [11]). Indeed, definition (14) is also applicable to loose talk, as the following consideration shows.

(16) Examples of hyperbole and loose talk:

- a) The speaker knows that Bill is a very nice person and says, "Bill is the nicest person there is".
- If g(the nicest person there is) = f(very nice person) and f(Bill) = g(Bill), then g verifies "Bill is the nicest person there is" by metaphorically interpreting "the nicest person there is", because $f(the nicest person there is) \subseteq g(the nicest person there is)$ and $[f(Bill) \in f(very nice person)] \Rightarrow [g(Bill) \in g(the nicest person there is)].$
- b) Marie lives in Issy-les-Moulineaus, a block away from the city limits of Paris. She says "I live in Paris".

If $f(in) \subseteq g(in)$ and $\langle Issy-les-Moulineaus, Paris \rangle \in g(in)$, then g verifies "Marie lives in Issy-les-Moulineaus" by metaphorically interpreting "in", because $[g(Marie lives in Issy-les-Moulineaus) = truth & \langle Issy-les-Moulineaus, Paris \rangle \in g(in)] \Rightarrow [g(Marie lives in Paris) = truth].$

2.6. Conditionals

In DRT, the conditional sentence, "If a farmer owns a donkey, he beats it", is expressed as $K_1 \Rightarrow K_2$, where $K_1 = \langle \{x_1, x_2\}, \{farmer(x_1), donkey(x_2), x_1 owns x_2\} \rangle$ and $K_2 = \langle \{x_3, x_4\}, \{x_3 = x_1, x_4 = x_2, x_3 beats x_4\} \rangle$. According to (2d), a DRT-function f verifies $K_1 \Rightarrow K_2$ iff for every f's extension g that verifies K_1 , there is g's extension h that verifies K_2 . In the classical DRT, it is not possible to interpret counterfactual conditionals. This is because any extension of f falsifies K_1 , whereas we are interested in the worlds in which K_1 is true. Through a use of DRT-functions, a description of meaning of counterfactuals becomes possible. Definition (17) describes truth conditions for conditionals; we presuppose here the meaning of "differs minimally from". Gärdenfors' notion of revision provides an interesting explication of minimal difference (cf. [3]). (17) Definition of DRT-functions for conditional sentences

f verifies $K_1 > K_2$ iff

for every DRT-function g that verifies K_1 and differs minimally from f, there is g's extension h that verifies K_2 .

Definition (17) integrates DRT-treatment of implication with Stalnaker's and Lewis' view of counterfactuals (cf. [6], [12]). According to Stalnaker, p > q is true in the world w_1 iff q is true in the world that is the closest p-world to w_1 . Here, p-world means a world in which p is true. According to Lewis, p > q is true in the world w_1 iff some $(p \land q)$ -world is closer to w_1 than any $(p \land \neg q)$ -world. Lewis' definition is a generalization of Stalnaker's one. Lewis considers the case in which there are many close p-worlds but no closest one. When f verifies p > q, interpretation functions that verify p and differ minimally from f correspond to interpretation functions for the $(p \land q)$ -worlds that are closer to w_1 than any $(p \land \neg q)$ -world.

(18) Examples for counterfactuals

a) "If Peter owned a donkey, he would beat it."

The statement can be expressed as $K_1 > K_2$, where $K_1 = \langle \{x_1, x_2\}, \{x_1 = Peter, donkey(x_2), x_1 owns x_2\} \rangle$ and $K_2 = \langle \{x_3, x_4\}, \{x_3 = x_1, x_4 = x_2, x_3 beats x_4\} \rangle$. Suppose that a standard DRT-function f verifies $K_1 > K_2$. According to (17), if there is a DRT-function g that verifies K_1 and differs minimally from f, then there is g's extension h that verifies K_2 . In the actual world, Peter owns no donkey. However, in a world that differs minimally from the actual world and in which Peter owns a donkey, Peter beats his donkey.

b) "If I were a millionaire, my VW would be a Rolls."

Let f be a standard interpretation function. We introduce an interpretation function g that differs minimally from f and satisfies the condition, $g(I) \in g(millionaire)$. Because (18b) is a counterfactual conditional, $f(I) \notin f(millionaire)$. Because of minimal difference, it is natural to assume: g(I) = f(I). Thus, $f(millionaire) \neq g(millionaire)$. When (18b) is true, we can find g's extension h such that $h(my VW) \in h(Rolls)$. Because of minimal difference, it should hold: h(my VW) = g(my VW) = f(my VW). Hence, $f(my VW) \in h(Rolls)$. Thus, the car I have in the actual world is a Rolls in the world in which I am a millionare.

According to Fauconnier, the phrase "if ... then ..." introduces a new mental space (cf. [2]). A counterfactual conditional introduces a counterfactual mental space. A counterfactual conditional "if p, then q" claims that q is true in a particular p-space, where p-space means a mental space in which p is true.

Fauconnier criticizes Lewis' view of counterfactuals as inflexible and gives the following examples:

- (19) Fauconnier's examples
- a) If Napoleon had been the son of Alexander, he would have been Macedonian.
- b) If Napoleon had been the son of Alexander, he would have won the battle.

The both statements might be accepted as reasonable. According to Fauconnier, these examples demonstrate that it has no sense to ask after the absolute truth of several counterfactual sentences.

However, his argument is not convincing. The vagueness of the antecedents of these sentences is the true source of confusions. The following disambiguation of the antecedents shows that the speaker states two different antecedents.

- (20) Disambiguation of (19)
- a) If Napoleon had been the son of Alexander and had lived before Christ, he would have been Macedonian.

b) If Napoleon had been the son of Alexander and had inherited all good properties of Alexander, he would have won the battle.

2.7. Summary

In application examples, we had two types of treatments. We needed a combination of several functions or a slight modification of a standard interpretation. Interpretation is a creative activity based on knowledge of meaning and the world. In DRT, interpretations of variables are modifiable, but this modification is often too weak to describe complex semantic phenomena. I proposed in this paper to make interpretations of any symbols modifiable. The proposed general semantic theory is applicable to many problems and it is still strictly formal.

CONCLUSIONS

We can speak about the reality through different representation systems. We need, therefore, a semantic theory that can unify different forms of representations. Extending an idea in [7], I proposed in this paper a general semantic theory based on a flexible treatment of functions. It has been briefly shown how to deal with dynamic semantics in DRT, integration of non-literal uses of language and semantics for pictorial representations.

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