Formal and Empirical Grammatical Inference

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Formal GI and learning theory

GI of Regular Patterns

Empirical GI and nonregular patterns

Outline of the tutorial

- I. Formal GI and learning theory (de la Higuera)
- II. Empirical approaches to regular and subregular natural language classes (Heinz)
- III. Empirical approaches to nonregular natural language classes (van Zaanen)

I Formal GI and learning theory

- What is grammatical inference?
- What does learning or having learnt imply?
- Reasons for considering formal learning
- Some criteria to study learning in a probabilistic and a non probabilistic setting

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A simple definition

Grammatical inference is about learning a grammar given information about a language

Vocabulary

- Learning = building, inferring
- Grammar= finite representation of a possibly infinite set of strings, or trees, or graphs
- Information=you can learn from text, from an informant, by actively querying
- Language= possibly infinite set of strings, or trees, or graphs

A DFA (Ack: Jeffrey Heinz)

The $(CV)^*$ language representing licit sequences of sounds in many languages in the world. Consonants and vowels must alternate; words must begin with C and must end with V. States show the regular expression indicating its "good tails".





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A finite state transducer (Ack: Jeffrey Heinz)

A subsequential transducer illustrating a common phonological rule of palatalization ($k \longrightarrow t \hat{J} / \underline{\qquad} i$). States are labelled with a number and then the output string given by the σ function for that state.



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So for example:	tak taki	t(w) kata t∫ita tak tat∫i	
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Our definition			
Grammatical in information abo			g <mark>a grammar</mark> given



Why not write "learn a language"?

Because you always learn a representation of a language

Paradox

Take two learners learning a context-free language, one is learning a quadratic normal form and the other a Greibach normal form, they cannot agree that they have learnt the same thing (*undecidable question*).

Worth thinking about... is it a paradox? Do two English speakers agree they speak the same language?

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Our definition

Grammatical inference is about learning a grammar given information about a language

How can <u>a become the?</u>

- Ask for the grammar to be the smallest, best (re a score). \rightarrow Combinatorial characterisation
- The learning problem becomes an optimisation problem!
- Then we often have theorems saying that
 - If our algorithm does solve the optimisation problem, what we have learnt is correct
 - If we can prove that we can't solve the optimisation problem, then the class is not learnable

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Naive example



- How are you convinced that it works?
 - Because it follows sound principles as defined by number theory specialists?
 - Because you have tested and the number 772356191 has been produced?
 - Because you have proved that the series of numbers that will be produced is incompressible?
- Empirical approach
- Experimental approach
- Formal approach





- Benchmarks
- Competitions
- Necessary but not sufficient
- How do we know that all the cases are covered?
- How do we know that we dont have a hidden bias?

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Formal approach: showing that the algorithm has converged

- Is impossible:
 - Just one run
 - Can't prove that 23 is random
- But we can say something about the algorithm:
 - That in the near future, given some string, we can predict if this string belongs to the language or not;
 - Choose between defining clearly "near future" and accepting probable truths (or error bounds) or leaving it undefined and using identification.

What else would we like to say?

That if the solution we have returned is not good, then that is because the initial data was bad (insufficient, biased)

Idea:

Blame the data, not the algorithm

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Suppose we cannot say anything of the sort?

- Then that means that we may be terribly wrong even in a favourable setting
- Thus there is a hidden bias
- Hidden bias: the learning algorithm is supposed to be able to learn anything inside class L₁, but can really only learn things inside class L₂, with L₂ ⊂ L₁

Saying something about the process itself

- Key idea: if there is something to learn and the data is not corrupt, then, given enough time, we will learn it
- Replace the notion of *learning* by that of *identifying*

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In practise, does it make sense?

- No, because we never know if we are in the ideal conditions (something to learn + good data + enough of it)
- Yes, because at least we get to blame the data, not the algorithm

Complexity issues

- Complexity theory should be used: the total or update runtime, the size of the data needed, the number of mind changes, the number and weight of errors...
- ... should be measured and limited.

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A linguistic criterion

- One argument appealing to linguists (we hope) is that if the criteria are not met for some class of languages that a human is supposed to know how to learn, something is wrong somewhere
- (preposterously, the maths can't be wrong...)

Non probabilistic settings

- Identification in the limit
- Resource bounded identification in the limit
- Active learning (query learning)

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Identification in the limit

- Information is presented to the learner who updates its hypothesis after inspecting each piece of data
- At some point, always, the learner will have found the correct concept and not change from it

(Gold 1967 & 1978)

Example								
	Number	Presentation	Analysis of hy-	New hypothesis				
			pothesis	(regexp)				
	1	a +		а				
	2	aaa +	inconsistent	a*				
	3	aaaa -	inconsistent	$a(aa)^*$				
	4	<i>аааааа -</i>	consistent	$a(aa)^*$				
	9234 aaaaaaaaa -		consistent	a(aa)*				
	45623416	aaaaaaaaa $+$	consistent	a(aa)*				

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A presentation is

- a function $\phi:\mathbb{N} \to X$
 - where X is some set,
 - and such that ϕ is associated to a language *L* through a function YIELDS : YIELDS(ϕ) = *L*
 - If $\phi(\mathbb{N}) = \psi(\mathbb{N})$ then $\operatorname{YIELDS}(\phi) = \operatorname{YIELDS}(\psi)$





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Example: presentations for $\{a^n b^n : n \in \mathbb{N}\}$

- Legal presentation from text: λ , a^2b^2 , a^7b^7 ,...
- Illegal presentation from text: *ab*, *ab*, *ab*,...
- Legal presentation from informant : (λ, +), (abab, −), (a²b², +), (a⁷b⁷, +), (aab, −), (abab, −),...

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Example: presentation for Spanish

Legal presentation from text: En un lugar de la Mancha...

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- Illegal presentation from text: Goooool
- Legal presentation from informant : (en,+), (whatever,-), (un,+), (lugar,+), (lugor,-), (xwszrrzt,-),

What happens before convergence?

On two occasions I have been asked [by members of Parliament], 'Pray, Mr. Babbage, if you put into the machine wrong figures, will the right answers come out?' I am not able rightly to apprehend the kind of confusion of ideas that could provoke such a question. **Charles Babbage**

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Further definitions

- Given a presentation ϕ , ϕ_n is the set of the first *n* elements in ϕ .
- A learning algorithm (learner) A is a function that takes as input a set ϕ_n and returns a grammar of a language.
- Given a grammar G, L(G) is the language generated/recognised/ represented by G.



Convergence to a hypothesis

A converges to G with ϕ if

- $orall n \in \mathbb{N}: \mathbf{A}(\phi_n)$ halts and gives an answer
- $\blacksquare \exists n_0 \in \mathbb{N} : n \ge n_0 \implies \mathbf{A}(\phi_n) = G$
- If furthermore $\mathbb{L}(G) = \text{YIELDS}(\phi)$ then we have identified.





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- We say that the learner A is consistent if φ_n is consistent with A(φ_n) ∀n
- A consistent learner is always consistent with the past

Consistency and conservatism

- We say that the learner **A** is *conservative* if whenever $\phi(n+1)$ is consistent with $\mathbf{A}(\phi_n)$, we have $\mathbf{A}(\phi_n) = \mathbf{A}(\phi_{n+1})$
- A conservative learner doesn't change his mind needlessly

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Learning from data

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- A learner is order dependent if it learns something different depending on the order in which it receives the data.
- Usually an order independent learner is better.



Definition: polynomial characteristic sample

 \mathcal{G} has polynomial characteristic samples for identification learner **A** if there exists a polynomial p() such that: given any G in \mathcal{G} , $\exists Y$ correct sample for G, such that whenever $Y \subset \phi_n$, $\mathbf{A}(\phi_n) \equiv G$ and $\|Y\| \leq p(\|G\|)$

- As soon as the CS is in the data, the result is correct;
- The CS is small.

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Polynomial queries

(Angluin 1987)

- Algorithm A learns with a polynomial number of queries if the number of queries made before halting with a correct grammar is polynomial in
 - the size of the target,
 - the size of the information received.



Learning a language from sampling

- We have a distribution over Σ^{\star}
- We sample twice:
 - once to learn,
 - once to see how well we have learned
- The PAC setting: Les Valiant, Turing award 2010



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PAC-learning

(Valiant 1984, Pitt 1989)

- \mathcal{L} a class of languages
- $\blacksquare \mathcal{G}$ a class of grammars
- $\epsilon > 0$ and $\delta > 0$
- m a maximal length over the strings
- n a maximal size of machines

H is ϵ -AC (approximately correct)* if $Pr_D[H(x) \neq G(x)] < \epsilon$



- Using cryptographic assumptions, we cannot PAC-learn DFA
- Cannot PAC-learn NFA, CFGs with membership queries either
- Learning can be seen as finding the encryption function from examples (Kearns & Vazirani)

Alternatively

- Instead of learning classifiers in a probabilistic world, learn directly the distributions!
- Learn probabilistic finite automata (deterministic or not)

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No error (Angluin 1988)

- This calls for identification in the limit with probability 1
- Means that the probability of not converging is 0
- Goal is to identify the structure and the probabilities
- Mainly a (nice) theoretic setting

Results

- If probabilities are computable, we can learn with probability 1 finite state automata (Carrasco and Oncina, 1994)
- But not with bounded (polynomial) resources (de la Higuera and Oncina, 2004)

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		· · ·		<u> </u>

- PAC definition applies
- But error should be measured by a distance between the target distribution and the hypothesis
- How do we measure the distance: L₁, L₂, L_∞, Kullback-Leibler?

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Results

- Too easy to learn with L_{∞}
- Too hard to learn with L_1
- Both results hold for the same algorithm! (de la Higuera and Oncina, 2004)
- Nice algorithms for biased classes of distributions

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Open problems

We conclude this section on "what is language learning about" with some open questions:

- What is a good definition of polynomial identification?
- How do we deal with shifting targets? (robustness issues)
- Alternative views on learnability?
- Is being learnable a good indicator of being linguistically reasonable?
- Can we learn transducers? Probabilistic transducers?

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II. GI of Regular Patterns

- Why regular?
- What are the general GI strategies?
- What are the main results?
- The main techniques?
- The main lessons?

Logically Possible Computable Patterns





#1. Define "learning" so that large regions can be learned



GI Strategies





- characterize the patterns we are interested in (strategy #2).
- 3 Hard problems are *easier to solve* with better characterizations because the instance space of the problem is smaller.

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Why Begin with Regular?

Insights obtained here can be (and have been) applied fruitfully to nonregular classes.

- Angluin 1982 showed a subclass of regular languages (the reversible languages) was identifiable in the limit from positive data by an incremental learner.
- Yokomori's (2004) Very Simple Languages are a subclass of the context-free languages, but draws on ideas from the reversible languages.
- Similarly, Clark and Eryaud's (2007) substitutable languages (also subclass of context-free) are also based on insights from this paper.

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Section Outline

- 1 Targets of Learning
- 2 Learning Frameworks
- 3 State-merging
- Results for learning regular languages, relations, and distributions

Targets of Learning: Regular Languages





Formal GI and learning theory GI of Regular Patterns Empirical GI and nonregular patterns Targets of Learning: Regular Relations Multiple grammars (i.e. representations) for regular relations: Regular expressions (for relations) Generalized regular expressions (for relations)

- Finite state transducers
-

Targets of Learning: Regular distributions



This tutorial: Finite State Automata

Acceptors and subsequential transducers admit canonical forms

- 1 The smallest deterministic acceptor, syntactic monoids, ...
- 2 Canonical forms relate to algebraic properties (Nerode equivalence relation, i.e. states represent sets of "good tails")
- In contrast, canonical regular expressions have yet to be determined. For example, there are no canonical (e.g. shortest) regular expressions for regular languages.

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All possible vs. some restricted set

i.e. "distribution-free" vs. "non distribution-free"

What kind of samples?

Positive data vs. postive and negative data

• Other choices (e.g. query learning) are not discussed here.



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Learning Frameworks: Main Results

"Distribution-free" with positive data only

- 1 No superfinite class (including regular, cf, etc.) is identifiable in the limit (Gold 1967)
- 2 Not even the finite class is PAC-learnable (Blumer et al. 1989)
- 3 No superfinite class is identifiable in the limit with probability p (p > 2/3) (Pitt 1985, Wiehagen et al. 1986, Angluin 1988)
- 4 But many subregular classes are learnable in this difficult setting.

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Learning Frameworks: Main Results



Learning Frameworks: Main Results



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Learning regular languages: Key technique

State-merging

- Angluin 1982 (reversible languages)
- Muggleton 1990 (contextual languages)
- Garcia et al. 1990 (strictly local languages)
- Oncina et al. 1993 (subsequential functions)
- Clark and Thollard 2004 (PDFA distributions)
- **.**..



Other techniques



Only so much can be covered...



It's impossible to be fair to all those who have contributed and to cover all the variants, even all the algorithms in a short tutorial. That's why there are books!
Overview of State-merging

1 Builds a FSA representation of the input

2 Generalize by merging states

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Illustrative Example: Stress pattern of Pintupi

a.	ра́па	'earth'	ό σ
b.	t ^j úťaya	'many'	ό σ σ
c.	málawàna	'through from behind'	ό σ ờ σ
d.	púliŋkàlat ^j u	'we (sat) on the hill'	ό σ δ σ σ
e.	t ^j ámulìmpat ^j ùŋku	'our relation'	ό σ ờ σ ờ σ
f.	tíliriŋulàmpat ^j u	'the fire for our benefit flared up'	ό σ ὸ σ ὸ σ σ
		•	
g.	kúran ^j ùlulìmpat ^j ù _l a	'the first one who is our relation'	ό σ ὸ σ ὸ σ ὸ σ
h.	yúmaılìŋkamàrat ^j ùıaka	'because of mother-in- law'	ό σ ὸ σ ὸ σ ὸ σ σ

Generalization (Hayes (1995:62) citing Hansen and Hansen (1969:163)):

Primary stress falls on the initial syllable

Secondary stress falls on alternating nonfinal syllables

Illustrative Example: Stress pattern of Pintupi

Generalization (Hayes (1995:62) citing Hansen and Hansen (1969:163)):

- Primary stress falls on the initial syllable
- Secondary stress falls on alternating nonfinal syllables

Minimal deterministic FSA for Pintupi Stress







3 Suffix Trees (reverse determinstic)

Examples of Prefix and Suffix Trees

$$S = \begin{cases} \dot{\sigma} & \dot{\sigma} \sigma \\ \dot{\sigma} \sigma \sigma & \dot{\sigma} \sigma \dot{\sigma} \sigma \\ \dot{\sigma} \sigma \dot{\sigma} \sigma \sigma & \dot{\sigma} \sigma \dot{\sigma} \sigma \dot{\sigma} \sigma \end{cases}$$



State-merging Informally



- States are identified as equivalent and then *merged*.
- All transitions are preserved.
- This is one way in which generalizations may occur—because the post-merged machine accepts everything the pre-merged machine accepts, possibly more.



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State-merging Formally

Definition

Given an acceptor $A = (Q, I, F, \delta)$ and a partition π of its states state-merging returns the acceptor $A/\pi = (Q', I', F', \delta')$:

1 $Q' = \pi$ (the states are the *blocks* of π)

2
$$I' = \{B \in \pi : I \cap B \neq \emptyset\}$$

$$F' = \{B \in \pi : F \cap B \neq \emptyset\}$$

4 For all $B \in \pi$ and $a \in \Sigma$, $\delta'(B, a) = \{B' \in \pi : \exists q \in B, q' \in B' \text{ such that } q' \in \delta(q, a)\}$

Theorem

Theorem

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Given any regular language L, let A(L) denote the minimal deterministic acceptor recognizing L. There exists a finite sample $S \subseteq L$ and a partition π over PT(S) such that $PT(S)/\pi = A(L)$.

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Notes

- The finite sample need only exercise every transition in A(L).
- What is π ?

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Illustrative Example

Let's merge states with the same incoming paths of length 2!



Summary of Algorithm

 States in the prefix tree are merged if they have the same k-length suffix.

$$u \sim v \stackrel{\text{def}}{\iff} \exists x, y, w \text{ such that } |w| = k, u = xw, v = yw$$

2 The algorithm then is simply:

$$G = PT(S)/\pi_{\sim}$$

3 This algorithm provably identifies in the limit from positive data the Strictly (k + 1)-Local class of languages (Garcia et al. 1990).

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Back to the Illustrative Example

Results for stress patterns more generally

- Out of 109 distinct stress patterns in the world's languages (encoded as FSAs), this state-merging strategy works for only 44 of them
- If we merge states with the same paths up to length 5(!), only 81 are learned.
- This is the case even permitting very generous input samples.

In other words, 44 attested stress patterns are Strictly 3-Local and 81 are Strictly 6-Local. 28 are not Strictly 6-Local In fact those 28 are not Strictly k-Local for any k (Edlefsen et al. 2008).

Other ways to merge states

If the current structure is "ill-formed" then merge states to eliminate source of ill-formedness



 2 Merge states indiscriminately unless "ill-formedness" arises

 Merge unless something tells us not to

 1 unless "onward subsequentiality" is lost (for transducers, Oncina et al. 1993)

 2 unless they are "μ-distinguishable" (Clark and Thollard 2004)

 3 ...

State-merging as inference rules

Strictly k-Local languages (Garcia et al. 1990)

merge states with same incoming paths of length k

$$orall u,v,w\in\Sigma^*$$
 : $uv,wv,\in \mathit{Prefix}(L)$ and $|v|=k$

$$\downarrow \\ Tails_L(uv) = Tails_L(wv) \in L$$





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Regular relations



OSTIA

- identifies subsequential functions in the limit from positive data.
- 2 Merges states greedily unless subsequentiality is violated
- 3 If the function is partial, exactness is guaranteed only where the function is defined.

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OSTIA (Oncina et al. 1993)

Subsequential relations

- **1** are a subclass of the regular relations, recognizing functions.
- 2 are those which are recognized by subsequential transducers, which are deterministic on the input and which have an "output" string associated with every state.
- 3 have a canonical form.
- 4 have been generalized to permit up to *p* outputs for each input (Mohri 1997).

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OSTIA for learning phonological rules

Gildea and Jurafsky 1996

- Show that OSTIA doesn't learn the English tapping rule or German word-final devoicing rule from data present in adapted dictionaries of English or German
- 2 Applied additional phonologically motivated heuristics to improve state-merging choices.

What about well-defined subclasses of subsequential relations?

Weighted finite-state automata

non-distribution-free with positive data

The problem

Given a finite multiset of words drawn independently from the target distribution, what grammar accurately describes the distribution?

Theorem

The class of distributions describable with Non-deterministic Probabilistic Finite-State Automata (NPFA) exactly matches the class of distributions describable with Hidden Markov Models (Vidal et al. 2005).

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(Vidal et. al 2005, de la Higuera 2010)

Strictly 2-Local Distributions are bigram models



Figure: The structure of a bigram model. The 16 parameters of this model are given by associating probabilities to each transition and to "ending" at each state.





1 *N*-gram models can't describe long-distance dependencies.



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Sibilant Harmony example from Samala (Ineseño Chumash)

[[tojonowonowa]] 'it stood upright' (Applegate 1972:72)

- cf. *[stojonowonowa]] and
- cf. *[ftojonowonowas]

Hypothesis: *[stojonowonowaʃ] and *[ʃtojonowonowas] are ill-formed because the *discontiguous subsequences sf* and *fs* are ill-formed.

Strictly Piecewise languages

Rogers et al. 2010

- 1 solely make distinctions on the basis of potentially discontiguous subsequences up to some length k
- 2 are mathematically natural. They have several chacterizations in terms of formal language theory, automata theory, logic, model theory, and the
- **3** algebraic theory of automata (Fu et al. 2011)

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Strictly Piecewise Distributions



- **1** are defined in terms of the factored automata-theoretic representations (Rogers et al. 2010)
- 2 along with the co-emission probability as the product (Vidal et al. 2005)
- 3 Estimation over the factors permits learnability of the patterns like the ones in Samala.

SP2 learning results for Chumash

Training corpus 4800 words from a dictionary of Samala

			2	X	
P	$P(x \mid y <)$	S	fs	ſ	tĴ
	S	0.0325	0.0051	0.0013	0.0002
v	ts	0.0212	0.0114	0.0008	0.
J	ſ	0.0011	0.	0.067	0.0359
	tſ	0.0006	0.	0.0458	0.0314

Table: SP2 probabilities of sibilant occuring sometime after another one (collapsing laryngeal distinctions)

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Learning larger classes of regular distributions

More non-distribution-free with positive data

The class of distributions describable with PDFA

- 1 are identifiable in the limit with probability one (de la Higuera and Thollard 2000).
- 2 are learnable in modified-PAC setting (Clark and Thollard 2004).

3 The algorithms presented employ state-merging methods.

- **1** This is a (much!) larger class than that which is describable with n-gram distributions or with SP distributions.
- 2 To my knowledge these approaches have not been applied to tasks in CL.

Summary

#1. Define "learning" so that large regions can be learned



Oncina et al. 1993, de la Higuera and Thollard 2000, Clark and Thollard 2004, . . .

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Angluin 1982, Muggleton 1990, Garcia et al. 1990, Heinz 2010, ...

Recursively Enumerable

Kwakiutl stress

Bach 1975

English consonant clusters

Clements and Keyser 1983

Have we put the cart before the horse?



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Have we put the cart before the horse?

Research strategy

 $Patterns \Rightarrow Characterizations \Rightarrow Learning algorithms$

- **1** Identify the range and kind of patterns (linguistics).
- 2 Characterize the range and kind of patterns (computational linguistics).
- 3 Create learning algorithms for these classes, prove their success in a variety of settings, and otherwise demonstrate their success (grammatical inference, formal learning theory, computational linguistics)

Subregular classes of regular sets



- State-merging is a well-studied strategy for inferring automata, including acceptors, transducers, and weighted acceptors and transducers.
- 2 It has yielded theoretical results in many learning frameworks including both distribution-free and non-distribution-free learning frameworks.

Conclusion to section 2 part 2



4 There is a rich literature in GI which speaks to these classes, and how such patterns in these classes can be learned.



Introduction

- Language learning
 - Starting from family of languages
 - Given set of samples
 - Identify language that is used to generate samples
- Formal grammatical inference
 - Identify family of languages that can be learned efficiently
 - Under certain restrictions
- Empirical grammatical inference
 - Exact underlying family of languages is unknown
 - Target language is approximation

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Empirical GI

- Try to identify language given samples
 - E.g. sentences (syntax), words (morphology), ...
- Underlying language class is unknown
 - For algorithm we still need to make a choice
- If identification is impossible, provide approximation
 - Evaluation of empirical GI is different from formal GI

Family of languages



- Use simple, fixed structures (n-grams)
 - Find probabilities
 - Extract structure from treebanks
 - Slightly more flexible structure
 - Find probabilities
 - Learn structure
 - Flexible structure
 - Find probabilities

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N-grams

- **1** Starting from a plain text or collection of texts (corpus)
- **2** Extract all subsequences of length *n* (*n*-grams)
- **3** Count occurrences of *n*-grams in texts
- 4 Assign probabilities to each *n*-gram based on counts

Issues

- Unseen *n*-grams
 - Back-off: use *n*-grams with smaller *n*
 - Smoothing: adjust probabilities for unseen *n*-grams









probabilities of structure

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Overview of systems

EMILE

- Alignment-Based Learning (ABL)
- ADIOS
- CCM+DMV
- U-DOP
- **.**..

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Underlying approach

- Given a collection of plain sentences
- On what basis are we going to assign structure?
- Should structure be linguistically motivated?
 - or similar to what linguists would assign?
- Perhaps we can use tests for constituency to find structure

Substitutability

Elements of the same type are substitutable

Test for constituency (Harris, 1951) What is (a family fare)_{NP} Replace noun phrase with another noun phrase What is (the payload of an African Swallow)_{NP}

Learning by reversing test

<u>What is</u> (a family fare)_X <u>What is</u> (the payload of an African Swallow)_X

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EMILE

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- Learns context-free grammars
- Using plain sentences
- Originally used to show formal learnability of (a form of) Categorial Grammars in a PAC learning setting

GI of Regular Patterns

(Adriaans 1992, Adriaans and Vervoort 2002, Vervoort 2000)

Approach

- Starting from simple sentences identify recurring subsequences
- 2 Store recurring subsequences and contexts
- 3 Introduce grammar rules when there is enough evidence
 - Practical implementation allows for several constraints
 - Context length, subsequence length, ...

	es Mary (.) walks	John (.)	(.) sees Mary		contexts
John	Х		Х		
walks		Х			
Mary	×				
sees					
:	:	:	÷	·	
terms					

Formal GI and learning theory

GI of Regular Patterns

Empirical GI and nonregular patterns

Learn grammar rules

- Terms that share (approximately) same context are clustered
 "John" and "Mary" are grouped together
- Occurrences of terms in cluster are replaced by new symbol
- Modified sequences may again contain terms/contexts
- Terms may consist of multiple words

Example

Mary slaps John $\Rightarrow X$ slaps X

"sees" and "slaps" now also share the same context

Alignment-Based Learning (ABL)

- Based on substitutability test
- Using plain sentences
- Similar to EMILE, but
 - Clustered terms are not explicitly replaced by symbol
 - Terms and contexts are *always* separated
 - All terms are considered (and only selected afterwards)
- Output is structured version of input or grammar
- (van Zaanen 2000a, b, 2002)



Alignment-Based Learning (ABL)

- Alignment learning
 - Align pairs of sentences
 - Unequal parts of sentences are stored as hypotheses

(Clustering)

- Group hypotheses in same context together
- Selection learning
 - Remove overlapping hypotheses

Formal GI and learning theory

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Alignment learning

- Align pairs of sentences
 - using edit distance (Wagner and Fischer 1974)
 - or suffixtrees (Geertzen and van Zaanen 2004, Ukkonen 1995)
- Unequal parts of sentences are stored as hypotheses
- Align all sentences in a corpus to all others

Example

- $(Y_1 | need (X_1 a dinner during the flight)_{X_1})_{Y_1}$
- $(Z_1 | \text{need})_{Z_1} (X_1 \text{ to return on } (Z_2 \text{ tuesday})_{Z_2})_{X_1}$
- $(Y_1(Z_1 \text{ he wants})_{Z_1} \text{ to return on } (Z_2 \text{ wednesday})_{Z_2})_{Y_1}$

Selection Learning

- Alignment learning can generate overlapping brackets
- Underlying grammar is considered context-free
- Structure describes parse according to underlying grammar
- "Wrong" brackets have to be removed
 - Based on e.g. chronological order or statistics

Example

from $(Y_1 \text{ Tilburg } (X_2 \text{ to})_{Y_1} \text{ Portland})_{X_2}$ from $(X_1 \text{ Portland } (Y_2 \text{ to})_{X_1} \text{ Tilburg})_{Y_2}$

Formal GI and learning theory

GI of Regular Patterns

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ADIOS

Automatic Distillation of Structure (ADIOS) (Solan 2005)

Idea

- 1 Represent language as a graph
- 2 Compress graph
- 3 As long as possible, find significant patterns in paths
 - Using substitutability and significance tests
- 4 (Recursion may be added as a post-processing step)









$$D_R(e_1; e_3) = \frac{P_R(e_1; e_4)}{P_R(e_1; e_3)}$$

- *P_R* describes path to the right similarly *P_L* describes path to the left
- Significance is computed based on D_R and D_L wrt parameter
 - Informally: find significant changes in number of paths
- Pick most significant pattern

Constituent-Context Model (CCM)

Consider all possible binary tree structures on POS sequences

- Define a probability distribution over the possible bracketings
 - A bracketing is a particular structure on a sequence

$$P(s,B) = P_{bin}(B)P(s|B)$$

$$P(s|B) = \prod_{i,j:i \leq j} P_{\mathsf{span}}(s_{ij}|B_{ij}) P_{\mathsf{Ctx}}(s_{i-1}, s_j|B_{ij})$$

- Run (iterative) Expectation-Maximization (EM) algorithm
 to maximize likelihood Π_{s∈S}P(s)
- (Klein 2002)



$\mathsf{CCM} + \mathsf{DMV}$

- CCM and DMV can be combined
- Both models have different view on structure
- Results of combined system are better than either systems
 - Strengths of both systems are combined
- (Klein 2004)

Formal GI and learning theory	GI of Regular Patterns	Empirical GI and nonregular patterns
U-DOP		
 uses POS U-DOP uses E Extends p Requires pract 	ability distribution over "all" sequences Data-Oriented Parsing (DO robabilistic model of context tical implementation choice ampling due to huge size of	DP) as formalism -free grammars es
Procedure		
2 Extract all <i>sul</i>	ossible binary trees on exa btrees abilities on subtrees using	



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Underlying idea

■ U-DOP works because span of subtrees reoccur in a corpus

- Likelihood of "useful" spans increase
- Hence, likelihood of contexts (also subtrees) increase
- Essentially, U-DOP uses implied substitutability
 - while system leans heavily on probabilities



Evaluation settings

- Air Travel Information System (ATIS)
- Taken from Penn Treebank II
- **568** English sentences

Example

list the flights from baltimore to seattle that stop in minneapolis does this flight serve dinner the flight should arrive at eleven a.m. tomorrow what airline is this

Formal GI and learning theory	GI of Regular F	GI of Regular Patterns		Empirical GI and nonregular patte	
Results on ATIS					
	Micro	Macro	Macro ²		
Preci	sion 47.01	46.18	46.18		
Recal	44.94	50.98	50.98		
F-Sco	ore 44.60	47.10	48.46		
Explanation					
Miene Count					
Micro Count	constituents,	weighted	average per	sentence	
Macro Count	constituents a	and avera	ge per senter	nce	
Macro ² Compu	te Macro Pre	ecision/Re	ecall, average	at end	

Results on ATIS						
		remove sentence	remove empty	remove both		
Micro Precision	47.01	47.67	77.10	79.07		
Micro Recall	44.94	45.30	44.95	45.29		
Micro F-Score	44.60	45.09	55.31	56.13		
Macro Precision	46.18	47.66	77.08	81.18		
Macro Recall	50.98	52.96	51.07	52.80		
Macro F-Score	47.10	48.62	60.00	62.47		
Macro ² F-Score	48.46	50.17	61.43	63.99		
Example						
(bla bla bla)→bla bla bla						
bla () bla ́→bla bla						
$(bla () bla) \rightarrow bla bla$						
(5)) / 514 51	-			

Formal GI and learning theory

GI of Regular Patterns

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Evaluation insights

- No standard evaluation exists but de facto evaluation datasets arise
 - ATIS (van Zaanen and Adriaans 2001)
 - WSJ10, WSJ40 (WSJ with sentence length limitations)
 - NEGRA10 (German)
 - CTB10 (Chinese)
- Systems have different input/output
- Evaluation settings influence results
 - Different metrics (micro/macro²)
 - Included constituents (sentence/empty)
- Formal grammatical inference does not have this problem
 - Evaluation performed through formal proofs





- Is learning context-sensitive grammars impossible?
 - That depends
 - To what degree is the grammar context-sensitive?
- We may not need "full" context-sensitiveness
 - Grammar rules: $\alpha A\beta \rightarrow \alpha \gamma \beta$
- Mildly context-sensitive grammars may be enough for NL (Huybrechts 1984, Shieber 1985)
- Perhaps the full power of context-freeness is not needed



Learning context-sensitive languages

Open research area

Some work has already been done

- Augmented Regular Expressions (Alquézar 1997)
- Variants of substitutability (Yoshinaka 2009)
- Distributional Lattice Grammars (Clark 2010)

GI of Regular Patterns

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Relationship between empirical and formal GI

- Is there a relationship between empirical GI and formal GI?
- Example: consider the case of substitutability
- There are situations in which substitutability breaks:
 - John eats <u>meat</u>
 - John eats <u>much</u>
- This suggests that learning based on substitutability learns a different family of languages (not CFG)
- Non-terminally separated (NTS) languages
 - Subclass of deterministic context-free grammars







Formal GI and empirical GI

Relation between formal GI and empirical GI

- Formal GI can show learnability
 - Under certain conditions

Emprical GI tries to learn structure from real data

Practically shows possibilities and limitations

Ultimate aim: Find family of languages that is

- learnable under different conditions
- fits natural languages

Formal GI and learning theory

GI of Regular Patterns

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CONCLUSIONS

- There have been new strong positive results in a recent past for all the cases mentioned (subclasses of regular, PFA, transducers, CFGs, MCSGs)
- 2 Look for ICGI! It's the conference where these exciting results happen (as well as exciting challenges, competitions, benchmarks etc.)
- 3 The use of GI techniques both in computational linguistics and natural language processing is taking place.
- **4** The future is bright!

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