

# LogicAsker: Evaluating and Improving the Logical Reasoning Ability of Large Language Models

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## Abstract

We introduce LogicAsker, a novel approach for evaluating and enhancing the logical reasoning capabilities of large language models (LLMs) such as ChatGPT and GPT-4. Despite LLMs' prowess in tasks like writing assistance, code generation, and machine translation, assessing their ability to reason has been challenging. Traditional evaluations often prioritize accuracy on downstream tasks over direct assessments of reasoning processes. LogicAsker addresses this gap by employing a set of atomic reasoning skills grounded in propositional and predicate logic to systematically examine and improve the reasoning prowess of LLMs. Our methodology reveals significant gaps in LLMs' learning of logical rules, with identified reasoning failures ranging from 29% to 90% across different models. Moreover, we leverage these findings to construct targeted demonstration examples and fine-tune data, notably enhancing logical reasoning in models like GPT-4o by up to 5%. To our knowledge, this is the first effort to utilize test case outcomes to effectively refine LLMs' formal reasoning capabilities. We make our code, data, and results publicly available<sup>1</sup> to facilitate further research and replication of our findings.

## 1 Introduction

Large language models (LLMs), such as OpenAI's GPT series have significantly impacted natural language processing, excelling in a variety of tasks including text generation, machine translation, and code generation (Gao et al., 2022, 2023a; Jiao et al., 2023).

Reasoning, defined as the cognitive process of using logic to draw conclusions from given facts (Wei et al., 2022b,a), is crucial for complex interactions that go beyond text generation. Accurately assessing this ability in LLMs is essen-

tial, yet challenging, as models may correctly perform tasks merely relying on shortcuts such as pattern recognition without truly engaging in logical reasoning (Huang and Chang, 2022; Huang et al., 2023; Liu et al., 2023a). Consider the following inference example: Either it is raining, or Tom will play football; if it rains, then the floor will be wet; the floor is dry; therefore, Tom will play football. We may encounter the following challenges: 1) It's unclear if a correct LLM response is due to reasoning or simple heuristics like word correlations (e.g., "dry floor" is more likely to correlate with "playing football"). 2) If an LLM fails, pinpointing the specific breakdown in reasoning is difficult (i.e., inferring not raining from the floor being dry or inferring playing football from not raining). 3) Current systems lack comprehensive test cases that encompass various formal reasoning types beyond implication, such as logical equivalence (e.g., A and B are true; therefore, B and A are true. 4) Evaluating an LLM's reasoning on such cases offers limited insight into enhancing its reasoning capabilities.

To better handle these challenges, a well-performing testing framework should be able to define a set of skills that a) directly correspond to the reasoning process, b) cannot be further divided, c) cover all formal logical reasoning scenarios, and d) can identify LLMs' weaknesses and facilitate improving LLMs' performance. Property a) ensures that the task cannot be accomplished by other approaches, such as inferring from the correlations of words, and the evaluation result directly reflects the model's reasoning ability. Property b) and c) ensure that the set of skills is fundamental and comprehensive, which can provide helpful insights to accomplish Property d).

We introduce LogicAsker, an automatic framework designed to evaluate and enhance LLMs' formal reasoning skills using Minimum Functionality Tests (MFTs) (Ribeiro et al., 2020), akin to

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<sup>1</sup><https://github.com/yxwan123/LogicAsker>

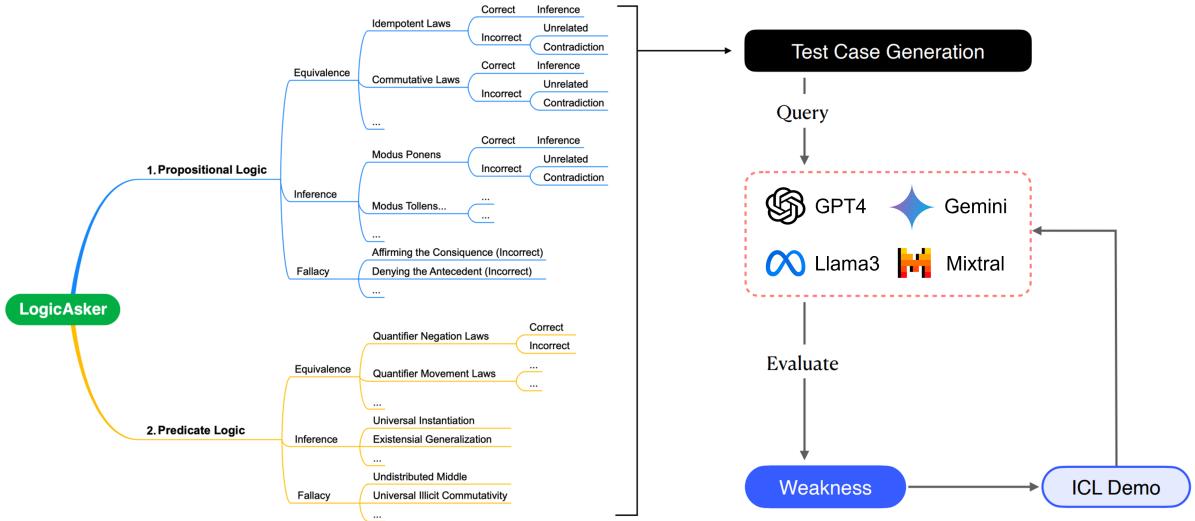


Figure 1: Overview of the LogicAsker framework.

software engineering’s unit tests, which utilize straightforward examples to assess specific behaviors. These tests help identify when models rely on shortcuts rather than genuinely mastering a skill (Ribeiro et al., 2020). Specifically, LogicAsker builds a set of atomic skills from foundational principles of propositional and predicate logic, two fundamental systems used to formalize reasoning procedures (Partee et al., 1990), together with common logical fallacies (Hurley and Watson, 2020). Based on the skill set, LogicAsker generates reasoning questions by translating standard logic expressions into natural language, assesses LLMs’ accuracy per skill, pinpoints weaknesses, and creates in-context-learning (Brown et al., 2020) examples and fine-tuning data to bolster reasoning abilities. In addition, for each skill, LogicAsker uses diverse vocabulary to frame various natural language queries, computing average performance to minimize biases from word correlations.

Table 1 demonstrates that LogicAsker complements existing frameworks by providing a comprehensive evaluation scope and utilizing outcomes to enhance LLMs’ reasoning capabilities, while other datasets often face data leakage and are scope-limited. LogicAsker serves as an extensive diagnostic tool for LLMs’ formal reasoning, significantly exceeding the coverage of comparable tools and enabling detailed assessments across diverse reasoning rules such as inferences, quantifiers, and fallacies. Scaling up the scope presents significant challenges due to the complexity of designing algorithms capable of processing various logical rules and translating them into natural language. Despite these complexities, LogicAsker uniquely integrates

all formal logical rules and common fallacies, facilitating robust testing and refinement of reasoning capabilities.

We evaluated LogicAsker’s performance through extensive testing on six state-of-the-art (SOTA) LLMs (Hugging Face, 2024), including four closed-source LLMs (GPT-4o, GPT-4, ChatGPT, and Gemini-1.5) and two open-source LLMs (Llama3 and Mixtral). Our findings reveal that LogicAsker’s test cases effectively pinpoint logical reasoning failures across these models, with error rates (i.e.,  $1 - \text{accuracy}$ ) between 29% and 90%. These test cases also facilitate the creation of in-context learning examples and fine-tuning data, thereby enhancing logical reasoning capabilities. For instance, applying LogicAsker’s cases to GPT-4o improved its reasoning accuracy from 92% to 97%. All resources are released for reproduction and further research<sup>2</sup>.

We summarize the main contributions of this work as follows:

- We are the first work that formally defines a comprehensive set of 34 atomic and 208 extended skills necessary for LLMs to execute formal reasoning based on propositional and predicate logic.
- We develop LogicAsker, a fully automatic tool that utilizes atomic skills to generate test cases to assess and enhance LLMs’ reasoning abilities, marking a first in utilizing test results to directly improve LLM performance.
- We conduct a thorough empirical evaluation of the logical reasoning abilities of six SOTA LLMs.

<sup>2</sup><https://github.com/yxwan123/LogicAsker>

We demonstrate that the test results by LogicAsker can be used to effectively evaluate and improve the performance of LLMs.

## 2 Preliminaries

### 2.1 Formal Analysis of Reasoning Abilities

“Reasoning” can be characterized into formal reasoning and informal reasoning. The former is a systematic and logical process that follows a set of rules and principles, and the reasoning within these systems will provide valid results as long as one follows the defined rules (e.g., all A are B, all B are C; therefore, all A are C). The latter is a less structured approach that relies on intuition, experience, and common sense to draw conclusions and solve problems (Huang and Chang, 2022; Bronkhorst et al., 2020) (e.g., Hong Kong residents have a high life expectancy; this is probably because they have healthy living habits). Generally, formal reasoning is more structured and reliable, whereas informal reasoning is more adaptable and open-ended but may be less reliable. In this paper, we focus on the formal reasoning process to systematically analyze LLMs’ reasoning abilities.

To formalize reasoning procedures, two fundamental systems are usually adopted, namely, propositional logic and predicate logic. The former one deals with propositions or statements that can be either true or false, and utilizes logical operators including  $\wedge$  (and),  $\vee$  (or),  $\neg$  (not),  $\rightarrow$  (inference), and  $\leftrightarrow$  (bidirectional) to connect these statements. The latter one, in contrast, extends propositional logic to deal with more complex statements that involve variables, quantifiers, and predicates. Both propositional logic and predicate logic contain various rules for the reasoning process. These rules can be categorized into equivalence rules and inference rules. Equivalent rules summarize the basic expressions that are equivalent in terms of truth value (e.g.,  $\neg(P \wedge Q) \Leftrightarrow (\neg P) \vee (\neg Q)$ ). Inference rules summarize the basic valid inference rules (e.g., from the premises:  $A \rightarrow B$ , and  $A$ , we can infer  $B$ ).

We refer to (Partee et al., 1990) for a more detailed explanation. Table 7-9 in Appendix A list common inference rules in predicate logic and propositional logic. Besides inference rules, formal logic systems can also express common logical fallacies, i.e., arguments that may sound convincing but are based on faulty logic and are, therefore, invalid. We list the common logical fallacies in Table 10.

### 2.2 Minimum Functionality Test

In this paper, we adopted the concept of Minimum Functionality Tests (MFTs), introduced in (Ribeiro et al., 2020), to evaluate the reasoning ability of LLMs. MFTs are analogous to unit tests in software engineering, where a collection of simple examples is used to check a specific behavior within a capability. These tests involve creating small and focused datasets that are particularly effective in detecting whether models resort to shortcuts to handle complex inputs, rather than truly mastering the capability.

To apply MFTs in evaluating the reasoning ability of LLMs, we treated each formal logical rule as an independent task and generated abundant test cases for each task. Each test case was designed to trigger logical failures in the LLMs, allowing us to assess the strengths and weaknesses of LLMs in the logical reasoning process, and providing a solid foundation for further analysis and improvement.

## 3 LogicAsker

In this section, we introduce the design and implementation of LogicAsker, a novel tool to trigger logical reasoning failures in large language models. Figure 1 overviews the workflow of LogicAsker, which consists of three main modules: test case generation, weakness identification and in-context learning (ICL) demonstration. In particular, the test case generation module utilizes atomic skills defined on the two formal logic systems and an inference synthesis approach to generate questions as test cases. Then, the generated cases are fed into the LLMs to reveal weaknesses and provide insights into the LLMs by the weakness identification process. Finally, LogicAsker utilizes these insights to construct ICL demonstrations to improve the reasoning abilities of the LLMs.

### 3.1 Reasoning Skills

**Atomic skills.** As described in Section 2.1, propositional and predicate logic are two fundamental systems that formalize the reasoning process. The inference rules and equivalence laws in these two systems are atomic and can cover all correct reasoning scenarios; therefore, we define these 34 rules as the set of atomic skills an LLM should possess to perform formal reasoning.

**Extended skills.** Predicate logic extends propositional logic to deal with more complex statements that involve variables, quantifiers, and predicates.

Table 1: Comparison with previous works.

|                                       | Fully Automatic | Atomic Skills | Formal Rules | Include Fallacies | Identify Weakness | Improve LLMs | LLMs* Tested | Example Testbed |
|---------------------------------------|-----------------|---------------|--------------|-------------------|-------------------|--------------|--------------|-----------------|
| CLUTRR (Sinha et al., 2019)           | ×               | ×             | ×            | ×                 | ✓                 | ×            | -            | BERT            |
| LogiQA (Liu et al., 2020)             | ×               | ×             | ×            | ×                 | ×                 | ×            | -            | BERT            |
| RECLOR (Yu et al., 2020)              | ×               | ×             | ×            | ×                 | ✓                 | ×            | 2            | GPT2            |
| Soft Reasoner (Clark et al., 2020)    | ✓               | ×             | 1            | ×                 | ✓                 | ×            | -            | RoBERTa         |
| LogicNLI (Tian et al., 2021)          | ×               | ×             | 7            | ×                 | ✓                 | ×            | -            | BERT            |
| FOLIO (Han et al., 2022)              | ×               | ×             | ×            | ×                 | ×                 | ×            | 4            | GPT3            |
| LogicInference (Ontañón et al., 2022) | ✓               | ×             | 19           | ×                 | ×                 | ×            | -            | T5              |
| ProntoQA-OOD (Saparov et al., 2023)   | ✓               | ×             | 6            | ×                 | ✓                 | ×            | 4            | GPT3.5          |
| <b>LogicAsker (Ours)</b>              | ✓               | ✓             | 34           | ✓                 | ✓                 | ✓            | 6            | GPT4            |

\* We consider language models with more than 1 billion parameters as LLMs.

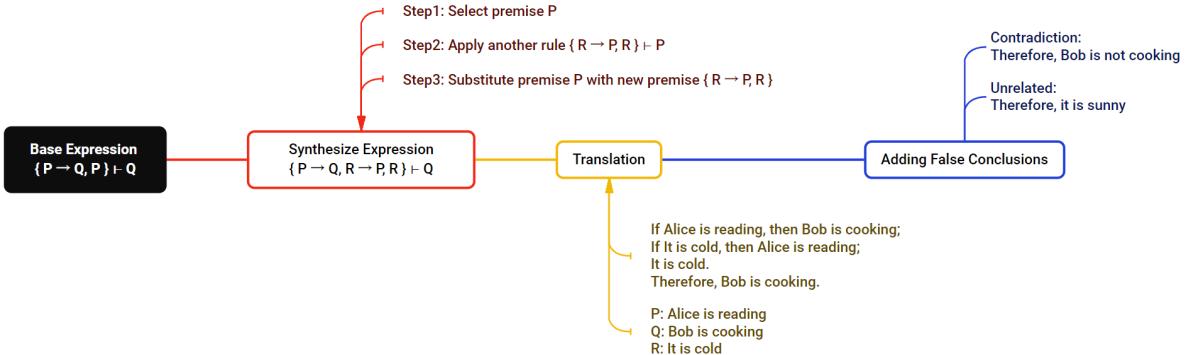


Figure 2: Test case generation procedure.

In this regard, besides the unique equivalence and inference laws in predicate logic, we add quantifiers and variables to every rule in propositional logic to form the predicate version of the laws. Using this approach, we expand the set of 34 atomic skills into a set of 208 extended skills. In Appendix B, we provide some concrete examples of these extended rules.

### 3.2 Test Case Generation

To generate logical questions, LogicAsker first adopts a rule-based method to generate logical expressions systematically based on reasoning skills and then translates the logical expressions into natural language. Figure 2 provides an overview of the procedure.

**Logic expression generation.** To better control the process of logic expression generation, we first define the length of an inference problem by the number of syllogisms it involves. We use the inference rules described in Section 2.1 to generate inference expressions with length one. When a longer inference ( $> 1$ ) is specified, we start with a base expression  $E_0 := P_1 \wedge P_2 \rightarrow C_1$  with length one and expand the inference chain. Specifically, we substitute the premises (either or both) of the first inference with the conclusion of some other

syllogism and append the premises of those syllogisms into the list of all premises. For example, we can find another syllogism  $E_1 := P_3 \wedge P_4 \rightarrow P_2$  with  $P_2$  as the conclusion and then obtain a new expression  $E_{new} := P_1 \wedge P_3 \wedge P_4 \rightarrow C_1$  with the inference length of two. We can obtain inference expressions of any length by recursively expanding the inference chain as above. During the generation process, one can specify the desired rules and length to allow complete control over expected test cases.

In addition to the correct inference expression created above, we generate three kinds of false inference expressions: contradiction, unrelated, and fallacy. A contradiction is generated by negating the conclusion of a correct inference expression and an unrelated is generated by replacing the conclusion of a valid inference expression with an irrelevant statement. For example, for  $E_0 := P_1 \wedge P_2 \rightarrow C_1$ , a contradiction is  $E_c := P_1 \wedge P_2 \rightarrow \neg C_1$ , an unrelated can be  $E_u := P_1 \wedge P_2 \rightarrow U_1$ . We create a fallacy by directly using the fallacy rules listed in Section 2.1 for an inference length of one. For a fallacy with a more extended length, we select a fallacy rule as the base expression and expand the inference chain using correct rules, ensuring the expression's incorrectness.

**Natural language translation.** Partially inspired by (Ontañón et al., 2022), translating a clause into natural language involves a series of patterns that depend on the structure of the clause. Simple propositions are transformed into one of the template patterns, such as “subject verb-action”, “subject predicate”, or “impersonal-action” with a predefined set of subjects, verbs, predicates, and impersonal actions that can be chosen randomly without repetition. For predicate clauses that involve constants or variables, we employ templates “subject verb-action”, “subject predicate” to translate them. Furthermore, each clause can be rendered in various modes, such as the present, past, or negated forms. Additionally, connectives like “or,” “and,” “implies,” and “if and only if” also adhere to their designated patterns. For quantified clauses, we adopt patterns like “for all  $x$ ,  $X$ ”, “there is at least one  $x$  for which  $X$ ”, and “some  $X$ s are  $Y$ .”. To facilitate the generation process, we curate extensive lists of potential subjects, including common names in English, and compile plausible predicates, actions, and impersonal actions. Compared to Ontanon et al. (Ontañón et al., 2022), our method provides a more natural and less ambiguous translation. We provide a detailed illustration of the translation process in Appendix C.

### 3.3 Weakness Identification

To measure the reasoning abilities of the LLMs, we calculate the accuracy of LLMs’ answers  $\text{Acc} = \frac{N_{\text{correct}}}{N_{\text{total}}}$ . Where  $N_{\text{total}}$  denotes the total number of responses, and  $N_{\text{correct}}$  denotes the number of responses that are correct. In particular, since all generated queries are formulated as yes-or-no questions, LogicAsker adopts an automatic approach that searches for pre-defined keywords (e.g., “yes” and “no”) in sentences to identify correct answers.

To reveal the weaknesses of LLMs, we generate  $n$  test cases for each leaf node in the rule tree depicted in Figure 1. Then, we calculated the response accuracy of an LLM of each leaf node. Based on the result, we can identify the weaknesses of LLMs by listing the leaf nodes that receive the lowest accuracy. In addition, by grouping the accuracy by different attributes in the rule tree, we can gain insights into the strengths and weaknesses of LLMs on these attributes (e.g., performance on predicate logic vs. propositional logic).

Table 2: Conversational LLMs used in the evaluation.

| Name                              | Rank |
|-----------------------------------|------|
| GPT-4o (OpenAI, 2024c)            | 1    |
| GPT-4 (OpenAI, 2024b)             | 4    |
| ChatGPT (OpenAI, 2024a)           | 37   |
| Gemini-1.5 (Google, 2024)         | 10   |
| Llama3-70b (Meta Platforms, 2024) | 12   |
| Mixtral-8x7b (Mistral AI, 2024)   | 42   |

### 3.4 Improving LLMs

**In-context learning (ICL)** is a paradigm that enables LLMs to learn tasks with examples in the form of demonstrations (Brown et al., 2020). It leverages task instructions and a few demonstration examples to convey the task semantics, which are then combined with query questions to create inputs for the language model to make predictions. ICL has demonstrated impressive performance in various natural language processing and code intelligence. However, the performance of ICL is known to rely on high-quality demonstrations (Gao et al., 2023b) strongly. To fully unleash the potential of ICL, LogicAsker utilizes the weak skills of each LLM to construct both correct and incorrect examples with expected answers and explanations as demonstrations to facilitate the reasoning of LLMs. The generation process follows a similar approach to the test case generation described in § 3.2, with the difference being that we append a brief explanation and the correct answer at the end of each case. We show an instance of the demonstration example in Appendix D.

**Fine-tuning** is another widely used technique to enhance model performance on specific tasks (Moslem et al., 2023; Wei et al., 2021). This process involves taking a pre-trained model and further training it on a smaller, task-specific dataset. The rationale behind fine-tuning is to leverage the learned features and knowledge of the pre-trained model, adapting it to particular nuances and characteristics of a targeted domain or task. In this paper, we directly utilize the data generated by LogicAsker to fine-tune LLMs to improve their reasoning ability.

## 4 Experiments

### 4.1 Experimental Setup

We apply LogicAsker to test six state-of-the-art (SOTA) LLMs, including four close-source and

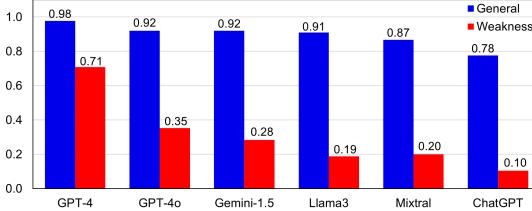


Figure 3: Overall accuracy.

two open-source models. Table 2 lists brief information on these systems. All of them are ranked within the top 50 in the LMSYS Chatbot Arena Leaderboard<sup>3</sup> according to the assessment results in June 2024. We leave details of how we access the model, the parameters used, and the prompt we used in Appendix E.

We conduct two iterations of experiments for a comprehensive assessment. In the first iteration, we follow the setting in § 3.3 and set  $n = 25$ , resulting in 5,200 cases. Statistics of the sampled data are described in Appendix F. The second iteration is based on the first one, which focuses on the identified weaknesses of each LLM, i.e., the ten leaf nodes in Figure 1 with the lowest accuracy. We generated 25 additional test cases for each weakness. These 250 test cases comprise our “weakness dataset,” which will be utilized for further evaluation in § 4.5.

## 4.2 Effectiveness of LogicAsker

We demonstrate the effectiveness of LogicAsker through the overall performance of LLMs on the test cases. The overall performance of LLMs in the first and second iteration is shown in Figure 3. The result reveals that our framework can effectively expose logical failures in the first iteration, with LLM’s accuracy ranging from 78%-98%. When focusing on the weak skills of LLMs in the second iteration, we further reduce the accuracy to 10%-71% for the LLMs. What’s surprising is that most of these LLMs show accuracy even lower than random guesses (i.e., 50% here) when confronted with logical questions involving specific logical rules. This contradicts their remarkable performance in various LLM benchmarks, for example, achieving top 50 ranks on the LLM Arena Leaderboard. It suggests that existing benchmark datasets are not comprehensive enough to assess the generalization ability of LLMs in reasoning.

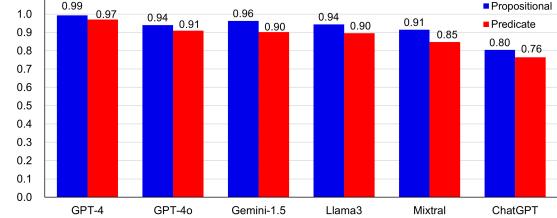


Figure 4: Propositional and predicate logic accuracy.

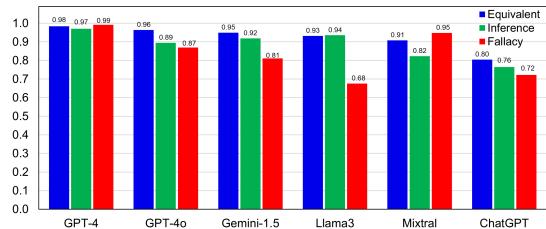


Figure 5: Accuracy of different rule categories.

## 4.3 Insights into Reasoning Abilities

We conducted a comprehensive analysis to gain insights from the failures exposed by LogicAsker, obtaining three key observations from the evaluation:

### Most LLMs are better at easier logical skills.

We compared the performance of LLMs on propositional logic and predicate logic, the former of which is simpler in form while the latter involves more complex quantifier manipulations. Figure 4 illustrates the difference between the accuracy obtained for the two logic systems. Notably, we observed that most LLMs are better at propositional logic, implying their limited ability in complex reasoning scenarios.

**Most LLMs are weak in recognizing logical fallacies.** Figure 5 presents the accuracy of LLMs under different skill categories. Interestingly, we discovered that among three types of skills, recognizing fallacies has the lowest accuracy for most LLMs, with GPT-4 and Mixtral-8x7B being the exceptions. It suggests that current LLMs are over-confident even in fallacies, which may be learned from the mistakes in pretraining data.

**Case study: GPT-4 did not learn all logic rules well.** To provide a direct impression of what skills LLMs cannot perform well, we list three atomic rules in which GPT-4 has the lowest accuracy in Table 3. While GPT-4 has an average accuracy of 98% over all skills, it only achieves 60% - 68% accuracy on these skills, indicating that it cannot perform these atomic skills smoothly.

We also discovered that longer inference chains

<sup>3</sup><https://huggingface.co/spaces/lmsys/chatbot-arena-leaderboard>

Table 3: Weakness of GPT-4

| Rule                       | Type      | Example  | Accuracy |
|----------------------------|-----------|--|----------|
| Existential resolution     | Incorrect | For all $v$ , $v$ will not play squash or $v$ will go running. There is at least one $v$ for which $v$ will play squash or $v$ will play tennis. Therefore, there is at least one $v$ for which $v$ will go running. | 0.60     |
| Universal resolution       | Correct   | For all $x$ , $x$ will climb a mountain or $x$ is a police officer. For all $x$ , $x$ will not climb a mountain or $x$ is rich. Therefore, for all $x$ , $x$ is a police officer or $x$ is rich.                     | 0.60     |
| Law of quantifier movement | Correct   | For all $x$ , if Joseph sleeps, then $x$ is a janitor. Therefore, if Joseph sleeps, then for all $x$ , $x$ is a janitor.   | 0.68     |

Table 4: Human evaluation results on the quality of test cases.

| Invalid Cases | a     | b     | c     | Total |
|---------------|-------|-------|-------|-------|
| Count         | 10    | 8     | 0     | 18    |
| Percentage    | 1.92% | 1.54% | 0.00% | 3.46% |

are more challenging for LLMs, the details are provided in Appendix G. These insights provide a valuable understanding of the strengths and weaknesses of each LLM when handling logical questions, allowing us to uncover specific areas that require improvement and potential avenues for enhancing overall performance. We provide a full breakdown list of the LLMs' performance on various skills in Appendix H.

#### 4.4 The Quality of Test Cases

Since the test cases are automatically generated, we conduct a human evaluation to measure the quality of the generated test cases by LogicAsker. To achieve this, we randomly sampled 10% (520) of the test cases generated during the first iteration of the experiment in 4.2 and conduct manual inspection. Two annotators with bachelor's degrees were recruited to answer the questions manually. Each test case was annotated as either valid or invalid based on the following three questions: **a)** Is the question grammatically correct? **b)** Is the question understandable and has only one interpretation? **c)** Can the target answer be derived from the question? A test case is considered valid only when both annotator's answer to the above questions are positive. The results of the annotation are presented in Table 4. This result is statistically sufficient to prove that the probability of LogicAsker generating understandable and solvable logical questions is larger than or equal to 0.94 (with p-value 0.05), indicating that **the test cases generated by LogicAsker are highly reliable and valid**.

#### 4.5 LogicAsker to Improve Reasoning

In this section, we explore the potential of LogicAsker in further improving the reasoning ability of LLMs through in-context learning (ICL) and fine-tuning.

We first employ LogicAsker to generate ICL demonstrations tailored to address the weaknesses dataset uncovered in the experiments in § 4.2. For each inference problem, we generated ICL demonstrations that provide both the expected answer and an explanation as described in § 3. We evaluate the effectiveness of the ICL demonstrations generated by LogicAsker by comparing the following prompting strategies: a) Zero-Shot: We provide only task instructions without any ICL demonstrations. b) Zero-Shot Chain-of-Thought (CoT): We use the instruction "Please think step-by-step" (Kojima et al., 2022) to elicit the zero-shot reasoning ability of the LLMs. c) Random ICL Demonstrations: In addition to the task instruction, we also include four ICL demonstrations selected randomly from the available rules with balanced answer labels, i.e., two correct and two incorrect. d) Weakness ICL Demonstration: Instead of random demonstrations, we include four ICL demonstrations using the weakness rules identified in § 4.2 with balanced answer labels.

We perform testing with the 5.2k sampled data on all models and list the result in Table 5. In general, the weakness ICL demonstrations are more effective than those random ICL demonstrations, and both ICL methods bring more performance gain than CoT, **indicating that the test cases generated by LogicAsker can improve reasoning**.

To further demonstrate the effectiveness of LogicAsker, we fine-tune ChatGPT on 5.2k separately generated data on all skills and 2.8k separately generated data on weaknesses of ChatGPT, respectively. We test the two fine-tuned model on both LogicAsker and another dataset, LogiQA, a challenging dataset for machine reading comprehension

Table 5: Performance of ICL demonstrations by LogicAsker (%).

| Model   | Zero-Shot | CoT   | ICL   | ICL(Weak)    |
|---------|-----------|-------|-------|--------------|
| GPT-4   | 97.75     | 96.60 | 97.98 | 99.48        |
| GPT-4o  | 91.92     | 92.94 | 95.77 | 97.23        |
| Gemini  | 92.06     | 93.62 | 96.13 | 96.67        |
| Llama-3 | 91.02     | 94.54 | 94.83 | 93.35        |
| Mixtral | 86.77     | 86.23 | 76.40 | 82.02        |
| ChatGPT | 77.62     | 78.19 | 82.90 | 81.04        |
| Average | 89.52     | 90.35 | 90.67 | <b>91.63</b> |

with logical reasoning (Liu et al., 2020). We use the "test" split of LogiQA which contains 651 test data. The results are presented in Table 6. We can observe that models fine-tuned on LogicAsker can effectively enhance the models' reasoning ability on both datasets, suggesting the generalizability of LogicAsker. **These findings demonstrate the effectiveness of LogicAsker in improving the reasoning ability of LLMs.**

Table 6: ChatGPT performance on LogiQA and LogicAsker after fine-tuning (%).

|            | Vanilla | FT (All)     | FT (Weak)    |
|------------|---------|--------------|--------------|
| LogicAsker | 77.62   | <b>99.50</b> | 97.83        |
| LogiQA     | 40.55   | 41.01        | <b>41.78</b> |

## 5 Discussion

A surprising result from our experiments is the lower performance of CoT prompting in some LLMs, such as GPT-4, despite the widespread belief in CoT's ability to enhance logical reasoning.

One possible explanation lies in the fundamental difference between how CoT operates and the structure of LogicAsker's test cases. CoT functions by breaking down complex reasoning tasks into intermediate, manageable steps, guiding the model toward a final conclusion through a series of natural language reasoning steps. However, LogicAsker tests focus on assessing LLMs' mastery of the most basic, indivisible reasoning skills. These fundamental reasoning skills are the building blocks for more complex reasoning processes like CoT. Since these basic skills cannot be further subdivided into intermediate steps, the benefits of CoT are limited when applied to LogicAsker's simpler test cases. In contrast, improving an LLM's understanding of these core skills can enhance its performance on more complex tasks where CoT is applicable, as demonstrated by ChatGPT's improved results on the LogiQA dataset (see Table 6).

Further analysis of failure cases reveals another intriguing observation. In certain CoT-prompted scenarios, GPT-4 tends to rely on common sense rather than logical deduction. Given that LogicAsker's data is synthetic and not aligned with common-sense reasoning, this tendency leads to performance degradation. This is illustrated by the following example, where CoT prompts GPT-4 to incorrectly infer weather conditions based on premises, while a zero-shot approach provides a more accurate deduction by sticking to strict logical interpretation:

**Q:** Consider the following premises: If Linda were sad, then it is sunny, and if it is sunny, then Linda is sad. Can we infer the following from them? Answer yes or no: It is not overcast.

**Ans (Zero-shot):** From these premises, we can infer that *Linda is sad* and *it is sunny* are equivalent; ... This means that Linda is sad if and only if it is sunny. However, without additional information about whether Linda is actually sad or not, we cannot definitively conclude the state of the weather (overcast or not). Therefore, based on the information given and without knowing whether Linda is sad, the answer is no.

**Ans (CoT):** ... The statement "*It is sunny*" directly contradicts the possibility of it being overcast at the same time, assuming a typical understanding of weather where sunny conditions and overcast conditions are mutually exclusive. ... Therefore, if either is true, it must be sunny, and thus it cannot be overcast. ... Since the premises lead us to conclude that it is sunny, we can infer that it is not overcast. Final answer: yes.

This observation underscores the effectiveness of LogicAsker in revealing situations where models default to heuristics or memorized knowledge rather than true logical reasoning. While CoT has been shown to improve logical reasoning in general tasks, LogicAsker's framework exposes when models fail to genuinely reason and instead fall back on familiar or remembered patterns. This insight suggests that strengthening LLMs' mastery of basic reasoning skills is a necessary foundation for improving performance on tasks that benefit from CoT strategies.

## 6 Related Work

Significant advancements in NLP reasoning have been achieved through methods such as Chain-of-Thoughts (CoT) prompting (Wei et al., 2022b), which enables models to generate reasoning steps with minimal training. Enhancing this, the Program of Thoughts (PoT) prompting (Chen et al., 2022) leverages external interpreters like Python to

handle complex mathematical problems. Further augmenting reasoning validity, the Logic Agent framework transforms LLMs into logic agents that can dynamically apply propositional logic rules to convert natural language inputs into structured logic forms (Liu et al., 2024).

Recent studies have focused on evaluating the reasoning capabilities of Large Language Models (LLMs) by measuring their performance across various reasoning tasks. These include arithmetic (Cobbe et al., 2021; Hendrycks et al., 2021; Amini et al., 2019; Patel et al., 2021; Miao et al., 2020; Ling et al., 2017; Roy and Roth, 2016), common-sense (Talmor et al., 2019; Geva et al., 2021; Clark et al., 2018), symbolic (Wei et al., 2022b), and table reasoning (Nan et al., 2021), as well as understanding words, dates, and causal relationships (Srivastava et al., 2022), and generalization (Lake and Baroni, 2017; Anil et al., 2022). Despite these efforts, it remains uncertain whether LLMs truly reason or rely on simple heuristics, since most assessments focus only on accuracy and do not thoroughly evaluate the reasoning processes.

Efforts to develop metrics for more formal reasoning analysis in LLMs include creating datasets with first-order logic problems (Han et al., 2022), generating test cases using a single predicate inference rule (Saparov and He, 2022), building instruction-tuning dataset designed for CoT reasoning with GPT-4 (Liu et al., 2023b), and employing propositional logic with randomized methods (Ontañón et al., 2022). These methods, however, often lack generalizability or focus on limited deduction rules. Saparov et al. (Saparov et al., 2023) introduced a comprehensive approach by using all deduction rules in propositional logic to assess LLMs' deductive reasoning across complex proofs. Our research expands further, incorporating all rules and equivalent laws in both propositional and predicate logic, aiming to enhance understanding of each rule's impact on LLM performance and using these insights for improvement.

## 7 Conclusion

In this paper, we present LogicAsker, an automated tool designed to comprehensively evaluate and improve the formal reasoning abilities of LLMs under a set of atomic skills.

Our research demonstrated the efficacy of LogicAsker in identifying logical reasoning failures in a diverse set of widely deployed LLMs, we achieved

a substantial error detection rate in revealing reasoning flaws in these models, ranging from 29% to 90%. Additionally, we utilized the test cases from LogicAsker to design in-context learning demonstrations, which effectively enhance the logical reasoning capabilities of LLMs, e.g., improving from 92% to 97% for GPT-4o.

By providing insights into the strengths and weaknesses of LLMs in reasoning, we are able to improve the reliability and trustworthiness of these models. The release of all the code and data aims to facilitate replication and encourage further research in this crucial area.

## Limitations

This paper identifies two primary limitations that highlight areas for future research:

- Although our ICL (In-Context Learning) method significantly enhances the logical reasoning capabilities of large language models (LLMs), there remains a performance gap compared to human-level reasoning. Further refinements and innovations in model training and architecture may be necessary to bridge this gap.
- Our method is currently applicable only to LLMs that possess robust in-context learning capabilities. LLMs lacking this feature may not benefit from our approach. Future studies could explore fine-tuning methods to extend the applicability of our improvements across a broader spectrum of LLMs, potentially enhancing models with weaker or no inherent in-context learning abilities.

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Table 7: Propositional logic equivalence laws.

| Law                | Logical Equivalence  | Example  |
|--------------------|--|--|
| Idempotent laws    | $P \wedge P \Leftrightarrow P$<br>$P \vee P \Leftrightarrow P$   | I am a teacher and I am a teacher $\Leftrightarrow$ I am a teacher.<br>It's raining or it's raining $\Leftrightarrow$ it's raining.  |
| Commutative laws   | $P \wedge Q \Leftrightarrow Q \wedge P$<br>$P \vee Q \Leftrightarrow Q \vee P$   | It is cold and it is winter $\Leftrightarrow$ It is winter and it is cold.<br>You can go to the party or you can study $\Leftrightarrow$ You can study or you can go to the party.   |
| Associative laws   | $(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$<br>$(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$                     | It is raining and it is cold, and also it is winter $\Leftrightarrow$ It is raining, and also, it is cold and it is winter.<br>Either I will go to the park or I will go to the library is true, or I will go to the cinema $\Leftrightarrow$ I will go to the park or either I will go to the library or I will go to the cinema is true.                       |
| Distributive laws  | $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$<br>$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$ | It is raining and either I have an umbrella or I have a raincoat $\Leftrightarrow$ It is raining and I have an umbrella, or it is raining and I have a raincoat.<br>Either I will go to the park, or it is cloudy and it is cold $\Leftrightarrow$ Either I will go to the park or it is cloudy is true, and either I will go to the park or it is cold is true. |
| DeMorgan's laws    | $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$<br>$\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$                             | It is not true that it's both cold and raining $\Leftrightarrow$ It's not cold or it's not raining.<br>It's not true that I will study or play $\Leftrightarrow$ I won't study and I won't play.   |
| Complement laws    | $\neg(\neg P) \Leftrightarrow P$   | It is not the case that it is not raining $\Leftrightarrow$ It is raining.   |
| Conditional laws   | $P \rightarrow Q \Leftrightarrow \neg P \vee Q$  | If it rains, then I'll stay at home $\Leftrightarrow$ It doesn't rain or I stay at home.   |
| Bidirectional laws | $(P \leftrightarrow Q) \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$   | I'll go to the park if and only if it's sunny $\Leftrightarrow$ Either it's sunny and I go to the park, or it's not sunny and I don't go to the park.  |

## A Logical Rules and Fallacies

We list all the logic equivalence rules in Table 7-8, logic inference rules in Table 9, and common logical fallacies in Table 10.

## B Extended Rules

### B.1 Equivalent Extension

The equivalent rule extension is based on the following fact:

$$\{A \Leftrightarrow B, \forall x(A)\} \vdash \{\forall x(B)\}$$

(i.e., if A and B are equivalent, and for all x, A is true, then for all x, B is also true), and

$$\{A \Leftrightarrow B, \exists x(A)\} \vdash \{\exists x(B)\}$$

(i.e., if A and B are equivalent, and there exist x such that A is true, then there exist x such that B is true). For example, the predicate version of the DeMorgan's law

$$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$$

will become

$$\forall x(\neg(P(x) \wedge Q(x))) \Leftrightarrow \forall x(\neg P(x) \vee \neg Q(x)),$$

and

$$\exists x(\neg(P(x) \wedge Q(x))) \Leftrightarrow \exists x(\neg P(x) \vee \neg Q(x)).$$

In this example, the goal is to extend the propositional equivalence law to its predicate version by adding quantifiers. To achieve this goal, we first note that DeMorgan's law states that "P and Q cannot both be true" (e.g., Alice is happy and Bob is happy cannot both be true) is equivalent to "either not P or not Q" (e.g., either Alice is not happy or Bob is not happy). Since the two expressions are equivalent, we can add the same quantifier to both sides and the equivalence will still hold. Therefore, by adding a "for all" quantifier to both sides, we obtain "for all x, P(x) and Q(x) cannot both be true" (for all persons in the room, the person likes Charley and the person likes David cannot both be true) is equivalent to "for all x, either not P(x) or not Q(x)" (e.g., for all person in the room, either the person doesn't like Charley or the person doesn't like David). Before the extension, the law can only be applied to simple propositions (e.g., P = "Alice is happy", Q = "Bob is happy"), but after extension, the law can be applied to predicates with variables and quantifiers (e.g., P(x) = "x likes Charley", Q(x) = "x likes David"). The same also applies to the

Table 8: Predicate logic quantifier laws.

| Law                     | Logical Equivalence  | Example  |
|-------------------------|--|--|
| Quantifier Negation     | $\neg \forall x P(x) \Leftrightarrow \exists x \neg P(x)$                          | It is not the case that all birds can fly $\Leftrightarrow$ There exists a bird that cannot fly.   |
|                         | $\neg \exists x P(x) \Leftrightarrow \forall x \neg P(x)$                          | There is no human that can live forever $\Leftrightarrow$ All humans cannot live forever.  |
| Quantifier Distribution | $\forall x(P(x) \wedge Q(x)) \Leftrightarrow \forall x P(x) \wedge \forall x Q(x)$ | Every student is smart and diligent $\Leftrightarrow$ Every student is smart, and every student is diligent.   |
|                         | $\exists x(P(x) \vee Q(x)) \Leftrightarrow \exists x P(x) \vee \exists x Q(x)$     | There is a person who is either a doctor or a lawyer $\Leftrightarrow$ There is a person who is a doctor, or there is a person who is a lawyer.  |
| Quantifier Movement     | $\forall x(P \rightarrow Q(x)) \Leftrightarrow (P \rightarrow \forall x Q(x))$     | For every child, if it is raining then they are inside $\Leftrightarrow$ If it is raining, then every child is inside when the notion of raining doesn't depend on the specific child.   |
|                         | $\exists x(P \wedge Q(x)) \Leftrightarrow (P \wedge \exists x Q(x))$               | There exists a student who is tall and a good basketball player $\Leftrightarrow$ There is a tall student and there exists a student who is a good basketball player when the notion of being tall doesn't depend on the specific student. |

"exist" quantifier.

## B.2 Inference Extension

The inference rule extension is based on the following fact:

$$\{A \wedge B \rightarrow C\} \vdash \{\forall x, (A) \wedge \forall x, (B) \rightarrow \forall x, (C)\},$$

(i.e., if A and B imply C, then for all x, A is true and for all x, B is true implies for all x, C is true)

$$\{A \wedge B \rightarrow C\} \vdash \{\exists x, (A) \wedge \forall x, (B) \rightarrow \exists x, (C)\}.$$

(i.e., if A and B imply C, there exists x such that A is true and for all x, B is true implies there exists x such that C is true). Since all propositional inference rules are of the form  $P \wedge Q \rightarrow C$ , we can transform them into their predicate form  $\forall x, P(x) \wedge \forall x, Q(x) \rightarrow \forall x, C(x)$  and  $\exists x, P(x) \wedge \forall x, Q(x) \rightarrow \exists x, C(x)$  following similar procedure in the previous section.

## C Natural Language Translation

### C.1 Algorithm

Given an input: a logic clause of the form [operator, Clause<sub>A</sub>, Clause<sub>B</sub>], where the clauses are also of the form [operator, Clause<sub>A</sub>, Clause<sub>B</sub>], the algorithm will do the following:

1. **Single Proposition Clause:** If the clause is just a single proposition, the algorithm finds this proposition's natural language form and returns it. The natural language form is obtained by combining vocabularies according to certain templates (e.g., subject + action).

2. **Negation:** If the clause starts with a “ $\neg$ ” operator, the algorithm then translates the rest of the clause based on a negation template, making sure to negate the statement.

3. **Quantifiers:** For clauses that start with “ $\forall$ ” (meaning for all items) or “ $\exists$ ” (meaning there is at least one item), it translates these into natural language, adjusting the phrasing based on whether we're asserting something positively or negating it.

4. **Logical Connectives:** If the clause combines propositions using logical operators like “ $\wedge$ ”, “ $\vee$ ”, “ $\rightarrow$ ” (implies), or “ $\leftrightarrow$ ” (if and only if), the function translates these into natural language phrases that express the relationship between the propositions.

### C.2 Example

Consider the expression:  $[\forall x, \rightarrow, A(x), B(x)]$ . Here's how the function would translate it:

1. It sees the “ $\forall x$ ” quantifier and adds “For all x,” to the sentence and continues to process the clause  $[\rightarrow, A(x), B(x)]$ .
2. It sees the “ $\rightarrow$ ” operator, which means “if...then...”. It connects the two operands with the operator and obtains “For all x, if A(x), then B(x)”. Then, it continues to process the clauses A(x), B(x).
3. Since A(x), B(x) are single proposition clauses, the function looks up the vocabulary and synthesizes the natural language versions

Table 9: Propositional and predicate logic inference rules.

| Inference Rule             | Logical Form  | Example  |
|----------------------------|---|--|
| Universal Instantiation    | $\forall x P(x) \vdash P(c)$                                      | All humans are mortal. Hence, Socrates is mortal.  |
| Existential Generalization | $P(c) \vdash \exists x P(x)$                                      | This apple is red. Hence, there exists a red apple.  |
| Universal Generalization   | $\{P(x)\} \vdash \forall y P(y)$                                  | Any particular human is mortal. Hence, all humans are mortal.  |
| Modus Ponens               | $\{P \rightarrow Q, P\} \vdash Q$                                 | If it rains, the street gets wet. It is raining. Hence, the street is wet.   |
| Modus Tollens              | $\{P \rightarrow Q, \neg Q\} \vdash \neg P$                       | If it rains, the street gets wet. The street is not wet. Hence, it is not raining.                                     |
|                            | $\{P \rightarrow \neg Q, Q\} \vdash \neg P$                       | If it rains, the street does not get wet. The street is wet. Hence, it is not raining.                                 |
| Transitivity               | $\{P \rightarrow Q, Q \rightarrow R\} \vdash P \rightarrow R$     | If I study, I will pass the test. If I pass the test, I will get a reward. Hence, if I study, I will get a reward.     |
| Disjunctive Syllogism      | $\{P \vee Q, \neg P\} \vdash Q$                                   | Either it's raining or it's snowing. It's not raining. Hence, it's snowing.  |
| Addition                   | $\{P\} \vdash P \vee Q$   | It is raining. Hence, it is raining or snowing.  |
| Simplification             | $\{P \wedge Q\} \vdash P$   | It is raining and it is cold. Hence, it is raining.  |
|                            | $\{P \wedge Q\} \vdash Q$   | It is raining and it is cold. Hence, it is cold.   |
| Conjunction                | $\{P, Q\} \vdash P \wedge Q$                                      | It is raining. It is cold. Hence, it is raining and it is cold.  |
| Resolution                 | $\{P \vee Q, \neg P \vee R\} \vdash Q \vee R$                     | Either it is raining or snowing. If it is not raining, then it is cloudy. Hence, either it is snowing or it is cloudy. |
| Disjunction Elimination    | $\{P \rightarrow R, Q \rightarrow R, P \vee Q\} \vdash R$         | If it rains, I will stay home. If it snows, I will stay home. Either it will rain or snow. Hence, I will stay home.    |
| Biconditional Introduction | $\{P \rightarrow Q, Q \rightarrow P\} \vdash P \leftrightarrow Q$ | If I study, I pass. If I pass, I studied. Hence, I study if and only if I pass.  |
| Biconditional Elimination  | $\{P \leftrightarrow Q\} \vdash P \rightarrow Q$                  | I study if and only if I pass. Hence, if I study, I pass.  |
|                            | $\{P \leftrightarrow Q, \neg P\} \vdash \neg Q$                   | I study if and only if I pass. I didn't study. Hence, I didn't pass.   |
|                            | $\{P \leftrightarrow Q, \neg Q\} \vdash \neg P$                   | I study if and only if I pass. I didn't pass. Hence, I didn't study.   |

of the proposition. For example,  $A(x) = "x$  drinks water",  $B(x) = "x$  is a cashier".

4. It constructs the sentence: "For all  $x$ , if  $x$  drinks water, then  $x$  is a cashier".

### C.3 Vocabulary

We list the vocabulary used in our experiment:

#### Subjects

- x, y, z, James, Mary, Robert, Patricia, John, Jennifer, Michael, Linda, William, Elisabeth, David, Barbara, Richard, Susan, Joseph, Jessica, Thomas, Sarah, Charles, Karen, Alice, Benjamin, Daniel, Emily, George, Helen, Ian, Julie.

#### Predicates

- a cashier, a janitor, a bartender, a server, an office clerk, a mechanic, a carpenter, an electrician, a nurse, a doctor, a police officer, a taxi driver, a soldier, a politician, a lawyer, a scientist, an astronaut, a poet, an artist, a sailor, a writer, a musician, poor, rich, happy, sad, fast, curious, excited, bored, tired, joyful, intelligent, skilled, efficient, meticulous, creative.

#### Actions

- make tea, makes tea, making tea, drink water, drinks water, drinking water, read a book, reads a book, reading a book, play tennis, plays tennis, playing tennis, play squash, plays squash, playing squash, play a game, plays a game, playing a game, go running, goes running, running, work, works, working, sleep, sleeps, sleeping, cook, cooks, cooking, listen to a song, listens to a song, listening to a song, write a letter, writes a letter, writing a letter, drive a car, drives a car, driving a car, climb a mountain, climbs a mountain, climbing a mountain, take a plane, takes a plane, taking a plane, paint a picture, paints a picture, painting a picture.

#### Impersonal Candidates

- snowing, snows, doesn't snow, snow, raining, rains, doesn't rain, rain, sunny, is sunny, is not sunny, be sunny, cloudy, is cloudy, is not cloudy, be cloudy, windy, is windy, is not windy, be windy, cold, is cold, is not cold, be cold, late, is late, is not late, be late, overcast, is overcast, is not overcast, be overcast, foggy, is foggy, is not foggy, be foggy, humid,

Table 10: Common fallacies.

| Name                     | Logical Form   | Example  |
|--------------------------|--|--|
| Affirming the Consequent | $p \rightarrow q, q \vdash p$  | If I study, I will pass the test. I passed the test. Therefore, I studied.                   |
| Denying the Antecedent   | $p \rightarrow q, \neg p \vdash \neg q$  | If it rains, the street gets wet. It is not raining. Therefore, the street is not wet.       |
| Affirming a Disjunct     | $p \vee q, p \vdash \neg q$  | Either I will study or I will fail the test. I studied. Therefore, I will not fail the test. |
| Denying a Conjunct       | $\neg(p \wedge q), \neg p \vdash q$  | I'm not both hungry and thirsty. I'm not hungry. Therefore, I'm thirsty.                     |
| Illicit Commutativity    | $p \rightarrow q \vdash q \rightarrow p$   | If I am in Paris, then I am in France. Therefore, if I am in France, I am in Paris.          |
| Existential Fallacy      | $\forall x(P(x) \rightarrow Q(x)), \neg \exists x(P(x)) \vdash \neg \exists x(Q(x))$                         | All birds can fly. No birds are present. Therefore, nothing can fly.                         |
| Illicit Major            | $\forall x(P(x) \rightarrow Q(x)), \exists x(Q(x)) \vdash \exists x(P(x))$                                   | All humans are mortal. Something is mortal. Therefore, something is human.                   |
| Illicit Minor            | $\forall x(P(x) \rightarrow Q(x)), \forall x(P(x) \rightarrow R(x)) \vdash \forall x(R(x) \rightarrow Q(x))$ | All men are mortal. All men are humans. Therefore, all humans are mortal.                    |
| Undistributed Middle     | $\forall x(P(x) \rightarrow Q(x)), Q(a) \vdash P(a)$   | All dogs are animals. My cat is an animal. Therefore, my cat is a dog.                       |

is humid, is not humid, be humid.

## D Prompting LLMs

For all GPT models, we set the system prompt of to blank.

### D.1 Zero-Shot Example

Consider the following premises: The claim that John is a poet and the claim that it is cloudy cannot both be true. Can we infer the following from them? Answer yes or no: Jessica is not listening to a song.

### D.2 Zero-Shot CoT

For Zero-Shot CoT, we add "Please think step-by-step and answer the following question." To zero-shot queries.

### D.3 ICL Example

Q: Consider the following premises: If it snows, then Joseph is a politician. Can we infer the following from them? Answer yes or no: It is snowing if and only if Joseph is a politician.

A: Let A be the claim that "it snows", B be

the claim that "Joseph is a politician", then the premises are "if A then B, if B then A", which is equivalent to "A if and only if B" by the biconditional introduction rule. Therefore, we can infer that It is snowing if and only if Joseph is a politician. The answer is ==yes==.

Q: Consider the following premises: It is late and it is windy. Can we infer the following from them? Answer yes or no: It is windy.

A: Let A be the claim that "it is late", B be the claim that "it is windy", then the premises are "A and B". By the simplification rule, we can infer B. Therefore, we can infer that it is windy. The answer is ==yes==.

Q: Consider the following premises: Jessica is not running. Jessica is running or it is raining. Can we infer the following from them? Answer yes or no: Sarah is not happy.

A: Let A be the claim that "Jessica is running", B be the claim that "it is raining", C be the claim that "Sarah is happy", then the premises are "not A, A or B". We cannot infer C from the premises. The answer is ==no==.

Q: Consider the following premises: It is not raining. It is raining or it is late. Can we infer the following from them? Answer yes or no: It is not late.

A: Let A be the claim that "it is raining", B be the claim that "it is late", then the premises are "not A, A or B". We can infer "B", which is "it is late". Therefore, we cannot infer "it is not late". The answer is ==no==.

Q: Consider the following premises: For all  $x$ ,  $x$  will write a letter, and  $x$  will climb a mountain and  $x$  is a musician. Can we infer the following from them? Answer yes or no: For all  $x$ ,  $x$  will write a letter and  $x$  will climb a mountain, and  $x$  is a musician.

## E Accessing LLMs

For commercial models, the specific models we accessed are gpt-4o-2024-05-13 for GPT-4o, gpt-4-turbo-2024-04-09 for GPT-4, gpt-3.5-turbo-0125 for ChatGPT, gemini-1.5-flash-latest (accessed June, 2024) for Gemini-1.5. All accesses are made via their official APIs<sup>4</sup>. For the open-source models, we use their respective Hugging Face<sup>5</sup> repository, i.e., meta-llama/Meta-Llama-3-70B for Llama-3 and mistralai/Mixtral-8x7B-v0.1 for Mixtral-8x7B.

For model parameters, we set the *temperature* to 0.0 and *max\_tokens* to 500 for all models. We keep other parameters to the models' respective default values.

## F Sampled Data Statistics

We analyzed various statistics of the sampled dataset, classified into different categories as presented in Table 11. The table summarizes the dis-

tribution of logic types, categorizes the rule types used in our analysis, and details the types of problems.

Table 11: Distribution of logic types, rule categories, and problem types.

| Logic Type    | Count |
|---------------|-------|
| Predicate     | 146   |
| Propositional | 62    |
| Rule Category | Count |
| Inference     | 108   |
| Equivalent    | 81    |
| Fallacy       | 19    |
| Problem Type  | Count |
| Inference     | 82    |
| Unrelated     | 63    |
| Contradiction | 63    |
| Total         | 208   |

## G Accuracy Versus Inference Length

To assess the impact of inference length, we generated test cases of varying lengths (i.e., ranging from 1 to 9) using randomly selected rules. For each length, we generated 100 test cases. Table 12 shows the performance of LLMs in these test cases. Generally, LLMs perform gradually worse as the inference length increases, indicating the increased complexity introduced by longer inference chains.

Table 12: LLMs' performance versus inference length.

| Len     | 1    | 3    | 5    | 7    | 9    |
|---------|------|------|------|------|------|
| GPT4    | 0.99 | 0.88 | 0.81 | 0.86 | 0.76 |
| GPT4o   | 0.94 | 0.81 | 0.79 | 0.72 | 0.63 |
| Gemini  | 0.93 | 0.80 | 0.73 | 0.74 | 0.64 |
| Llama3  | 0.91 | 0.78 | 0.74 | 0.81 | 0.67 |
| Mixtral | 0.79 | 0.67 | 0.64 | 0.66 | 0.62 |
| ChatGPT | 0.80 | 0.77 | 0.71 | 0.61 | 0.44 |

## H Complete Break-Down Result for All LLMs

In this section, we list the complete results of all LLMs on the set of 208 atomic rules in Table 13 through Table 18. The results are sorted in ascending order according to the zero-shot performance of the models.

<sup>4</sup><https://platform.openai.com/docs/> and <https://ai.google.dev/gemini-api>

<sup>5</sup><https://huggingface.co/>

Table 13: Break-down of the accuracy of GPT-4 on all rules (sorted by zero-shot accuracy).

| Logic         | Rule category | Rule                                   | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|---------------|---------------|--|---------------|-----------|---------------|------------|------|
| Predicate     | Inference     | Universal generalization               | Inference     | 0.52      | 0.52          | 0.52       | 0.52 |
| Predicate     | Inference     | Existential resolution                 | Unrelated     | 0.60      | 0.76          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal resolution                   | Inference     | 0.60      | 0.36          | 0.08       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier movement             | Inference     | 0.68      | 0.60          | 0.60       | 0.72 |
| Predicate     | Inference     | Existential resolution                 | Inference     | 0.72      | 0.52          | 0.96       | 0.96 |
| Predicate     | Inference     | Existential biconditional introduction | Inference     | 0.76      | 0.44          | 0.96       | 1.00 |
| Predicate     | Inference     | Existential biconditional introduction | Contradiction | 0.80      | 0.80          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential conditional laws           | Inference     | 0.80      | 0.80          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential transitivity               | Inference     | 0.80      | 0.76          | 0.96       | 1.00 |
| Predicate     | Equivalent    | Universal distributive laws            | Inference     | 0.80      | 0.36          | 0.20       | 0.96 |
| Propositional | Fallacy       | Denying a conjunct                     | Inference     | 0.88      | 0.92          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential distributive laws          | Inference     | 0.88      | 0.88          | 0.96       | 0.92 |
| Predicate     | Equivalent    | Law of quantifier movement             | Contradiction | 0.92      | 1.00          | 1.00       | 0.96 |
| Predicate     | Inference     | Existential biconditional elimination  | Inference     | 0.92      | 0.92          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential biconditional introduction | Unrelated     | 0.92      | 0.96          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential biconditional laws         | Contradiction | 0.92      | 0.92          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential conjunction                | Unrelated     | 0.92      | 0.92          | 0.92       | 0.96 |
| Predicate     | Inference     | Existential modus tollens              | Inference     | 0.92      | 0.88          | 1.00       | 1.00 |
| Propositional | Inference     | Resolution                             | Inference     | 0.92      | 0.64          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal disjunction elimination      | Inference     | 0.92      | 0.92          | 0.44       | 1.00 |
| Predicate     | Inference     | Universal generalization               | Contradiction | 0.92      | 0.88          | 1.00       | 1.00 |
| Propositional | Equivalent    | De morgan's laws                       | Contradiction | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier distribution         | Inference     | 0.96      | 0.96          | 1.00       | 1.00 |
| Propositional | Equivalent    | Associative laws                       | Unrelated     | 0.96      | 0.96          | 1.00       | 1.00 |
| Propositional | Equivalent    | Conditional laws                       | Contradiction | 0.96      | 0.96          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential associative laws           | Unrelated     | 0.96      | 0.96          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential biconditional laws         | Inference     | 0.96      | 0.96          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Existential denying a conjunct         | Inference     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunction elimination    | Inference     | 0.96      | 0.92          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential modus ponens               | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential transitivity               | Contradiction | 0.96      | 0.92          | 1.00       | 1.00 |
| Propositional | Equivalent    | Idempotent laws                        | Inference     | 0.96      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Modus tollens                          | Inference     | 0.96      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Simplification                         | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal biconditional elimination    | Inference     | 0.96      | 0.96          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal biconditional laws           | Inference     | 0.96      | 0.88          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal conditional laws             | Contradiction | 0.96      | 0.92          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal conjunction                  | Unrelated     | 0.96      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal disjunction elimination      | Unrelated     | 0.96      | 0.96          | 0.92       | 1.00 |
| Predicate     | Inference     | Universal disjunctive syllogism        | Unrelated     | 0.96      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal resolution                   | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | De morgan's laws                       | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | De morgan's laws                       | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier distribution         | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier distribution         | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier movement             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier negation             | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier negation             | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier negation             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Addition                               | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Addition                               | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Addition                               | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Fallacy       | Affirming a disjunct                   | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Fallacy       | Affirming the consequent               | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Associative laws                       | Contradiction | 1.00      | 0.92          | 0.96       | 1.00 |
| Propositional | Equivalent    | Associative laws                       | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional elimination              | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional elimination              | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional elimination              | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional introduction             | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional introduction             | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional introduction             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Biconditional laws                     | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Biconditional laws                     | Inference     | 1.00      | 1.00          | 0.96       | 1.00 |
| Propositional | Equivalent    | Biconditional laws                     | Unrelated     | 1.00      | 0.96          | 1.00       | 1.00 |
| Propositional | Equivalent    | Commutative laws                       | Contradiction | 1.00      | 0.96          | 1.00       | 1.00 |
| Propositional | Equivalent    | Commutative laws                       | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Commutative laws                       | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Commutative laws                       | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Commutative laws                       | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Commutative laws                       | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Complement laws                        | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Complement laws                        | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Complement laws                        | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Complement laws                        | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Complement laws                        | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Complement laws                        | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Conditional laws                       | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Conditional laws                       | Inference     | 1.00      | 0.96          | 1.00       | 1.00 |
| Propositional | Inference     | Conjunction                            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Conjunction                            | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Conjunction                            | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Fallacy       | Denying the antecedent                 | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunction elimination                | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunction elimination                | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunction elimination                | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |

Continued on next page

Table 13: Break-down of the accuracy of GPT-4 on all rules (sorted by zero-shot accuracy).

| Logic         | Rule category | Rule                                  | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|---------------|---------------|---------------------------------------|---------------|-----------|---------------|------------|------|
| Propositional | Inference     | Disjunctive syllogism                 | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunctive syllogism                 | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunctive syllogism                 | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Distributive laws                     | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Distributive laws                     | Inference     | 1.00      | 0.96          | 1.00       | 1.00 |
| Propositional | Equivalent    | Distributive laws                     | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential de morgan's laws          | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential de morgan's laws          | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential de morgan's laws          | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential affirming a disjunct      | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Existential affirming the consequent  | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential associative laws          | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential associative laws          | Inference     | 1.00      | 1.00          | 1.00       | 0.96 |
| Predicate     | Inference     | Existential biconditional elimination | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential biconditional elimination | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential biconditional laws        | Unrelated     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential commutative laws          | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential commutative laws          | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential commutative laws          | Unrelated     | 1.00      | 0.84          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential complement laws           | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential complement laws           | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential complement laws           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential conditional laws          | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential conditional laws          | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential conjunction               | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential conjunction               | Inference     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Existential denying the antecedent    | Inference     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunction elimination   | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunction elimination   | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunctive syllogism     | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunctive syllogism     | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunctive syllogism     | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential distributive laws         | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential distributive laws         | Unrelated     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Existential fallacy                   | Inference     | 1.00      | 1.00          | 0.96       | 1.00 |
| Predicate     | Inference     | Existential generalization            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential generalization            | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential generalization            | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential idempotent laws           | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential idempotent laws           | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential idempotent laws           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Existential illicit commutativity     | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential modus ponens              | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential modus ponens              | Inference     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential modus tollens             | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential modus tollens             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential resolution                | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential simplification            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential simplification            | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential simplification            | Unrelated     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential transitivity              | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Idempotent laws                       | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Idempotent laws                       | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Fallacy       | Illicit commutativity                 | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Illicit major                         | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Illicit minor                         | Inference     | 1.00      | 0.96          | 1.00       | 0.96 |
| Propositional | Inference     | Modus ponens                          | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Modus ponens                          | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Modus ponens                          | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Modus tollens                         | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Modus tollens                         | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Resolution                            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Resolution                            | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Simplification                        | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Simplification                        | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Transitivity                          | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Transitivity                          | Inference     | 1.00      | 1.00          | 0.96       | 1.00 |
| Propositional | Inference     | Transitivity                          | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Undistributed middle                  | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal de morgan's laws            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal de morgan's laws            | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal de morgan's laws            | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal addition                    | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal addition                    | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal addition                    | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Universal affirming a disjunct        | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Universal affirming the consequent    | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal associative laws            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |

Continued on next page

Table 13: Break-down of the accuracy of GPT-4 on all rules (sorted by zero-shot accuracy).

| Logic     | Rule category | Rule                                 | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|-----------|---------------|--------------------------------------|---------------|-----------|---------------|------------|------|
| Predicate | Equivalent    | Universal associative laws           | Inference     | 1.00      | 0.92          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal associative laws           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal biconditional elimination  | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal biconditional elimination  | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal biconditional introduction | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal biconditional introduction | Inference     | 1.00      | 0.80          | 1.00       | 1.00 |
| Predicate | Inference     | Universal biconditional introduction | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal biconditional laws         | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal biconditional laws         | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal commutative laws           | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal commutative laws           | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal commutative laws           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal complement laws            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal complement laws            | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal complement laws            | Unrelated     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal conditional laws           | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal conditional laws           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal conjunction                | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal conjunction                | Inference     | 1.00      | 1.00          | 0.92       | 1.00 |
| Predicate | Fallacy       | Universal denying a conjunct         | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Fallacy       | Universal denying the antecedent     | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal disjunction elimination    | Contradiction | 1.00      | 1.00          | 0.72       | 1.00 |
| Predicate | Inference     | Universal disjunctive syllogism      | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal disjunctive syllogism      | Inference     | 1.00      | 0.88          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal distributive laws          | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal distributive laws          | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal generalization             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal idempotent laws            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal idempotent laws            | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal idempotent laws            | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Fallacy       | Universal illicit commutativity      | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal instantiation              | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal instantiation              | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal instantiation              | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal modus ponens               | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal modus ponens               | Inference     | 1.00      | 1.00          | 0.88       | 1.00 |
| Predicate | Inference     | Universal modus ponens               | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal modus tollens              | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal modus tollens              | Inference     | 1.00      | 1.00          | 0.92       | 1.00 |
| Predicate | Inference     | Universal modus tollens              | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal resolution                 | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal simplification             | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal simplification             | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal simplification             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal transitivity               | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal transitivity               | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal transitivity               | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |

Table 14: Break-down of the accuracy of GPT-4o on all rules (sorted by zero-shot accuracy).

| Logic         | Rule category | Rule                                   | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|---------------|---------------|--|---------------|-----------|---------------|------------|------|
| Propositional | Inference     | Resolution                             | Inference     | 0.04      | 0.64          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal resolution                   | Inference     | 0.08      | 0.00          | 0.16       | 0.88 |
| Predicate     | Inference     | Existential biconditional introduction | Unrelated     | 0.20      | 0.96          | 1.00       | 0.56 |
| Propositional | Inference     | Biconditional introduction             | Unrelated     | 0.40      | 1.00          | 1.00       | 1.00 |
| Propositional | Fallacy       | Denying a conjunct                     | Inference     | 0.40      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Existential denying the antecedent     | Inference     | 0.40      | 1.00          | 0.96       | 0.88 |
| Predicate     | Inference     | Universal biconditional introduction   | Unrelated     | 0.40      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential resolution                 | Inference     | 0.52      | 0.00          | 0.00       | 0.56 |
| Predicate     | Inference     | Universal generalization               | Inference     | 0.52      | 0.52          | 0.52       | 0.52 |
| Predicate     | Inference     | Universal simplification               | Inference     | 0.56      | 0.68          | 0.96       | 0.96 |
| Predicate     | Inference     | Universal disjunction elimination      | Inference     | 0.60      | 0.32          | 0.44       | 0.92 |
| Predicate     | Equivalent    | Law of quantifier distribution         | Inference     | 0.64      | 0.56          | 0.60       | 0.80 |
| Predicate     | Equivalent    | Law of quantifier movement             | Inference     | 0.64      | 0.44          | 0.72       | 0.84 |
| Propositional | Equivalent    | Conditional laws                       | Contradiction | 0.64      | 0.96          | 0.96       | 0.96 |
| Predicate     | Equivalent    | Existential conditional laws           | Contradiction | 0.64      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential transitivity               | Inference     | 0.64      | 0.00          | 0.20       | 0.60 |
| Predicate     | Inference     | Universal transitivity                 | Inference     | 0.68      | 0.68          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunctive syllogism                  | Contradiction | 0.72      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunctive syllogism      | Contradiction | 0.72      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Existential fallacy                    | Inference     | 0.72      | 1.00          | 0.96       | 1.00 |
| Predicate     | Equivalent    | Universal associative laws             | Inference     | 0.72      | 0.96          | 0.88       | 1.00 |
| Predicate     | Inference     | Universal disjunctive syllogism        | Inference     | 0.72      | 0.88          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential biconditional introduction | Inference     | 0.76      | 0.04          | 0.16       | 0.28 |
| Predicate     | Inference     | Existential resolution                 | Unrelated     | 0.76      | 1.00          | 0.96       | 0.64 |
| Predicate     | Fallacy       | Illicit minor                          | Inference     | 0.76      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal biconditional introduction   | Inference     | 0.76      | 0.84          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal disjunctive syllogism        | Contradiction | 0.76      | 1.00          | 1.00       | 1.00 |
| Propositional | Fallacy       | Denying the antecedent                 | Inference     | 0.80      | 1.00          | 1.00       | 0.96 |
| Predicate     | Inference     | Existential biconditional elimination  | Unrelated     | 0.80      | 0.96          | 1.00       | 1.00 |

Table 14: Break-down of the accuracy of GPT-4o on all rules (sorted by zero-shot accuracy).

| Logic         | Rule category | Rule                                   | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|---------------|---------------|--|---------------|-----------|---------------|------------|------|
| Predicate     | Inference     | Existential disjunction elimination    | Inference     | 0.80      | 0.08          | 0.12       | 0.76 |
| Predicate     | Inference     | Universal biconditional elimination    | Inference     | 0.80      | 0.88          | 0.92       | 1.00 |
| Predicate     | Inference     | Existential biconditional elimination  | Contradiction | 0.84      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential conjunction                | Unrelated     | 0.84      | 0.88          | 0.92       | 0.92 |
| Predicate     | Fallacy       | Existential denying a conjunct         | Inference     | 0.84      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential modus tollens              | Inference     | 0.84      | 0.64          | 0.80       | 1.00 |
| Predicate     | Inference     | Existential transitivity               | Contradiction | 0.84      | 1.00          | 1.00       | 0.92 |
| Predicate     | Fallacy       | Universal denying the antecedent       | Inference     | 0.84      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal distributive laws            | Inference     | 0.84      | 0.04          | 0.44       | 0.60 |
| Predicate     | Inference     | Universal modus tollens                | Inference     | 0.84      | 0.64          | 0.88       | 1.00 |
| Predicate     | Equivalent    | Existential de morgan's laws           | Inference     | 0.88      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential biconditional introduction | Contradiction | 0.88      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunctive syllogism      | Inference     | 0.88      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential modus tollens              | Unrelated     | 0.88      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential transitivity               | Unrelated     | 0.88      | 1.00          | 1.00       | 0.88 |
| Predicate     | Fallacy       | Universal denying a conjunct           | Inference     | 0.88      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal disjunction elimination      | Unrelated     | 0.88      | 0.80          | 0.96       | 0.92 |
| Predicate     | Inference     | Universal modus ponens                 | Inference     | 0.88      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal modus tollens                | Unrelated     | 0.88      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal transitivity                 | Unrelated     | 0.88      | 0.92          | 1.00       | 1.00 |
| Propositional | Equivalent    | De morgan's laws                       | Contradiction | 0.92      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier distribution         | Contradiction | 0.92      | 1.00          | 1.00       | 0.92 |
| Predicate     | Equivalent    | Law of quantifier movement             | Contradiction | 0.92      | 0.84          | 0.84       | 0.96 |
| Propositional | Equivalent    | Associative laws                       | Contradiction | 0.92      | 0.92          | 1.00       | 1.00 |
| Propositional | Equivalent    | Biconditional laws                     | Unrelated     | 0.92      | 0.96          | 1.00       | 0.96 |
| Propositional | Inference     | Disjunction elimination                | Inference     | 0.92      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential biconditional elimination  | Inference     | 0.92      | 0.72          | 0.56       | 1.00 |
| Predicate     | Equivalent    | Existential commutative laws           | Unrelated     | 0.92      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential conjunction                | Inference     | 0.92      | 0.96          | 1.00       | 1.00 |
| Propositional | Inference     | Modus tollens                          | Unrelated     | 0.92      | 1.00          | 1.00       | 0.96 |
| Predicate     | Equivalent    | Universal de morgan's laws             | Inference     | 0.92      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal disjunctive syllogism        | Unrelated     | 0.92      | 0.92          | 0.96       | 0.96 |
| Propositional | Equivalent    | De morgan's laws                       | Inference     | 0.96      | 0.96          | 1.00       | 0.96 |
| Predicate     | Equivalent    | Law of quantifier negation             | Contradiction | 0.96      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Biconditional laws                     | Inference     | 0.96      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunction elimination                | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunctive syllogism                  | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Distributive laws                      | Inference     | 0.96      | 0.68          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential de morgan's laws           | Contradiction | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Existential affirming a disjunct       | Inference     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Existential affirming the consequent   | Inference     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential associative laws           | Unrelated     | 0.96      | 0.96          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential biconditional laws         | Contradiction | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunctive syllogism      | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential distributive laws          | Inference     | 0.96      | 0.12          | 0.52       | 0.80 |
| Predicate     | Inference     | Existential generalization             | Inference     | 0.96      | 1.00          | 1.00       | 0.96 |
| Predicate     | Inference     | Existential modus ponens               | Unrelated     | 0.96      | 0.96          | 1.00       | 0.96 |
| Predicate     | Inference     | Existential modus tollens              | Contradiction | 0.96      | 1.00          | 1.00       | 0.96 |
| Predicate     | Inference     | Existential simplification             | Inference     | 0.96      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Simplification                         | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Transitivity                           | Inference     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Universal affirming a disjunct         | Inference     | 0.96      | 1.00          | 1.00       | 0.96 |
| Predicate     | Inference     | Universal biconditional elimination    | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal biconditional laws           | Inference     | 0.96      | 0.56          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal complement laws              | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal conjunction                  | Inference     | 0.96      | 0.96          | 1.00       | 1.00 |
| Propositional | Equivalent    | De morgan's laws                       | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier distribution         | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier movement             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier negation             | Inference     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier negation             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Addition                               | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Addition                               | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Addition                               | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Fallacy       | Affirming a disjunct                   | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Fallacy       | Affirming the consequent               | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Associative laws                       | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Associative laws                       | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional elimination              | Contradiction | 1.00      | 1.00          | 1.00       | 0.96 |
| Propositional | Inference     | Biconditional elimination              | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional elimination              | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional introduction             | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional introduction             | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Biconditional laws                     | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Commutative laws                       | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Commutative laws                       | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Commutative laws                       | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Complement laws                        | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Complement laws                        | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Complement laws                        | Unrelated     | 1.00      | 0.96          | 1.00       | 1.00 |
| Propositional | Equivalent    | Conditional laws                       | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |

Continued on next page

Table 14: Break-down of the accuracy of GPT-4o on all rules (sorted by zero-shot accuracy).

| Logic         | Rule category | Rule                                 | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|---------------|---------------|--------------------------------------|---------------|-----------|---------------|------------|------|
| Propositional | Equivalent    | Conditional laws                     | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Conjunction                          | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Conjunction                          | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Conjunction                          | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunction elimination              | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunctive syllogism                | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Distributive laws                    | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Distributive laws                    | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential de morgan's laws         | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential addition                 | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential addition                 | Inference     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential addition                 | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential associative laws         | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential associative laws         | Inference     | 1.00      | 0.88          | 1.00       | 0.92 |
| Predicate     | Equivalent    | Existential biconditional laws       | Inference     | 1.00      | 0.40          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential biconditional laws       | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential commutative laws         | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential commutative laws         | Inference     | 1.00      | 0.92          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential complement laws          | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential complement laws          | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential complement laws          | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential conditional laws         | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential conditional laws         | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential conjunction              | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunction elimination  | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunction elimination  | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential distributive laws        | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential distributive laws        | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential generalization           | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential generalization           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential idempotent laws          | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential idempotent laws          | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential idempotent laws          | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Existential illicit commutativity    | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential modus ponens             | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential modus ponens             | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential resolution               | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential simplification           | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential simplification           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Idempotent laws                      | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Idempotent laws                      | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Idempotent laws                      | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Fallacy       | Illicit commutativity                | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Illicit major                        | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Modus ponens                         | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Modus ponens                         | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Modus ponens                         | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Modus tollens                        | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Modus tollens                        | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Resolution                           | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Resolution                           | Unrelated     | 1.00      | 0.84          | 0.92       | 0.88 |
| Propositional | Inference     | Simplification                       | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Simplification                       | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Transitivity                         | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Transitivity                         | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Undistributed middle                 | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal de morgan's laws           | Contradiction | 1.00      | 1.00          | 0.96       | 1.00 |
| Predicate     | Equivalent    | Universal de morgan's laws           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal addition                   | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal addition                   | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal addition                   | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Universal affirming the consequent   | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal associative laws           | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal associative laws           | Unrelated     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal biconditional elimination  | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal biconditional introduction | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal biconditional laws         | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal biconditional laws         | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal commutative laws           | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal commutative laws           | Inference     | 1.00      | 0.92          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal commutative laws           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal complement laws            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal complement laws            | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal conditional laws           | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal conditional laws           | Inference     | 1.00      | 0.92          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal conditional laws           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal conjunction                | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal conjunction                | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal disjunction elimination    | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal distributive laws          | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal distributive laws          | Unrelated     | 1.00      | 0.96          | 1.00       | 1.00 |

Continued on next page

Table 14: Break-down of the accuracy of GPT-4o on all rules (sorted by zero-shot accuracy).

| Logic     | Rule category | Rule                            | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|-----------|---------------|---------------------------------|---------------|-----------|---------------|------------|------|
| Predicate | Inference     | Universal generalization        | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal generalization        | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal idempotent laws       | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal idempotent laws       | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal idempotent laws       | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Fallacy       | Universal illicit commutativity | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal instantiation         | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal instantiation         | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal instantiation         | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal instantiation         | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal modus ponens          | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal modus ponens          | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal modus tollens         | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal resolution            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal resolution            | Unrelated     | 1.00      | 1.00          | 0.92       | 0.76 |
| Predicate | Inference     | Universal simplification        | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal simplification        | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal transitivity          | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |

Table 15: Break-down of the accuracy of Gemini-1.5 on all rules (sorted by zero-shot accuracy).

| Logic         | Rule category | Rule                                   | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|---------------|---------------|--|---------------|-----------|---------------|------------|------|
| Predicate     | Fallacy       | Existential denying the antecedent     | Inference     | 0.00      | 0.60          | 0.48       | 0.84 |
| Predicate     | Inference     | Existential biconditional introduction | Unrelated     | 0.08      | 0.28          | 0.12       | 0.32 |
| Predicate     | Inference     | Existential biconditional introduction | Contradiction | 0.16      | 0.52          | 0.96       | 0.88 |
| Predicate     | Inference     | Existential resolution                 | Unrelated     | 0.20      | 0.40          | 0.44       | 0.80 |
| Propositional | Fallacy       | Denying the antecedent                 | Inference     | 0.32      | 0.80          | 0.92       | 0.72 |
| Predicate     | Fallacy       | Existential denying a conjunct         | Inference     | 0.32      | 0.40          | 0.64       | 0.88 |
| Predicate     | Inference     | Universal instantiation                | Inference     | 0.32      | 1.00          | 0.64       | 0.64 |
| Predicate     | Inference     | Universal disjunctive syllogism        | Contradiction | 0.44      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential transitivity               | Unrelated     | 0.48      | 0.96          | 0.84       | 0.68 |
| Predicate     | Equivalent    | Existential conditional laws           | Contradiction | 0.52      | 0.88          | 0.84       | 0.72 |
| Predicate     | Fallacy       | Universal denying the antecedent       | Inference     | 0.52      | 0.88          | 0.80       | 0.76 |
| Predicate     | Inference     | Universal generalization               | Inference     | 0.52      | 0.52          | 0.52       | 0.52 |
| Predicate     | Equivalent    | Existential conditional laws           | Inference     | 0.56      | 0.36          | 0.84       | 0.88 |
| Predicate     | Inference     | Universal resolution                   | Contradiction | 0.56      | 0.68          | 0.80       | 0.88 |
| Predicate     | Inference     | Universal biconditional introduction   | Contradiction | 0.64      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Existential affirming a disjunct       | Inference     | 0.68      | 0.64          | 0.44       | 0.72 |
| Predicate     | Equivalent    | Universal conditional laws             | Contradiction | 0.68      | 0.72          | 0.88       | 0.92 |
| Propositional | Equivalent    | Conditional laws                       | Contradiction | 0.72      | 0.64          | 0.88       | 0.88 |
| Predicate     | Equivalent    | Existential biconditional laws         | Contradiction | 0.72      | 0.96          | 0.88       | 0.88 |
| Predicate     | Equivalent    | Law of quantifier movement             | Inference     | 0.76      | 0.48          | 0.56       | 0.84 |
| Predicate     | Equivalent    | Law of quantifier negation             | Contradiction | 0.76      | 0.96          | 0.80       | 0.88 |
| Propositional | Inference     | Disjunctive syllogism                  | Contradiction | 0.76      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential associative laws           | Unrelated     | 0.76      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential conjunction                | Unrelated     | 0.76      | 0.88          | 0.96       | 0.92 |
| Predicate     | Fallacy       | Existential fallacy                    | Inference     | 0.76      | 0.84          | 0.80       | 0.76 |
| Predicate     | Inference     | Universal transitivity                 | Contradiction | 0.76      | 1.00          | 0.96       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier movement             | Contradiction | 0.80      | 0.76          | 0.88       | 0.96 |
| Propositional | Inference     | Biconditional introduction             | Contradiction | 0.80      | 0.96          | 1.00       | 0.92 |
| Predicate     | Inference     | Existential biconditional elimination  | Unrelated     | 0.80      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Resolution                             | Unrelated     | 0.80      | 0.68          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal de morgan's laws             | Inference     | 0.80      | 0.64          | 0.88       | 1.00 |
| Predicate     | Inference     | Universal generalization               | Contradiction | 0.80      | 0.96          | 1.00       | 1.00 |
| Propositional | Equivalent    | Conditional laws                       | Inference     | 0.84      | 0.64          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential modus tollens              | Unrelated     | 0.84      | 1.00          | 0.96       | 1.00 |
| Predicate     | Equivalent    | Universal biconditional laws           | Contradiction | 0.84      | 1.00          | 0.96       | 0.96 |
| Predicate     | Equivalent    | Universal idempotent laws              | Inference     | 0.84      | 0.76          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal simplification               | Inference     | 0.84      | 0.60          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential biconditional laws         | Unrelated     | 0.88      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential distributive laws          | Unrelated     | 0.88      | 0.96          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential idempotent laws            | Unrelated     | 0.88      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Transitivity                           | Contradiction | 0.88      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Universal affirming a disjunct         | Inference     | 0.88      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal disjunction elimination      | Inference     | 0.88      | 1.00          | 1.00       | 0.96 |
| Predicate     | Equivalent    | Law of quantifier distribution         | Contradiction | 0.92      | 1.00          | 1.00       | 0.96 |
| Propositional | Equivalent    | Biconditional laws                     | Contradiction | 0.92      | 0.96          | 0.96       | 0.92 |
| Propositional | Inference     | Disjunctive syllogism                  | Inference     | 0.92      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential biconditional elimination  | Inference     | 0.92      | 0.96          | 0.96       | 0.88 |
| Predicate     | Equivalent    | Existential commutative laws           | Unrelated     | 0.92      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal biconditional elimination    | Inference     | 0.92      | 0.84          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal conjunction                  | Inference     | 0.92      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal disjunctive syllogism        | Inference     | 0.92      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal modus ponens                 | Inference     | 0.92      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal modus tollens                | Inference     | 0.92      | 0.80          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier distribution         | Inference     | 0.96      | 0.96          | 0.92       | 0.96 |
| Propositional | Equivalent    | Associative laws                       | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Commutative laws                       | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunction elimination                | Contradiction | 0.96      | 1.00          | 1.00       | 0.96 |
| Predicate     | Equivalent    | Existential de morgan's laws           | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential addition                   | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |

Table 15: Break-down of the accuracy of Gemini-1.5 on all rules (sorted by zero-shot accuracy).

| Logic         | Rule category | Rule                                   | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|---------------|---------------|--|---------------|-----------|---------------|------------|------|
| Predicate     | Inference     | Existential biconditional introduction | Inference     | 0.96      | 0.84          | 0.96       | 0.92 |
| Predicate     | Inference     | Existential disjunctive syllogism      | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential modus tollens              | Inference     | 0.96      | 0.92          | 0.92       | 0.96 |
| Predicate     | Inference     | Existential simplification             | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Illicit minor                          | Inference     | 0.96      | 1.00          | 1.00       | 0.96 |
| Propositional | Inference     | Resolution                             | Contradiction | 0.96      | 0.96          | 1.00       | 1.00 |
| Propositional | Inference     | Resolution                             | Inference     | 0.96      | 0.96          | 1.00       | 1.00 |
| Propositional | Inference     | Simplification                         | Unrelated     | 0.96      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal biconditional elimination    | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal biconditional introduction   | Inference     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Universal denying a conjunct           | Inference     | 0.96      | 0.96          | 0.96       | 0.92 |
| Predicate     | Inference     | Universal disjunctive syllogism        | Unrelated     | 0.96      | 0.96          | 0.96       | 1.00 |
| Predicate     | Inference     | Universal simplification               | Contradiction | 0.96      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | De morgan's laws                       | Contradiction | 1.00      | 0.80          | 0.76       | 0.72 |
| Propositional | Equivalent    | De morgan's laws                       | Inference     | 1.00      | 0.92          | 1.00       | 1.00 |
| Propositional | Equivalent    | De morgan's laws                       | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier distribution         | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier movement             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier negation             | Inference     | 1.00      | 0.88          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier negation             | Unrelated     | 1.00      | 0.96          | 1.00       | 1.00 |
| Propositional | Inference     | Addition                               | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Addition                               | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Addition                               | Unrelated     | 1.00      | 0.96          | 1.00       | 1.00 |
| Propositional | Fallacy       | Affirming a disjunct                   | Inference     | 1.00      | 1.00          | 1.00       | 0.92 |
| Propositional | Fallacy       | Affirming the consequent               | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Associative laws                       | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Associative laws                       | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional elimination              | Contradiction | 1.00      | 0.96          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional elimination              | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional elimination              | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional introduction             | Inference     | 1.00      | 0.92          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional introduction             | Unrelated     | 1.00      | 1.00          | 0.96       | 0.88 |
| Propositional | Equivalent    | Biconditional laws                     | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Biconditional laws                     | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Commutative laws                       | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Commutative laws                       | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Complement laws                        | Contradiction | 1.00      | 0.96          | 1.00       | 1.00 |
| Propositional | Equivalent    | Complement laws                        | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Complement laws                        | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Conditional laws                       | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Conjunction                            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Conjunction                            | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Conjunction                            | Unrelated     | 1.00      | 0.96          | 1.00       | 1.00 |
| Propositional | Fallacy       | Denying a conjunct                     | Inference     | 1.00      | 0.80          | 0.96       | 0.96 |
| Propositional | Inference     | Disjunction elimination                | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunction elimination                | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunctive syllogism                  | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Distributive laws                      | Contradiction | 1.00      | 0.96          | 1.00       | 0.96 |
| Propositional | Equivalent    | Distributive laws                      | Inference     | 1.00      | 0.92          | 1.00       | 1.00 |
| Propositional | Equivalent    | Distributive laws                      | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential de morgan's laws           | Contradiction | 1.00      | 1.00          | 0.96       | 0.92 |
| Predicate     | Equivalent    | Existential de morgan's laws           | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential addition                   | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential addition                   | Inference     | 1.00      | 0.76          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Existential affirming the consequent   | Inference     | 1.00      | 1.00          | 0.96       | 0.88 |
| Predicate     | Equivalent    | Existential associative laws           | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential associative laws           | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential biconditional elimination  | Contradiction | 1.00      | 1.00          | 1.00       | 0.96 |
| Predicate     | Equivalent    | Existential biconditional laws         | Inference     | 1.00      | 0.88          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential commutative laws           | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential commutative laws           | Inference     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential complement laws            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential complement laws            | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential complement laws            | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential conditional laws           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential conjunction                | Contradiction | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential conjunction                | Inference     | 1.00      | 0.96          | 0.92       | 0.96 |
| Predicate     | Inference     | Existential disjunction elimination    | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunction elimination    | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunction elimination    | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunctive syllogism      | Contradiction | 1.00      | 1.00          | 0.96       | 0.96 |
| Predicate     | Inference     | Existential disjunctive syllogism      | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential distributive laws          | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential distributive laws          | Inference     | 1.00      | 0.80          | 1.00       | 0.96 |
| Predicate     | Inference     | Existential generalization             | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential generalization             | Inference     | 1.00      | 0.84          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential generalization             | Unrelated     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential idempotent laws            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential idempotent laws            | Inference     | 1.00      | 0.92          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Existential illicit commutativity      | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential modus ponens               | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |

Continued on next page

Table 15: Break-down of the accuracy of Gemini-1.5 on all rules (sorted by zero-shot accuracy).

| Logic         | Rule category | Rule                                 | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|---------------|---------------|--------------------------------------|---------------|-----------|---------------|------------|------|
| Predicate     | Inference     | Existential modus ponens             | Inference     | 1.00      | 0.96          | 0.96       | 1.00 |
| Predicate     | Inference     | Existential modus ponens             | Unrelated     | 1.00      | 1.00          | 1.00       | 0.96 |
| Predicate     | Inference     | Existential modus tollens            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential resolution               | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential resolution               | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential simplification           | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential simplification           | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential transitivity             | Contradiction | 1.00      | 0.76          | 0.84       | 0.92 |
| Predicate     | Inference     | Existential transitivity             | Inference     | 1.00      | 0.92          | 0.96       | 0.96 |
| Propositional | Equivalent    | Idempotent laws                      | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Idempotent laws                      | Inference     | 1.00      | 0.88          | 1.00       | 1.00 |
| Propositional | Equivalent    | Idempotent laws                      | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Fallacy       | Illicit commutativity                | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Illicit major                        | Inference     | 1.00      | 1.00          | 0.96       | 0.92 |
| Propositional | Inference     | Modus ponens                         | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Modus ponens                         | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Modus ponens                         | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Modus tollens                        | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Modus tollens                        | Inference     | 1.00      | 0.96          | 1.00       | 1.00 |
| Propositional | Inference     | Modus tollens                        | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Simplification                       | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Simplification                       | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Transitivity                         | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Transitivity                         | Unrelated     | 1.00      | 1.00          | 0.96       | 1.00 |
| Predicate     | Fallacy       | Undistributed middle                 | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal de morgan's laws           | Contradiction | 1.00      | 1.00          | 0.80       | 0.84 |
| Predicate     | Equivalent    | Universal de morgan's laws           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal addition                   | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal addition                   | Inference     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal addition                   | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Universal affirming the consequent   | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal associative laws           | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal associative laws           | Inference     | 1.00      | 0.84          | 1.00       | 0.96 |
| Predicate     | Equivalent    | Universal associative laws           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal biconditional elimination  | Contradiction | 1.00      | 1.00          | 0.92       | 0.96 |
| Predicate     | Inference     | Universal biconditional introduction | Unrelated     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal biconditional laws         | Inference     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal biconditional laws         | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal commutative laws           | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal commutative laws           | Inference     | 1.00      | 0.92          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal commutative laws           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal complement laws            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal complement laws            | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal complement laws            | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal conditional laws           | Inference     | 1.00      | 0.84          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal conditional laws           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal conjunction                | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal conjunction                | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal disjunction elimination    | Contradiction | 1.00      | 1.00          | 0.92       | 0.96 |
| Predicate     | Inference     | Universal disjunction elimination    | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal distributive laws          | Contradiction | 1.00      | 0.88          | 0.92       | 1.00 |
| Predicate     | Equivalent    | Universal distributive laws          | Inference     | 1.00      | 0.68          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal distributive laws          | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal generalization             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal idempotent laws            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal idempotent laws            | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Universal illicit commutativity      | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal instantiation              | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal instantiation              | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal modus ponens               | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal modus ponens               | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal modus tollens              | Contradiction | 1.00      | 1.00          | 0.92       | 0.96 |
| Predicate     | Inference     | Universal modus tollens              | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal resolution                 | Inference     | 1.00      | 0.88          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal resolution                 | Unrelated     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal simplification             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal transitivity               | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal transitivity               | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |

Table 16: Break-down of the accuracy of Llama3 on all rules (sorted by zero-shot accuracy).

| Logic         | Rule category | Rule                                   | Problem   | Zero shot | Zero shot cot | Random icl | Weak |
|---------------|---------------|--|-----------|-----------|---------------|------------|------|
| Predicate     | Fallacy       | Existential denying the antecedent     | Inference | 0.00      | 0.72          | 0.44       | 0.40 |
| Predicate     | Inference     | Existential resolution                 | Unrelated | 0.04      | 0.28          | 0.16       | 0.12 |
| Predicate     | Inference     | Existential biconditional introduction | Unrelated | 0.08      | 0.64          | 0.20       | 0.04 |
| Predicate     | Fallacy       | Universal denying the antecedent       | Inference | 0.12      | 0.76          | 0.76       | 0.52 |
| Predicate     | Fallacy       | Existential denying a conjunct         | Inference | 0.16      | 0.16          | 0.60       | 0.48 |
| Predicate     | Fallacy       | Universal denying a conjunct           | Inference | 0.16      | 0.76          | 0.52       | 0.12 |
| Propositional | Fallacy       | Denying the antecedent                 | Inference | 0.28      | 0.88          | 0.52       | 0.16 |
| Predicate     | Fallacy       | Existential fallacy                    | Inference | 0.28      | 0.80          | 0.28       | 0.28 |

Table 16: Break-down of the accuracy of Llama3 on all rules (sorted by zero-shot accuracy).

| Logic         | Rule category | Rule                                   | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|---------------|---------------|--|---------------|-----------|---------------|------------|------|
| Predicate     | Inference     | Existential transitivity               | Unrelated     | 0.36      | 0.76          | 0.76       | 0.88 |
| Propositional | Inference     | Transitivity                           | Contradiction | 0.40      | 1.00          | 0.96       | 1.00 |
| Predicate     | Equivalent    | Existential distributive laws          | Inference     | 0.44      | 0.60          | 0.64       | 0.76 |
| Predicate     | Equivalent    | Existential conditional laws           | Contradiction | 0.52      | 0.92          | 0.68       | 0.76 |
| Predicate     | Inference     | Universal generalization               | Inference     | 0.52      | 0.40          | 0.52       | 0.44 |
| Propositional | Equivalent    | Conditional laws                       | Contradiction | 0.56      | 0.76          | 0.92       | 1.00 |
| Predicate     | Equivalent    | Existential biconditional laws         | Contradiction | 0.60      | 0.92          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal conditional laws             | Contradiction | 0.60      | 0.76          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal transitivity                 | Contradiction | 0.60      | 1.00          | 1.00       | 1.00 |
| Propositional | Fallacy       | Denying a conjunct                     | Inference     | 0.64      | 0.84          | 0.60       | 0.60 |
| Predicate     | Equivalent    | Existential conditional laws           | Inference     | 0.64      | 0.48          | 0.96       | 1.00 |
| Propositional | Equivalent    | De morgan's laws                       | Contradiction | 0.68      | 0.88          | 1.00       | 0.52 |
| Predicate     | Fallacy       | Illicit major                          | Inference     | 0.68      | 1.00          | 0.84       | 0.60 |
| Predicate     | Equivalent    | Law of quantifier movement             | Inference     | 0.72      | 0.48          | 0.56       | 1.00 |
| Predicate     | Inference     | Universal instantiation                | Inference     | 0.72      | 0.76          | 0.92       | 1.00 |
| Predicate     | Inference     | Existential transitivity               | Contradiction | 0.76      | 0.72          | 1.00       | 0.96 |
| Predicate     | Inference     | Universal disjunctive syllogism        | Contradiction | 0.76      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier movement             | Contradiction | 0.80      | 0.84          | 0.96       | 0.88 |
| Predicate     | Equivalent    | Law of quantifier negation             | Contradiction | 0.80      | 1.00          | 0.76       | 0.96 |
| Propositional | Inference     | Disjunctive syllogism                  | Inference     | 0.80      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential conjunction                | Unrelated     | 0.80      | 0.96          | 0.96       | 0.96 |
| Predicate     | Inference     | Universal resolution                   | Unrelated     | 0.80      | 0.96          | 0.80       | 0.56 |
| Propositional | Equivalent    | Conditional laws                       | Inference     | 0.84      | 0.96          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential associative laws           | Unrelated     | 0.84      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential biconditional elimination  | Contradiction | 0.84      | 1.00          | 0.92       | 0.96 |
| Predicate     | Equivalent    | Existential distributive laws          | Unrelated     | 0.84      | 1.00          | 1.00       | 0.96 |
| Predicate     | Fallacy       | Illicit minor                          | Inference     | 0.84      | 0.92          | 0.48       | 0.20 |
| Propositional | Inference     | Resolution                             | Unrelated     | 0.84      | 0.72          | 0.76       | 0.72 |
| Predicate     | Equivalent    | Universal distributive laws            | Inference     | 0.84      | 0.64          | 0.64       | 0.92 |
| Predicate     | Equivalent    | Existential de morgan's laws           | Contradiction | 0.88      | 1.00          | 1.00       | 0.64 |
| Predicate     | Fallacy       | Existential affirming the consequent   | Inference     | 0.88      | 1.00          | 0.80       | 0.48 |
| Predicate     | Inference     | Existential biconditional introduction | Inference     | 0.88      | 0.92          | 0.96       | 1.00 |
| Predicate     | Equivalent    | Existential commutative laws           | Unrelated     | 0.88      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunctive syllogism      | Contradiction | 0.88      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential generalization             | Inference     | 0.88      | 0.88          | 0.96       | 0.92 |
| Predicate     | Fallacy       | Universal affirming a disjunct         | Inference     | 0.88      | 1.00          | 0.96       | 0.92 |
| Predicate     | Inference     | Existential biconditional elimination  | Unrelated     | 0.92      | 0.96          | 1.00       | 0.96 |
| Predicate     | Inference     | Existential resolution                 | Contradiction | 0.92      | 0.76          | 0.96       | 0.80 |
| Predicate     | Inference     | Universal biconditional introduction   | Contradiction | 0.92      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal biconditional introduction   | Inference     | 0.92      | 0.96          | 0.88       | 1.00 |
| Predicate     | Equivalent    | Universal biconditional laws           | Contradiction | 0.92      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal biconditional laws           | Inference     | 0.92      | 0.88          | 0.92       | 1.00 |
| Predicate     | Equivalent    | Universal conditional laws             | Inference     | 0.92      | 0.80          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal distributive laws            | Unrelated     | 0.92      | 1.00          | 1.00       | 0.96 |
| Predicate     | Inference     | Universal resolution                   | Contradiction | 0.92      | 0.72          | 0.72       | 0.60 |
| Predicate     | Inference     | Universal transitivity                 | Unrelated     | 0.92      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | De morgan's laws                       | Inference     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier negation             | Unrelated     | 0.96      | 1.00          | 1.00       | 0.96 |
| Propositional | Equivalent    | Biconditional laws                     | Unrelated     | 0.96      | 1.00          | 1.00       | 0.96 |
| Propositional | Equivalent    | Commutative laws                       | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Conditional laws                       | Unrelated     | 0.96      | 0.96          | 1.00       | 1.00 |
| Propositional | Equivalent    | Distributive laws                      | Unrelated     | 0.96      | 1.00          | 1.00       | 0.96 |
| Predicate     | Equivalent    | Existential de morgan's laws           | Unrelated     | 0.96      | 0.96          | 0.96       | 1.00 |
| Predicate     | Equivalent    | Existential biconditional laws         | Inference     | 0.96      | 0.96          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential conditional laws           | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunctive syllogism      | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential distributive laws          | Contradiction | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential modus ponens               | Unrelated     | 0.96      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential modus tollens              | Unrelated     | 0.96      | 1.00          | 0.96       | 0.92 |
| Propositional | Inference     | Modus ponens                           | Inference     | 0.96      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Simplification                         | Unrelated     | 0.96      | 0.96          | 1.00       | 1.00 |
| Propositional | Inference     | Transitivity                           | Inference     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal de morgan's laws             | Contradiction | 0.96      | 0.84          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal disjunction elimination      | Unrelated     | 0.96      | 0.96          | 0.96       | 0.96 |
| Predicate     | Inference     | Universal disjunctive syllogism        | Unrelated     | 0.96      | 0.96          | 0.96       | 0.96 |
| Predicate     | Inference     | Universal modus ponens                 | Inference     | 0.96      | 0.96          | 1.00       | 1.00 |
| Propositional | Equivalent    | De morgan's laws                       | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier distribution         | Inference     | 1.00      | 0.92          | 0.96       | 0.88 |
| Predicate     | Equivalent    | Law of quantifier distribution         | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier movement             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier negation             | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Addition                               | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Addition                               | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Addition                               | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Fallacy       | Affirming a disjunct                   | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Fallacy       | Affirming the consequent               | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |

Continued on next page

Table 16: Break-down of the accuracy of Llama3 on all rules (sorted by zero-shot accuracy).

| Logic         | Rule category | Rule                                   | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|---------------|---------------|--|---------------|-----------|---------------|------------|------|
| Propositional | Equivalent    | Associative laws                       | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional elimination              | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional elimination              | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional elimination              | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional introduction             | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional introduction             | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional introduction             | Unrelated     | 1.00      | 1.00          | 1.00       | 0.92 |
| Propositional | Equivalent    | Biconditional laws                     | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Biconditional laws                     | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Commutative laws                       | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Commutative laws                       | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Complement laws                        | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Complement laws                        | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Complement laws                        | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Conjunction                            | Contradiction | 1.00      | 1.00          | 0.92       | 0.92 |
| Propositional | Inference     | Conjunction                            | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Conjunction                            | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunction elimination                | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunction elimination                | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunction elimination                | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunctive syllogism                  | Contradiction | 1.00      | 1.00          | 0.96       | 0.92 |
| Propositional | Inference     | Disjunctive syllogism                  | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Distributive laws                      | Contradiction | 1.00      | 1.00          | 0.92       | 0.96 |
| Propositional | Equivalent    | Distributive laws                      | Inference     | 1.00      | 0.92          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential de morgan's laws           | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential addition                   | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential addition                   | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential addition                   | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential associative laws           | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential associative laws           | Inference     | 1.00      | 0.96          | 0.96       | 1.00 |
| Predicate     | Inference     | Existential biconditional elimination  | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential biconditional introduction | Contradiction | 1.00      | 0.76          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential biconditional laws         | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential commutative laws           | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential commutative laws           | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential complement laws            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential complement laws            | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential conjunction                | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential conjunction                | Inference     | 1.00      | 0.96          | 0.92       | 0.92 |
| Predicate     | Inference     | Existential disjunction elimination    | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunction elimination    | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunction elimination    | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunctive syllogism      | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential generalization             | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential generalization             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential idempotent laws            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential idempotent laws            | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential idempotent laws            | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Existential illicit commutativity      | Inference     | 1.00      | 1.00          | 1.00       | 0.96 |
| Predicate     | Inference     | Existential modus ponens               | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential modus ponens               | Inference     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential modus tollens              | Contradiction | 1.00      | 0.96          | 1.00       | 0.92 |
| Predicate     | Inference     | Existential modus tollens              | Inference     | 1.00      | 1.00          | 0.92       | 1.00 |
| Predicate     | Inference     | Existential resolution                 | Inference     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential simplification             | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential simplification             | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential simplification             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential transitivity               | Inference     | 1.00      | 0.96          | 1.00       | 1.00 |
| Propositional | Equivalent    | Idempotent laws                        | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Idempotent laws                        | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Idempotent laws                        | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Fallacy       | Illicit commutativity                  | Inference     | 1.00      | 1.00          | 1.00       | 0.92 |
| Propositional | Inference     | Modus ponens                           | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Modus ponens                           | Unrelated     | 1.00      | 0.96          | 1.00       | 1.00 |
| Propositional | Inference     | Modus tollens                          | Contradiction | 1.00      | 1.00          | 0.92       | 1.00 |
| Propositional | Inference     | Modus tollens                          | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Modus tollens                          | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Resolution                             | Contradiction | 1.00      | 1.00          | 1.00       | 0.96 |
| Propositional | Inference     | Resolution                             | Inference     | 1.00      | 0.84          | 1.00       | 1.00 |
| Propositional | Inference     | Simplification                         | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Simplification                         | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Transitivity                           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Undistributed middle                   | Inference     | 1.00      | 1.00          | 1.00       | 0.96 |
| Predicate     | Equivalent    | Universal de morgan's laws             | Inference     | 1.00      | 1.00          | 1.00       | 0.92 |
| Predicate     | Equivalent    | Universal de morgan's laws             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal addition                     | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal addition                     | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal addition                     | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Universal affirming the consequent     | Inference     | 1.00      | 1.00          | 1.00       | 0.80 |
| Predicate     | Equivalent    | Universal associative laws             | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |

Continued on next page

Table 16: Break-down of the accuracy of Llama3 on all rules (sorted by zero-shot accuracy).

| Logic     | Rule category | Rule                                 | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|-----------|---------------|--------------------------------------|---------------|-----------|---------------|------------|------|
| Predicate | Equivalent    | Universal associative laws           | Inference     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal associative laws           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal biconditional elimination  | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal biconditional elimination  | Inference     | 1.00      | 0.88          | 0.96       | 1.00 |
| Predicate | Inference     | Universal biconditional elimination  | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal biconditional introduction | Unrelated     | 1.00      | 0.96          | 1.00       | 0.96 |
| Predicate | Equivalent    | Universal biconditional laws         | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal commutative laws           | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal commutative laws           | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal commutative laws           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal complement laws            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal complement laws            | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal complement laws            | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal conditional laws           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal conjunction                | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal conjunction                | Inference     | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate | Inference     | Universal conjunction                | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal disjunction elimination    | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal disjunction elimination    | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal disjunctive syllogism      | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal distributive laws          | Contradiction | 1.00      | 0.96          | 1.00       | 0.88 |
| Predicate | Inference     | Universal generalization             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal idempotent laws            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal idempotent laws            | Inference     | 1.00      | 0.92          | 1.00       | 1.00 |
| Predicate | Equivalent    | Universal idempotent laws            | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Fallacy       | Universal illicit commutativity      | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal instantiation              | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal instantiation              | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal modus ponens               | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal modus ponens               | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal modus tollens              | Contradiction | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate | Inference     | Universal modus tollens              | Inference     | 1.00      | 0.88          | 1.00       | 1.00 |
| Predicate | Inference     | Universal modus tollens              | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal resolution                 | Inference     | 1.00      | 0.76          | 0.96       | 1.00 |
| Predicate | Inference     | Universal simplification             | Contradiction | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate | Inference     | Universal simplification             | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal simplification             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate | Inference     | Universal transitivity               | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |

Table 17: Break-down of the accuracy of Mixtral on all rules (sorted by zero-shot accuracy).

| Logic         | Rule category | Rule                                   | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|---------------|---------------|--|---------------|-----------|---------------|------------|------|
| Predicate     | Inference     | Universal disjunction elimination      | Inference     | 0.04      | 0.16          | 0.32       | 0.32 |
| Predicate     | Inference     | Universal modus ponens                 | Inference     | 0.04      | 0.12          | 0.08       | 0.16 |
| Predicate     | Equivalent    | Existential conditional laws           | Inference     | 0.08      | 0.20          | 0.00       | 0.00 |
| Predicate     | Inference     | Universal modus tollens                | Inference     | 0.08      | 0.16          | 0.12       | 0.20 |
| Predicate     | Equivalent    | Universal conditional laws             | Inference     | 0.20      | 0.16          | 0.20       | 0.24 |
| Predicate     | Inference     | Universal addition                     | Inference     | 0.24      | 0.08          | 0.04       | 0.00 |
| Predicate     | Inference     | Universal disjunctive syllogism        | Inference     | 0.28      | 0.20          | 0.20       | 0.32 |
| Predicate     | Inference     | Existential addition                   | Inference     | 0.32      | 0.28          | 0.04       | 0.00 |
| Propositional | Equivalent    | Conditional laws                       | Inference     | 0.36      | 0.40          | 0.16       | 0.32 |
| Predicate     | Inference     | Existential modus ponens               | Inference     | 0.36      | 0.60          | 0.56       | 0.68 |
| Predicate     | Inference     | Existential transitivity               | Inference     | 0.40      | 0.36          | 0.20       | 0.36 |
| Predicate     | Inference     | Universal resolution                   | Inference     | 0.40      | 0.76          | 0.40       | 0.32 |
| Predicate     | Equivalent    | Law of quantifier negation             | Inference     | 0.44      | 0.44          | 0.72       | 0.64 |
| Predicate     | Inference     | Existential biconditional elimination  | Inference     | 0.44      | 0.60          | 0.68       | 0.76 |
| Predicate     | Inference     | Universal generalization               | Inference     | 0.44      | 0.44          | 0.36       | 0.04 |
| Predicate     | Inference     | Universal instantiation                | Inference     | 0.44      | 0.44          | 0.20       | 0.24 |
| Predicate     | Inference     | Universal transitivity                 | Inference     | 0.44      | 0.40          | 0.56       | 0.32 |
| Predicate     | Inference     | Universal biconditional introduction   | Contradiction | 0.48      | 0.76          | 0.80       | 0.84 |
| Predicate     | Equivalent    | Law of quantifier movement             | Inference     | 0.52      | 0.48          | 0.20       | 0.16 |
| Propositional | Inference     | Biconditional introduction             | Contradiction | 0.52      | 0.92          | 1.00       | 1.00 |
| Propositional | Inference     | Modus ponens                           | Inference     | 0.52      | 0.48          | 0.88       | 0.96 |
| Propositional | Inference     | Resolution                             | Unrelated     | 0.56      | 0.76          | 0.84       | 0.96 |
| Predicate     | Inference     | Universal biconditional elimination    | Inference     | 0.56      | 0.56          | 0.72       | 0.64 |
| Predicate     | Inference     | Existential disjunctive syllogism      | Inference     | 0.60      | 0.48          | 0.48       | 0.80 |
| Predicate     | Inference     | Existential biconditional introduction | Inference     | 0.64      | 0.52          | 0.32       | 0.44 |
| Predicate     | Inference     | Existential biconditional introduction | Unrelated     | 0.64      | 0.64          | 0.64       | 0.68 |
| Predicate     | Inference     | Existential resolution                 | Unrelated     | 0.64      | 0.64          | 0.64       | 0.56 |
| Predicate     | Equivalent    | Universal de morgan's laws             | Inference     | 0.64      | 0.60          | 0.60       | 0.88 |
| Predicate     | Equivalent    | Universal biconditional laws           | Inference     | 0.64      | 0.56          | 0.96       | 0.96 |
| Predicate     | Inference     | Universal conjunction                  | Inference     | 0.64      | 0.68          | 0.48       | 0.72 |
| Predicate     | Inference     | Existential biconditional introduction | Contradiction | 0.68      | 0.36          | 0.92       | 1.00 |
| Predicate     | Inference     | Existential biconditional introduction | Inference     | 0.68      | 0.56          | 0.44       | 0.68 |
| Predicate     | Inference     | Existential modus tollens              | Inference     | 0.68      | 0.64          | 0.80       | 0.88 |
| Predicate     | Inference     | Universal biconditional introduction   | Inference     | 0.68      | 0.64          | 0.80       | 0.88 |
| Propositional | Equivalent    | Biconditional laws                     | Contradiction | 0.72      | 0.92          | 0.92       | 0.92 |
| Predicate     | Equivalent    | Existential biconditional laws         | Inference     | 0.72      | 0.64          | 0.48       | 0.72 |
| Propositional | Inference     | Modus tollens                          | Inference     | 0.72      | 0.80          | 0.28       | 0.64 |
| Propositional | Inference     | Transitivity                           | Inference     | 0.72      | 0.68          | 0.48       | 0.36 |
| Predicate     | Inference     | Universal transitivity                 | Contradiction | 0.72      | 0.48          | 0.84       | 1.00 |

Table 17: Break-down of the accuracy of Mixtral on all rules (sorted by zero-shot accuracy).

| Logic         | Rule category | Rule                                  | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|---------------|---------------|---------------------------------------|---------------|-----------|---------------|------------|------|
| Predicate     | Equivalent    | Law of quantifier distribution        | Inference     | 0.76      | 0.76          | 0.60       | 0.80 |
| Predicate     | Fallacy       | Existential affirming a disjunct      | Inference     | 0.76      | 0.88          | 0.84       | 0.84 |
| Predicate     | Inference     | Existential conjunction               | Inference     | 0.76      | 0.80          | 0.44       | 0.40 |
| Predicate     | Inference     | Existential transitivity              | Unrelated     | 0.76      | 0.96          | 0.88       | 0.88 |
| Propositional | Inference     | Transitivity                          | Contradiction | 0.76      | 0.52          | 0.80       | 0.88 |
| Predicate     | Inference     | Universal simplification              | Inference     | 0.76      | 0.76          | 0.52       | 0.36 |
| Predicate     | Equivalent    | Law of quantifier movement            | Contradiction | 0.80      | 0.72          | 0.92       | 1.00 |
| Propositional | Inference     | Disjunction elimination               | Inference     | 0.80      | 0.88          | 0.80       | 0.76 |
| Propositional | Inference     | Disjunctive syllogism                 | Inference     | 0.80      | 0.84          | 0.64       | 0.64 |
| Predicate     | Inference     | Existential conjunction               | Unrelated     | 0.80      | 0.96          | 0.84       | 0.96 |
| Predicate     | Inference     | Universal biconditional elimination   | Contradiction | 0.80      | 0.92          | 0.88       | 0.88 |
| Predicate     | Equivalent    | Universal conditional laws            | Contradiction | 0.80      | 0.80          | 0.76       | 0.84 |
| Predicate     | Inference     | Universal disjunctive syllogism       | Contradiction | 0.80      | 0.84          | 0.72       | 0.92 |
| Predicate     | Equivalent    | Universal distributive laws           | Inference     | 0.80      | 0.76          | 0.16       | 0.16 |
| Predicate     | Equivalent    | Universal idempotent laws             | Inference     | 0.80      | 0.84          | 0.28       | 0.68 |
| Propositional | Inference     | Addition                              | Inference     | 0.84      | 0.76          | 0.96       | 0.32 |
| Propositional | Inference     | Biconditional elimination             | Inference     | 0.84      | 0.88          | 0.56       | 0.80 |
| Propositional | Inference     | Biconditional introduction            | Unrelated     | 0.84      | 0.96          | 0.32       | 0.08 |
| Propositional | Equivalent    | Biconditional laws                    | Inference     | 0.84      | 0.88          | 0.96       | 1.00 |
| Propositional | Fallacy       | Denying the antecedent                | Inference     | 0.84      | 0.88          | 0.76       | 0.44 |
| Predicate     | Equivalent    | Existential distributive laws         | Inference     | 0.84      | 0.76          | 0.24       | 0.48 |
| Predicate     | Inference     | Existential transitivity              | Contradiction | 0.84      | 0.52          | 0.96       | 1.00 |
| Propositional | Inference     | Resolution                            | Inference     | 0.84      | 0.96          | 0.80       | 0.64 |
| Propositional | Inference     | Simplification                        | Inference     | 0.84      | 0.80          | 1.00       | 0.84 |
| Predicate     | Equivalent    | Universal biconditional laws          | Contradiction | 0.84      | 0.88          | 0.92       | 0.92 |
| Predicate     | Equivalent    | Law of quantifier distribution        | Contradiction | 0.88      | 0.88          | 0.92       | 1.00 |
| Propositional | Fallacy       | Affirming a disjunct                  | Inference     | 0.88      | 0.96          | 0.64       | 0.88 |
| Propositional | Fallacy       | Denying a conjunct                    | Inference     | 0.88      | 0.92          | 0.92       | 0.84 |
| Predicate     | Equivalent    | Existential de morgan's laws          | Inference     | 0.88      | 0.80          | 0.72       | 0.88 |
| Predicate     | Equivalent    | Existential biconditional laws        | Contradiction | 0.88      | 0.36          | 0.88       | 1.00 |
| Predicate     | Equivalent    | Existential conditional laws          | Contradiction | 0.88      | 0.76          | 0.84       | 1.00 |
| Predicate     | Inference     | Existential generalization            | Inference     | 0.88      | 0.88          | 0.68       | 0.08 |
| Predicate     | Equivalent    | Universal complement laws             | Inference     | 0.88      | 0.80          | 0.48       | 0.80 |
| Predicate     | Inference     | Universal generalization              | Contradiction | 0.88      | 0.96          | 0.64       | 0.96 |
| Predicate     | Inference     | Universal modus ponens                | Contradiction | 0.88      | 1.00          | 0.96       | 0.96 |
| Propositional | Inference     | Biconditional elimination             | Contradiction | 0.92      | 0.88          | 0.80       | 0.96 |
| Propositional | Equivalent    | Distributive laws                     | Inference     | 0.92      | 0.96          | 0.76       | 0.84 |
| Predicate     | Equivalent    | Existential associative laws          | Inference     | 0.92      | 0.88          | 0.84       | 0.72 |
| Predicate     | Inference     | Existential biconditional elimination | Contradiction | 0.92      | 0.92          | 0.96       | 0.92 |
| Predicate     | Inference     | Existential biconditional elimination | Unrelated     | 0.92      | 1.00          | 0.96       | 0.92 |
| Predicate     | Equivalent    | Existential commutative laws          | Inference     | 0.92      | 0.88          | 0.84       | 0.68 |
| Predicate     | Fallacy       | Existential denying a conjunct        | Inference     | 0.92      | 0.92          | 0.76       | 0.80 |
| Predicate     | Fallacy       | Existential denying the antecedent    | Inference     | 0.92      | 0.88          | 0.56       | 0.64 |
| Predicate     | Inference     | Existential disjunction elimination   | Inference     | 0.92      | 0.80          | 0.72       | 0.72 |
| Predicate     | Fallacy       | Existential fallacy                   | Inference     | 0.92      | 0.84          | 0.68       | 0.84 |
| Predicate     | Inference     | Existential resolution                | Inference     | 0.92      | 0.64          | 0.60       | 0.56 |
| Predicate     | Equivalent    | Universal associative laws            | Inference     | 0.92      | 0.88          | 0.48       | 0.64 |
| Predicate     | Fallacy       | Universal denying the antecedent      | Inference     | 0.92      | 0.92          | 0.92       | 0.80 |
| Propositional | Equivalent    | De morgan's laws                      | Inference     | 0.96      | 0.92          | 1.00       | 0.96 |
| Predicate     | Equivalent    | Law of quantifier negation            | Contradiction | 0.96      | 0.96          | 0.76       | 0.72 |
| Propositional | Inference     | Biconditional introduction            | Inference     | 0.96      | 0.92          | 0.92       | 1.00 |
| Propositional | Equivalent    | Biconditional laws                    | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Commutative laws                      | Inference     | 0.96      | 0.96          | 0.76       | 0.92 |
| Propositional | Inference     | Conjunction                           | Contradiction | 0.96      | 0.96          | 0.96       | 0.92 |
| Propositional | Inference     | Conjunction                           | Inference     | 0.96      | 0.96          | 0.92       | 0.88 |
| Predicate     | Inference     | Existential disjunctive syllogism     | Unrelated     | 0.96      | 1.00          | 0.76       | 0.96 |
| Predicate     | Equivalent    | Existential distributive laws         | Contradiction | 0.96      | 1.00          | 0.88       | 1.00 |
| Predicate     | Equivalent    | Existential distributive laws         | Unrelated     | 0.96      | 1.00          | 0.80       | 0.96 |
| Predicate     | Inference     | Existential modus tollens             | Unrelated     | 0.96      | 1.00          | 0.96       | 0.96 |
| Predicate     | Inference     | Existential simplification            | Inference     | 0.96      | 0.96          | 0.60       | 0.80 |
| Predicate     | Fallacy       | Undistributed middle                  | Inference     | 0.96      | 1.00          | 0.84       | 0.88 |
| Predicate     | Equivalent    | Universal de morgan's laws            | Contradiction | 0.96      | 0.88          | 0.76       | 0.92 |
| Predicate     | Equivalent    | Universal commutative laws            | Inference     | 0.96      | 0.96          | 0.80       | 0.88 |
| Predicate     | Inference     | Universal disjunction elimination     | Contradiction | 0.96      | 1.00          | 0.96       | 1.00 |
| Predicate     | Inference     | Universal resolution                  | Contradiction | 0.96      | 0.96          | 0.88       | 0.92 |
| Propositional | Equivalent    | De morgan's laws                      | Contradiction | 1.00      | 1.00          | 1.00       | 0.88 |
| Propositional | Equivalent    | De morgan's laws                      | Unrelated     | 1.00      | 1.00          | 0.84       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier distribution        | Unrelated     | 1.00      | 1.00          | 0.80       | 0.84 |
| Predicate     | Equivalent    | Law of quantifier movement            | Unrelated     | 1.00      | 1.00          | 0.80       | 0.96 |
| Predicate     | Equivalent    | Law of quantifier negation            | Unrelated     | 1.00      | 1.00          | 0.72       | 0.76 |
| Propositional | Inference     | Addition                              | Contradiction | 1.00      | 1.00          | 0.72       | 1.00 |
| Propositional | Inference     | Addition                              | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Fallacy       | Affirming the consequent              | Inference     | 1.00      | 1.00          | 0.96       | 0.92 |
| Propositional | Equivalent    | Associative laws                      | Contradiction | 1.00      | 1.00          | 1.00       | 0.88 |
| Propositional | Equivalent    | Associative laws                      | Inference     | 1.00      | 0.96          | 0.92       | 0.88 |
| Propositional | Equivalent    | Associative laws                      | Unrelated     | 1.00      | 1.00          | 0.88       | 1.00 |
| Propositional | Inference     | Biconditional elimination             | Unrelated     | 1.00      | 1.00          | 0.92       | 1.00 |
| Propositional | Equivalent    | Commutative laws                      | Contradiction | 1.00      | 1.00          | 0.96       | 0.96 |
| Propositional | Equivalent    | Commutative laws                      | Unrelated     | 1.00      | 1.00          | 0.84       | 0.96 |
| Propositional | Equivalent    | Complement laws                       | Contradiction | 1.00      | 1.00          | 0.72       | 1.00 |
| Propositional | Equivalent    | Complement laws                       | Inference     | 1.00      | 1.00          | 0.60       | 0.88 |

Continued on next page

Table 17: Break-down of the accuracy of Mixtral on all rules (sorted by zero-shot accuracy).

| Logic         | Rule category | Rule                                 | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|---------------|---------------|--------------------------------------|---------------|-----------|---------------|------------|------|
| Propositional | Equivalent    | Complement laws                      | Unrelated     | 1.00      | 1.00          | 0.88       | 1.00 |
| Propositional | Equivalent    | Conditional laws                     | Contradiction | 1.00      | 0.96          | 0.92       | 0.96 |
| Propositional | Equivalent    | Conditional laws                     | Unrelated     | 1.00      | 1.00          | 0.88       | 1.00 |
| Propositional | Inference     | Conjunction                          | Unrelated     | 1.00      | 1.00          | 0.96       | 0.96 |
| Propositional | Inference     | Disjunction elimination              | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunction elimination              | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunctive syllogism                | Contradiction | 1.00      | 0.92          | 0.92       | 0.96 |
| Propositional | Inference     | Disjunctive syllogism                | Unrelated     | 1.00      | 1.00          | 0.96       | 1.00 |
| Propositional | Equivalent    | Distributive laws                    | Contradiction | 1.00      | 0.96          | 0.72       | 0.88 |
| Propositional | Equivalent    | Distributive laws                    | Unrelated     | 1.00      | 1.00          | 0.88       | 1.00 |
| Predicate     | Equivalent    | Existential de morgan's laws         | Contradiction | 1.00      | 1.00          | 0.92       | 0.96 |
| Predicate     | Equivalent    | Existential de morgan's laws         | Unrelated     | 1.00      | 1.00          | 0.84       | 0.92 |
| Predicate     | Inference     | Existential addition                 | Contradiction | 1.00      | 1.00          | 0.80       | 0.84 |
| Predicate     | Inference     | Existential addition                 | Unrelated     | 1.00      | 1.00          | 0.80       | 0.92 |
| Predicate     | Fallacy       | Existential affirming the consequent | Inference     | 1.00      | 1.00          | 0.96       | 1.00 |
| Predicate     | Equivalent    | Existential associative laws         | Contradiction | 1.00      | 1.00          | 0.80       | 1.00 |
| Predicate     | Equivalent    | Existential associative laws         | Unrelated     | 1.00      | 1.00          | 0.96       | 0.96 |
| Predicate     | Equivalent    | Existential biconditional laws       | Unrelated     | 1.00      | 1.00          | 0.96       | 1.00 |
| Predicate     | Equivalent    | Existential commutative laws         | Contradiction | 1.00      | 1.00          | 0.72       | 1.00 |
| Predicate     | Equivalent    | Existential commutative laws         | Unrelated     | 1.00      | 1.00          | 0.88       | 0.88 |
| Predicate     | Equivalent    | Existential complement laws          | Contradiction | 1.00      | 1.00          | 0.76       | 0.96 |
| Predicate     | Equivalent    | Existential complement laws          | Inference     | 1.00      | 0.92          | 0.72       | 0.84 |
| Predicate     | Equivalent    | Existential complement laws          | Unrelated     | 1.00      | 0.96          | 0.88       | 0.88 |
| Predicate     | Equivalent    | Existential conditional laws         | Unrelated     | 1.00      | 1.00          | 0.92       | 0.96 |
| Predicate     | Inference     | Existential conjunction              | Contradiction | 1.00      | 0.96          | 0.96       | 1.00 |
| Predicate     | Inference     | Existential disjunction elimination  | Contradiction | 1.00      | 1.00          | 0.92       | 1.00 |
| Predicate     | Inference     | Existential disjunction elimination  | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunctive syllogism    | Contradiction | 1.00      | 1.00          | 0.92       | 0.96 |
| Predicate     | Inference     | Existential generalization           | Contradiction | 1.00      | 1.00          | 0.52       | 0.80 |
| Predicate     | Inference     | Existential generalization           | Unrelated     | 1.00      | 1.00          | 0.88       | 0.88 |
| Predicate     | Equivalent    | Existential idempotent laws          | Contradiction | 1.00      | 1.00          | 0.80       | 0.92 |
| Predicate     | Equivalent    | Existential idempotent laws          | Inference     | 1.00      | 1.00          | 0.52       | 0.84 |
| Predicate     | Equivalent    | Existential idempotent laws          | Unrelated     | 1.00      | 1.00          | 0.92       | 0.96 |
| Predicate     | Fallacy       | Existential illicit commutativity    | Inference     | 1.00      | 0.96          | 0.96       | 1.00 |
| Predicate     | Inference     | Existential modus ponens             | Contradiction | 1.00      | 1.00          | 0.92       | 1.00 |
| Predicate     | Inference     | Existential modus ponens             | Unrelated     | 1.00      | 1.00          | 0.96       | 1.00 |
| Predicate     | Inference     | Existential modus tollens            | Contradiction | 1.00      | 1.00          | 0.80       | 1.00 |
| Predicate     | Inference     | Existential resolution               | Contradiction | 1.00      | 1.00          | 0.92       | 1.00 |
| Predicate     | Inference     | Existential simplification           | Contradiction | 1.00      | 1.00          | 0.64       | 0.68 |
| Predicate     | Inference     | Existential simplification           | Unrelated     | 1.00      | 1.00          | 0.96       | 0.96 |
| Propositional | Equivalent    | Idempotent laws                      | Contradiction | 1.00      | 1.00          | 0.88       | 0.80 |
| Propositional | Equivalent    | Idempotent laws                      | Inference     | 1.00      | 0.96          | 0.60       | 0.96 |
| Propositional | Equivalent    | Idempotent laws                      | Unrelated     | 1.00      | 1.00          | 0.96       | 0.96 |
| Propositional | Fallacy       | Illicit commutativity                | Inference     | 1.00      | 1.00          | 0.92       | 1.00 |
| Predicate     | Fallacy       | Illicit major                        | Inference     | 1.00      | 1.00          | 0.96       | 0.96 |
| Predicate     | Fallacy       | Illicit minor                        | Inference     | 1.00      | 0.96          | 0.96       | 1.00 |
| Propositional | Inference     | Modus ponens                         | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Modus ponens                         | Unrelated     | 1.00      | 1.00          | 0.96       | 1.00 |
| Propositional | Inference     | Modus tollens                        | Contradiction | 1.00      | 1.00          | 0.92       | 0.96 |
| Propositional | Inference     | Modus tollens                        | Unrelated     | 1.00      | 1.00          | 0.84       | 1.00 |
| Propositional | Inference     | Resolution                           | Contradiction | 1.00      | 1.00          | 0.96       | 1.00 |
| Propositional | Inference     | Simplification                       | Contradiction | 1.00      | 1.00          | 1.00       | 0.96 |
| Propositional | Inference     | Simplification                       | Unrelated     | 1.00      | 1.00          | 0.92       | 0.96 |
| Propositional | Inference     | Transitivity                         | Unrelated     | 1.00      | 1.00          | 0.88       | 0.88 |
| Predicate     | Equivalent    | Universal de morgan's laws           | Unrelated     | 1.00      | 1.00          | 0.84       | 0.96 |
| Predicate     | Inference     | Universal addition                   | Contradiction | 1.00      | 0.96          | 0.88       | 0.88 |
| Predicate     | Inference     | Universal addition                   | Unrelated     | 1.00      | 1.00          | 0.80       | 0.72 |
| Predicate     | Fallacy       | Universal affirming a disjunct       | Inference     | 1.00      | 1.00          | 0.88       | 0.96 |
| Predicate     | Fallacy       | Universal affirming the consequent   | Inference     | 1.00      | 0.96          | 0.92       | 1.00 |
| Predicate     | Equivalent    | Universal associative laws           | Contradiction | 1.00      | 1.00          | 0.96       | 1.00 |
| Predicate     | Equivalent    | Universal associative laws           | Unrelated     | 1.00      | 1.00          | 0.72       | 0.88 |
| Predicate     | Inference     | Universal biconditional elimination  | Unrelated     | 1.00      | 0.88          | 0.80       | 0.96 |
| Predicate     | Inference     | Universal biconditional introduction | Unrelated     | 1.00      | 1.00          | 0.88       | 0.92 |
| Predicate     | Equivalent    | Universal biconditional laws         | Unrelated     | 1.00      | 1.00          | 0.80       | 0.92 |
| Predicate     | Equivalent    | Universal commutative laws           | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal commutative laws           | Unrelated     | 1.00      | 1.00          | 0.76       | 0.84 |
| Predicate     | Equivalent    | Universal complement laws            | Contradiction | 1.00      | 1.00          | 0.96       | 0.88 |
| Predicate     | Equivalent    | Universal complement laws            | Unrelated     | 1.00      | 1.00          | 0.76       | 0.80 |
| Predicate     | Equivalent    | Universal conditional laws           | Unrelated     | 1.00      | 0.96          | 0.72       | 0.96 |
| Predicate     | Inference     | Universal conjunction                | Contradiction | 1.00      | 1.00          | 0.80       | 1.00 |
| Predicate     | Inference     | Universal conjunction                | Unrelated     | 1.00      | 0.88          | 0.64       | 0.68 |
| Predicate     | Fallacy       | Universal denying a conjunct         | Inference     | 1.00      | 1.00          | 0.96       | 0.96 |
| Predicate     | Inference     | Universal disjunction elimination    | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal disjunctive syllogism      | Unrelated     | 1.00      | 0.96          | 0.92       | 1.00 |
| Predicate     | Equivalent    | Universal distributive laws          | Contradiction | 1.00      | 0.96          | 0.96       | 1.00 |
| Predicate     | Equivalent    | Universal distributive laws          | Unrelated     | 1.00      | 1.00          | 0.88       | 0.96 |
| Predicate     | Inference     | Universal generalization             | Unrelated     | 1.00      | 1.00          | 0.80       | 0.80 |
| Predicate     | Equivalent    | Universal idempotent laws            | Contradiction | 1.00      | 0.92          | 1.00       | 0.88 |
| Predicate     | Equivalent    | Universal idempotent laws            | Unrelated     | 1.00      | 0.84          | 0.88       | 0.84 |
| Predicate     | Fallacy       | Universal illicit commutativity      | Inference     | 1.00      | 1.00          | 0.92       | 1.00 |
| Predicate     | Inference     | Universal instantiation              | Contradiction | 1.00      | 0.96          | 0.68       | 0.84 |

Continued on next page

Table 17: Break-down of the accuracy of Mixtral on all rules (sorted by zero-shot accuracy).

| Logic     | Rule category | Rule                     | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|-----------|---------------|--------------------------|---------------|-----------|---------------|------------|------|
| Predicate | Inference     | Universal instantiation  | Unrelated     | 1.00      | 1.00          | 0.44       | 0.84 |
| Predicate | Inference     | Universal modus ponens   | Unrelated     | 1.00      | 0.96          | 0.84       | 1.00 |
| Predicate | Inference     | Universal modus tollens  | Contradiction | 1.00      | 1.00          | 0.96       | 1.00 |
| Predicate | Inference     | Universal modus tollens  | Unrelated     | 1.00      | 1.00          | 0.96       | 0.92 |
| Predicate | Inference     | Universal resolution     | Unrelated     | 1.00      | 1.00          | 0.88       | 0.96 |
| Predicate | Inference     | Universal simplification | Contradiction | 1.00      | 1.00          | 0.80       | 0.84 |
| Predicate | Inference     | Universal simplification | Unrelated     | 1.00      | 0.96          | 0.72       | 0.84 |
| Predicate | Inference     | Universal transitivity   | Unrelated     | 1.00      | 1.00          | 0.88       | 0.96 |

Table 18: Break-down of the accuracy of ChatGPT on all rules (sorted by zero-shot accuracy).

| Logic         | Rule category | Rule                                   | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|---------------|---------------|--|---------------|-----------|---------------|------------|------|
| Predicate     | Inference     | Existential resolution                 | Unrelated     | 0.00      | 0.40          | 0.72       | 0.52 |
| Predicate     | Equivalent    | Universal de morgan's laws             | Contradiction | 0.04      | 0.40          | 0.24       | 0.12 |
| Predicate     | Equivalent    | Existential de morgan's laws           | Contradiction | 0.08      | 0.96          | 0.64       | 1.00 |
| Predicate     | Inference     | Existential biconditional introduction | Unrelated     | 0.08      | 0.52          | 0.40       | 0.52 |
| Predicate     | Inference     | Universal instantiation                | Inference     | 0.08      | 0.44          | 0.04       | 0.12 |
| Predicate     | Fallacy       | Existential denying the antecedent     | Inference     | 0.12      | 0.88          | 0.76       | 0.68 |
| Predicate     | Inference     | Universal disjunctive syllogism        | Contradiction | 0.12      | 0.60          | 0.20       | 0.04 |
| Predicate     | Inference     | Universal transitivity                 | Contradiction | 0.12      | 0.32          | 1.00       | 0.68 |
| Propositional | Equivalent    | De morgan's laws                       | Contradiction | 0.20      | 0.80          | 0.56       | 0.56 |
| Predicate     | Fallacy       | Existential denying a conjunct         | Inference     | 0.20      | 0.60          | 0.76       | 0.76 |
| Predicate     | Equivalent    | Universal distributive laws            | Inference     | 0.24      | 0.04          | 0.24       | 0.20 |
| Predicate     | Inference     | Universal transitivity                 | Inference     | 0.24      | 0.00          | 0.44       | 1.00 |
| Propositional | Inference     | Biconditional introduction             | Unrelated     | 0.28      | 0.72          | 0.00       | 0.64 |
| Predicate     | Equivalent    | Existential de morgan's laws           | Unrelated     | 0.28      | 0.92          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Existential affirming a disjunct       | Inference     | 0.28      | 0.72          | 0.84       | 0.48 |
| Predicate     | Equivalent    | Universal conditional laws             | Inference     | 0.28      | 0.08          | 0.28       | 0.52 |
| Predicate     | Inference     | Universal disjunction elimination      | Inference     | 0.28      | 0.00          | 0.72       | 0.96 |
| Propositional | Equivalent    | Conditional laws                       | Inference     | 0.32      | 0.16          | 0.12       | 0.44 |
| Predicate     | Inference     | Existential transitivity               | Unrelated     | 0.32      | 0.60          | 0.92       | 0.24 |
| Predicate     | Inference     | Universal conjunction                  | Inference     | 0.32      | 0.24          | 0.64       | 0.48 |
| Propositional | Inference     | Resolution                             | Unrelated     | 0.36      | 0.72          | 0.96       | 0.64 |
| Predicate     | Fallacy       | Universal denying the antecedent       | Inference     | 0.36      | 0.96          | 0.72       | 0.32 |
| Predicate     | Inference     | Universal generalization               | Inference     | 0.36      | 0.28          | 0.00       | 0.04 |
| Predicate     | Equivalent    | Law of quantifier movement             | Inference     | 0.40      | 0.32          | 0.00       | 0.20 |
| Propositional | Equivalent    | Biconditional laws                     | Contradiction | 0.40      | 0.68          | 0.68       | 0.04 |
| Propositional | Inference     | Transitivity                           | Inference     | 0.40      | 0.16          | 0.80       | 1.00 |
| Predicate     | Inference     | Universal resolution                   | Contradiction | 0.40      | 0.92          | 0.96       | 0.40 |
| Predicate     | Equivalent    | Existential conditional laws           | Inference     | 0.44      | 0.04          | 0.16       | 0.44 |
| Propositional | Inference     | Conjunction                            | Inference     | 0.48      | 0.56          | 0.84       | 0.84 |
| Predicate     | Fallacy       | Existential fallacy                    | Inference     | 0.48      | 0.80          | 0.76       | 0.60 |
| Predicate     | Inference     | Universal modus tollens                | Inference     | 0.48      | 0.16          | 0.48       | 0.52 |
| Propositional | Inference     | Addition                               | Inference     | 0.52      | 0.16          | 0.08       | 0.12 |
| Predicate     | Equivalent    | Existential biconditional laws         | Contradiction | 0.52      | 0.92          | 0.68       | 0.64 |
| Predicate     | Equivalent    | Existential conditional laws           | Unrelated     | 0.52      | 0.92          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunctive syllogism      | Unrelated     | 0.52      | 0.88          | 0.92       | 1.00 |
| Predicate     | Inference     | Existential modus tollens              | Unrelated     | 0.52      | 1.00          | 0.96       | 1.00 |
| Predicate     | Inference     | Existential transitivity               | Inference     | 0.52      | 0.12          | 0.32       | 0.80 |
| Propositional | Inference     | Modus tollens                          | Inference     | 0.52      | 0.44          | 0.84       | 0.96 |
| Propositional | Inference     | Transitivity                           | Unrelated     | 0.52      | 0.68          | 0.76       | 0.44 |
| Predicate     | Inference     | Universal addition                     | Inference     | 0.52      | 0.08          | 0.00       | 0.00 |
| Predicate     | Inference     | Universal disjunctive syllogism        | Inference     | 0.52      | 0.24          | 0.48       | 0.56 |
| Propositional | Equivalent    | Associative laws                       | Unrelated     | 0.56      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential biconditional elimination  | Unrelated     | 0.56      | 0.92          | 0.96       | 1.00 |
| Predicate     | Equivalent    | Existential conditional laws           | Contradiction | 0.56      | 0.80          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal generalization               | Contradiction | 0.56      | 0.88          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal modus ponens                 | Inference     | 0.56      | 0.08          | 0.60       | 0.88 |
| Propositional | Inference     | Disjunctive syllogism                  | Unrelated     | 0.60      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential associative laws           | Unrelated     | 0.60      | 0.96          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential commutative laws           | Unrelated     | 0.60      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential modus ponens               | Unrelated     | 0.60      | 0.96          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal conditional laws             | Contradiction | 0.60      | 0.84          | 1.00       | 0.64 |
| Predicate     | Fallacy       | Universal denying a conjunct           | Inference     | 0.60      | 0.96          | 1.00       | 0.72 |
| Propositional | Equivalent    | Conditional laws                       | Contradiction | 0.64      | 0.76          | 1.00       | 0.68 |
| Propositional | Fallacy       | Denying a conjunct                     | Inference     | 0.64      | 1.00          | 1.00       | 0.76 |
| Predicate     | Inference     | Existential conjunction                | Contradiction | 0.64      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential conjunction                | Unrelated     | 0.64      | 0.92          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Illicit major                          | Inference     | 0.64      | 0.84          | 0.96       | 0.60 |
| Predicate     | Equivalent    | Universal biconditional laws           | Contradiction | 0.64      | 0.96          | 0.48       | 0.24 |
| Propositional | Inference     | Disjunctive syllogism                  | Inference     | 0.68      | 0.48          | 0.88       | 0.96 |
| Predicate     | Equivalent    | Existential biconditional laws         | Unrelated     | 0.68      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential simplification             | Unrelated     | 0.68      | 0.84          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal disjunctive syllogism        | Unrelated     | 0.68      | 0.96          | 0.96       | 0.92 |
| Predicate     | Inference     | Universal simplification               | Inference     | 0.68      | 0.44          | 0.04       | 0.12 |
| Propositional | Equivalent    | De morgan's laws                       | Unrelated     | 0.72      | 0.92          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional elimination              | Inference     | 0.72      | 0.60          | 0.84       | 1.00 |
| Propositional | Equivalent    | Distributive laws                      | Unrelated     | 0.72      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential addition                   | Unrelated     | 0.72      | 0.96          | 1.00       | 1.00 |
| Propositional | Inference     | Transitivity                           | Contradiction | 0.72      | 0.92          | 1.00       | 1.00 |

Table 18: Break-down of the accuracy of ChatGPT on all rules (sorted by zero-shot accuracy).

| Logic         | Rule category | Rule                                   | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|---------------|---------------|--|---------------|-----------|---------------|------------|------|
| Predicate     | Equivalent    | Universal associative laws             | Inference     | 0.72      | 0.44          | 0.96       | 0.64 |
| Predicate     | Inference     | Universal biconditional elimination    | Contradiction | 0.72      | 0.72          | 0.68       | 0.36 |
| Predicate     | Inference     | Universal biconditional elimination    | Inference     | 0.72      | 0.24          | 0.60       | 0.92 |
| Propositional | Fallacy       | Affirming a disjunct                   | Inference     | 0.76      | 0.92          | 0.84       | 0.48 |
| Propositional | Equivalent    | Biconditional laws                     | Unrelated     | 0.76      | 1.00          | 0.96       | 0.92 |
| Propositional | Equivalent    | Distributive laws                      | Inference     | 0.76      | 0.32          | 0.20       | 0.60 |
| Predicate     | Inference     | Existential conjunction                | Inference     | 0.76      | 0.72          | 0.60       | 0.52 |
| Predicate     | Equivalent    | Existential distributive laws          | Unrelated     | 0.76      | 0.88          | 1.00       | 1.00 |
| Propositional | Equivalent    | Idempotent laws                        | Unrelated     | 0.76      | 1.00          | 0.96       | 0.96 |
| Predicate     | Inference     | Universal biconditional elimination    | Unrelated     | 0.76      | 1.00          | 1.00       | 0.96 |
| Predicate     | Equivalent    | Universal distributive laws            | Contradiction | 0.76      | 0.96          | 1.00       | 0.68 |
| Predicate     | Inference     | Universal modus tollens                | Unrelated     | 0.76      | 0.96          | 1.00       | 1.00 |
| Propositional | Equivalent    | Associative laws                       | Inference     | 0.80      | 0.92          | 1.00       | 1.00 |
| Propositional | Equivalent    | Commutative laws                       | Unrelated     | 0.80      | 1.00          | 1.00       | 0.96 |
| Propositional | Inference     | Disjunctive syllogism                  | Contradiction | 0.80      | 0.96          | 0.28       | 0.00 |
| Predicate     | Inference     | Existential biconditional elimination  | Inference     | 0.80      | 0.32          | 0.56       | 0.72 |
| Predicate     | Equivalent    | Existential complement laws            | Unrelated     | 0.80      | 0.96          | 1.00       | 1.00 |
| Propositional | Inference     | Simplification                         | Unrelated     | 0.80      | 0.96          | 0.96       | 0.96 |
| Predicate     | Inference     | Universal biconditional introduction   | Unrelated     | 0.80      | 0.92          | 0.68       | 0.96 |
| Predicate     | Inference     | Universal disjunction elimination      | Contradiction | 0.80      | 0.48          | 1.00       | 0.52 |
| Predicate     | Equivalent    | Law of quantifier distribution         | Unrelated     | 0.84      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Conditional laws                       | Unrelated     | 0.84      | 0.88          | 0.96       | 0.96 |
| Propositional | Inference     | Conjunction                            | Unrelated     | 0.84      | 1.00          | 1.00       | 1.00 |
| Propositional | Fallacy       | Denying the antecedent                 | Inference     | 0.84      | 0.80          | 0.48       | 0.08 |
| Predicate     | Inference     | Existential disjunctive syllogism      | Contradiction | 0.84      | 0.88          | 0.96       | 1.00 |
| Predicate     | Inference     | Existential generalization             | Inference     | 0.84      | 0.60          | 0.28       | 0.40 |
| Predicate     | Equivalent    | Existential idempotent laws            | Unrelated     | 0.84      | 0.96          | 1.00       | 1.00 |
| Propositional | Inference     | Modus ponens                           | Unrelated     | 0.84      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Modus tollens                          | Unrelated     | 0.84      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal conjunction                  | Contradiction | 0.84      | 0.80          | 1.00       | 0.96 |
| Predicate     | Inference     | Universal conjunction                  | Unrelated     | 0.84      | 1.00          | 1.00       | 1.00 |
| Predicate     | Fallacy       | Existential affirming the consequent   | Inference     | 0.88      | 0.88          | 1.00       | 0.88 |
| Predicate     | Equivalent    | Existential biconditional laws         | Inference     | 0.88      | 0.40          | 0.88       | 0.92 |
| Predicate     | Inference     | Existential transitivity               | Contradiction | 0.88      | 0.96          | 1.00       | 0.96 |
| Predicate     | Equivalent    | Universal de morgan's laws             | Unrelated     | 0.88      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal addition                     | Unrelated     | 0.88      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal idempotent laws              | Unrelated     | 0.88      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal modus ponens                 | Unrelated     | 0.88      | 0.96          | 1.00       | 0.96 |
| Predicate     | Inference     | Universal modus tollens                | Contradiction | 0.88      | 1.00          | 1.00       | 0.92 |
| Predicate     | Inference     | Universal resolution                   | Unrelated     | 0.88      | 1.00          | 1.00       | 0.88 |
| Predicate     | Equivalent    | Law of quantifier distribution         | Contradiction | 0.92      | 0.92          | 0.80       | 0.72 |
| Predicate     | Equivalent    | Law of quantifier movement             | Contradiction | 0.92      | 0.80          | 0.96       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier movement             | Unrelated     | 0.92      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional introduction             | Contradiction | 0.92      | 0.84          | 0.08       | 0.28 |
| Propositional | Equivalent    | Commutative laws                       | Inference     | 0.92      | 0.92          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential associative laws           | Contradiction | 0.92      | 0.96          | 1.00       | 0.96 |
| Predicate     | Equivalent    | Existential distributive laws          | Inference     | 0.92      | 0.24          | 0.40       | 0.40 |
| Propositional | Inference     | Modus ponens                           | Inference     | 0.92      | 0.80          | 1.00       | 0.84 |
| Predicate     | Equivalent    | Universal associative laws             | Contradiction | 0.92      | 1.00          | 1.00       | 0.84 |
| Predicate     | Inference     | Universal biconditional introduction   | Contradiction | 0.92      | 0.96          | 0.60       | 0.84 |
| Predicate     | Equivalent    | Universal distributive laws            | Unrelated     | 0.92      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal instantiation                | Contradiction | 0.92      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal simplification               | Unrelated     | 0.92      | 0.96          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier distribution         | Inference     | 0.96      | 0.76          | 0.72       | 0.80 |
| Predicate     | Equivalent    | Law of quantifier negation             | Unrelated     | 0.96      | 0.96          | 1.00       | 1.00 |
| Propositional | Inference     | Addition                               | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Propositional | Fallacy       | Affirming the consequent               | Inference     | 0.96      | 0.88          | 0.96       | 0.92 |
| Propositional | Equivalent    | Associative laws                       | Contradiction | 0.96      | 1.00          | 0.96       | 0.92 |
| Propositional | Inference     | Biconditional elimination              | Contradiction | 0.96      | 1.00          | 0.92       | 0.88 |
| Propositional | Inference     | Biconditional elimination              | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Biconditional introduction             | Inference     | 0.96      | 0.44          | 1.00       | 1.00 |
| Propositional | Equivalent    | Biconditional laws                     | Inference     | 0.96      | 0.60          | 1.00       | 1.00 |
| Propositional | Equivalent    | Commutative laws                       | Contradiction | 0.96      | 0.76          | 0.96       | 0.60 |
| Predicate     | Equivalent    | Existential associative laws           | Inference     | 0.96      | 0.88          | 0.96       | 0.84 |
| Predicate     | Inference     | Existential biconditional introduction | Inference     | 0.96      | 0.40          | 0.96       | 1.00 |
| Predicate     | Equivalent    | Existential commutative laws           | Contradiction | 0.96      | 1.00          | 1.00       | 0.96 |
| Predicate     | Inference     | Existential modus ponens               | Inference     | 0.96      | 0.28          | 0.76       | 0.96 |
| Predicate     | Inference     | Existential modus tollens              | Inference     | 0.96      | 0.32          | 0.32       | 0.32 |
| Predicate     | Fallacy       | Illicit minor                          | Inference     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal addition                     | Contradiction | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal associative laws             | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal biconditional introduction   | Inference     | 0.96      | 0.52          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal biconditional laws           | Inference     | 0.96      | 0.36          | 0.84       | 0.96 |
| Predicate     | Inference     | Universal disjunction elimination      | Unrelated     | 0.96      | 1.00          | 0.96       | 0.80 |
| Predicate     | Inference     | Universal generalization               | Unrelated     | 0.96      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal instantiation                | Unrelated     | 0.96      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal modus ponens                 | Contradiction | 0.96      | 0.88          | 1.00       | 0.80 |
| Propositional | Equivalent    | De morgan's laws                       | Inference     | 1.00      | 0.16          | 0.52       | 1.00 |
| Predicate     | Equivalent    | Law of quantifier negation             | Contradiction | 1.00      | 1.00          | 0.88       | 0.80 |
| Predicate     | Equivalent    | Law of quantifier negation             | Inference     | 1.00      | 0.48          | 0.48       | 0.68 |
| Propositional | Inference     | Addition                               | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |

Continued on next page

Table 18: Break-down of the accuracy of ChatGPT on all rules (sorted by zero-shot accuracy).

| Logic         | Rule category | Rule                                   | Problem       | Zero shot | Zero shot cot | Random icl | Weak |
|---------------|---------------|--|---------------|-----------|---------------|------------|------|
| Propositional | Equivalent    | Complement laws                        | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Complement laws                        | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Complement laws                        | Unrelated     | 1.00      | 0.92          | 1.00       | 1.00 |
| Propositional | Inference     | Conjunction                            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Disjunction elimination                | Contradiction | 1.00      | 1.00          | 1.00       | 0.20 |
| Propositional | Inference     | Disjunction elimination                | Inference     | 1.00      | 0.60          | 0.92       | 1.00 |
| Propositional | Inference     | Disjunction elimination                | Unrelated     | 1.00      | 1.00          | 1.00       | 0.96 |
| Propositional | Equivalent    | Distributive laws                      | Contradiction | 1.00      | 0.84          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential de morgan's laws           | Inference     | 1.00      | 0.76          | 0.24       | 0.80 |
| Predicate     | Inference     | Existential addition                   | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential addition                   | Inference     | 1.00      | 0.12          | 0.20       | 0.00 |
| Predicate     | Inference     | Existential biconditional elimination  | Contradiction | 1.00      | 1.00          | 1.00       | 0.96 |
| Predicate     | Inference     | Existential biconditional introduction | Contradiction | 1.00      | 0.84          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential commutative laws           | Inference     | 1.00      | 0.64          | 0.92       | 0.88 |
| Predicate     | Equivalent    | Existential complement laws            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential complement laws            | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential disjunction elimination    | Contradiction | 1.00      | 1.00          | 1.00       | 0.84 |
| Predicate     | Inference     | Existential disjunction elimination    | Inference     | 1.00      | 0.72          | 0.68       | 1.00 |
| Predicate     | Inference     | Existential disjunction elimination    | Unrelated     | 1.00      | 1.00          | 1.00       | 0.96 |
| Predicate     | Inference     | Existential disjunctive syllogism      | Inference     | 1.00      | 0.36          | 0.48       | 0.44 |
| Predicate     | Equivalent    | Existential distributive laws          | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential generalization             | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential generalization             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential idempotent laws            | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Existential idempotent laws            | Inference     | 1.00      | 1.00          | 0.88       | 0.88 |
| Predicate     | Fallacy       | Existential illicit commutativity      | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential modus ponens               | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential modus tollens              | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential resolution                 | Contradiction | 1.00      | 0.96          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential resolution                 | Inference     | 1.00      | 0.64          | 0.52       | 0.84 |
| Predicate     | Inference     | Existential simplification             | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Existential simplification             | Inference     | 1.00      | 0.88          | 0.96       | 0.68 |
| Propositional | Equivalent    | Idempotent laws                        | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Equivalent    | Idempotent laws                        | Inference     | 1.00      | 1.00          | 0.56       | 0.52 |
| Propositional | Fallacy       | Illicit commutativity                  | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Modus ponens                           | Contradiction | 1.00      | 1.00          | 1.00       | 0.76 |
| Propositional | Inference     | Modus tollens                          | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Resolution                             | Contradiction | 1.00      | 0.72          | 1.00       | 0.92 |
| Propositional | Inference     | Resolution                             | Inference     | 1.00      | 0.60          | 1.00       | 1.00 |
| Propositional | Inference     | Simplification                         | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Propositional | Inference     | Simplification                         | Inference     | 1.00      | 1.00          | 1.00       | 0.92 |
| Predicate     | Fallacy       | Undistributed middle                   | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal de morgan's laws             | Inference     | 1.00      | 0.32          | 0.68       | 0.88 |
| Predicate     | Fallacy       | Universal affirming a disjunct         | Inference     | 1.00      | 1.00          | 1.00       | 0.92 |
| Predicate     | Fallacy       | Universal affirming the consequent     | Inference     | 1.00      | 1.00          | 1.00       | 0.96 |
| Predicate     | Equivalent    | Universal biconditional laws           | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal commutative laws             | Contradiction | 1.00      | 1.00          | 1.00       | 0.96 |
| Predicate     | Equivalent    | Universal commutative laws             | Inference     | 1.00      | 0.56          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal commutative laws             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal complement laws              | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal complement laws              | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal complement laws              | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal conditional laws             | Unrelated     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal idempotent laws              | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Equivalent    | Universal idempotent laws              | Inference     | 1.00      | 0.88          | 0.68       | 0.80 |
| Predicate     | Fallacy       | Universal illicit commutativity        | Inference     | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal resolution                   | Inference     | 1.00      | 0.16          | 0.24       | 0.88 |
| Predicate     | Inference     | Universal simplification               | Contradiction | 1.00      | 1.00          | 1.00       | 1.00 |
| Predicate     | Inference     | Universal transitivity                 | Unrelated     | 1.00      | 1.00          | 0.96       | 0.36 |