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Semantically-Aligned Equation Generation for Solving and Reasoning Math Word Problems

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https://github.com/MiuLab/E2EMathSolver

Math Word Problem

Each notebook takes ***$0.5*** *and each pen takes* ***$1****. Tom has* ***$10****. How many notebooks can he buy after buying* ***5*** *pens?*

Reasoning & Solving

$$x = 10 - 1 \times 5 \div 0.5$$

Prior Work

Non-neural approaches

- Template-based
  (Kushman et al., Upadhyay and Chang)

$x = (? + ?) \times ? - ?$

fill

$x = (1 + 2) \times 3 - 4$

Rely on hand-crafted features!

Deep learning

- Seq2Seq
  (Wang et al., Ling et al.)

Problem

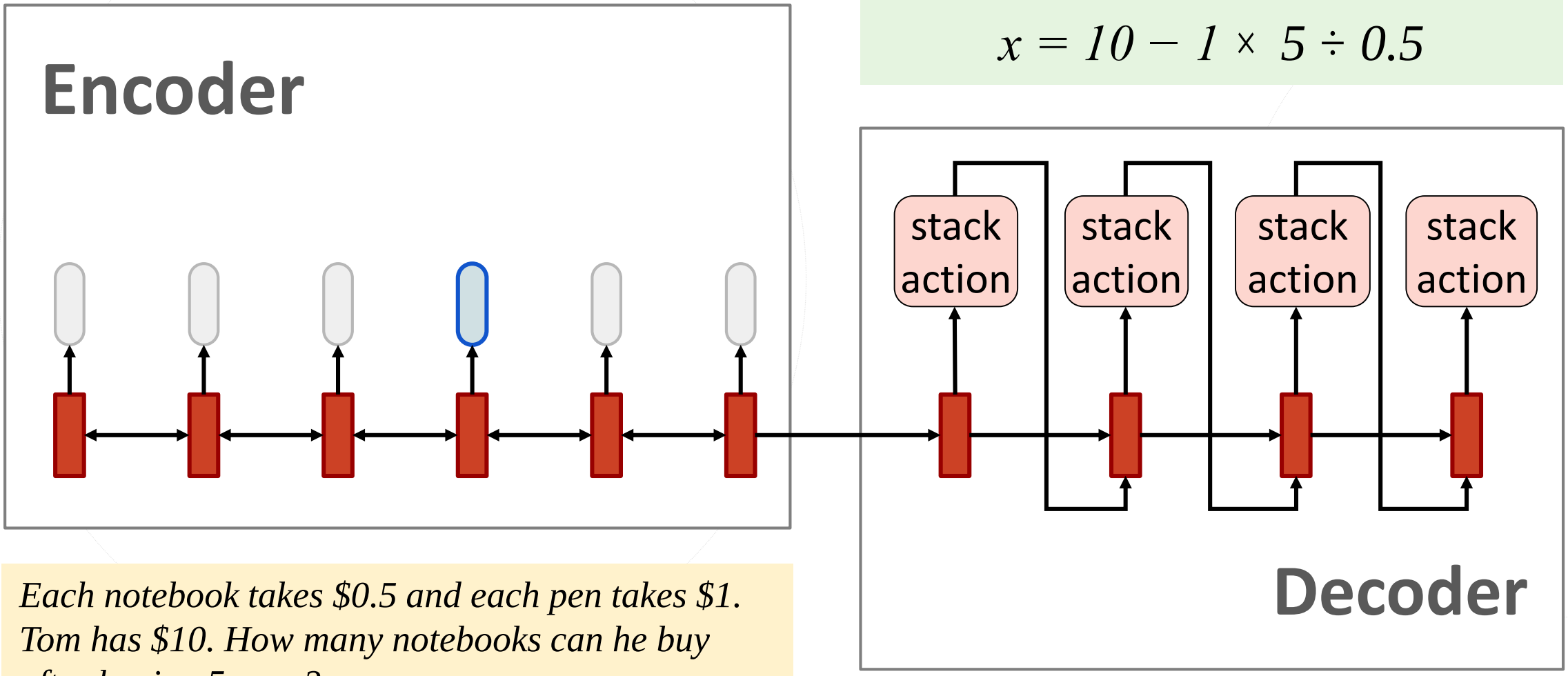
generate

$x = (1 + 2) \times 3 - 4$

Does not use the structure of math expression.

Our model is **end-to-end** and **structural**!

Overview of the Proposed Model



Each notebook takes $0.5 and each pen takes $1. Tom has $10. How many notebooks can he buy after buying 5 pens?

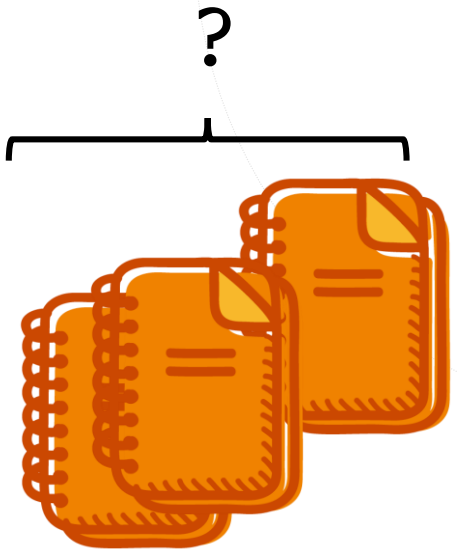
Look Again at the Problem

*Each notebook takes $**0.5** and each pen takes $**1**. Tom has $**10**. How many notebooks can he buy after buying **5** pens?*

$0.5

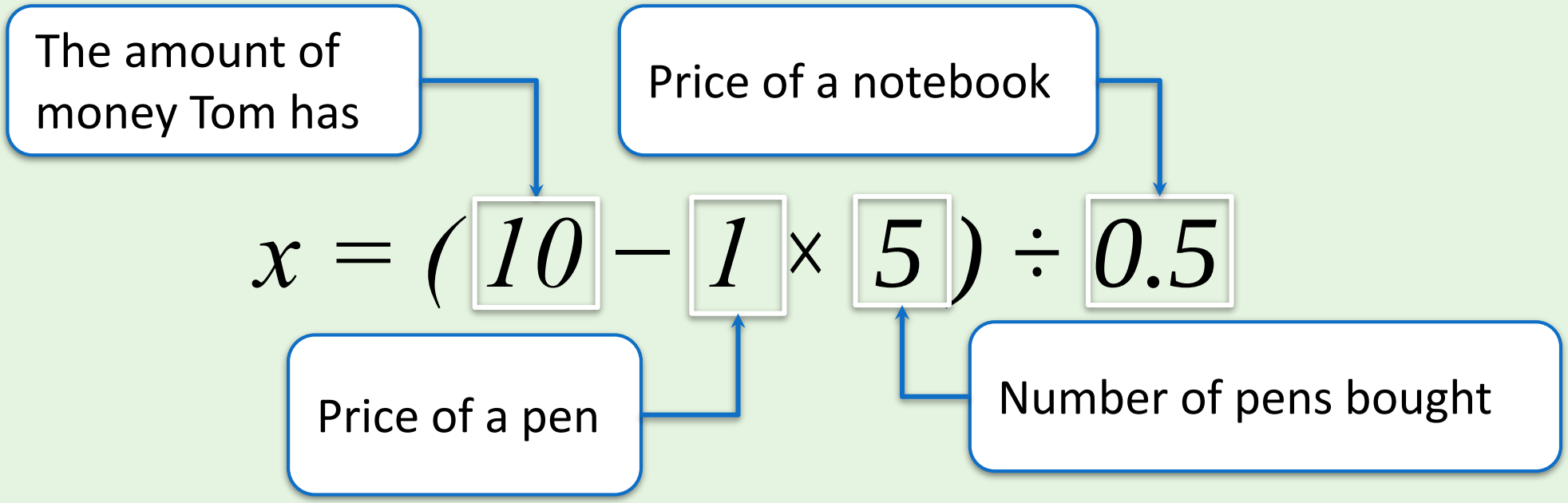
$1

$10

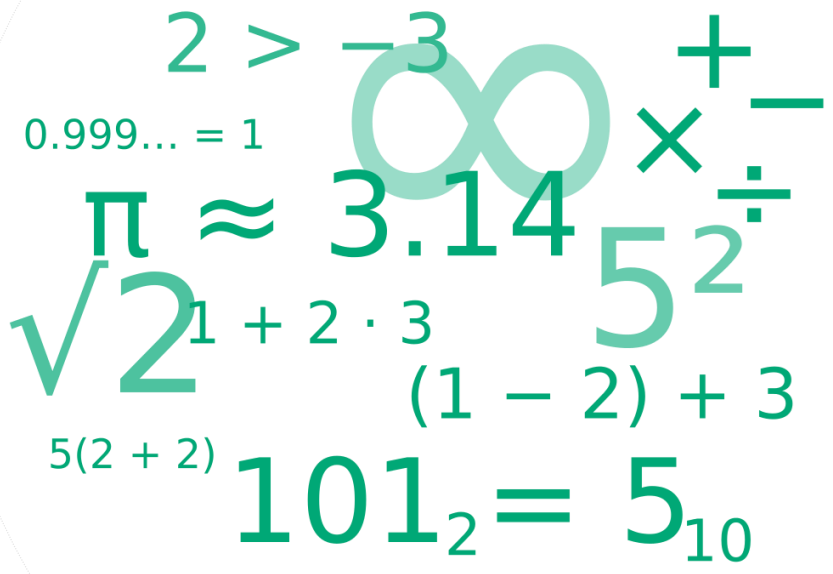


Semantic Meaning of the Operands

*Each notebook takes $**0.5** and each pen takes $**1**. Tom has $**10**. How many notebooks can he buy after buying **5** pens?*



Idea: Bridging Symbolic and Semantic Worlds



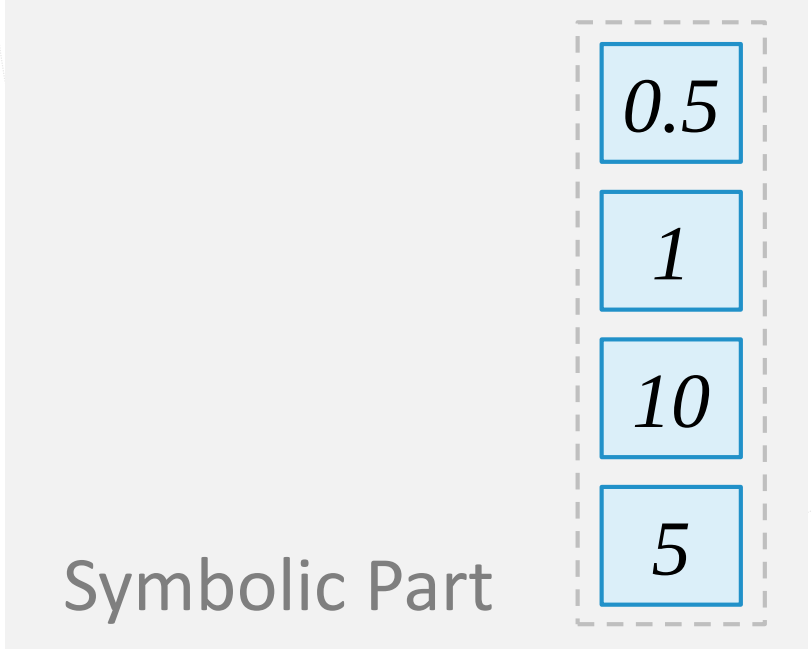
Symbolic World

Semantic World

Preprocess

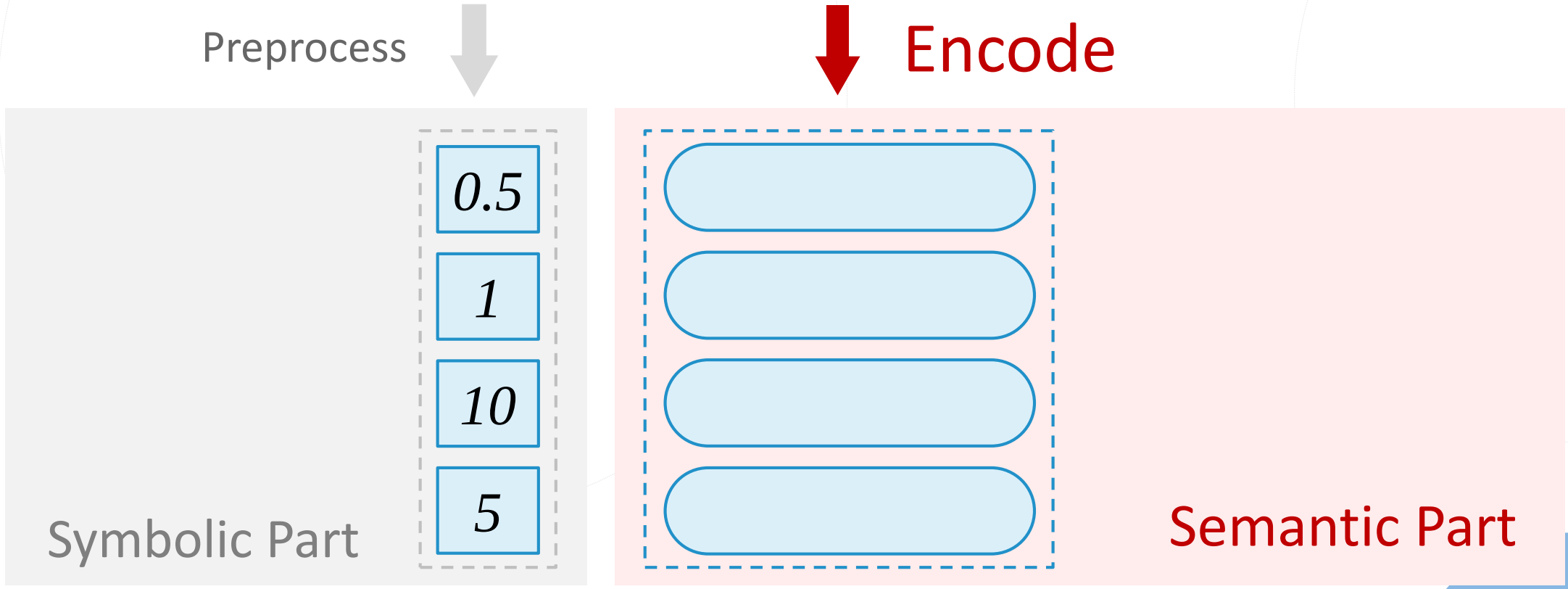
*Each notebook takes $**0.5** and each pen takes $**1**. Tom has $**10**. How many notebooks can he buy after buying **5** pens?*

Preprocess

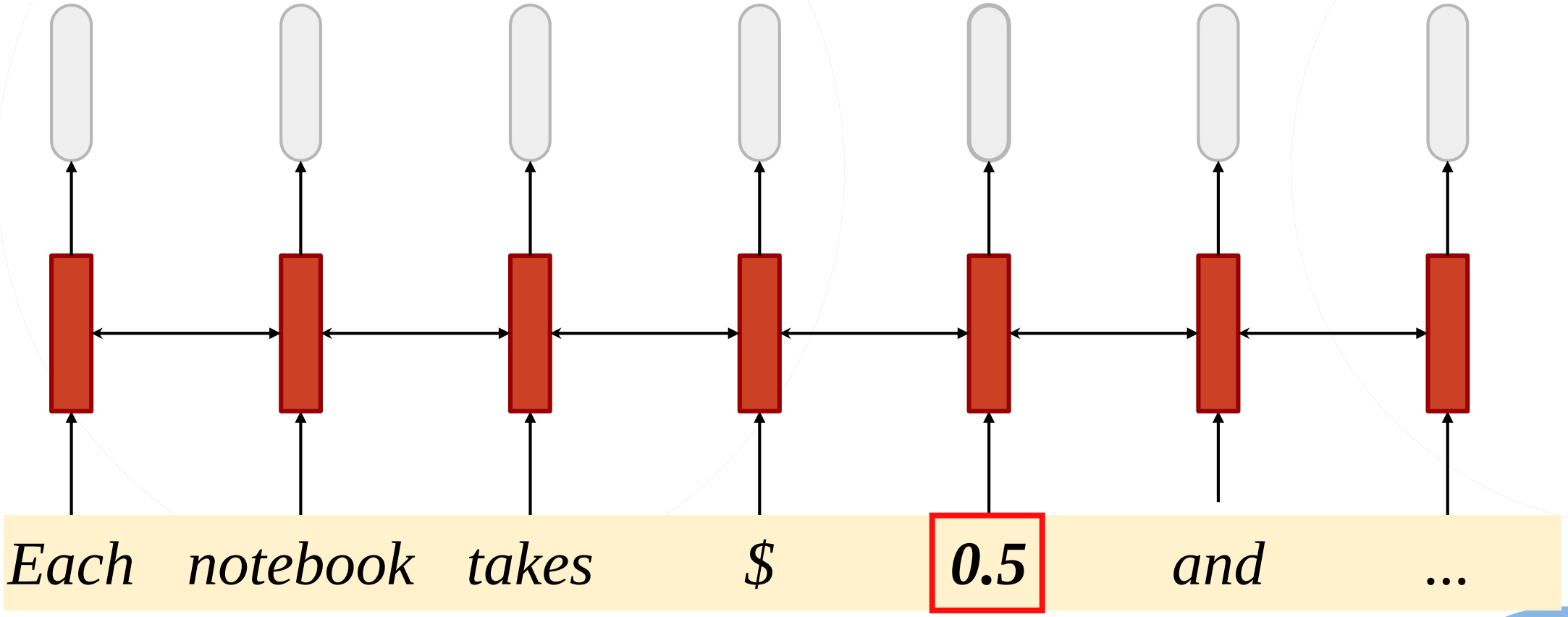


Symbol Encoding

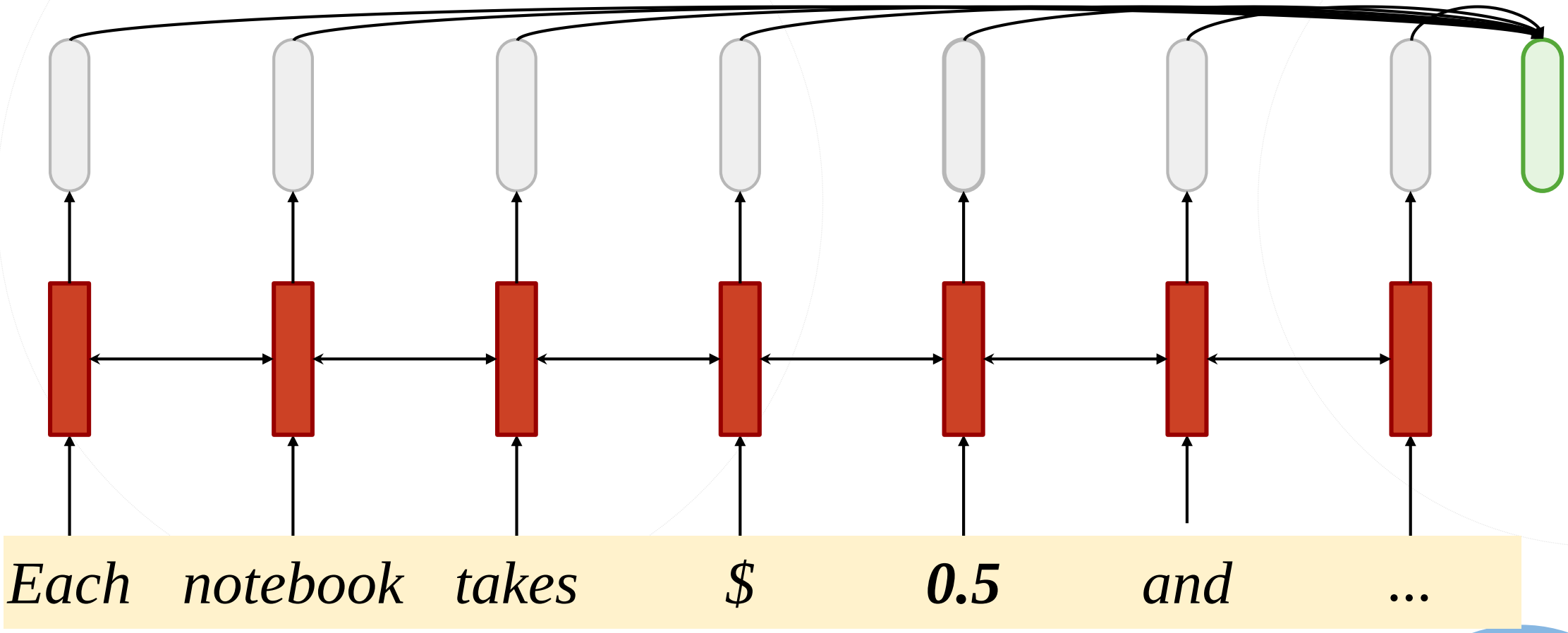
*Each notebook takes $**0.5** and each pen takes $**1**. Tom has $**10**. How many notebooks can he buy after buying **5** pens?*



Inside Encoder



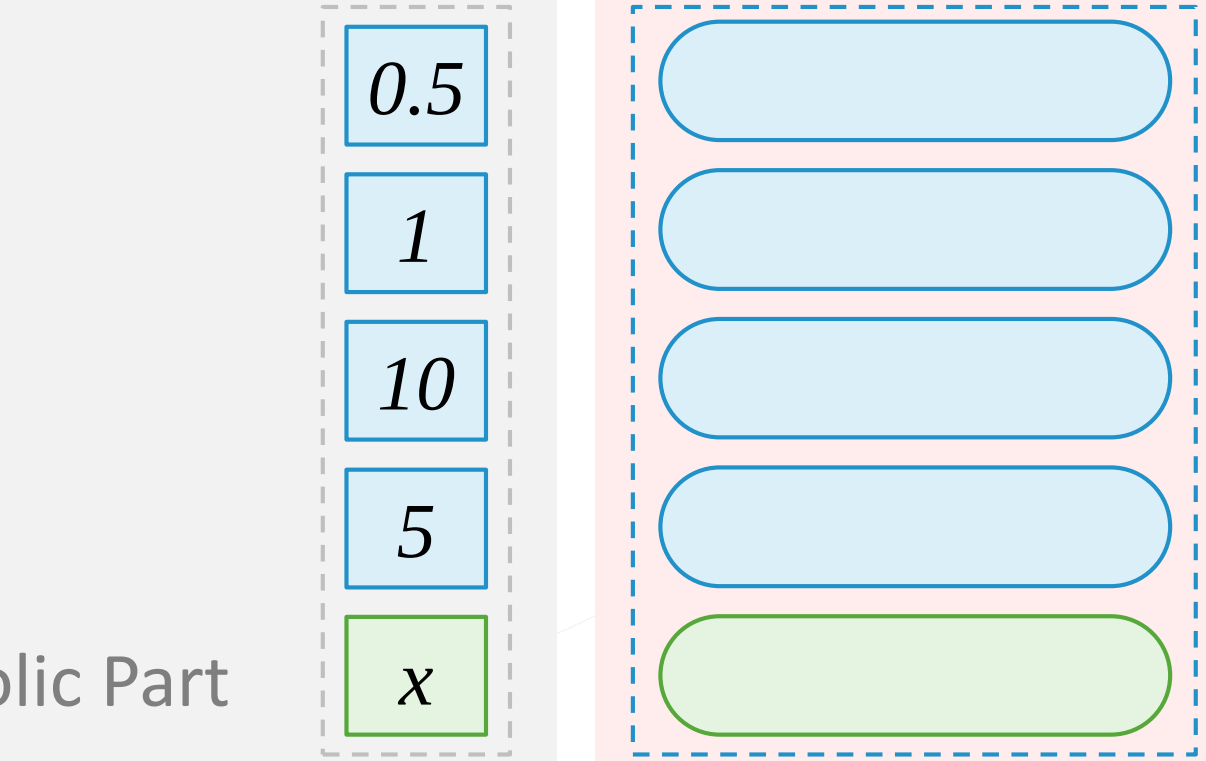
Semantic Generation for Unknown *x*



* This part is actually done when decoding, but is present at this place for illustration. Check our paper for more information.

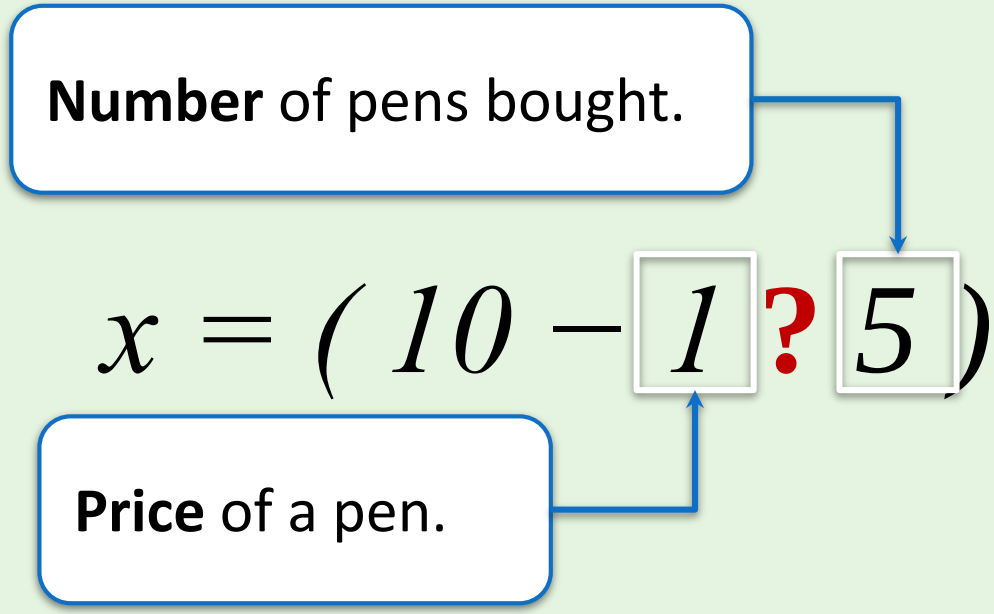
Operands & Their Semantics

*Each notebook takes $**0.5** and each pen takes $**1**. Tom has $**10**. How many notebooks can he buy after buying **5** pens?*



Intuition of Using Semantics

*Each notebook takes $**0.5** and each pen takes $**1**. Tom has $**10**. How many notebooks can he buy after buying **5** pens?*



Equation Generation in Postfix

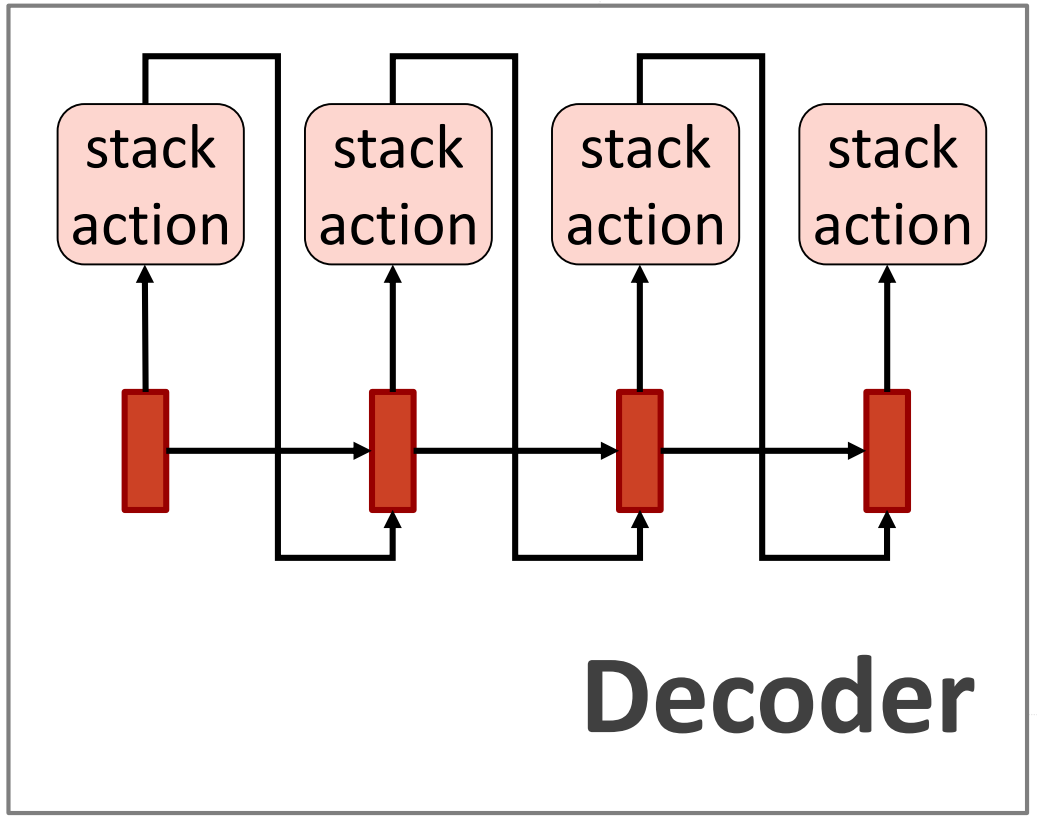
*Each notebook takes $**0.5** and each pen takes $**1**. Tom has $**10**. How many notebooks can he buy after buying **5** pens?*

$$x \ 10 \ 1 \ 5 \ \times \ - \ 0.5 \ \div \ =$$

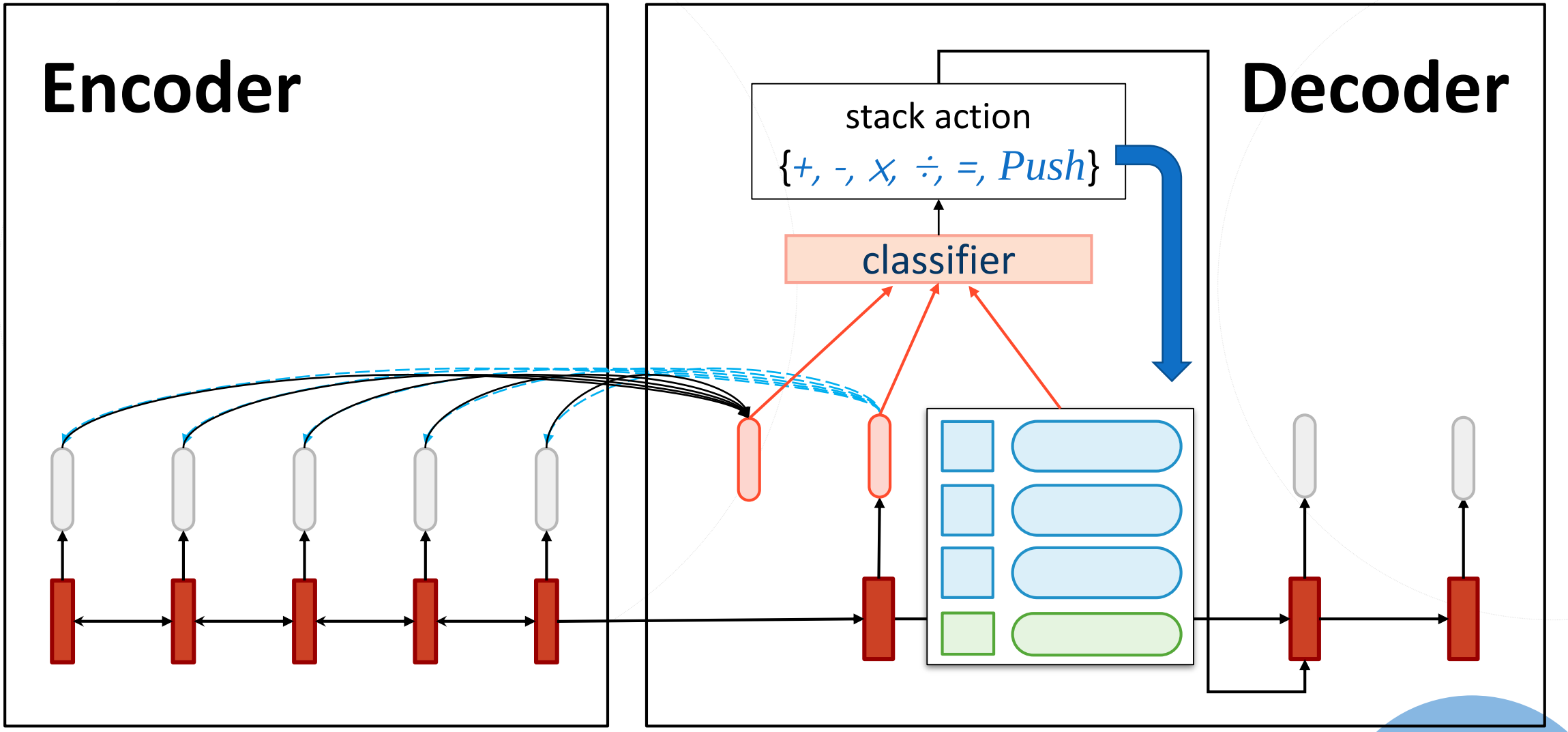
Equation Generation by Stack Actions

- Stack is used
- The decoder generates stack actions.
- An equation is generated with actions on stack.

$$x = 10 - 1 \times 5 \div 0.5$$



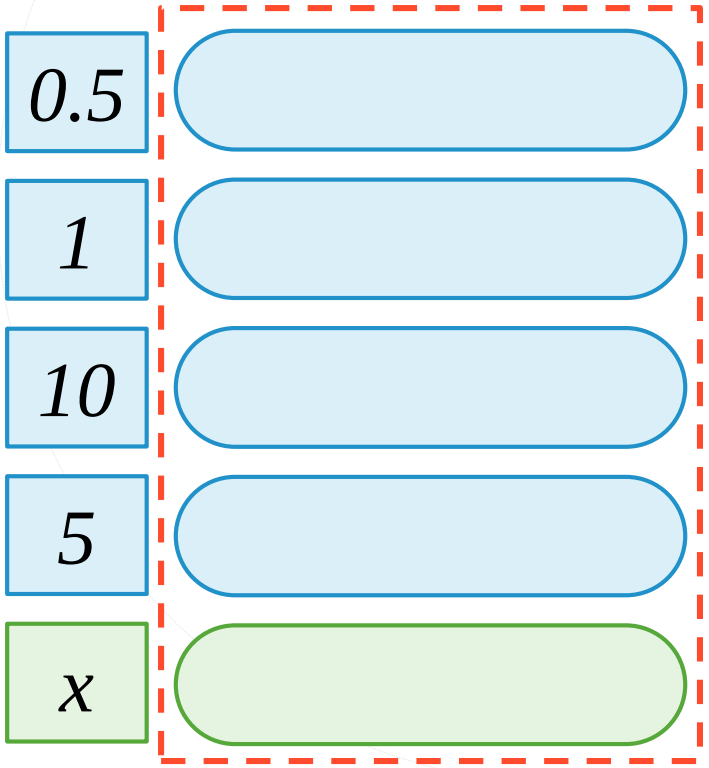
Action Selection in Each Step



Equation Generation by Stack Actions

Target Equation: $x = 10 - 1 \times 5 \div 0.5$

Generated Actions:

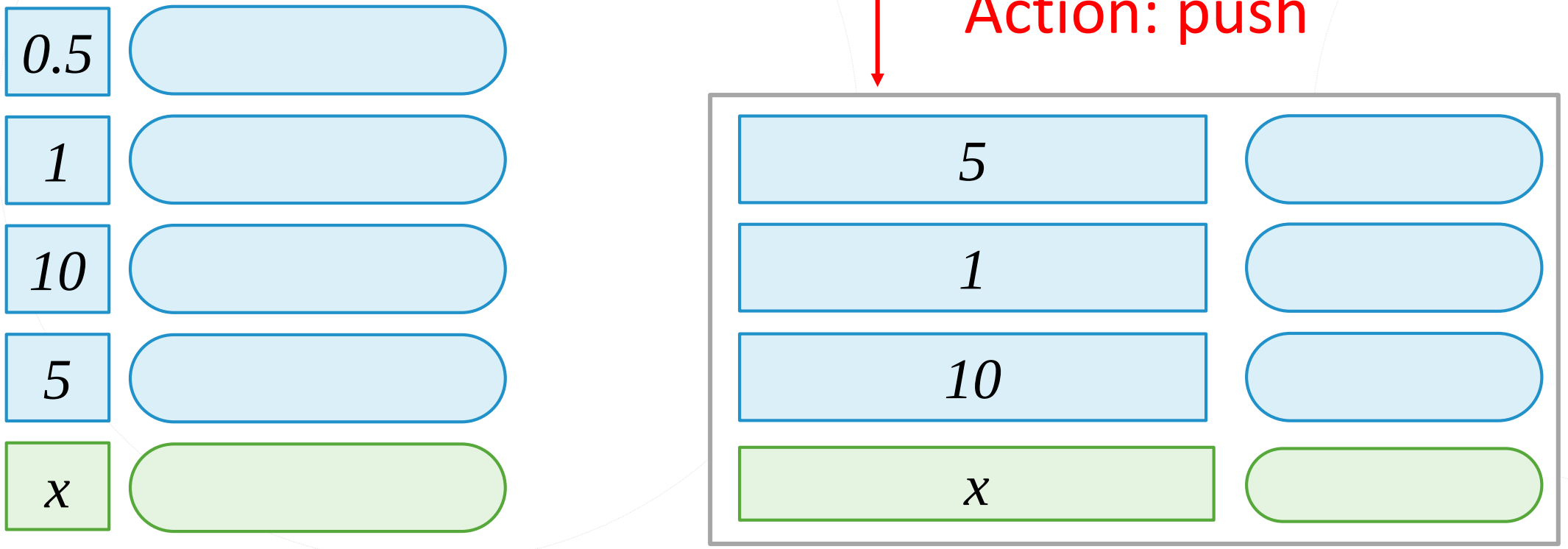


Action: push

Equation Generation by Stack Actions

Target Equation: $x = 10 - 1 \times 5 \div 0.5$

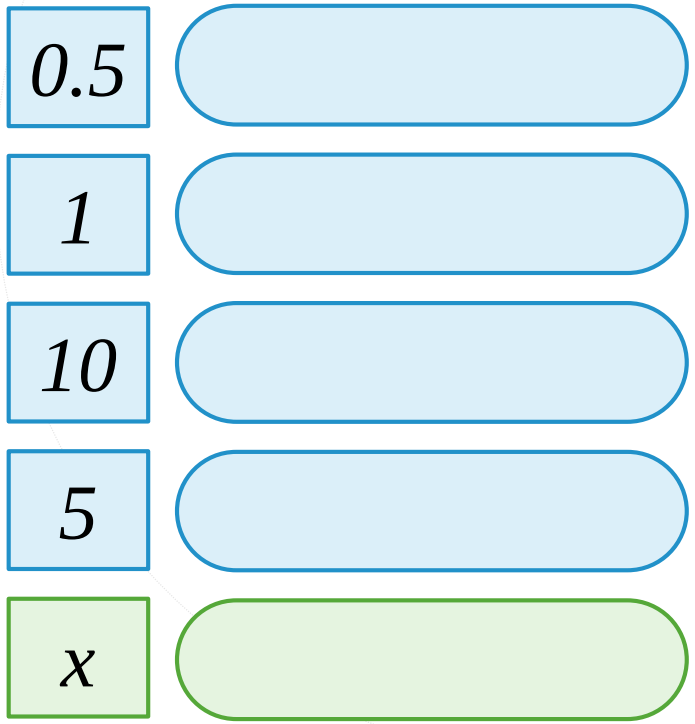
Generated Actions: *x 10 1* ***5***



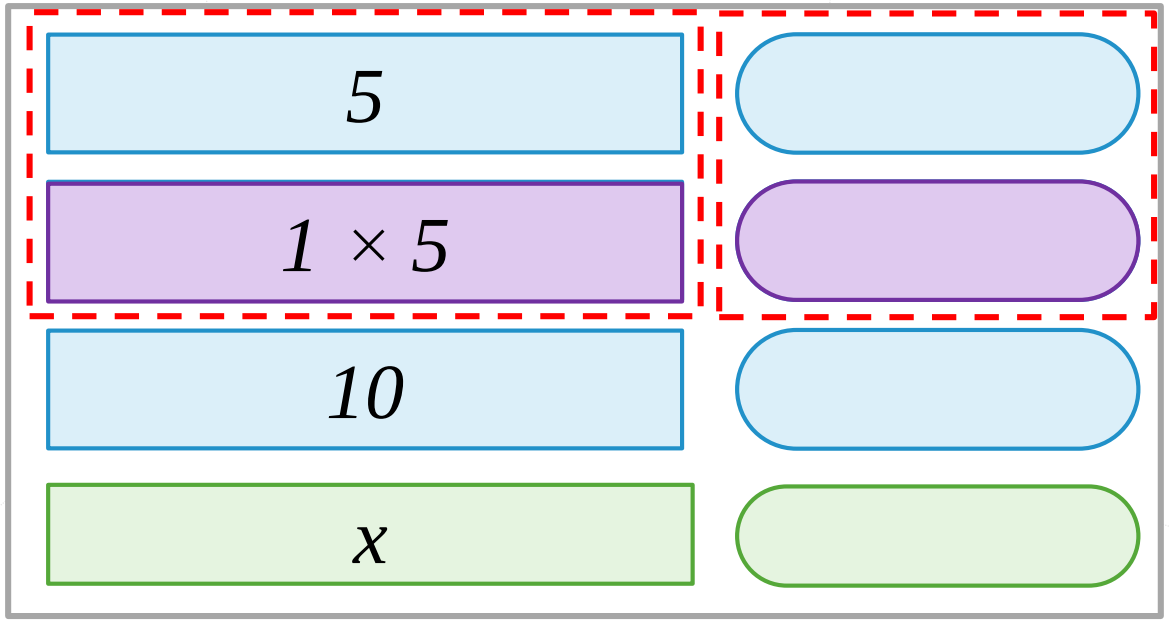
Equation Generation by Stack Actions

Target Equation: $x = 10 - 1 \times 5 \div 0.5$

Generated Actions: *x 10 1* ***5***



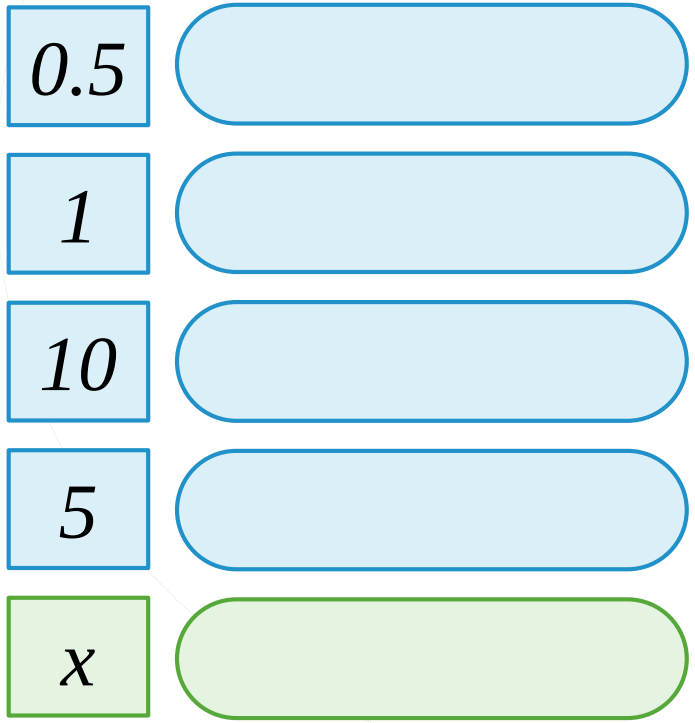
Action: ×



Equation Generation by Stack Actions

Target Equation: $x = 10 - 1 \times 5 \div 0.5$

Generated Actions: $x\ 10\ 1\ 5\ \times\ 0.5\ \div\ =$

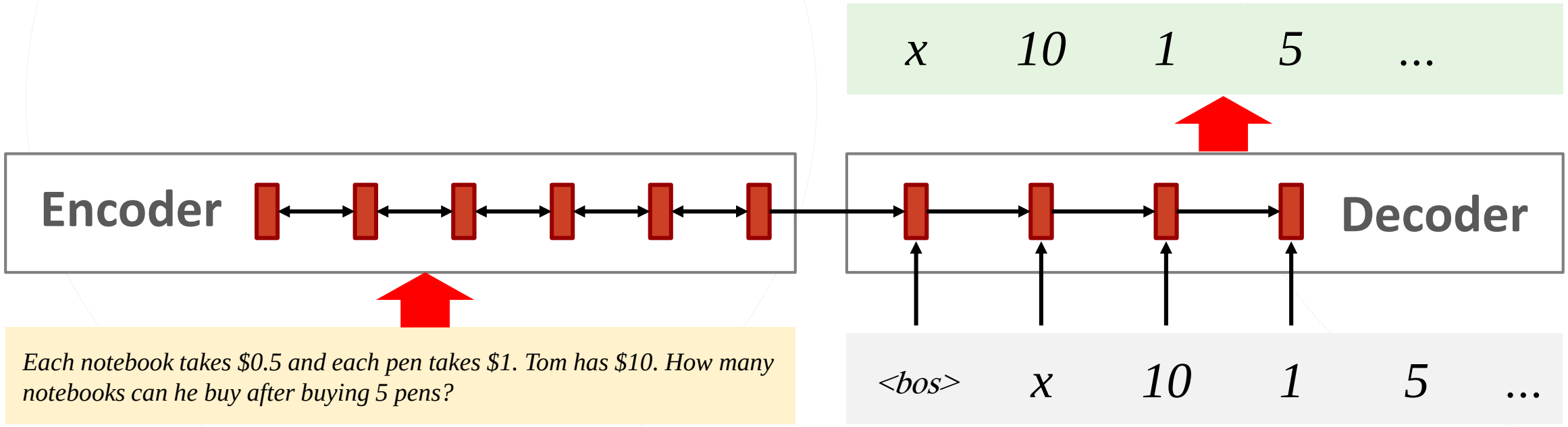


After many steps...

$x=(10-1\times 5)\div 0.5$

Training Process

- Target equation is given.
- Trained as Seq2Seq.



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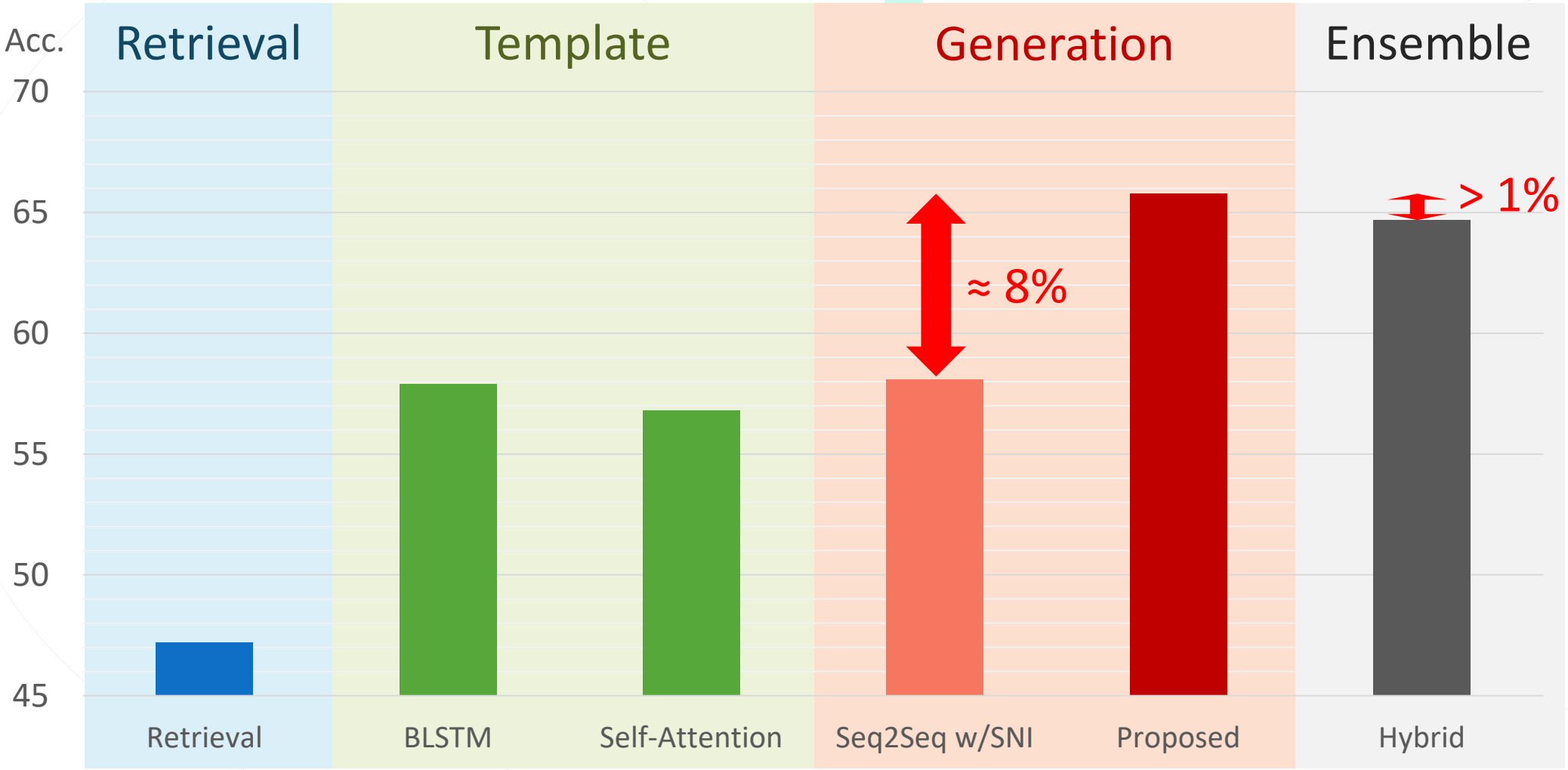
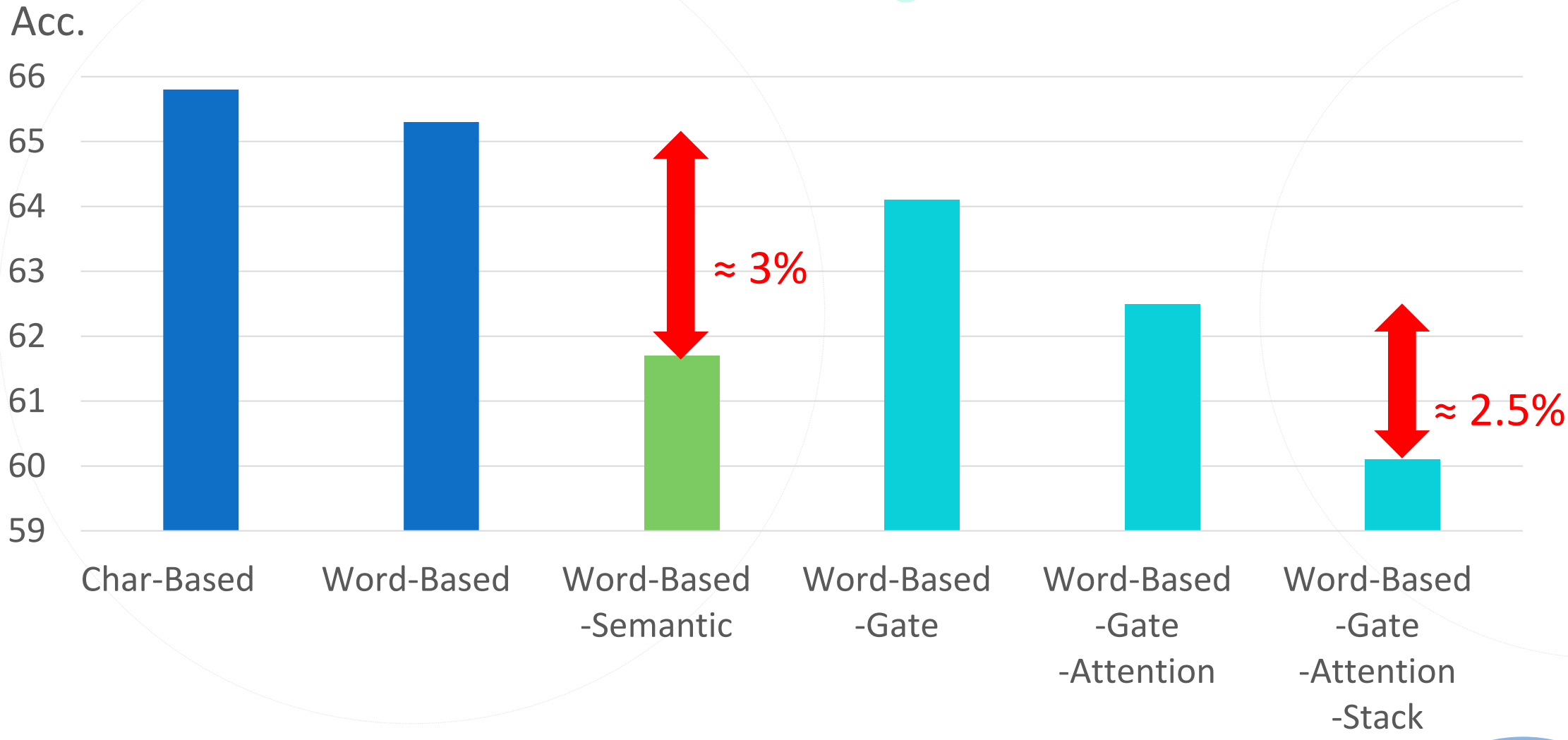
Experiments

- Dataset: Math23k
  - In Chinese
  - 23000 math word problems.
  - Operators: +, -, ×, ÷

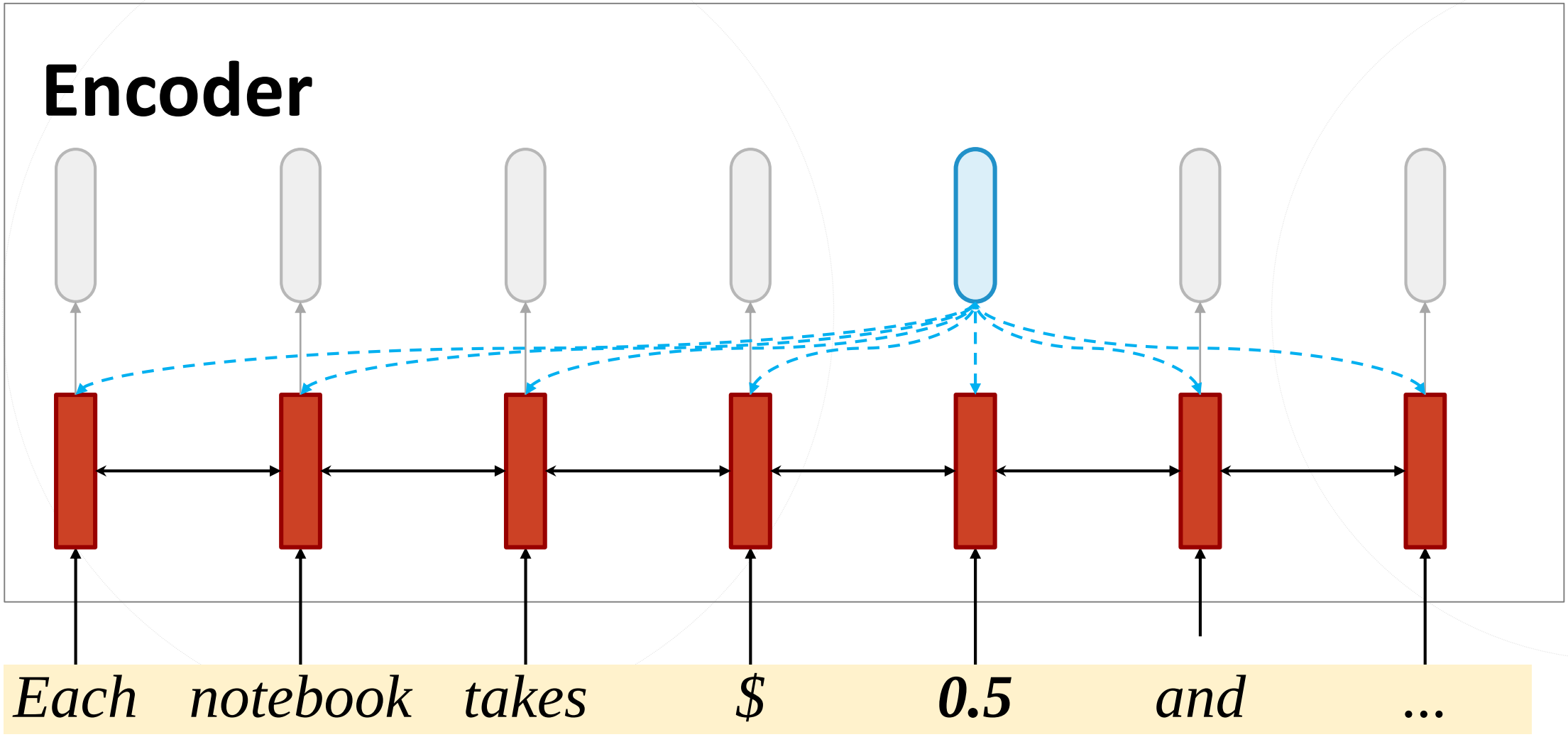
Results


Acc.
70
65
60
55
50
45
Retrieval
Template
Generation
Ensemble
≈ 8%
> 1%
Retrieval
BLSTM
Self-Attention
Seq2Seq w/SNI
Proposed
Hybrid

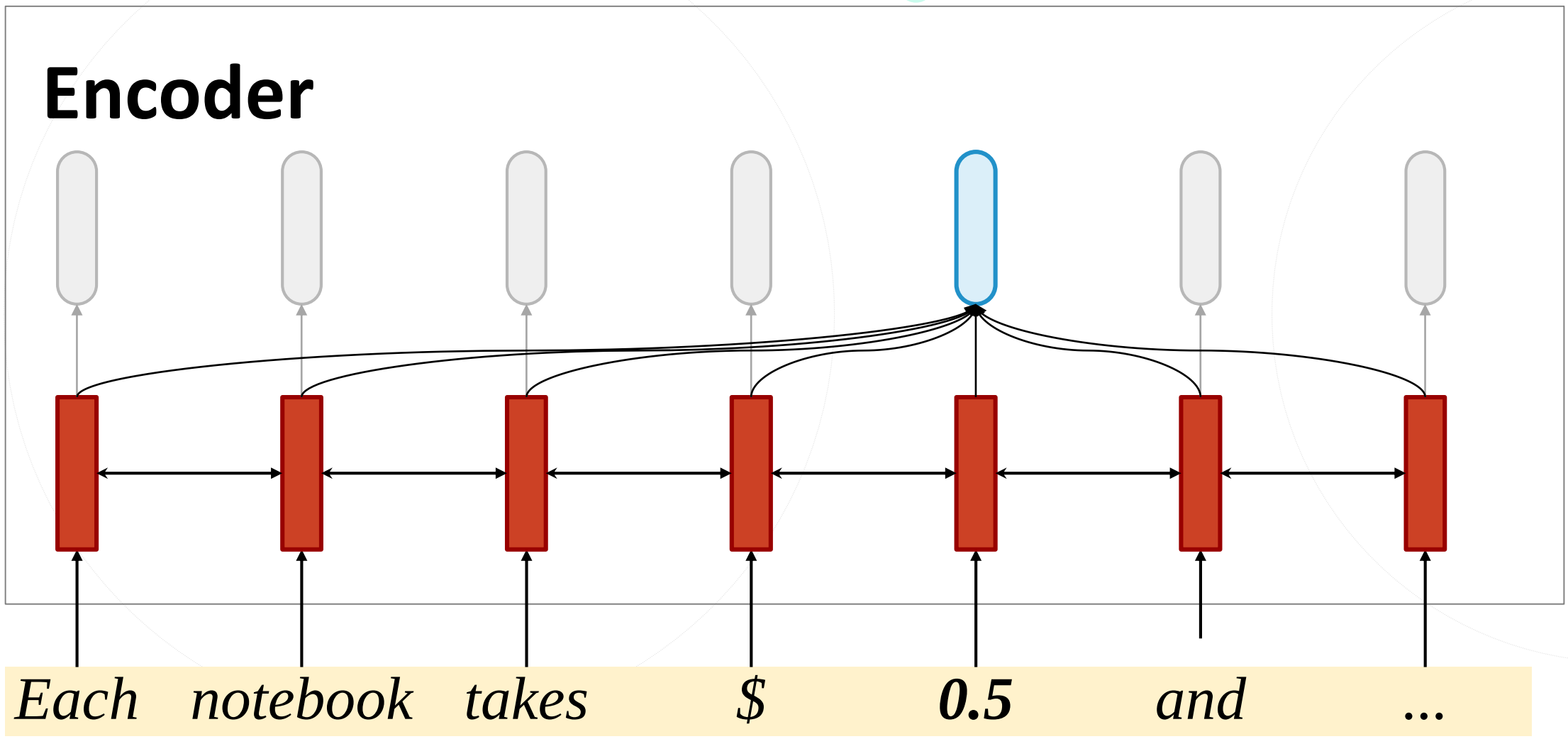
Ablation Test



Self-Attention for Qualitative Analysis



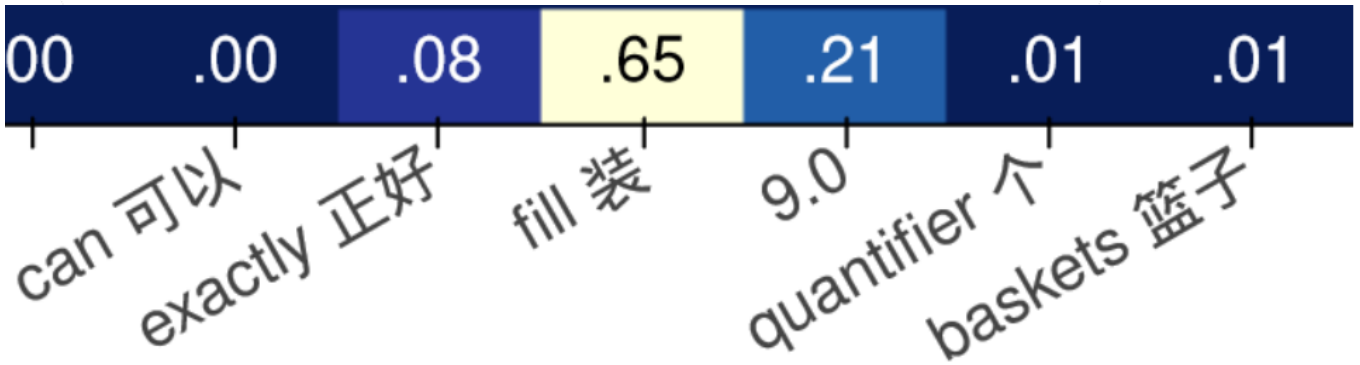
Self-Attention for Qualitative Analysis



Attention for Operand Semantics

The attention focuses on:

- Informative verbs
  - “*gain*”, “*get*”, “*fill*”, etc.
- Quantifier-related words
  - “*every*”, “*how many*”, etc.



Conclusion

Three main contributions

- **Approach:** equation generation with stack
- **Originality:** automatic extraction of operand semantics
- **Performance:** a SOTA end-to-end neural model on Math23k

Code Available @

https://github.com/MiuLab/E2EMathSolver