

A Appendix

A.1 Full Derivation of KL-divergence in Eq. (8)

$$\begin{aligned}
& KL(q_\phi(z|x)||p(z)) \\
&= \int q_\phi(z|x) \log \frac{q_\phi(z|x)}{p(z)} dz \\
&= \left\langle \log \frac{q_\phi(z|x)}{p(z)} \right\rangle_{q_\phi(z|x)} \\
&= \left\langle \log \Gamma\left(\sum_{k=1}^K \alpha_k\right) - \sum_{k=1}^K \log \Gamma(\alpha_k) + \sum_{k=1}^K (\alpha_k - 1) \log z_k - \log \Gamma\left(\sum_{k=1}^K \beta_k\right) + \sum_{k=1}^K \log \Gamma(\beta_k) - \sum_{k=1}^K (\beta_k - 1) \log z_k \right\rangle_{q_\phi(z|x)} \\
&= \log \Gamma\left(\sum_{k=1}^K \alpha_k\right) - \sum_{k=1}^K \log \Gamma(\alpha_k) - \log \Gamma\left(\sum_{k=1}^K \beta_k\right) + \sum_{k=1}^K \log \Gamma(\beta_k) + \sum_{k=1}^K (\alpha_k - \beta_k) \left\langle \log z_k \right\rangle_{q_\phi(z|x)} \\
&= \log \Gamma\left(\sum_{k=1}^K \alpha_k\right) - \sum_{k=1}^K \log \Gamma(\alpha_k) - \log \Gamma\left(\sum_{k=1}^K \beta_k\right) + \sum_{k=1}^K \log \Gamma(\beta_k) + \sum_{k=1}^K [(\alpha_k - \beta_k) \int \log z_k \frac{\Gamma(\sum_{j=1}^K \alpha_j)}{\prod_{j=1}^K \Gamma(\alpha_j)} \prod_{j=1}^K z_j^{\alpha_j-1} dz_{1:K}] \\
&= \log \Gamma\left(\sum_{k=1}^K \alpha_k\right) - \sum_{k=1}^K \log \Gamma(\alpha_k) - \log \Gamma\left(\sum_{k=1}^K \beta_k\right) + \sum_{k=1}^K \log \Gamma(\beta_k) + \sum_{k=1}^K (\alpha_k - \beta_k) (\psi(\alpha_k) - \psi(\sum_{k=1}^K \alpha_k))
\end{aligned} \tag{9}$$