CoT-Valve: Length-Compressible Chain-of-Thought Tuning

Xinyin Ma*, **Guangnian Wan***, **Runpeng Yu**, **Gongfan Fang**, **Xinchao Wang**†
National University of Singapore
{maxinyin, guangnian}@u.nus.edu, xinchao@nus.edu.sg

Abstract

Chain-of-Thought significantly enhances a model's reasoning capability, but it also comes with a considerable increase in inference costs due to long chains. With the observation that the reasoning path can be easily compressed under easy tasks but struggles on hard tasks, we explore the feasibility of elastically controlling the length of reasoning paths with only one model, thereby reducing the inference overhead of reasoning models dynamically based on task difficulty. We introduce a new tuning and inference strategy named CoT-Valve, designed to allow models to generate reasoning chains of varying lengths. To achieve this, we propose to identify a direction in the parameter space that, when manipulated, can effectively control the length of generated CoT. Moreover, we show that this property is valuable for compressing the reasoning chain. We construct datasets with chains from long to short for the same questions and explore two enhanced strategies for CoT-Valve: (1) a precise length-compressible CoT tuning method, and (2) a progressive chain length compression approach. Our experiments show that CoT-Valve successfully enables controllability and compressibility of the chain and shows better performance than the prompt-based control. We applied this method to QwQ-32B-Preview, reducing reasoning chains on GSM8K from 741 to 225 tokens with a minor performance drop (95.07% to 94.92%) and on AIME from 6827 to 4629 tokens, with only one additional incorrect answer.

1 Introduction

Chain-of-Thought (CoT) reasoning (Wei et al., 2022) has emerged as a powerful technique for enhancing the reasoning capabilities of large language models (Jaech et al., 2024; Dubey et al., 2024; Abdin et al., 2024), particularly in complex tasks such as mathematics and coding (Sprague et al., 2024) that require multi-step inference. By simulating the process of human-like thought progression, CoT enables models to break down complex problems into sub-questions, improving accuracy and interpretability (Joshi et al., 2023). Those reasoning abilities have also been tested in different domains, such as image generation (Ma et al., 2025) and visual understanding (Shao et al., 2024).

Training reasoning models often involves generating extensive reasoning paths through methods such as sampling (Wang et al., 2023), tree search (Yao et al., 2023; Guan et al., 2025; Zhang et al., 2024) or reinforcement learning (DeepSeek-AI, 2025) to reach the correct answer ultimately. However, these long chains often incorporate redundant intermediate steps that can be unnecessary or too complex (Lightman et al., 2024), and the redundancy in the reasoning paths for training leads to inefficiencies in token usage and increased inference costs. However, crafting an optimal reasoning chain that omits extraneous details is challenging due to the limited availability of intermediate rewards to guide the process and human annotations (Zhang et al., 2025a). Removing some or all of the intermediate steps and then training or distilling the model (Liu et al., 2024b; Yu et al., 2024) will degrade the performance. Alternative approaches employ information-theoretic measures (Ton et al., 2024) or identify an "overthinking" solution in QwQ (Team, 2024b) to evaluate the contribution of each sentence to the final answer.

We observe that current reasoning models, such as QwQ (Team, 2024b) and DeepSeek-R1 (DeepSeek-AI, 2025) allocate an excessive number of tokens to simple tasks, while potentially providing insufficient tokens for complex tasks. *Thus, a long reasoning path is still essential, while maintaining the ability to compress reasoning paths for simpler questions is equally important.*

*Equal contribution
†Corresponding Author

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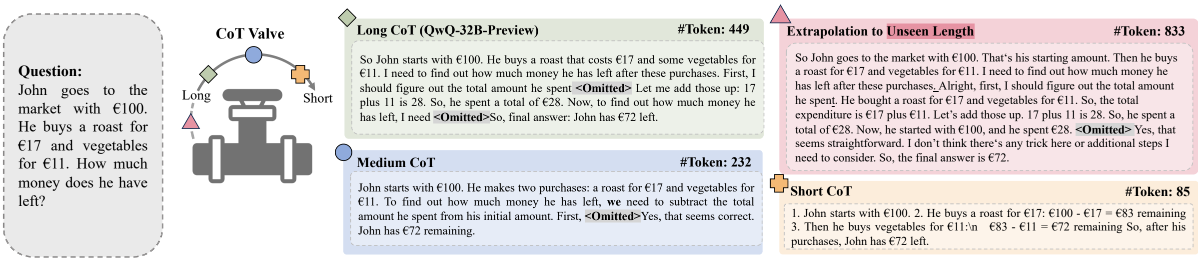


Figure 1: The reasoning model, after the length-compressible CoT tuning, can generate reasoning paths from long to short, leveraging LoRA as a 'Valve'. We show one example from our constructed dataset MixChain.

To solve this, our goal is to fine-tune a model capable of generating both long and short reasoning paths, rather than being restricted to a compressed form. We offer a new way to control the length of CoT, which we refer to as Length-Compressible Chain-of-Thought Tuning.

A central component of the proposed method is to identify an update direction in the parameter space, which, by manipulating it, acts as increasing or decreasing the length of CoT. Taking a large step in this direction leads the model to generate a short sequence, while a small step still produces a long and complex reasoning trajectory. We choose to incorporate this update direction by LoRA (Hu et al., 2022), enabling it to function as an additional branch that facilitates easy modulation of intensity while imposing minimal extra parameters on the model. We explore methods to identify this direction and demonstrate that it offers superior controllability compared to prompt-based approaches, which enables the generation of short CoT that prompt-based methods are unable to achieve. Besides, we observe that the direction can be extrapolated, allowing the reasoning chains to be extended beyond or shortened to lengths unseen in the training set. Leveraging this compressibility, we construct a dataset that pairs long and short reasoning chains for each question. This dataset is then utilized in two ways: (1) to refine the direction for more precise tuning, and (2) to compress the reasoning path progressively.

We evaluate our method across different types of models, ranging from a pre-trained LLM with little reasoning ability, LLaMA-3.1-8B and LLaMA-3.2-1.5B-Instruct (Dubey et al., 2024), to post-trained reasoning models, QwQ-32B-Preview (Team, 2024b), and distilled reasoning models, DeepSeek-R1 (DeepSeek-AI, 2025). Our results demonstrate that, with training for one time, our approach enables a model to generate reasoning paths of varying lengths, and we can achieve better results than previous chain compression baselines. Besides, our study highlights several interesting findings: (1) Short reasoning paths can sometimes outperform longer ones, underscoring the significance of CoT-Valve in enhancing model efficiency. (2) Not every reasoning chain, despite all leading to the correct final answer, is conducive to model optimization. Excessively long or short chains complicate the distillation of CoT, posing challenges to the model training.

In summary, our contributions are: (1) **CoT-Valve**: Enables elastic control of length for CoT within the parameter space, allowing a single model to generate CoT from short to long. (2) **MixChain Dataset**: A dataset with reasoning paths of varying lengths for each question. (3) **Improved Tuning** & **Progressive Compression**: Refines the direction-tuning process based on MixChain and introduces progressive compression for inference efficiency. (4) **Performance & Controllability**: Achieves controllable reasoning generation and state-of-the-art results for compressed CoT.

2 Related Work

Chain-of-Thought. Chain-of-thought (Wei et al., 2022) reasoning has shown promising progress in recent years, especially the success of OpenAi-O1 (Jaech et al., 2024) and Deepseek-R1 models (DeepSeek-AI, 2025). This introduces the test-time scaling law, apart from the traditional scaling law for training (Hoffmann et al., 2022). Several approaches have been proposed to boost the language model to have better problem-solving abilities, including the model has its self-reasoning abilities (Team, 2024b) or use Best-of-N (Nakano et al., 2021), beam search and Monte Carlo Tree Search (Kocsis and Szepesvari, 2006; Guan et al., 2025) to search and refine the solution without further finetune the large language models. The outcome reward model and process reward models are also introduced to evaluate the score for the entire solution, especially the final answer (Cobbe et al., 2021a) and the quality of the reasoning path (Wang et al., 2024; Luo et al., 2024)

Chain Compression in reasoning model. Due to the high computational cost associated with inference in reasoning models, particularly for long-chain reasoning, chain compression has become a critical area of research. (Yu et al., 2024) attempts to distill the chain-of-thought into System 1 but fails to observe improvements when intermediate steps are omitted. (Jin et al., 2024) conducts a comprehensive empirical study between length and performance. (Deng et al., 2023) proposes internalizing reasoning steps within the hidden states of models, while several implicit-based approaches(Deng et al., 2024; Hao et al., 2024; Cheng and Durme, 2024) aim to compress token-wise generation by transitioning from language space to hidden space. Other studies focus on skipping intermediate reasoning steps (Liu et al., 2024b) or using summarization techniques to generate shorter reasoning chains (Kang et al., 2024). Additionally, (Chen et al., 2024) addresses the overthinking issue in QwQ (Team, 2024b) and employs SimPO (Meng et al., 2024) for optimization. Kimi K1.5 (Team et al., 2025) proposes merging long-CoT models with short-CoT models in a training-free manner. O1-Pruner (Luo et al., 2025) adopts reinforcement learning to shorten responses.

3 Method

In this section, we provide an in-depth discussion of our method. Section 3.1 introduces a simple yet effective approach that enables a single tuning process to generate models with CoT with different lengths. This stage also serves as an initial step for subsequent refinements. Next, in Section 3.2, we explore multiple scenarios in which we can apply CoT-Valve to construct the dataset MixChain. In Section 3.3, we propose several advanced methods that take advantage of long-to-short datasets to improve precision and control over the generated reasoning paths in compressible fine-tuning.

3.1 Length-Compressible CoT Tuning

Our primary objective is to achieve a new way to control the length of reasoning paths after training a reasoning model. Existing approaches, such as prompt-based control, explicitly define sequence length in the prompt (Han et al., 2024) or utilize summary tokens (Ding et al., 2024) for guidance. However, these methods offer only limited control over the length of CoT generated. For instance, requesting a sequence of less than 20 tokens may result in the model generating over 350 tokens (see Table 14 in the Appendix), and these methods struggle to produce answers with very short lengths. To address these limitations, we introduce CoT-Valve for training one model but can adjust the length of reasoning paths.

Consider a reasoning model defined by the parameter θ. For a given question q in the dataset $\mathcal{D}$, the probability of generating an answer a and its reasoning thoughts $\{t_i\}_{i=1}^n$ given the question q can be described by:

$$p\left(a \mid t_1, \ldots, t_n, q ; \theta\right) \prod_{i=1}^n p\left(t_i \mid t_{<i}, q ; \theta\right) \quad (1)$$

where $\{t_i\}_{i=1}^n$ might include errors or unnecessary details. With short synthesized or human-annotated explanations $\{t_i\}_{i=1}^m$ with $m < n$, the training objective is to adjust the parameter in such a way that the chain is shortened while still yielding the correct answer:

$$\max_{\Delta\theta} \mathbb{E}_{(q,a)\sim\mathcal{D}} p\left(a \mid t_1, \ldots, t_m, q ; \theta+\Delta\theta\right) \prod_{i=1}^m p\left(t_i \mid t_{<i}, q ; \theta+\Delta\theta\right) \quad (2)$$

and $\Delta\theta$ denotes the change in the parameter space that steers the model towards generating a more concise chain.

Since the model, with and without $\Delta\theta$, outputs the same final answer, $\Delta\theta$ can be interpreted as a task vector (Ilharco et al., 2023). The task here is to control the length of the CoT, provided that the only difference in the training set lies in intermediate reasoning steps $\{t_i\}_{i=1}^n$. Those reasoning paths are different in length but ultimately lead to the same final answer. Thus, we can control the task vector to achieve the goal of adjusting the length of CoT. $\Delta\theta$ is designed within a parameter-efficient space, functioning as an external branch for inference that incurs minimal overhead. Controlling this external branch enables the manipulation of the length of the reasoning path.

Task Arithmetic: Interpolation and Extrapolation of $\Delta\theta$. To manipulate this update within the parameter space, we can control the magnitude of a

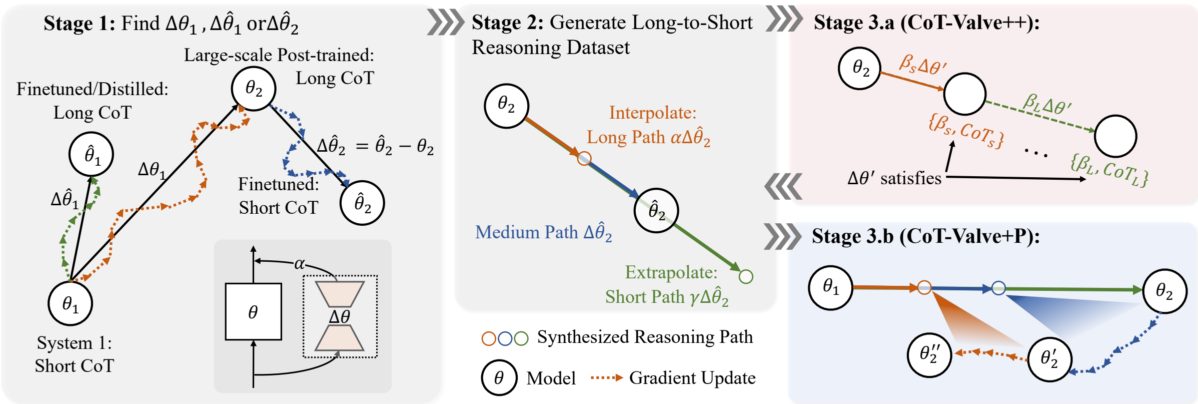


Figure 2: Illustration of CoT-Valve. In Stage 1, we first determine $\Delta\theta$ from distilling or post-training. Then, the trained $\Delta\theta$ is utilized to construct the MixChain dataset. Using this dataset, we can then apply two enhanced training methods to achieve more precise control over reasoning paths or to shorten the reasoning paths as needed.

$\Delta\theta$ as an arithmetic operation. We use two primary operations on $\Delta\theta$ here: interpolation and extrapolation. Let α denote the magnitude of $\Delta\theta$ for LoRA. When α falls within the range of (0,1), the model smoothly transitions between longer and shorter reasoning paths, similar to weight interpolation between two models (Frankle et al., 2020; Team et al., 2025). When $\alpha > 1$, extrapolation is introduced, further shortening the reasoning path beyond what was observed during training. This enables an exploration of the minimal reasoning length required to arrive at a given answer. Thus, by adjusting α at inference, we can modulate the model's behavior, with each value of α corresponding to different CoT lengths.

Application Unlike prompt-based approaches that can only regulate the overall length of the reasoning process using prompt words, $\Delta\theta$ provides finer granularity control. $\Delta\theta$ is served in the external parameter space. This allows for greater flexibility in adjusting the reasoning trajectory. Specifically, it facilitates the selective retention of long-chain reasoning in certain thoughts while applying stronger compression to simpler reasoning segments. As a result, reductions in chain length can be localized to specific portions of the inference process rather than being uniformly applied across the entire reasoning path. We remain the design of this segment selection in future work.

3.2 Construct the MixChain Dataset

A crucial thing for the above process is the construction of the training dataset, especially the reasoning chain $\{t_i\}_{i=1}^n$. To have reasoning chains with different lengths, previous approaches rely on multiple rounds of sampling, selecting reasoning paths under different random seeds, or using some handcrafted way to remove parts of the answer (Chen et al., 2024).

We introduce MixChain, a dataset inherently generated by our method that contains reasoning paths of varying lengths. This dataset is structured such that each question is associated with multiple reasoning paths, with lengths progressively decreasing from long to short. By simply adjusting the parameter α, our approach avoids the need for repeated sampling and achieves this diverse set of reasoning paths. In contrast to multi-sampling techniques, MixChain enables a more reliable and consistent generation of shorter reasoning paths while simultaneously capturing a spectrum of reasoning lengths. To construct MixChain, we consider two possible scenarios:

- If a well-annotated dataset with human-labeled solutions is available, such as GSM8K (Cobbe et al., 2021b) or PRM800k (Lightman et al., 2024), it can be leveraged to fine-tune the model for generating shorter reasoning chains as a cold start ($\theta_1 \rightarrow \tilde{\theta}_1$ and $\theta_2 \rightarrow \tilde{\theta}_2$ in Figure 2).

- In the absence of a dataset containing explicit reasoning paths, or when only final answers are available without full explanations, training solely on final answers is unlikely to enable the model to generate reasoning steps. To address this limitation, we propose an alternative method for constructing MixChain. Specifically, we leverage an existing base LLM (e.g., LLaMA-3.1-8B or Qwen-

32B-Instruct) as θ_1 and use its corresponding reasoning model (e.g., DeepSeek-R1-Distill-Llama-8B or QwQ-Preview) to derive $\Delta\theta$. The parameter update between these models serves as a form of linear interpolation, enabling the transition from θ_1 to θ_2. This transition is then used to construct the dataset, as illustrated in Figure 2, where the parameter shift is represented by $\theta_1 \rightarrow \theta_2$.

3.3 Improved Tuning for CoT-Valve

In this section, we present two enhanced variants of CoT-Valve: one aimed at achieving improved controllability and the other focused on optimizing the compression ratio of the reasoning paths.

A More Precise CoT-Valve Paradigm: CoT-Valve++. In the previously proposed CoT-Valve framework, the training process only constrained $\Delta\theta$ to satisfy the final objective with $\alpha = 1$. However, during inference, we expect all positions along this direction to exhibit reasoning trajectories of varying lengths. This leads to the inconsistency between training and inference. With MixChain, we can explicitly incorporate this requirement during training by introducing an additional constraint, ensuring that the model can adapt to reasoning chains of different lengths across all positions in this direction. For each training sample, in addition to the question, answer, and solution, we have introduced a normalized term β, which represents the factor for the length of the reasoning path. Under this dataset, our training objective is modified to find a parameter update $\Delta\theta'$ such that it satisfies:

$$\max_{\Delta\theta'} \mathbb{E}_{(q,a)\sim\mathcal{D}'} p\left(a \mid t_{<m}, q; \theta + \beta\Delta\theta'\right) \prod_{i=1}^{m} p(t_i | t_{<i}, q; \theta + \beta\Delta\theta') \quad (3)$$

Where $\mathcal{D}'$ is the Mixchain dataset. Each sample consists of the question q, the answer a, the solution $\{t_i\}_{i=1}^{m}$ and β, where β is calculated as:

$$\beta = 1 - \frac{m - m_{min}}{m_{max} - m_{min}} \quad (4)$$

Here, m_{min} and m_{max} is the length of the shortest solution and longest solution for this question. Based on synthetic samples, we introduce additional constraints that enable us to better identify the updated parameter $\Delta\theta'$, facilitating more precise compressibility and controllability.

Progressive Chain Compression: CoT-Valve+P. The structure of MixChain, which features progressively shorter reasoning paths for each question, facilitates a progressive chain-length compression strategy. This approach is similar to iterative pruning in model compression (Molchanov et al., 2017). In this process, the model is trained with a shorter reasoning path from the dataset at each iteration, rather than training directly with the shortest reasoning CoT. This gradual compression method allows the model to progressively reduce the length of its reasoning paths.

4 Experiments

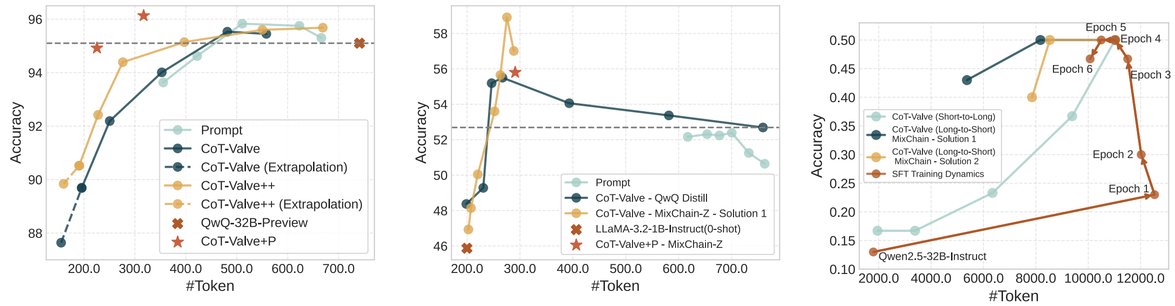
4.1 Experimental Setup

Models. We evaluate our method under several models: QwQ-32B-Preview (Team, 2024b), DeepSeek-R1-Distill-Llama-8B (DeepSeek-AI, 2025), LLaMA-3.1-8B (Dubey et al., 2024), LLaMA-3.2-1B (Dubey et al., 2024) and Qwen-32B-Instruct (Team, 2024a) with LIMO (Ye et al., 2025). We tested different scenarios for CoT-Valve:

- **(Long to Short CoT)** For QwQ-32B-Preview (QwQ for abbreviation) and DeepSeek-R1-Distill-Llama-8B (R1-Distill), we used our method to control and compress the length of the reasoning chain.
- **(Short to Long CoT)** For LLaMA-3.1-8B and LLaMA-3.2-1B-Instruct, we applied our method to distill reasoning abilities from QwQ-32B-Preview and incorporated CoT-Valve in the distillation process.
- **(Short-Long-Short CoT)** We tested another setting to first post-train a short-CoT LLM, Qwen-2.5-32B-Instruct (Team, 2024a), to generate Long CoT and then compress it to Short CoT. CoT-Valve can be applied in both two stages.

Metrics. We report both accuracy and the number of tokens in the answer for each experiment. Given the trade-off between reasoning path length, model size, and performance, we use a new metric, Accuracy per Computation Unit(ACU), to better capture this balance and evaluate model efficiency. It is defined as:

$$\text{ACU} = \frac{\text{Accuracy}}{\#\text{Params} \times \#\text{Tokens}} \quad (5)$$



(a) GSM8K, QwQ-32B-Preview (b) GSM8K, Llama-3.2-1B-Instruct (c) AIME, Qwen2.5-32B-I w/ LIMO

Figure 3: Token length and accuracy for different methods, datasets and reasoning models. Points connected by curves in (a) and (b) represent results from one model.

Since the ACU value typically falls within the range of 10^{-5} to 10^{-2}, we report it in units of 10^2 for improved readability.

Training and Evaluation. For training the model, we use LoRA (Hu et al., 2022) in most of our experiments, except in the experiment for LIMO on Qwen-2.5-32B-Instruct we use full parameter fine-tuning. We also show the results using DoRA (Liu et al., 2024a) in the Appendix. The hyper-parameters for each experiment are shown in Appendix A. We select two math datasets to evaluate the performance, for one easy math dataset, GSM8K (Cobbe et al., 2021b) and one hard math dataset, AIME24.

4.2 Datasets

We find in our experiments that the quality of the solution is important to the performance, even if all the human-annotated solutions or synthesized solutions reach the final answer. In our experiments, we use the question from the train set of GSM8K, the math split of PRM800K or the question from LIMO, and we employ three types of datasets with those questions in our experiments:

- Ground-truth Dataset: The dataset provides a human-annotated or model-synthesized solution. We use this as the cold start.
- MixChain from cold-start (MixChain-C): After taking the ground-truth dataset to train the model, we can get the first model to generate solutions from short to long. Then we use it to generate the dataset.
- MixChain from zero-shot (MixChain-Z): We employ CoT-Valve between a reasoning model (θ_2) and a base LLM (θ_1) to generate the solutions.

Method	Accuracy	#Token	ACU ↑
Llama-3.3-70B-Instruct	92.6	235.4	0.56
Llama-3.1-405B-Instruct	95.6	186.7	0.13
Qwen2.5-32B-Instruct	93.1	269.3	1.09
Qwen2.5-Math-72B-Instruct	95.8	312.1	0.43
QwQ-32B-Preview	95.1	741.1	0.40
Prompt (Han et al., 2024)	93.6	355.5	0.82
Prompt (Ding et al., 2024)	95.5	617.7	0.48
In-domain Train Set: GSM8K			
CoT-Valve - Ground-Truth	94.0	352.8	0.83
CoT-Valve++ - MixChain-C	94.4	276.3	1.07
CoT-Valve+P - MixChain-Z	96.1	317.1	0.95
CoT-Valve+P - MixChain-Z	94.9	225.5	1.32
Out-of-Domain Train Set: PRM12K			
Overthink(Chen et al., 2024) - SFT	94.8	749.5	0.40
Overthink(Chen et al., 2024) - SimPO	94.8	326.2	0.91
O1-Pruner(Luo et al., 2025) - SFT	95.7	717	0.42
O1-Pruner(Luo et al., 2025)	96.5	534	0.56
CoT-Valve+P - MixChain-Z	95.4	288.5	1.03

Table 1: Results of QwQ-32B-Preview on GSM8K. Values of ACU are scaled by 10^2 for readability. We list the dataset we use after the method name.

For each dataset, we filter out all the solutions with incorrect answers. We show the statistics of the dataset in Table 11 in the Appendix.

4.3 From Long-CoT to Short-CoT.

Controllable Results. We illustrate the result in Figure 3a. First, using ground-truth samples as a cold start, we develop a model capable of generating reasoning paths of various lengths, as demonstrated in 'CoT-Valve' in Figure 3a. CoT-Valve already matches the performance of prompt-based control but can generate shorter reasoning chains. We then extrapolate $\Delta\theta$ to produce even shorter reasoning paths. Then, building on MixChain-C from this first model, we conduct further training by CoT-Valve++. CoT-Valve++ substantially surpasses the baseline and shows greater generalization capabilities in cases of extrapolation.

Method	AIME24	#Token	ACU↑
Qwen2.5-32B-Instruct	4/30	1794.2	0.023
Qwen2.5-Math-72B-Instruct	7/30	1204.5	0.061
Gemini-Flash-Thinking (Team et al., 2023)	15/30	10810.5	-
QwQ-32B-Preview.Train set: GSM8K			
QwQ-32B-Preview	14/30	6827.3	0.021
Prompt (Han et al., 2024)	13/30	6102.5	0.022
Prompt (Ding et al., 2024)	13/30	5562.3	0.024
Overthink (Chen et al., 2024)	13/30	5154.5	0.026
CoT-Valve - GSM8K	14/30	5975.0	0.024
CoT-Valve++ - MixChain-C	13/30	5360.5	0.025
CoT-Valve+P - MixChain-Z	13/30	4629.6	0.029
Qwen-32B-Instruct. Train set: LIMO			
Qwen-32B-LIMO	15/30	10498.2	0.015
CoT-Valve	11/30	6365.2	0.018
SFT - MixChain - Solution 1	13/30	5368.0	0.025
CoT-Valve - MixChain - Solution 1	15/30	8174.8	0.019

Table 2: Results of QwQ-32B-Preview and Qwen-32B-Instruct w/ LIMO on AIME 24.

Model	GSM8k		AIME24	
	Acc	#Token	Acc	# Token
Llama-3.1-8B (0-shot)	15.7	915.0	0/30	1517.6
R1-Distill-Llama-8B	87.1	1636.6	14/30	12359.9
CoT-Valve	87.3	1315.2	6/30	7410.5
CoT-Valve+P - MixChain-Z	84.0	755.2	11/30	9039.0

Table 3: Result of DeepSeek-R1-Distill-Llama-8B.

Method	Accuracy	#Tokens	ACU↑
LLaMA-3.2-1B-Instruct(8-shot)	45.9	104.3	44.008
LLaMA-3.2-1B-Instruct(0-shot)	45.9	199.8	22.973
SFT-Full Finetune - GSM8k	46.1	139.4	33.070
SFT - GSM8k	43.8	137.7	31.808
Prompt	46.7	209.9	22.249
SFT - QwQ Distill	52.7	759.3	6.941
CoT-Valve - QwQ Distill	55.5	267.0	20.786
CoT-Valve+P - MixChain-Z	55.8	291.0	19.175
SFT - MixChain-Z - Solution 1	57.0	288.4	19.764
CoT-Valve - MixChain-Z - Solution 1	58.9	275.4	21.387

Table 4: Results on LLaMA-3-2-1B-Instruct. We report the result of Flexible Match here. QwQ Distill means we use QwQ to synthesize the solution and distill it.

Method	Accuracy	#Tokens	ACU↑
LLaMA-3.1-8B (8-shot)	56.9	282.1	2.521
LLaMA-3.1-8B (0-shot)	15.7	915.0	0.214
SFT-LoRA - GSM8k	59.0	191.9	3.843
SFT-LoRA - QwQ Distill	76.3	644.8	1.479
CoT-Valve - QwQ Distill	77.5	569.8	1.700
CoT-Valve+P - MixChain-Z	77.1	371.2	2.596
CoT-Valve + MixChain-Z - Solution 1	75.7	264.1	3.583

Table 5: Result on LLaMA-3.1-8B. We report the result of Strict Match here.

Compression Results. We evaluated our method against previous chain compression approaches, with the results detailed in Table 1, Table 2, and Table 3. For GSM8K, we adhered to the baseline setup to train with PRM12K. Utilizing progressive compression, our method surpassed the baseline by producing shorter reasoning paths and improved performance.

We also report experimental results on AIME, where the model was trained using MixChain-Z derived from GSM8K. To minimize the impact of randomness on performance, we employed greedy decoding in our AIME experiments. Compared to the baseline (Chen et al., 2024), our method reduced the token count from 5155 to 4630 while maintaining the same accuracy, despite being trained on an easier dataset.

4.4 From Short-CoT to Long-CoT & Short-Long-Short CoT

Our method can also be applied if a short-CoT model is distilled or post-trained to be a Long-CoT model. The results are shown in Figure 3b, Table 4 and Table 5. We found that CoT-Valve can also effectively control the length of the chains in this setting. Notably, we observed that shorter chains could achieve higher accuracy on GSM8K. Moreover, if the model is trained using the MixChain-Z dataset, the results are significantly better, whether using CoT-Valve (55.5 to 58.9) or just simply SFT (52.7 to 57.0). Additionally, after training a long-chain model, we can employ the MixChain dataset to reduce the length of its reasoning chains further. As illustrated in Figure 3c, the results suggest that initially training the chains to be long and subsequently compressing them to be shorter (Results with Long-to-Short) can yield better performance than directly using CoT-Valve in the short-to-long stage (Results with Short-to-Long). This demonstrates significant potential for compressing the reasoning chains. We can also surpass the result of Gemini-Flash-Thinking, with the same accuracy but fewer tokens (10810.5 v.s. 8174.8)

Training dynamics does not have the same effect as CoT-Valve. We also explore whether intermediate training steps can achieve similar effects. As depicted in Figure 3c, during the early training phases, the length of the CoT increases but does not correspond with the same rapid improvement in performance. As training progresses, the token length begins to decrease while performance improves. CoT-Valve exhibits a distinct pattern, smoothly bridging the gap between the length of CoT and performance.

Solution	Solution Length	Accuracy	#Token
Ground-Truth (Solution 0)	116.0	43.8	139.4
Solution 1	279.6	57.0	288.4
Solution 2	310.7	55.1	330.0
Solution 3	386.7	56.5	414.6
Solution 4	497.2	52.5	558.3

Table 6: Train LLaMA-3.2-1B-Instruct with solutions in MixChain-Z of different lengths on GSM8K.

4.5 Observations

Based on the results from LLaMA-3.1-8B, LLaMA-3.2-1.5B, QwQ, DeepSeek-R1-Distill-Llama-8B and Qwen2.5-32B-Instruct with LIMO, we summarize the following observations:

- **Longer reasoning chains are not always the best on simple datasets**. Across nearly all models, we find that those directly trained on long CoT data typically do not show the best performance. These models often underperform compared to those generated through CoT-Valve, which results in shorter but more accurate reasoning chains. This trend is particularly pronounced in smaller models. For instance, in the LLaMA-3.2-1B model, training on QwQ synthesized data yields an accuracy of 52.69 with 759.3 tokens. However, using CoT-Valve, we can achieve an accuracy of 55.50 with only 267.0 tokens. However, we do not observe this phenomenon in more complex datasets, indicating that while the reasoning model may be redundant for simple datasets, it still requires test-time scaling to effectively handle complex datasets.

- **Some reasoning chains are difficult for the model to learn, especially for small LLMs.** We fine-tuned LLaMA-3.2-1B-Instruct using only one solution from MixChain, where all solutions lead to the same final answer but involve different intermediate reasoning steps. The results, presented in Table 6, indicate that neither the shortest nor the longest chains are optimal for learning. Instead, the model most effectively learns from moderately short chains, achieving the highest accuracy while maintaining a relatively low token count. This phenomenon is particularly evident in smaller models, but it is not observed in larger models. We believe this could be beneficial for the distillation of CoT in small LLMs.

Solution Used	#Epoch	#Samples	Accuracy	#Tokens	ACU↑
-	-	-	95.07	741.1	0.40
4	1	6.8k	95.68	597.3	0.50
4+3	1	13.7k	94.84	458.4	0.65
4+3+2	1	20.5k	94.84	339.9	0.87
4+3+2+1	1	27.4k	96.13	317.1	0.95
4+3+2+1+0	1	34.2k	94.92	225.5	1.32
0	5	37.4k	92.19	250.5	1.15

Table 7: Ablation of Progressive Compression on QwQ. Here, solution 0 is the human-annotated solution from the original dataset.

Method	QwQ-32B-Preview Acc	QwQ-32B-Preview #Token	Llama-3.2-1B-I Acc	Llama-3.2-1B-I #Token
Prompt (Shortest)	93.6	355.5	52.5	621.0
Ours (Best)	94.4	276.3	55.5	267.0
Ours (Shortest)	87.5	133.8	50.4	247.0

Table 8: CoT-Valve can achieve shorter chains than prompts with better performance.

4.6 Analysis

Ablation on Progressive Compression. Table 7 demonstrates the effect of progressive compression. We compare two settings: training directly with the ground-truth solution for five epochs and applying progressive compression for five epochs in total, with the final epoch using the ground-truth data. Our results show that progressive compression significantly improves the performance of short CoT (from 92.19 to 94.92). For each turn, progressive compression gradually reduces the token number while maintaining accuracy.

CoT-Valve achieves shorter chains compared to prompt control We also present in Table 8 the shortest chain achieved by our method and compare these with those obtained using prompt control. Our method outperforms prompt control methods at shorter chain lengths. Additionally, we explored the limits of chain length for both methods and found that our approach can generate substantially shorter chains than what can be achieved through prompt control.

General Performance after Finetuning We tested the fine-tuned model (The best model of our method in Table 1) on the MMLU benchmark and two common-sense datasets (PIQA and Hellaswag) and the results are shown in Table 9. Since we only finetune the model with 6k samples with 5 epochs (The training takes 1k steps), the finetuning does not harm the performance on these tasks but can significantly shorten the reasoning chain.

	MMLU	HellaSwag	PIQA
Before CoT-Valve	80.87	84.51	82.15
After CoT-Valve	81.15	84.51	82.48

Table 9: The results of QwQ-32B-Preview before and after CoT-Valve+P - MixChain-Z, using GSM8K.

	# Tokens	Qwen2.5-Math-PRM	Skywork-o1-PRM
GSM8K	121.8	88.02	77.93
QwQ-32B-Preview	737.3	98.04	89.16
Solution 4	497.2	99.00 (+0.96)	91.76 (+2.60)
Solution 3	386.7	99.53 (+1.49)	92.66 (+3.50)
Solution 2	310.7	99.82 (+1.78)	93.79 (+4.63)
Solution 1	279.6	99.85 (+1.81)	93.47 (+4.31)

Table 10: Step Correctness of MixChain-Z - GSM8K dataset.

Step Correctness of MixChain dataset We validate the MixChain dataset by two process reward models, Qwen2.5-Math-PRM-72B (Zhang et al., 2025b) and Skywork-o1-Open-PRM-7B (He et al., 2024), to identify intermediate errors in the reasoning processes. The score in the table represents the average reward for each step, indicating step correctness. The results show that our proposed dataset, which features shorter reasoning paths, improves the step-level score compared to the baseline model (QwQ-32B-Preview) and the official chain in the dataset. This improvement is due to the reduction of redundant or noisy chains in reasoning, which enhances the overall step correctness in reasoning and increases the score.

5 Conclusion

In this paper, we propose a method that enables a model to generate reasoning chains of varying lengths instead of the prompt control. Based on this approach, we construct a dataset containing both long and short reasoning chains to further enhance controllability and compression efficiency. Experimental results demonstrate the effectiveness of our method in dynamic reasoning chain control and the compression of CoT. Future research can further explore finer-grained control strategies to improve reasoning efficiency and model controllability.

6 Limitations

While CoT-Valve effectively shortens reasoning chains with minimal impact on performance, extreme compression can still result in accuracy losses, particularly affecting complex tasks. Moreover, performance remains limited by the quality of the original model; if the model cannot generate high-quality reasoning chains, constructing and tuning a short-chain dataset becomes challenging. Additionally, the interpretability of this mechanism is limited. Although CoT-Valve enables a controlled reasoning chain, the theoretical understanding of this remains insufficiently explored.

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