

A new formal tool: Functorial variables  
representing assertions and presuppositions

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Abstract

The language of classical propositional logic is extended by functorial variables as a new syntactical category. Functorial variables render to be a possible integrating representation of both assertion and presupposition in one and the same logical formula different from such using classical conjunction.

0. Introduction

From a computational point of view there is an important difficulty of an adequate formalization of both assertion and presupposition. It rests on dividing utterances into an explicit part (assertion) and an implicit one (presupposition).

Frege /1892/ refused any explication of this implicit part, because of his idealization that logic deals only with correct statements. At least in regard of definite descriptions Russell /1905/ pleaded for such an explication. But classical conjunction was one of its essential formal tools. To a certain extent assertion and presupposition were represented at one and the same level. Strawson /1950/ claimed that presuppositions have to be explicated, but not at the same level like the implicit part. Many linguists - e. g. Kiefer /1973/ - believe that logical means can be used to represent both parts of utterances separately, but these representations cannot be put together in one and the same logical expression, because Russell's solution is unsatisfactory.

My intention is to show that functorial variables render to be possible tools for integrating representation of both assertion and presupposition in one and the same logical formula. Moreover - unlike Bergmann /1981/, Jung/Küstner /1986/ - this representation is a syntactical one and different from that given by means of classical conjunction (cp. Max /1986/, /forthcoming/).

1. The logical apparatus

1.1. Functors as classical functions

Let  $\phi_i^n$  be the form of n-placed propositional functors;  $1 \leq i \leq 2^{2^n}$ . I will use 1- and 2-placed functors only. These functors are interpreted as classical (i. e. 2-valued and extensional) functions. The value-tables are:

	$\phi_1^1$	$\phi_2^1$	$\phi_3^1$	$\phi_4^1$
1	1	1	0	0
0	1	0	1	0

	$\phi_1^2$	$\phi_2^2$	$\phi_3^2$	$\phi_4^2$	$\phi_5^2$	$\phi_6^2$	$\phi_7^2$	$\phi_8^2$
1,1	1	1	1	1	1	1	1	1
1,0	1	1	1	1	0	0	0	0
0,1	1	1	0	0	1	1	0	0
0,0	1	0	1	0	1	0	1	0

	$\phi_8^2$	$\phi_9^2$	$\phi_{10}^2$	$\phi_{11}^2$	$\phi_{12}^2$	$\phi_{13}^2$	$\phi_{14}^2$	$\phi_{15}^2$	$\phi_{16}^2$
1,1	1	0	0	0	0	0	0	0	0
1,0	0	1	1	1	1	0	0	0	0
0,1	0	1	1	0	0	1	1	0	0
0,0	0	1	0	1	0	1	0	1	0

1.2. Functorial variables

I introduce functorial variables as a new syntactical category, and take the classical functors as values of these variables. Introducing such variables we get a whole class of syntactical extensions of the classical propositional logic. Only functorial variables of the following form are considered:

$$G_{f,g}^2; 1 \leq f, g \leq 16.$$

The f's and g's are called components. The f-component (g-component) is called first (second) component.

Semantically,  $\phi_f^2$  and  $\phi_g^2$  are the values of functorial variables  $G_{f,g}^2$ . Therefore these functorial variables (abbreviation: FV's) represent sets of functors with exact two (not necessarily different) elements. With respect to an intuitive interpretation FV's "unite" the properties of both functors.

1.3. The language G

Primitive symbols:

- 1)  $p_1, q_1, p_2, q_2, \dots$  (propositional variables)
- 2)  $\phi_1^1, \phi_2^1, \phi_3^1, \phi_4^1, \phi_1^2, \dots, \phi_{16}^2$  (functors)
- 3)  $G_{1,1}^2, \dots, G_{1,16}^2, G_{2,1}^2, \dots, G_{16,16}^2$  (FV's)

Formation rules:

- (1) A propositional variable standing alone

is a formula of G.

- (2) If A, B and C are formulas of G, then  $\wp_1^1 A$  and  $\wp_j^2 BC$  are formulas of G.
- (3)  $G_{4,6}^2 p_1 q_1$  is a formula of G.
- (4) If A is a formula of G formed without reference to the formation rules (1) and (2), then  $G_{f,g}^2 AA$  is a formula of G.
- (5) A is a formula of G iff its being so follows from the formation rules (1) - (4).

Definitions and types of formulas:

- D1:  $\sim A =_{df} \wp_3^1 A$   
 D2:  $(A \vee B) =_{df} \wp_2^2 AB$     D4:  $(A \equiv B) =_{df} \wp_7^2 AB$   
 D3:  $(A \supset B) =_{df} \wp_5^2 AB$     D5:  $(A \wedge B) =_{df} \wp_8^2 AB$   
 D6:  $(p_1 q_1) =_{df} G_{4,6}^2 p_1 q_1$

A K-formula  $A^K$  (i. e. classical formula) is that formula which was exclusively formed by means of formation rules (1) and (2).

A G-formula  $A^G$  is that formula which was exclusively formed by means of formation rules (3) and (4).

The rules of substitution of G are formulated in such a manner that (a) the p-propositional variables and the q-propositional variables occurring in G-formulas act as variables of a different sort, i. e. in any case the former occur on the left, and the latter on the right in formulas of the form  $G_{4,6}^2 p_1 q_1$ ; (b) they have the same index; and (c) there is no rule of substitution which allows the substitution of more complex formulas for propositional variables within G-formulas. In the case of K-formulas we have the usual rule of substitution.

Connection conditions of FV's:

Now I explain how several FV's occurring in the same formula are connected. The conditions of connection are chosen in such a way that every formula containing FV's represents exactly two formulas without FV's:

- (1) Let  $A^G$  be a G-formula of G. I define both FV-free formulas in two steps:
  - (a) Let  $G_{f,g}^2$  be the main-FV of  $A^G$ . Then  $A^{G1} =_{df} A^G(G_{f,g}^2/\wp_f^2)$ , i. e.  $A^{G1}$  is the formula which results by substitution of  $\wp_f^2$  for  $G_{f,g}^2$ .
  - (b) Let  $K^{i2}$  be a functor or a FV of a well-formed part of  $A^{G1}$  ( $A^{G2}$ ), and let this well-formed part be more complex than a formula of the form  $G_{4,6}^2 p_1 q_1$ . With re-

spect to all well-formed parts of  $A^{G1}$  ( $A^{G2}$ ) I generate the formulas  $A^{GE1}$  and  $A^{GE2}$  by substitution of the functor indicated by the first component of the main-FV of the first argument of  $K^2$  for this main-FV, and by substitution of the functor indicated by the second component of the main-FV of the second argument of  $K^2$  for this main-FV.

The formulas  $A^{GE1}$  and  $A^{GE2}$  generated by this method differ only in the main-functor. They are both K-formulas.  $A^G/A^{GE1}$  and  $A^G/A^{GE2}$  are abbreviations for all substitutions in  $A^G$  which generate  $A^{GE1}$  and  $A^{GE2}$ , respectively.

- (2) Let A be a formula of G which can contain both functors and FV's. A well-formed part of A is called G-maximum iff
  - (i) A is a G-formula, and
  - (ii) its governed connective is not a FV.
 Let  $A_1, \dots, A_u$  be all G-maximum well-formed parts of A. Then

$$A^{E1} =_{df} A(A_1/A_1^{GE1}, \dots, A_u/A_u^{GE1})$$

$$A^{E2} =_{df} A(A_1/A_1^{GE2}, \dots, A_u/A_u^{GE2}).$$

#### 1.4. Validity of formulas with FV's

- (1) A G-formula  $A^G$  is valid iff both  $A^{GE1}$  and  $A^{GE2}$  are valid in the classical sense.
- (2) A formula A is valid iff both  $A^{E1}$  and  $A^{E2}$  are valid in the classical sense.

#### 2. Relations to classical logic

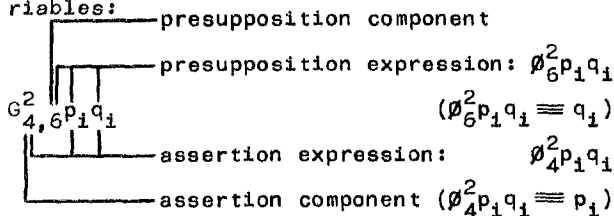
My system is semantically equivalent with the classical propositional logic in the sense that all FV's can be eliminated by replacing every formula A of G by the conjunction of its both closed substitutions, i. e.  $A^{E1}$  and  $A^{E2}$ . In this manner we get a complete and consistent system of classical logic. It holds: A formula A of G is valid iff its corresponding classical formula  $A^{E1} \wedge A^{E2}$  is valid.

There are some specific differences between the starting formula with FV's and its analogous formula without FV's. One important difference is the following: After replacing the propositional variables by values 1 or 0 the formula A gets none of these values and it remains unsaturated. Only if this formula is transmitted in one of its both closed substitutions -  $A^{E1}$  or  $A^{E2}$  -, then it gets a value. With respect to formulas with FV's which are neither tautologies nor contradictions there is another important difference: Let A be

such a formula. Then often  $A \equiv (A^{E1} \wedge A^{E2})$  is not valid.

### 3. Assertion and presupposition

The introduction of expressions of the form  $G_{4,6}^2 p_1 q_1$  renders to be a possible unconventional approach to assertion and presupposition. I postulate that the p-propositional variables represent elements of a set of assertions, and the corresponding q-variables represent elements of a set of presuppositions. The FV  $G_{4,6}^2$  constitutes an ordered sequence of both sorts of propositional variables:



Concerning logical relations between several sentences both assertions and presuppositions can influence this relation. In order to form a correct translation of such compound sentences their simple parts should be translated into expressions of the form  $G_{4,6}^2 p_1 q_1$ . Let  $A^G$  be a G-formula of G. Then we can put on the following generalization of our interpretation:

- $A^{GE1}$  - assertion expression
- $A^{GE2}$  - presupposition expression.

So we get a new syntactical method to explicate assertion and presupposition in one and the same formula. Unlike 4-valued/2-dimensional approaches our language possesses an enrichment of syntactical expressive power.

### 4. FV's and functors

The explication of both assertion and presupposition by means of formulas of the form  $(p_1 q_1)$  differs from that one by means of classical conjunction, because  $(p_1 q_1) \equiv (p_1 \wedge q_1)$  is not valid.

Because of

- T1:  $(p_1 \wedge q_1) \supset (p_1 q_1)$ , and
- T2:  $(p_1 q_1) \supset (p_1 \vee q_1)$

the representation by means of  $G_{4,6}^2$  is stronger than that one by conjunction, but it is weaker than that one by disjunction.

### 5. Negations

Because of

- T3:  $G_{11,11}^2(p_1 q_1)(p_1 q_1) \equiv \sim q_1$

the 2-placed FV  $G_{11,11}^2$  can be interpreted as presupposition-rejecting negation.

$G_{13,6}^2$  can be interpreted as presupposition-preserving negation, because

$$G_{13,6}^2(p_1 q_1)(p_1 q_1) \left\{ \begin{array}{l} \emptyset_{13}^2 \emptyset_4^2 p_1 q_1 \emptyset_6^2 p_1 q_1 \equiv \sim p_1 \\ \emptyset_6^2 \emptyset_4^2 p_1 q_1 \emptyset_6^2 p_1 q_1 \equiv q_1. \end{array} \right.$$

### 6. Extensions of the language G

Starting point:  $G_{4,6}^2 p_1 q_1$ .

Step\_1: Dropping index-equality of propositional variables:  $G_{4,6}^2 p_i q_j$ . Hence it follows a more direct formalization of sentences with the same presupposition:

$G_{4,6}^2 p_1 q_1$  and  $G_{4,6}^2 p_2 q_1$ .

Step\_2: Admitting of representation of several presuppositions (of one simple sentence) connected by propositional functors:  $G_{4,6}^2 p_1(q_1 \wedge q_2)$ .

Step\_3: Admitting of functional dependence between propositional variables (i. e. admitting of an interaction between assertion and presupposition of simple sentences):  $G_{3,5}^2 p_1 q_1$ .

Step\_4: Admitting of more than two components (e. g. 2 presupposition components):

$$G_{16,52,86}^3 p_1 q_1 r_1 \left\{ \begin{array}{l} \emptyset_{16}^3 p_1 q_1 r_1 \equiv p_1 \text{ (assertion)} \\ \emptyset_{52}^3 p_1 q_1 r_1 \equiv q_1 \text{ (presupposition 1)} \\ \emptyset_{86}^3 p_1 q_1 r_1 \equiv r_1 \text{ (presupposition 2)}. \end{array} \right.$$

Step\_5: Combinations of several steps.

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